FUZZY NEUTROSOPHIC MODELS FOR SOCIAL SCIENTISTS

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PREFACE

In this book, authors give the notion of different neutrosophic models like, neutrosophic cognitive maps (NCMs), neutrosophic relational maps (NEMs), neutrosophic relational equations (NREs), neutrosophic bidirectional associative memories (NBAMs) and neutrosophic associative memories (NAMs) for socio scientists.

This book has six chapters. The first chapter introduces the basic concepts of neutrosophic numbers and notions about neutrosophic graphs which are essential to construct these neutrosophic models. In chapter two we describe the concept of neutrosophic matrices and the essential operations related with them which are used in the study and working of these neutrosophic models.

However the reader must be familiar with the notions of Fuzzy Cognitive Maps (FCMs) model, Fuzzy Relational Maps (FRMs) model, Fuzzy Relational Equations (FREs), Fuzzy Associative Memories (FAMs) and Bidirectional Associative Memories (BAMs) to follow this book without any difficulty.

In chapter three we introduce the notion of Neutrosophic Cognitive Maps (NCMs) model and Neutrosophic Relational Maps (NRMs) model. Definitions and examples of this concept are given. Further modified NCMs, modified NRMs, combined disjoint block NRMs and NCMs, overlap block NCMs and

NRMs and linked NRMs are defined, described and developed in this chapter.

In chapter four the notion of Neutrosophic Relational Equations (NREs) are introduced and illustrated with examples. In chapter five we introduce the Neutrosophic Bidirectional Associative Memories (NBAMs) model. Using the concept of n-adaptive neutrosophic models the authors shift from NRMs to NBRMs and vice versa.

In the final chapter the authors deal with the notion of NAMs and show they are also n-adaptive structures.

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W.B.VASANTHA KANDASAMY
FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

In this chapter we introduce the notion of neutrosophic matrices, neutrosophic graphs and some simple operations on them. We know I the indeterminate such that $I^2 = I$ [4]. Further $\langle Z \cup I\rangle$ is the neutrosophic integer ring, $\langle R \cup I\rangle$ is the neutrosophic real ring, $\langle Q \cup I\rangle$ is the neutrosophic rational ring and $\langle C \cup I\rangle$ is the neutrosophic complex ring.

We see $\langle Z \cup I\rangle \subseteq \langle Q \cup I\rangle \subseteq \langle R \cup I\rangle \subseteq \langle C \cup I\rangle$.

Further $\langle Z_n \cup I\rangle$ is the neutrosophic modulo integer ring and $\langle C(Z_n) \cup I\rangle$ is the neutrosophic complex modulo integer ring where $C(Z_n) = \{a + bi_F \mid a, b \in Z_n, \text{ with } i_F^2 = (n-1)\}$ is the finite complex modulo integer ring.

If we replace in any matrix $a = (a_{ij})$ the real elements by elements in $\langle Z \cup I\rangle$ or $\langle R \cup I\rangle$ or $\langle Q \cup I\rangle$ or $\langle C \cup I\rangle$ or $\langle Z_n \cup I\rangle$ or $\langle C(Z_n)\cup I\rangle$ we call that A as a neutrosophic matrix.

Let $A = \begin{bmatrix} I & 5 & 6 \\ 0 & -I & 2 \\ 5 & 7 & 3I \end{bmatrix}$ be a 3×3 matrix.

We call A a neutrosophic square matrix.

DEFINITION 1.1: *Let A = (a_{ij}) be a n × n square matrix. If atleast one of the a_{ij}'s is I then we call A to be a neutrosophic square matrix or A is a square neutrosophic matrix.*

We illustrate this by an example.

Example 1.1: Let

$$A = \begin{bmatrix} 6 & -2 & 5I & 7 \\ 3 & 0 & 1 & 2 \\ 7I & 0 & 9 & -I \\ 4 & 2I & 5 & 8 \end{bmatrix},$$

A is a neutrosophic square matrix.

Example 1.2: Let

$$A_1 = \begin{bmatrix} I & 0 \\ 1 & 1 \end{bmatrix},$$

A_1 is also a 2 × 2 neutrosophic square matrix.

Example 1.3: Let

$$A = \begin{bmatrix} I \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix},$$

A is a neutrosophic matrix called the column neutrosophic matrix.

DEFINITION 1.2: *Let*

$$A = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix};$$

if atleast one of the a_i's is I then we call A to be a column neutrosophic matrix.

Example 1.4: Let

$$A = \begin{bmatrix} 0 \\ 1 \\ 2 \\ I \\ -1 \\ 0 \end{bmatrix},$$

A is a 6 × 1 column neutrosophic matrix.

DEFINITION 1.3: *Let $a = (a^1, \ldots, a^n)$ where at least one of the a^i is the indeterminate I, then a is a row neutrosophic matrix, $1 \leq i \leq n$.*

Example 1.5: Let A = (I, 3I, 0, 0, 1, 2); A is a 1 × 6 row neutrosophic matrix.

Example 1.6: Let B = (0 1 2 –3 I 0 I –7), B is a 1 × 8 row neutrosophic matrix.

DEFINITION 1.4: *Let $A = \{(a_{ij}) \mid 1 \leq i \leq n, 1 \leq j \leq m\}$ $m \neq n$ if atleast one of the a_{ij}'s is the indeterminant I then we call A to be a neutrosophic rectangular $n \times m$ matrix.*

We illustrate this by some examples.

Example 1.7: Let

$$A = \begin{pmatrix} 1 & 2 & I & -4 \\ 0 & 5 & 7 & I \\ 8 & 9 & -2 & -9 \end{pmatrix},$$

A is a 3 × 4 rectangular neutrosophic matrix.

Example 1.8: Let

$$A = \begin{pmatrix} I & 1 & 2 & 3 & -4 & 5 & 7I \\ 0 & 5 & 3 & 4 & 7 & I & 2 \end{pmatrix},$$

A is a 2 × 7 rectangular neutrosophic matrix.

Now we proceed onto define a neutrosophic diagonal matrix.

DEFINITION 1.5: *Let A = (a_{ii}) be a diagonal matrix i.e., A is a n × n square matrix. If atleast one of the a_{ii} is I then we call A to be a neutrosophic diagonal matrix, $1 \leq i \leq n$.*

Example 1.9: Let

$$A = \begin{pmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 2I & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{pmatrix},$$

A is a neutrosophic diagonal matrix.

Example 1.10: Let

$$A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & I-4 \end{pmatrix};$$

A is a neutrosophic diagonal matrix.

It is pertinent to mention here that only the n × n unit matrix i.e., diagonal matrix with entries in the diagonal as one alone acts as the identity matrix even for square neutrosophic matrices.

Now as in case of elementary or usual matrices we get the transpose of a neutrosophic matrix and the determinant of a square neutrosophic matrix.

We illustrate this by some examples.

Example 1.11: Let

$$A = \begin{bmatrix} 5I & 2 \\ -1 & 7 \end{bmatrix}$$

be a 2 × 2 square matrix |A| i.e., determinant of A is 35I + 2.

The following facts are not only interesting but also important. The determinant of a neutrosophic matrix need not always be a neutrosophic value.

This is shown by a simple example.

Example 1.12: Let

$$A = \begin{bmatrix} 7I & 3 \\ -7 & 0 \end{bmatrix}$$

be a neutrosophic 2 × 2 square matrix. |A| = 21, which is clearly a real number and not a neutrosophic number.

Thus a square neutrosophic matrix may have its determinant to be a real number or a neutrosophic number.

Example 1.13: Let

$$A = \begin{bmatrix} 5 & I \\ -2 & 0 \end{bmatrix}$$

be a 2 × 2 square neutrosophic matrix. |A| = 2I which is a pure neutrosophic number.

Now we see the transpose of a neutrosophic matrix is defined in the same way as the transpose of usual matrices from Q or Z or R or C or Z_n or $C(Z_n)$.

However the transpose of a neutrosophic matrix is a neutrosophic matrix.

We illustrate this by some examples.

Example 1.14: Let A = [4 –I 0 –7 8 4 5] be a neutrosophic 1 × 7 row matrix.

The transpose of A denoted by

$$A^t = \begin{bmatrix} 4 \\ -I \\ 0 \\ -7 \\ 8 \\ 4 \\ 5 \end{bmatrix}$$

is a 7 × 1 column neutrosophic matrix.

Example 1.15: Let

$$B = \begin{bmatrix} 13 \\ 0 \\ 2 \\ I \\ 7I \end{bmatrix}$$

be a 5 × 1 column neutrosophic matrix. Transpose of B, B^t = [13 0 2 I 7I] is a 1 × 5 row neutrosophic matrix.

However transpose of a square neutrosophic matrix is a square neutrosophic matrix and the transpose of a rectangular neutrosophic matrix is a rectangular neutrosophic matrix.

