# THE ISSUE OF NEGATIVE ABSOLUTE TEMPERATURES

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**Abstract.**
Here attention is drawn to a recent article in the journal *Nature* which claimed that researchers had cooled a system to a temperature *below* absolute zero. It is pointed out that this was *not* the claim in the original paper and is incorrect as far as the accepted definition of temperature is concerned. It is pointed out also that, if the claim were true, then it would imply that both the Third and Second Laws of Thermodynamics had been violated.

**Keywords:** Negative temperatures, second law of thermodynamics, third law of thermodynamics, Carnot cycles.

**Introduction.**

Recently, an extremely interesting article concerned with negative absolute temperatures appeared in the journal *Science*, entitled 'Negative absolute temperature for motional degrees of freedom' [1]. This article made no untoward statements or claims. Indeed, this article contained an exceptionally lucid account of the physics with which the authors were concerned. However, it was followed by an article in the journal *Nature* purporting to explain the aforementioned article in detail for the interested layman. In this latter article http://www.nature.com/news/quantum-gas-goes-below-absolute-zero-1.12146 the author, Zeeya Merali, stated that the authors of the *Science* article had succeeded in cooling a system to a temperature *below* absolute zero and even went by the title 'Quantum gas goes below absolute zero'. It should be noted from the outset that this was definitely *not* what was claimed in the said *Science* article. The article actually claimed to have achieved negative temperatures in a system but with the concept of negative temperatures being defined in accordance with the accepted principles of thermodynamics; that is, where negative temperatures are *higher* than positive temperatures. This point was explained very carefully in the opening section of the *Science* article; there was *no* mention of any system achieving a temperature *below* absolute zero

As Ramsey [2] explained, 'from a thermodynamic point of view, the only requirement for the existence of a negative temperature is that the entropy $S$ should not be restricted to a monotonically increasing function of the internal energy $U$. At any point for which the slope of the entropy as a function of $U$ becomes negative', the temperature is negative since temperature $T$ is given by

$$1/T = \left(\partial S/\partial U\right)_X$$

where $(\quad)_X$ indicates that the partial differentiation should be carried out with the thermodynamic variables $X$ which appear as additional differentials in the equation relating $TdS$ and $dU$ held constant. If entropy is a monotonically increasing function of the internal energy and achieves a maximum value then, beyond that maximum value, the entropy will decrease if the internal energy continues to increase. Beyond such a maximum $\left(\partial S/\partial U\right)_X$ will be negative and the region will correspond to negative absolute temperatures. It is seen immediately that, in passing from positive to negative temperatures, the system passes through infinite temperature or, in cooling from negative to positive temperatures, the system passes through ∞K instead of through absolute zero. This characteristic is illustrated extremely well by the negative temperature cooling curves shown in the work of Purcell and Pound [3].

The important difference reported by the *Science* article is that earlier experimental work usually involved examining spin systems, although some other relevant examples were mentioned by Lavenda [4], whereas this latest work seems to involve more directly understood physical systems. It might be noted that Lavenda's article raises the pertinent question of whether, or not, negative temperatures truly exist.

It should be noted also that, as well as misrepresenting what the authors of the *Science* article wrote, the apparent claims by Zeeya Merali do not accord with accepted thermodynamics. As is implied by the above, the absolute temperature scale runs by definition from +0K to ∞K and on *upwards* to - 0K. Temperatures below absolute zero simply cannot exist. This point is vitally important and cannot be overstressed.

**Implications of the Claims in Nature.**

While all the above comments are true and effectively negate the claims of the writer in *Nature*, it is important to realise the meaning hidden behind those claims and to reflect on whether, or not, the writer realised this and even truly understood the concept of negative absolute temperatures in thermodynamics. If, as was claimed in the *Nature* article, a system had been cooled to a temperature below absolute zero, it would have been necessary for that system to have passed through absolute zero. That, in turn, would have meant that, at some point, the said system had been at absolute zero in violation of one popular form of the Third Law of Thermodynamics. Possibly more importantly, however, if absolute zero is in fact an accessible temperature, that could lead to violations of the Second Law. This is seen easily by considering a Carnot cycle in which one of the isotherms is at absolute zero. However, the isotherm at absolute zero is also an adiabatic and so, the mentioned Carnot cycle could be viewed as consisting of one isotherm at some positive temperature and three adiabatics. In such a set-up, it is straightforward to see that the end result would be the conversion of a quantity of heat entirely into work in a cyclic process in the absence of other effects; that is, a direct violation of Kelvin's form of the Second Law.

However, this latter argument does show a very definite link between the Third and Second Laws, a fact which has been noted in the past [5].

It might be noted further that quite a body of work exists on negative absolute temperatures and Carnot cycles. It has been found [6] that it is impossible to operate a Carnot cycle when one of the heat reservoirs has an infinite temperature and also that such cycles may be run only in particular cases when the temperatures of the reservoirs are such that one is positive and the other negative. Further comments and a much longer list of useful references may be found in another article which appeared in the American Journal of Physics [7].

**A Final Point.**

While the mentioned Nature article is freely available to scientist and non-scientist alike, it is possibly even more disturbing to find this claiming of negative absolute temperatures to be below zero appearing in what might be described as a popular scientific book by an apparently distinguished professor. However, such is the case. This grave error appears quite openly in the book *The Laws of Thermodynamics: A Very Short Introduction* by Peter Atkins[8]. There are also several other highly questionable points in this text and what makes it even more disturbing if reviews on the Amazon web site are to be believed is that students are using this text to help in their understanding of the subject. Using this book can result in their understanding being incorrect.

**References.**

**[1]** Braun, S., et al., Science **339**(2013)52

[2] Ramsey, N.F., Phys.Rev.**103**(1956)20

[3] Purcell,E. M. and Pound, R. V., Phys. Rev.**81**(1951)279

[4] Lavenda, B. H., 1999, J. Phys. A**32**(1999)4279

[5] Dunning-Davies, J., Nuovo Cimento **10**B(1972)407

[6] Dunning-Davies, J., J. Phys. A. **9**(1976)605

[7] Dunning-Davies, J., Am, J. Phys. **46(5)**(1978)583

[8] Atkins, P., *The Laws of Thermodynamics: A Very Short Introduction*, (Oxford U. P., Oxford, 2010)