Example 1.16: Let

$$A = \begin{bmatrix} 3 & 0 & I & 7 & 2 & 1 \\ I & 8 & -2 & 1 & 3 & 7 \\ 9 & 1 & 3 & -I & 0 & 2 \end{bmatrix}$$

be a 3 × 6 rectangular neutrosophic matrix.

A^t, the transpose of

$$A \text{ is } \begin{bmatrix} 3 & I & 9 \\ 0 & 8 & 1 \\ I & -2 & 3 \\ 7 & 1 & -I \\ 2 & 3 & 0 \\ 1 & 7 & 2 \end{bmatrix}$$

is a 6 × 3 neutrosophic rectangular matrix.

Example 1.17: Let

$$A = \begin{bmatrix} 3 & I & 5 & 2 \\ -I & 4 & 8 & -1 \\ 7 & 5 & 3I & 0 \\ 2 & 4I & 2 & 5I \end{bmatrix}$$

be a 4 × 4 square neutrosophic matrix.

The transpose of A denoted by

$$A^t = \begin{bmatrix} 3 & -I & 7 & 2 \\ I & 4 & 5 & 4I \\ 5 & 8 & 3I & 2 \\ 2 & -1 & 0 & 5I \end{bmatrix}$$

is also a 4 × 4 square neutrosophic matrix.

Now having seen the notion of determinant and transpose of a neutrosophic matrix we now proceed onto define the notion of sum and product of neutrosophic matrices when ever they are defined.

THEOREM 1.1: *The sum of two neutrosophic matrices whose sum is defined need not be always a neutrosophic matrix.*

Proof: We prove this by an example.

Consider

$$A = \begin{bmatrix} 3 & I & 7 & 2 & 11 & -1 \\ 4 & 2 & 5 & -3I & 4 & 7 \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 7 & -I+4 & -7 & 4 & 1 & 0 \\ 2 & 4 & 2 & 3I-1 & 0 & 2 \end{bmatrix}$$

two 2 × 6 neutrosophic matrices.

$$A + B = \begin{bmatrix} 10 & 4 & 0 & 6 & 12 & -1 \\ 6 & 6 & 7 & -1 & 4 & 9 \end{bmatrix}$$

is also a 2 × 6 matrix but is not a neutrosophic matrix.

Thus we have shown the sum of two neutrosophic matrices in general is not a neutrosophic matrix.

We give yet another example.

Example 1.18: Let

$$A = \begin{bmatrix} 2 & 1 & -I \\ 7 & 5 & 4 \\ -2 & I+1 & 0 \end{bmatrix}$$

$$\text{and } B = \begin{bmatrix} 3 & 5 & I+7 \\ 8 & 0 & 2 \\ 1 & -I & -2 \end{bmatrix}$$

be two 3 × 3 square neutrosophic matrices.

The sum

$$A + B = \begin{bmatrix} 5 & 6 & 7 \\ 15 & 5 & 6 \\ -1 & 1 & -2 \end{bmatrix}$$

is only a 3 × 3 real matrix and not a neutrosophic matrix.

Example 1.19: Let

$$A = \begin{bmatrix} 2 & 1 & 5 & 1 & 0 \\ 0 & I & -2 & 0 & 7I \end{bmatrix}$$

and

$$B = \begin{bmatrix} 3 & 0 & 2 & 1 & 6 \\ 5 & -I & 0 & 1 & -7I \end{bmatrix}$$

be any two 2 × 5 neutrosophic matrices.

The sum of A and B denoted by

$$A + B = \begin{bmatrix} 5 & 1 & 7 & 2 & 6 \\ 5 & 0 & -2 & 1 & 0 \end{bmatrix}.$$

Clearly A + B is not a neutrosophic matrix.

Now we show that the product of two neutrosophic matrices in general is not a neutrosophic matrix whenever the product is defined.

We will illustrate this by some examples.

Example 1.20: Let

$$A = \begin{bmatrix} 3 & 0 & I \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 5 & 7 & 1 \\ 2I & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

be two 3 × 3 neutrosophic square matrices.

$$AB = \begin{bmatrix} 15 & 21 & 3 \\ 5 & 7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is only a real matrix.

Example 1.21: Let

$$A = \begin{bmatrix} I & 1 \\ 2I & 2 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 \\ -I & 2 \end{bmatrix}$$

be any two neutrosophic 2 × 2 matrices.

$$AB = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}$$

is the product of A and B which is clearly not a neutrosophic matrix but only a real matrix.

Now we proceed onto describe the notion of neutrosophic graphs and their properties.

If in the usual graph some of the edges are replaced by the indeterminancy I then the resulting graph is a neutrosophic graph.

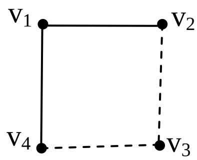
That is a neutrosophic graph is a usual graph in which at least one edge is a neutrosophic edge. The neutrosophic edge is denoted in this book only by dotted lines.

If the vertices are v_1 and v_2 connected by a neutrosophic edge then it is denoted by

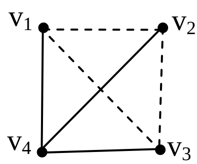
v_1 ● - - - - ● v_2 .

We proceed onto give examples of neutrosophic graphs.

Example 1.22: Let G be a neutrosophic graph with four vertices.

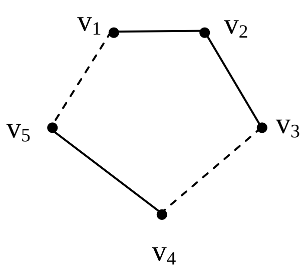


G_1 is a neutrosophic connected planar graph.
Let G_1 be the graph with four vertices.



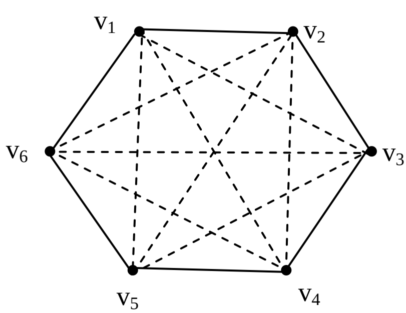
G_1 is a connected neutrosophic non planar graph.

Example 1.23: Let G be a neutrosophic graph which is as follows.



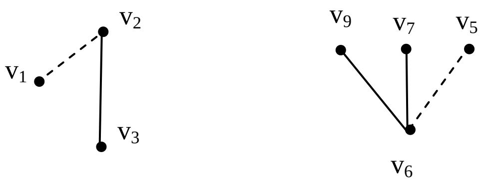
G is a connected planar neutrosophic graph with five vertices $\{v_1, v_2, \ldots, v_5\}$.

Example 1.24: Let G be a neutrosophic graph which is as follows:



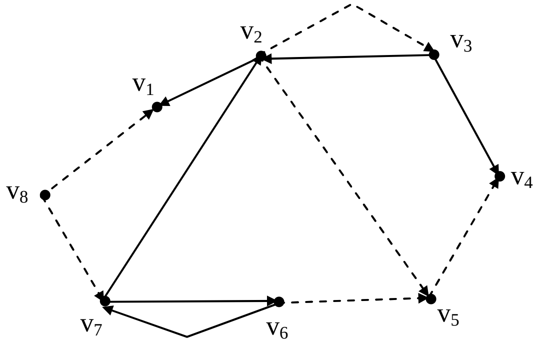
G is a neutrosophic complete graph.

Example 1.25: Let G be a neutrosophic graph which is as follows:



G is a neutrosophic graph which is not connected.

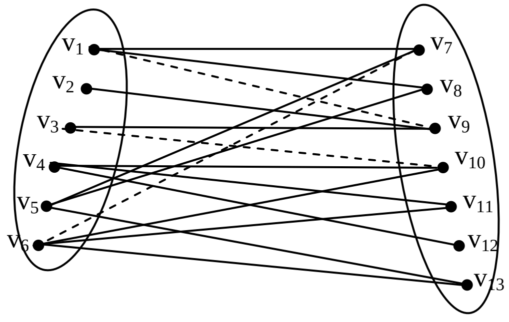
Example 1.26: Let G be a neutrosophic graph which is as follows:



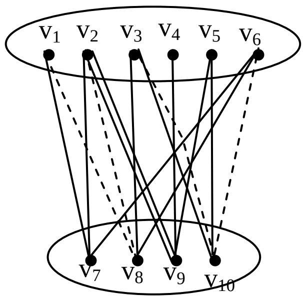
V is a neutrosophic directed graph.

We also use the concept of bipartite neutrosophic graph in fuzzy models we will describe them with one or two examples.

Example 1.27: Let G be a bipartile graph which is as follows:

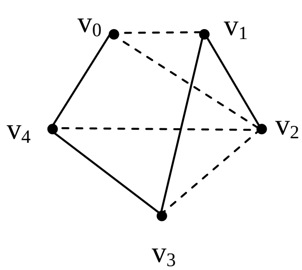


Example 1.28: Let G be a bipartite graph which is as follows:



Now we give the connection matrix or the adjacency matrix associated with the graph.

Example 1.29: Let G be the graph given in the following:

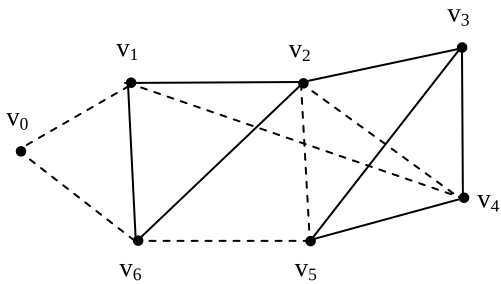


The adjacency matrix of this graph is a 5 × 5 neutrosophic matrix A given by

$$A = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} v_0 & v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & I & I & 0 & 1 \\ I & 0 & 1 & 1 & 0 \\ I & 1 & 0 & I & I \\ 0 & 1 & I & 0 & 1 \\ 1 & 0 & I & 1 & 0 \end{bmatrix} \end{array}.$$

A is 5 × 5 neutrosophic matrix whose diagonal elements are zero.

Example 1.30: Let G be a neutrosophic graph which is as follows:



The adjacency neutrosophic matrix associated with G is as follows:

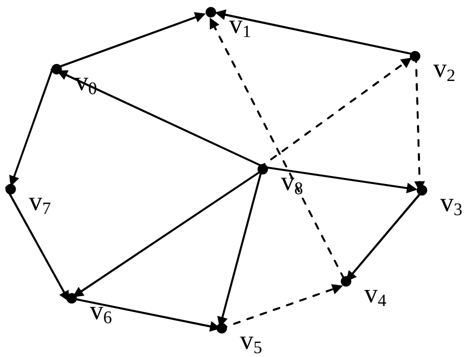
$$A = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & I \\ I & 0 & 1 & 0 & I & 0 & 1 \\ 0 & 1 & 0 & 1 & I & I & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & I & I & 1 & 0 & 1 & 0 \\ 0 & 0 & I & 1 & 1 & 0 & I \\ I & 1 & 1 & 0 & 0 & I & 0 \end{bmatrix} \end{array}$$

A is a 6 × 6 neutrosophic matrix.

This type of matrices will be used in the construction of NCM with a small change these neutrosophic graphs must necessarily be directed neutrosophic graphs with edge weights from the set {0, 1, –1, I}.

We will describe this situation by an example or two.

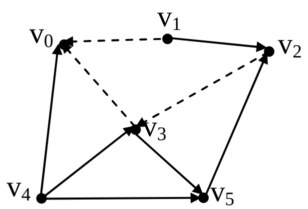
Example 1.31: Let G be a directed neutrosophic graph which is as follows:



The adjacency matrix associated with the neutrosophic graph is as follows:

$$
A = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\
\left[\begin{array}{ccccccccc}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & I & 1 & 0 & 1 & 1 & 0 & 0
\end{array}\right]
\end{array}.
$$

Example 1.32: Let G be a neutrosophic directed graph which is as follows:

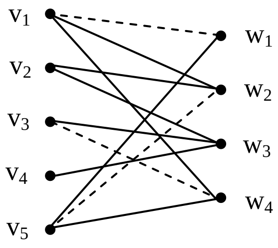


The adjacency matrix A of this directed neutrosophic graph is as follows:

$$A = \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We now proceed onto describe the matrix associated with bipartite neutrosophic graph.

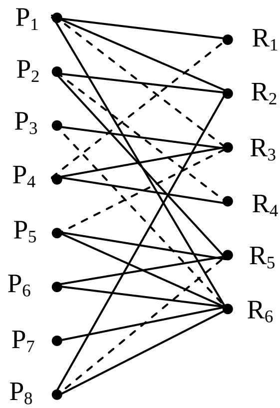
Example 1.33: Let G be a neutrosophic bipartite graph which is as follows:



The neutrosophic matrix associated with the neutrosophic graph G is the connection matrix which is as follows:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 \end{array} \\ \begin{bmatrix} I & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & I \\ 0 & 0 & 1 & 0 \\ 1 & I & 0 & 1 \end{bmatrix} \end{array}.$$

Example 1.34: Let G be a neutrosophic bipartite graph which is as follows:



The related matrix of this neutrosophic bipartite graph is as follows:

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{array}\begin{array}{c} \begin{array}{cccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 \end{array} \\ \begin{bmatrix} 1 & 1 & I & 0 & 0 & 1 \\ 0 & 1 & 0 & I & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & I \\ I & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & I & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & I & 1 \end{bmatrix} \end{array}$$

Now we proceed onto describe in the following chapter the notion of fuzzy neutrosophic matrices and their properties.

Chapter Two

FUZZY NEUTROSOPHIC MATRICES AND SOME BASIC OPERATIONS ON THEM

In this chapter we introduce the notion of fuzzy neutrosophic matrices and define on them some special operations which are essential for the functioning of neutrosophic models to be described in the following chapters.

DEFINITION 2.1: *Let A be a matrix if it entries are from [0, 1] and [0, I] then we call A to be a fuzzy neutrosophic matrix.*

We denote the combined interval by N = [0, 1] $\cup$ [0, I]. So N denotes the special fuzzy neutrosophic interval.

We give examples of fuzzy neutrosophic matrices.

Example 2.1: Let A = [0, 1, 0.2I, 0.02, 0.7I, I, 0.7]; A be a 1×7 row fuzzy neutrosophic matrix whose entries are from N.

DEFINITION 2.2 : *Let A = [x_1, …, x_n] be a row matrix. If each $x_i \in N = [0, 1] \cup [0, I]$ then we call A to be a fuzzy neutrosophic row matrix.*

DEFINITION 2.3: *Let*

$$B = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

be a column matrix. If each $y_i \in N$ then we call B to be a fuzzy neutrosophic column matrix, $(1 \leq i \leq m)$.

We now give examples of such matrices.

Example 2.2: Let

$$B = \begin{bmatrix} 0.3 \\ I \\ 0.8I \\ 1 \\ 0 \\ 0.6I \\ 0.32 \\ 0.5 \end{bmatrix}$$

be 8×1 column matrix. B is a fuzzy neutrosophic column matrix as every element of B is in N.

DEFINITION 2.4: *Let*

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

be a m $\times$ n rectangular matrix with $x_{ij} \in N$; $1 \leq i \leq m$; $1 \leq j \leq n$. X is defined to be a fuzzy neutrosophic rectangular matrix ($m \neq n$).

We illustrate this by the following examples.

Example 2.3: Let

$$A = \begin{bmatrix} 0.7 & 1 & 0.5 & 0.6+I & 1 \\ I & 0.3I & 1 & 0.2 & 0.I \\ 4I+I & 1+I & 0 & 1 & 0.I+1 \end{bmatrix}$$

be a 3 × 5 matrix. Clearly A is a rectangular fuzzy neutrosophic matrix.

Example 2.4: Let

$$B = \begin{bmatrix} 1 & 0 & I & 1 \\ 0.I & 1 & 0.2 & 0 \\ 0 & 0.3I & 0.4I & 0 \\ 0.7 & 1 & I & 1 \\ 0.2I & I & 0.21 & I \\ I & 0 & 0.7I & 1 \\ 0.9 & 0.2I & I & 0.2 \\ 0.3I & 1 & 1 & I \end{bmatrix}$$

be a 8 × 4 fuzzy neutrosophic rectangular matrix.

Now we proceed onto define fuzzy neutrosophic square matrix.

DEFINITION 2.5: *Let*

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix}$$

where $y_{ij} \in [0, 1] \cup [0, I]$. *Y is a* $n \times n$ *square fuzzy neutrosophic matrix,* $1 \leq i, j \leq n$.

We shall illustrate this by some examples.

Example 2.5: Let

$$X = \begin{bmatrix} 0..3 & 0.8I & I & 0.1 & 0.2I \\ 0.5I & 0 & 0 & 1 & 0.3 \\ I & 1 & 0.1 & 0.2I & 1 \\ 1 & 0.2I & 0.3I & 0 & 0.3I \\ 0.7I & 0.75 & 0.5 & I & 1 \end{bmatrix};$$

X is a 5 × 5 square fuzzy neutrosophic matrix.

Example 2.6: Let

$$Y = \begin{bmatrix} 0.2I & I & 1 \\ 0.3 & 0.5 & 0.I \\ 0.75 & 0.3I & 0 \end{bmatrix},$$

Y is a 3 × 3 square fuzzy neutrosophic matrix.

The following facts can easily be proved.

1. Every fuzzy neutrosophic matrix is not a fuzzy matrix and the collection of all fuzzy matrices is contained in the class of all fuzzy neutrosophic matrices.

2. All fuzzy matrices are real matrices but all real matrices are not in general fuzzy matrices.

3. Thus the collection of all fuzzy neutrosophic matrices happen to be the larger class containing the class of fuzzy matrices and real matrices.

   In case of fuzzy neutrosophic matrix we define a square matrix to be diagonal if every entry other than the main diagonal elements are zero.

DEFINITION 2.6: *Let*

$$D = \begin{bmatrix} d_1 & & & (0) \\ & d_2 & & \\ & & \ddots & \\ (0) & & & d_n \end{bmatrix}$$

be a $n \times n$ *matrix where* $d_i \in [0, 1] \cup [0, I]$; $1 \leq i \leq n$. *Then D is defined to be the fuzzy neutrosophic diagonal matrix.*

We illustrate this by the following example.

Example 2.7: Let

$$D = \begin{bmatrix} I+1 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.I+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7+I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2I \end{bmatrix}$$

be a 6 × 6 square diagonal matrix. D is a fuzzy neutrosophic diagonal matrix.

We have defined I.I = I.

Now the choice of I or a real value while working with any neutrosophic model is entirely in the hands of the expert. If the expert wishes to know even a smallest degree of indeterminancy she/he choose I to the real value if on the otherhand the expert wishes to neglect small degrees of indeterminancy to real values he / she can choose the real values to I.

We now proceed onto describe some special operations using fuzzy neutrosophic matrices.

In case of fuzzy neutrosophic matrices the addition of matrices as in case of fuzzy matrices will not yield compatibility.

For instance we show this by a simple example.

Example 2.8: Let

$$A = \begin{bmatrix} 0.3 & I & 1 & 0.7I+1 \\ 0.2+I & 1 & 1 & 0.3 \\ 0 & 0.7 & 0.6I & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 & 1 & I & 0.8I+1 \\ I+1 & 0 & 1+0.7I & 1 \\ 0.7I & 1 & 0.5 & 0.2I \end{bmatrix}$$

be two 3 × 4 fuzzy neutrosophic matrices.

Of course addition is defined. But the usual addition does not yield back a fuzzy neutrosophic matrix.

For

$$A + B = \begin{bmatrix} 0.3 & I+1 & 1+I & 0.5I+2 \\ 2I+1.2 & 1 & 2+0.7I & 1.3 \\ 0.7I & 1.7 & 0.5+0.6I & 1+0.2I \end{bmatrix},$$

clearly the elements of A + B are not in N = [0, 1] $\cup$ [0, I], for 1.3 $\notin$ N. 1.5I + 2 $\notin$ N and so on.

So we define the new type of addition.

We define for any fuzzy neutrosophic m $\times$ n matrices

$$A = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \text{ and } B = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \vdots & \vdots & & \vdots \\ y_{m1} & y_{m2} & \dots & y_{mn} \end{bmatrix}$$

$$\min(A, B) = \begin{bmatrix} \min(x_{11}, y_{11}) & \dots & \min(x_{1n}, y_{1n}) \\ \min(x_{21}, y_{21}) & \dots & \min(x_{2n}, y_{2n}) \\ \vdots & & \vdots \\ \min(x_{m1}, y_{m1}) & \dots & \min(x_{mn}, y_{mn}) \end{bmatrix}$$

where min (x_{ij}, y_{ij}) = x_{ij} if $x_{ij} < y_{ij}$ and x_{ij}, $y_{ij} \in$ [0, 1]

min (x_{ij} y_{ij}) = x_{ij} if $x_{ij} < y_{ij}$ x_{ij}, $y_{ij} \in$ [0, I]

min (x_{ij}, y_{ij}) = x_{ij} if the real coefficient of $x_{ij} \in$ [0, I] is less than the value of y_{ij} in [0, 1].

min $(x_{ij}\ y_{ij}) = x_{ij}$ if coefficient of $x_{ij} = y_{ij}$ and the expert prefers indeterminancy.

min $(x_{ij}\ y_{ij}) = y_{ij}$ if the expert prefers real value to indeterminance $(1 \leq i \leq m, 1 \leq j \leq n)$.

We illustrate this by examples.

$$\min (0.3, 0.7) = 0.3$$

$$\min (0.8I, 0.5I) = 0.5I$$

$$\min (0.3, 0.7I) = 0.3$$

$$\min (0.2I, 0.5) = 0.2I.$$

min (0.2I, 0.2) = 0.2 if the expert prefers real values to indeterminate values.

min (0.3I, 0.3) = 0.3I if the expert prefers indeterminate value to real value.

Now we can define

$$\max (A, B) = \begin{bmatrix} \max(x_{11}, y_{11}) & \ldots & \max(x_{1n}, y_{1n}) \\ \max(x_{21}, y_{21}) & \ldots & \max(x_{2n}, y_{2n}) \\ \vdots & & \vdots \\ \max(x_{m1}, y_{m1}) & \ldots & \max(x_{mn}, y_{mn}) \end{bmatrix}.$$

Now we define max $(x_{ij}\ y_{ij}) = x_{ij}$ if $x_{ij}, y_{ij} \in [0, 1]$ and $x_{ij} \geq y_{ij}$

i.e., max (0.7, 0.3) = 0.7.

max $(x_{ij}, y_{ij}) = x_{ij}$; if $x_{ij}, y_{ij} \in [0, I]$ and $x_{ij} \geq y_{ij}$, i.e., max (0.8I, 0.5I) = 0.8I.

max (0.9I, 0.3) = 0.9I i.e.,

max (x_{ij}, y_{ij}) = x_{ij} if $x_{ij} \in [0, I]$ and $y_{ij} \in [0, 1]$ with real coefficient of $x_{ij} \geq y_{ij}$ in case max (0.9I, 0.9) = 0.9I expert wishes to take an indeterminate value.

If he feels that his model would be sensitive to indeterminancy he / she can choose 0.9I otherwise 0.9.

The major adaptability of the model is the flexibility of it.

Thus in case of fuzzy neutrosophic matrices the addition and subtraction is replaced by max and min explained earlier.

Now we proceed onto define yet some more operations.

Let A be any n × n matrix with entries from the set {0, ± 1, ± I} we define a special type of operation on A with 1 × n row vector whose entries are from the set {0, 1, I} exhibiting the on or off state of the node.

Let

$$A = \begin{bmatrix} 0 & 1 & I & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & I & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -I & 1 & 0 \end{bmatrix}.$$

Let X = (1 0 0 0 0) be the state vector.

Now we find X o A (o is the usual multiplication of X with A).

$$X \text{ o } A = (1\ 0\ 0\ 0\ 0) \begin{bmatrix} 0 & 1 & I & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & I & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -I & 1 & 0 \end{bmatrix}$$

= (0 1 I 0 1) = Y, we threshold and update the state vector Y which is known as resultant vector.

Updating means keeping the on state of the given vector to remain in the onstate otherwise it will be made into on state which will be known as the updating the resultant vector.

Y = (0 1 I 0 1) is updated and Y_1 = (1 1 I 0 1).

The thresholding is, in the resultant every negative value is made as 0 and positive value is made as 1 and positive coefficient indeterminant value is made as I.

The combination of 1 + I is made into I or 1 according to the wishes of the expert depending on the problem.

$$Y_1 \text{ o } A = (1\ 1\ I\ 0\ 1) \text{ o } \begin{bmatrix} 0 & 1 & I & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & I & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -I & 1 & 0 \end{bmatrix}$$

$$= (0\ 1{+}I\ 0\ I\ 1{-}I) \rightarrow (1\ 1\ 0\ I\ 1) = Y_1.$$

This is taken as the final answer.

Such special operations are used in neutrosophic cognitive maps which will be used in chapter III of this book.

Next we proceed onto describe yet another new type of special product used in case of rectangular fuzzy neutrosophic matrices.

Let

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}$$

be a $m \times n$ matrix $m \neq n$ and $X = (x_1, \ldots, x_m)$ be the state vector, the elements $x_i \in \{0, 1, I\}$; $1 \leq i \leq m$ and $a_{ij} \in \{0, \pm 1, \pm I\}$; $1 \leq i \leq m$ and $1 \leq j \leq n$.

We find X o A if X o A = Y after thresholding Y we find Y_1, and calculate Y_1 o A^t.

If Y_1 o $A^t = X_1$ after thresholding and updating X_1 say to X_2 we find X_2 o A and so on.

We will illustrate this by some examples.

Example 2.9: Let

$$A = \begin{bmatrix} 1 & 0 & -1 & I & 0 \\ 0 & -I & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -I \\ 1 & 0 & 0 & 0 & -1 \\ I & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

be a fuzzy neutrosophic matrix.

Let X = (0 1 0 0 0 0 1) be the given state vector we find

$$X \text{ o } A = (0\ 1\ 0\ 0\ 0\ 0\ 1) \text{ o } \begin{bmatrix} 1 & 0 & -1 & I & 0 \\ 0 & -I & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -I \\ 1 & 0 & 0 & 0 & -1 \\ I & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$= (0, -I -1, 0\ 0\ 1) = (0\ 0\ 0\ 0\ 1).$$

Suppose X = (1 0 0 0 1 0 0) to find X o A.

Now

$$X \text{ o } A = (1\ 0\ 0\ 0\ 1\ 0\ 0) \text{ o } \begin{bmatrix} 1 & 0 & -1 & I & 0 \\ 0 & -I & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & -I \\ 1 & 0 & 0 & 0 & -1 \\ I & 1 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$= (1+I, 1, -1, I, 0)$$

after thresholding we get $Y_1 = (1, 1, 0, I, 0)$.

(Some expert who wants to give more weightage to the indeterminate may give $Y_1 = (I\ 1\ 0\ I\ 0)$)

Now we find Y_1 o A^t

$$= (I\ 1\ 0\ I\ 0) \text{ o } \begin{bmatrix} 1 & 0 & -1 & 1 & I & 0 & 0 \\ 0 & -I & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -I & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$= (2I,\ -I,\ -I,\ I,\ 1+I,\ I,\ -I) = (I, 0, 0, I, 1, I, 0).$$

After updating and thresholding we can get

$X_2 = (I\ 0\ 0\ I\ 1\ I\ 0)$ or ((I 0 0 I 1 I 0) according to the experts wishes).

We find X_2 o A = (I 0 0 I 1 I 0) × A

$$= (3I\ 1\ 0\ 2I\ -I) = Y_2.$$

After thresholding Y_2 we get

$$Y_3 = (I\ 1\ 0\ I\ 0).$$

Now Y_3 o $A^t = (2I\ \ -I\ \ -I\ \ I\ \ 1+I\ \ I\ -1) = X_3$.

We can update X_3 and proceed onto work in a similar manner.

After updating and thresholding X_3 we get
$X_4 = (I\ 1\ 0\ I\ 1\ I\ 1)$.

We work on in this manner.

Now we proceed onto define the notion of min max operations as the product of two fuzzy neutrosophic matrices.

Let

$$A = \begin{bmatrix} 0.3I & 0.5 & 0.8I \\ 0 & 0.7 & 1 \\ 0.4 & 0.6I & 0.5 \end{bmatrix}$$

be a 3 × 3 fuzzy neutrosophic matrix.

$$B = \begin{bmatrix} 0.9 & 0.5I & 0.7 & 0.7 \\ 0.3I & 0.2 & 0 & 0.9 \\ I & 0 & 0.5I & 0.5 \end{bmatrix}$$

be a 3 × 4 fuzzy neutrosophic matrix.

We have to find A o B using max min product.

$$\text{max min } \{A, B\} = A \text{ o } B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \end{bmatrix}.$$

We find max {min (0.3I, .9), min (0.5, 0.3I) , min (0.8I, I)}

= max {0.3I, 0.3I, 0.8I}

= 0.8I = r_{11} and so on.

max {min {0.4, 0.5I}, min {0.6I, 0.2}, min {0.5, 0}}

= max {0.4, 0.2, 0} = 0.4 = r_{32} and so on.

Thus if

$$A \text{ o } B = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \end{bmatrix} = \begin{bmatrix} 0.8I & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & 0.4 & r_{33} & r_{34} \end{bmatrix}.$$

On similar lines r_{ij}'s can be calculated $1 \leq i \leq 3$ and $1 \leq j \leq 4$.

Now we proceed onto define yet another type of new operation on new type of matrices which will be used in NBAM (Neutrosophic Bidirectional Associative Memories).

Consider a neutrosophic interval $N = [–n, n] \cup [–nI, nI]$; $n > 1$ an integer any element $x \in N$ is of the form $x + yI$; $x, y \in [–n, n]$, $x = 0$ or $y = 0$ can also occur in case $x = 0$; $yI \in [–nI, nI]$, when $y = 0$, $x \in [–n, n]$.

Let A be any $s \times t$ matrix with elements from N; i.e., $A = (a_{ij})$; $a_{ij} \in N$, $1 \leq i \leq s$, $1 \leq j \leq t$.

Take any $s \times 1$ row vector X with entries from N.

These collections of X with entries from N denoted by F_x are special type of vectors with which we work in this book. Likewise all vectors Y with entries from N denoted by F_Y will also be defined in chapter five.

Now we use yet another product on fuzzy neutrosophic matrices which we would explain briefly.

Let $A = (a_{ij})$ be a $n \times m$ fuzzy neutrosophic matrix with entries from $[0, 1] \cup [0, I]$ i.e., $a_{ij} \in [0, 1] \cup [0, I]$ and $1 \leq i \leq n$ and $1 \leq j \leq m$.

Let $X = (x_1, …, x_m)$ where $x_i \in \{0, 1, I\}$.

Now we find $Y = A \text{ o } X$.

$Y = (y_1, \ldots, y_n)$ where $y_i = \max \min (a_{ij}, x_j)$ and $1 \leq j \leq m$, $1 \leq i \leq n$.

We find $Y \text{ o } A = X$ where $x_j = \max (y_i, m_{ij})$, $1 \leq j \leq m$, $i = 1, 2, \ldots, n$.

This type of operations would be described and used in Neutrosophic Associative Memories (NAM) model in chapter six of this book.

Chapter Three

NEUTROSOPHIC COGNITIVE MAPS MODEL AND NEUTROSOPHIC RELATIONAL MAPS MODEL

In this chapter we just recall the definitions of two neutrosophic models, Neutrosophic Cognitive Maps (NCMs) model and Neutrosophic Relational Maps (NRMs) model and illustrate them with examples.

The reader is expected to know the concept of Fuzzy Cognitive Maps model and Fuzzy Relational Maps model [5-6, 8].

We just recall the definition of Neutrosophic Cognitive Maps model and describe how they function.

DEFINITION 3.1: *A Neutrosophic Cognitive Maps (NCMs) is a neutrosophic directed graph with concepts like policies or events etc; as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts. Let* C_1, C_2, …, C_n *denote n-nodes further we assume each node is a neutrosophic vector from the neutrosophic vector space V. So a node* C_1 *will be represented by* $(x_1, x_2, \ldots, x_n)$ *where* x_k*'s are zero* *or 1 or I (I is the indeterminate) and $x_k = 1$ means the node is in the ON state, $x_k = I$ implies the node is in the indeterminate state at that time or in that context and $x_k = 0$ the node is in the OFF state.*

Let C_i and C_j denote the two nodes of an NCM. The directed edge from C_i to C_j denotes the causality of C_i on C_j called connections. Every edge in the NCM is weighted with a number in the set {–1, 0, 1, I}. Let e_{ij} be the weight of the directed edge C_iC_j of the graph; $e_{ij} \in \{–1, 0, 1, I\}$. $e_{ij} = 0$ if C_i does not have any effect on C_j, $e_{ij} = 1$ if increase (or decrease) in C_i causes increase (or decrease) in C_j. $e_{ij} = –1$ if increase (or decrease) in C_i causes decrease (or increase) in C_j. $e_{ij} = I$ if the relation or effect of C_i on C_j is an indeterminate. NCMs with edge weight from {–1, 0, 1, I} are called simple NCMs.

Let $C_1, C_2, \ldots, C_n$ be nodes of a NCM. Let the neutrosophic matrix N(E) be defined as $N(E) = (e_{ij})$ (N(E) is the matrix associated with the neutrosophic directed graph), where e_{ij} is the weight of the directed edge $C_i C_j$, where $e_{ij} \in \{–1, I, 1, 0\}$. N(E) is called the adjacency matrix of the NCM.

Let $C_1, C_2, \ldots, C_n$ be the nodes of the NCM.

$A = (a_1, a_2, \ldots, a_n)$ where $a_i \in \{0, 1, I\}$. A is called the instantaneous state neutrosophic vector and it denotes the ON or OFF or indeterminate state/position of the node at that instant:

$a_i = 0$ if a_i is in OFF state (no effect)

$a_i = 1$ if a_i in the ON state (that is has effect or influence on the system)

$a_i = I$ if a_i is in the indeterminate state that is one cannot say the effect or the effect cannot be determined; $i = 1, 2, \ldots, n$.

Let $C_1, \ldots, C_n$ be the nodes of an NCM. $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \ldots, \overrightarrow{C_iC_j}$ be the edges of the NCM. Then the edges form a directed cycle. An NCM is said to be cyclic if it possesses a directed cycle. An NCM is said to be acyclic if it does not possess any directed cycle.

An NCM with cycles is said to have a feed back. When there is a feedback in the NCM; i.e., when the causal relations flow through a cycle in a revolutionary way / manner the NCM is called a dynamical system.

Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, …, $\overrightarrow{C_iC_j}$, …, $\overrightarrow{C_{n-1}C_n}$ be a cycle when C_i is switched on and if the causality flow through the edges of a cycle and it again causes C_i, we say that the dynamical system goes round and round. This is true for any node C_i, for i = 1, 2, …, n. The equilibrium state for this dynamical system is called the hidden pattern.

If the equilibrium state of a dynamical system is a unique state vector, then it is called the fixed point. Consider the NCM with C_1, C_2, …, C_n as nodes. For example let us start the dynamical system by switching on C_1. Let us assume that the NCM settles down with C_1 and C_n on, that is the state vector remain as (1, 0, …, 1) this neutrosophic state vector (1, 0, …, 0, 1) is called the fixed point.

If the NCM settles with a neutrosophic state vector repeating in the form

$A_1 \to A_2 \to \ldots \to A_i \to A_1$ then this equilibrium is called a limit cycle of the NCM.

We just indicate the methods of determining the hidden pattern of a neutrosophic dynamical system E.

Let C_1, C_2, …, C_n be the nodes of an NCM. Let N(E) be the associated adjacency matrix.

Let us find the hidden pattern when C_1 is switched on when an input is given as the vector A_1 = (1, 0, …, 0), this data should pass through the neutrosophic matrix N(E), this is done by multiplying A_1 by the matrix N(E). Let $A_1N(E) = (a_1, a_2, \ldots, a_n)$ with threshold operation, that is replacing a_i by 1 $a_i > k$ and a_i by 0 if $a_i \leq k$ (k a suitable positive integer) and a_i by I if a_i is not a integer but a neutrosophic number. We update $A_1N(E)$ by keeping C_i in the on state in the resulting vector that is the first coordinate of $A_1N(E)$ is 1. Suppose $A_1N(E) \hookrightarrow A_2$ ('$\hookrightarrow$' denotes the resultant vector has been updated and thresholded) we find $A_2N(E)$ and repeat the same procedure until we arrive at a fixed point or a limit cycle which is known as the hidden pattern of the dynamical system.

Now we know a NCM or an FCM works on the experts opinion. Suppose we have more than one expert and we should give importance to each and every expert in the same way. We achieve this by defining the notion of combined NCMs.

Suppose we have p experts working on the same problem with same set of nodes say $C_1, C_2, \ldots, C_n$ and all of them work with NCM model. We combine the p experts opinions as follows. Let $N(E_1), N(E_2), \ldots, N(E_p)$ be the neutrosophic adjacency matrices of the NCMs with nodes $C_1, C_2, \ldots, C_n$ then the combined NCM is got by adding all the neutrosophic adjacency matrices $N(E_1), N(E_2), \ldots, N(E_p)$. We denote the combined NCMs neutrosophic adjacency matrix by

$$N(E) = N(E_1) + N(E_2) + \ldots + N(E_p).$$

The following facts are important so enumerated.

Note 1: Here in this NCM model the nodes $C_1, C_2, \ldots, C_n$ are not indeterminate nodes because they indicate the concepts which are well known. But edges connecting C_i and C_j may be an indeterminate i.e., an expert may not be in a position to say that C_i has some causality on C_j either will be in a position to state that C_i has no relation with C_j in such cases the relation between C_i and C_j when an indeterminate is denoted by I.

Note 2: The method of finding the combined effect of the opinions of all the experts has both advantages as well as disadvantages.

The main advantage being the law of large numbers when several experts give opinion of about the problem one feels the solution is more valid or authenticated but the disadvantage is mutually opposite views cancel to give no effect on the system.

We now illustrate this situation by an example.

Example 3.1: We study in this example the child labour problem prevalent in India. We use the NCM model to study this problem.

We briefly describe the attributes related with the child labour problem.

C_1 : Child labour when we say it includes all types of labour or work or errand given to children below the age of 14 years. This also includes cleaning the house, watering the plants, running petty errands, domestic workers, working in tea shops, hotels etc., and rag picking.

C_2 : Political leaders. We include the political leaders in our study for the following reasons. For if the political leaders are stern and punish the industries or companies or hostels which practice child labour certainly they can reduce the number of child labourers in this country.

They help in the flourishing of the child labour for they know in the first place children are not vote banks and second these industries are the financiers for the politicians.

C_3 : Good teachers. Good teachers can stop the school dropout which directly can minimize the child labourers. If teachers are good and take special care of these children certainly child labour can be reduced though not eradicated.

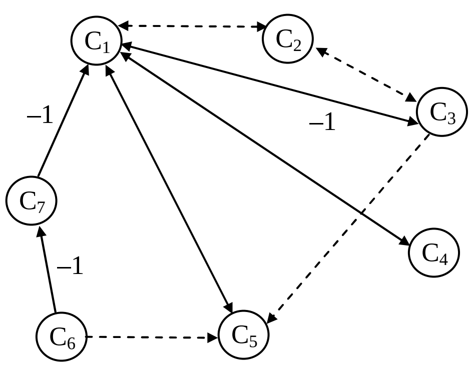
C_4 : Poverty. Poverty is one of the major cause for child labour. For parents feel when they have nothing to eat what is the use of educating their child.

C_5: Industries. We include the industries for they are the ones who practice child labour. For we know, match factories, big grocery stores, bedi factories, garment industries etc. employ more children and women than adults for two reasons. (1) They are sincere and will not ask even if they ask them to work for more hours and they do not know to while away the time. Thus they are made to work from 11 to 12 hours a day mostly as daily wagers. Secondly they are paid less than men so that is also advantageous to the industries. So in our study we are forced to induct industrialists as one of the nodes.

C_6 : Public. We are forced to include public for few reasons. They would not bother about the child labours even if they know some one is practicing it. Second they also encourage child labours as domestic servants, sweepers etc.

C_7: NGO's who take care of these school dropouts misuse them they belong to the category of bad NGOs and others help these children to study and continue their education, these NGOs will be termed as good NGOs.

Now we give the directed neutrosophic graph of an expert which is as follows:



The corresponding adjacency matrix N(E) related with the neutrosophic directed graph is as follows:

$$N(E) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & I & -1 & 1 & 1 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 \\ -1 & I & 0 & 0 & I & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Suppose we take the state vector $A_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0)$, that is the node child labour alone is in the on state and all other nodes are in the off state.

We will study the effect of the state vector A_1 on N(E). $A_1N(E) = (0\ I\ –1\ 1\ 1\ 0\ 0) \hookrightarrow (1\ \ I\ 0\ 1\ 1\ 0\ 0) = A_2$ say ($\hookrightarrow$ denotes the state vector has been updated and thresholded).

We now study the effect of A_2 on N(E).

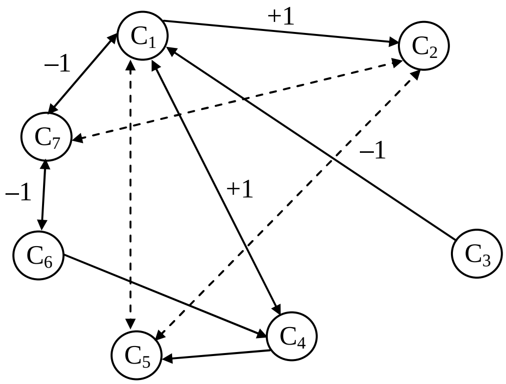
$A_2N(E) = (I + 2, I, –1+I, 1, 1, 0, 0) \hookrightarrow (1, I, 0, 1, 1, 0, 0) = A_3$ (say). It is clearly $A_2 = A_3$.

Thus the hidden pattern of the neutrosophic dynamical system N(E) is a fixed point given by $A_2 = (1\ I\ 0\ 1\ 1\ 0\ 0)$.

Thus we see if child labour is in vogue then the role of political leaders is an indeterminate. Poverty is a reason for child labour and the other reason is industrialist promote child labour.

Thus using NCMs we can work with the on state of any state vector.

Suppose another experts gives the following directed neutrosophic graph.



The corresponding neutrosophic connection matrix $N(E_1)$ is as follows:

$$N(E_1) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 1 & -1 & 1 & I & 0 & -1 \\ 0 & 0 & 0 & 0 & I & 0 & I \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & I & 0 & 0 & 0 & -1 & 0 \end{array}\right] \end{array}.$$

Let us study the only on state of the node C_1 that is child labour is present.

Suppose $A_1 = (1, 0, 0, 0, 0, 0, 0)$ is the state vector, to find the effect of A_1 on the neutrosophic dynamical system $N(E_1)$.

$$A_1N(E_1) = (0\ 1\ {-1}\ 1\ I\ 0\ {-1}) \hookrightarrow (1\ 1\ 0\ 1\ I\ 0\ 0) = A_2 \text{ (say)}$$

$$A_2\, N(E_1) = (1 + I,\ 1 + I,\ -1,\ 1,\ 2I+1,\ 0,\ -1+I)$$
$$\hookrightarrow\ (1\ 1\ 0\ 1\ 1\ 0\ 0) = A_3 \text{ (say)}.$$

We see $A_3N(E_1) \hookrightarrow A_3$.

We see the thresholding is done such that if we have $a_i = k + tI$ then we replace a_i by 1 if $k > 0$ and by 0 if $k < 0$.

The hidden pattern of the neutrosophic dynamical system is a fixed point given by A_3.

It is pertinent to keep on record that thresholding can be carried out in a different way by replacing 1+I by I and so on. The thresholding is mainly dependent on the nature of the problem and the experts' wishes.

We give one more illustration of the use of NCM model.

Example 3.2: Suppose we are interested in studying about Hacking of e-mail by students. The main problem of concern of the present day is; "How safe are the messages sent by e-mail?

Is there enough privacy? For if a letter is sent by post one can by certain say that it cannot be read by any other person, other than the receiver. Even tapping or listening (over hearing) of phone call from an alternate location / extension is only a very uncommon problem.

Here we study the problem using NCM.

However compared to these modes of communication even though e-mail guarantees a lot of privacy is highly a common practice to hack e-mail. Hacking is legally a cyber crime but is also one of the crimes that do not leave any trace of the hacker. Hacking another persons mail can be carried out for many reasons.

Here we study the variety of purposes and factors which are root cause of such crimes.

The following are the nodes associated with the problem of hacking e-mail.

C_1 : Curiosity. The curiosity to know what is his / her friend doing is one of the major reasons for hacking. Some times hacking is done to know the question paper for the test or assignment at times to know the private information of their teachers or girl friends in case of boys and boy friends in case of girls.

C_2 : Professional rivalry. The mail of a person can also be hacked due to professional rivalry to know what the friends do in case of research scholars or teachers and so on.

C_3 : Jealously / enmity. Many a times the e-mail get hacked due to rivalry or enmity or jealousy from a collegue who works or studies with them. This is mainly done to harm them and feel insecure.

C_4: Sexual satisfaction. This is mainly done by a friend of either the girl or a boy to go thro' the messages between two friends this is mainly done for some sexual satisfaction or so.

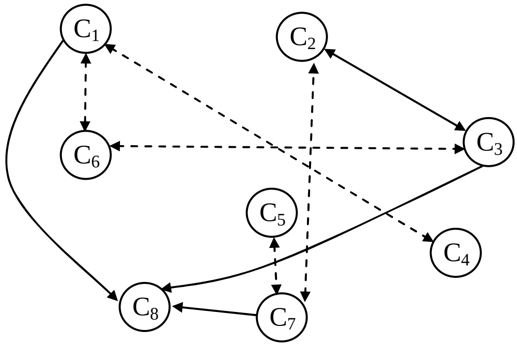
C_5: Fun / pastime. They hack e-mails only to show that they are very intelligent and they take this as a pastime or part time job.

C_6 : To satisfy ego. This is a very small group who just challenge and to get a satisfaction of their ego they go about to hack the e-mail.

C_7: Women students. Male students usually hack the e-mail accounts of the women students to know their private life. This is a biggest crime as they are invading the privacy of women students.

C_8: Breach of trust. Finally hacking of e-mail is a breach of trust and infact a criminal act.

Now using these eight nodes we proceed onto get the experts opinion. The neutrosophic directed graph given by the expert is as follows:



The corresponding neutrosophic connection matrix N(E) is as follows:

$$N(E) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I & 0 & I & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & I & 0 \\ 0 & 1 & 0 & 0 & 0 & I & 0 & 1 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & I & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Suppose we take the instantaneous state vector A_1 = (0 0 0 0 0 0 1 0) that women students e-mail alone hacked to be in the on state and all other nodes remain in the off state.

To find the effect of A_1 on the neutrosophic dynamical system N(E) is given by

A_1 N(E) = (0 I 0 0 I 0 0 1) $\hookrightarrow$ (0 I 0 0 I 0 1 1) = A_2 (say).

Now we study the effect of A_2 on the system.

$$A_2N(E) \hookrightarrow (0\ I\ I\ 0\ I\ 0\ 1\ 1\ ) = A_3 \text{ (say)}$$
$$A_3N(E) \hookrightarrow (0\ I\ I\ 0\ I\ I\ 1\ 1\ ) = A_4 \text{ (say)}$$
$$A_4N(E) \hookrightarrow (I\ I\ I\ 0\ I\ I\ 1\ 1\ ) = A_5 \text{ (say)}$$
$$A_5N(E) \hookrightarrow (I\ I\ I\ 0\ I\ I\ 1\ 1\ ) = A_6 \text{ (say)}.$$

We see $A_5 = A_6$. Thus the hidden pattern of the neutrosophic cognitive maps model is a fixed point given by $A_6 = (I\ I\ I\ 0\ I\ I\ 1\ 1)$.

So when women students e-mail are attacked alone is in the on state we see curiosity is an indeterminate, professional rivalry is an indeterminate, jealously / enmity also is an indeterminate; fun / pastime and to satisfy the ego are also indeterminate only the node which is not an indeterminate is Breach of trust.

Thus we can work with any on state on the node and find the resultant.

This is the way NCMs functions.

In the book Periyars views on untouchability we have worked with NCMs models to study the problem [12].

NCMs can also be used in the analysis of strategic planning simulation based on NCMs knowledge and differential game. This model will also be ideal in studying share market problem. In case the resultant is an indeterminate we need not invest for that particular node or keep the investment in abeyance.

Next we can also use the NCMs to study the hyper knowledge representation in strategy formation process [8]. Several other properties about NCMs and special types of NCMs are dealt in [8].

However in this book as we are mainly interested in constructing fuzzy neutrosophic models for socio scientist so we have restrained from describing them. But interested reader can refer [8] for more information about NCMs.

Now we proceed onto describe the Neutrosophic Relational Maps (NRM) model which are built analogous to the Fuzzy Relational Maps model.

Neutrosophic Cognitive Maps promote the causal relationship between concurrently active units or decides the absence of any relation between two units or the indeterminacies of any relation between two units.

But in Neutrosophic Relational Maps (NRMs) we divide the very causal nodes into two disjoint units. Thus for the modeling of the NRM we need a domain space and a range space which are disjoint in the sense of concepts.

We further assume no intermediate relation exists between the nodes with in the range space or with in the domain space. However if even some relation exists within the range space or with in the domain space the expert is not interested in studying them we can use this model.

The number of elements or nodes in the range space need not be equal to the number of elements or nodes in the domain space.

Throughout this book we assume the elements of a domain space are taken from the neutrosophic vector space of dimension n and that of the range space are neutrosophic of dimension m (m in general is not equal to n).

We denote the nodes of R by $R_1, \ldots, R_m$ of the range space where $R = \{(x_1, \ldots, x_m) \mid x_j = 0 \text{ or } 1 \text{ for } j = 1, 2, \ldots, m\}$.

If $x_i = 1$ it means that node R_i is in the on state and if $x_i = 0$ it means node R_i is in the off state and if $x_i = I$ in the resultant vector it means the effect if the node x_i is an indeterminate or whether it will be off or on cannot be predicted by the neutrosophic dynamical system.

Now we proceed onto define neutrosophic relational maps model.

A Neutrosophic Relational Maps (NRMs) is a neutrosophic directed graph or a map from D to R with concepts like policies or events etc as nodes and causalities as edges (Here by causalities we mean or include the indeterminate causalities also). It represents Neutrosophic Relations and Casual Relations between spaces D and R.

Let D_i and R_j denote the nodes of an NRM. The directed edge from D_i to R_j denotes the causality of D_i on R_j called relations.

Every edge in the NRM is weighed with a number from the set {0, 1, –1, I}. Let e_{ij} be the weight of the edge D_iR_j, $e_{ij} \in \{0, 1, -1, I\}$.

The weight of the edge is positive if increase in D_i implies increase in R_j or decrease in D_i implies decrease in R_j that is causality of D_i on R_j is 1. If $e_{ij} = -1$ then increase (or decrease) in D_i implies decrease (or increase) in R_j. If $e_{ij} = 0$ then D_i does not have any effect on R_j.

If $e_{ij} = I$ it implies that we are not in a position to determine the effect of D_i on R_j i.e., the effect of D_i on R_j is an indeterminate so we denote it by I.

When the nodes of the NRM take edge values from {0, 1, I, –1} we say the NRMs are simple NRMs.

Let $D_1, D_2, \ldots, D_n$ be the nodes of the domain space D of the NRM and let $R_1, R_2, \ldots, R_m$ be the nodes of the range space R of the same NRM, Let the matrix N(E) be defined as $N(E) = (e_{ij})$ where e_{ij} is the weight of the directed edge D_iR_j

(or R_j D_i) and $e_{ij} \in \{0, 1, -1, I\}$. N(E) is called the Neutrosophic relational matrix of the NRM.

Unlike NCMs, NRMs can also be rectangular matrices with rows corresponding to the range space. This is one of the marked differences between NRMs and NCMs. Further the number of entries for a particular model which can be treated as disjoint sets when dealt as NRM has less entries when the same model is treated as a NCM.

Thus when the unsupervised data can be dealt as two disjoint units it is always better to use NRMs than NCMs for certainly NRM will save not only time and economy but give a pair of state vectors as a hidden pattern which indicates the effect of one space on the other.

Let $D_1, D_2, \ldots, D_n$ and $R_1, R_2, \ldots, R_m$ denote the nodes of a NRM. Let $A = (a_1, a_2, \ldots, a_n)$ with $a_i \in \{0, 1, I\}$ is called the neutrosophic instantaneous state vector of the domain space and it denotes the on-off or the indeterminate state of the node at the instant. Similarly $B = \{(b_1, b_2, \ldots, b_m)$ where $b_i \in \{0, 1, I\}$; $(1 \leq i \leq m)\}$ denotes the instantaneous state of the nodes of the range space, that is the on or off or the indeterminate state of the node at that time.

If $a_i = 0$, then a_i is off, if $a_i = 1$ then a_i is in the on state and if $a_i = I$ at that time we cannot determine the state of the node or the node is in the indeterminate state. Similar reasoning for the nodes b_j; $1 \leq j \leq m$, $b_j \in \{0, 1, I\}$.

Let $D_1, D_2, \ldots, D_n$ and $R_1, R_2, \ldots, R_m$ be the nodes of the NRM. Let D_iR_j (or R_j D_i) be the edge of an NRM, j = 1, 2, …, m and i= 1, 2, …, n. The edges form a directed cycle. An NRM is said to be a cycle if it process a directed cycle. An NRM is said to be acyclic if it does not possess any directed cycle.

A NRM with cycles is said to be a NRM with a feed back.

When there is a feedback in the NRM, i.e., when the causal relations flow through a cycle in a revolutionary manner the NRM is called a neutrosophic dynamical system.

Let D_iR_j (or R_jD_i), $1 \leq j \leq m$, $1 \leq i \leq n$, when R_j (or D_i) is switched on and if causality flows through edges of a cycle and it again causes R_j (or D_i) we say that the neutrosophic dynamical system goes round and round. This is true for any node R_j (or D_i), $1 \leq j \leq m$ (or $1 \leq i \leq n$). The equilibrium state of the dynamical system is called or defined as the hidden pattern of the dynamical system.

If the equilibrium state of a dynamical system is a unique neutrosophic vector then it is defined as a fixed point.

Consider an NRM with $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ as nodes. For example let us start the dynamical system by switching on R_1 (or D_1). Let us assume that the NRM settles down with R_1 and R_m (or D_1 and D_m) on, or indeterminate or off, that is the neutrosophic state vector remains as (1 0 0 … 0 1) or (1 0 … 0 I) (or (1 0 0 … 1) or (1 0 … 0 I)); this state vector is called the fixed point.

If the NRM settles down with a state vector repeating in the form $A_1 \to A_2 \to A_3 \to \ldots \to A_i \to A_1$ or $(B_1 \to B_2 \to \ldots \to B_i \to B_1)$ then this equilibrium is defined as the limit cycle.

Now we describe the method of determining the hidden pattern of an NRM.

Let $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ be the nodes of the NRM with feed back. Let N(E) be the neutrosophic relational matrix. Let us find the hidden pattern when D_1 is switched on that is when an input is given as a vector; $A_1 = (1, 0 \ldots, 0)$ in D the data should pass through the neutrosophic relational matrix N(E). This is done by multiplying A_1 with the neutrosophic relational matrix N(E). Let $A_1N(E) = (r_1, \ldots, r_m)$ after thresholding and updating the resultant vector; we get $A_1N(E) \in$ R.

Now let $B = A_1N(E)$ we pass on B into the system $(N(E))^T$ and obtain $B(N(E))^T$. We update and threshold the vector $B(N(E))^T$ so that $B(N(E)^T) \in D$. This procedure is repeated until we arrive at a fixed point or a limit cycle. We see in case of NRMs we see the hidden pattern is always a pair of points.

We will illustrate this situation by an example or two.

Example 3.3: Now we study the teacher student problems. We take a few nodes associated with the teacher as the domain space and the nodes related with the student as the range space.

We take D_1, D_2, …, D_5 as the nodes related with the teacher. The concepts D_i are self explanatory; $1 \leq i \leq 5$.

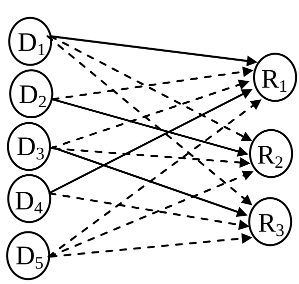
D_1 - Teacher is good
D_2 - Teaching is good
D_3 - Teaching is mediocre
D_4 - Teacher is kind
D_5 - Teacher is harsh (rude).

These five nodes are taken as the domain space.

Consider the following three nodes as the range space related with the student.

R_1 - Good student
R_2 - Bad student
R_3 - Average student.

The neutrosophic directed graph of the teacher student model is as follows:



The associated connection matrix of the neutrosophic directed graph is as follows:

$$N(E) = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array} \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{bmatrix} 1 & I & I \\ I & 1 & 0 \\ I & I & 1 \\ 1 & 0 & I \\ I & I & I \end{bmatrix} \end{array}.$$

Let A_1 = (1 0 0 0 0) be the instantaneous state vector in which only the node; teacher is good is in the on state and all other nodes are in the off state.

To find the effect of A_1 on the neutrosophic dynamical system N(E).

$$\begin{aligned}
A_1N(E) &= (1\ I\ I) = A_2 \text{ (say)} \\
A_2(N(E))^T &= (1 + I, 1+I, I, 1+I, I) \text{ (we replace nI by I)} \\
&\hookrightarrow (1\ 1\ I\ I\ I) = B_1 \text{ (say)} \\
B_1\ N(E) &= (2 + I, I+1, I) \\
&\hookrightarrow (1, I, I) = A_3 \text{ (say)} \\
A_3(N(E))^T &= (1 + I, I, I, 1+I, I) \\
&\hookrightarrow (1\ I\ I\ 1\ I) = B_2 \text{ say} \\
B_2(N(E)) &\hookrightarrow (1\ I\ I) = A_4\ (= A_3).
\end{aligned}$$

Thus the hidden pattern of the dynamical system is a fixed point given by the pair {(1, 1, I, 1, I), (1, I, I)}.

Thus when the teacher is good teaching is good. Teacher is mediocre is in the indeterminate state.

Teacher is kind comes to on state, whether the teacher is harsh is also indeterminate.

However if teacher is good certainly the student is good at studies however the student being bad and student being average remains as an indeterminate.

However the authors want to keep on record that by no means we have collected the data for this example, just we have only given as an illustration.

Now we present one more example of how the NRM works.

Example 3.4: Suppose one is interested in studying the problem of how the public interact with HIV/AIDS patience. For in the first place public may fear that they may get the disease and they associate a social stigma and so on.

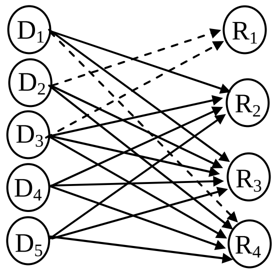
However we first enumerate the feelings of HIV/AIDS patients which are taken as the domain space of the system.

D_1 - Feeling of loneliness
D_2 - Feeling of guilt
D_3 - Desperation / fear of public
D_4 - Suffering both mental / physical
D_5 - Public disgrace.

The concepts / nodes related with the public taken as the nodes of the range space.

R_1 - Fear of getting the disease
R_2 - No mind to forgive the HIV/AIDS patients sin
R_3 - Social stigma to have HIV/AIDS patient as a friend
R_4 - No sympathy.

We now proceed onto give the neutrosophic graph using the experts opinion.



The corresponding neutrosophic connection matrix of the neutrosophic graph is as follows:

$$N(E) = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{array} \begin{array}{c} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & I \\ I & 0 & I & 1 \\ I & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & I & 1 & 1 \end{bmatrix} \end{array}.$$

Suppose one is interested in studying the impact of the state vector $A_1 = (0\ 1\ 0\ 0\ 0)$, that is the feeling of guilt is in the on state.

To find the effect of A_1 on the neutrosophic dynamical system N(E).

$A_1N(E) = (I\ 0\ I\ 1) = B_1$.

We find $B_1(N(E))^T = (1, 1+I, 1+I, 1+I, 1+I)$.

After updating and thresholding we get

$B_1\ (N(E))^T \hookrightarrow (I\ 1\ 1\ 1\ 1) = A_2$

$A_2N(E) = (I, 1+I, 1+I, 1+I)$

$\hookrightarrow (I\ 1\ 1\ 1) = B_2$ (say)

$$B_2(N(E))^T = (2+I, 1+2I, 3+I, 3, 2 + I)$$
$$\hookrightarrow (1, 1, 1, 1, 1) = A_3 \text{ (say)}$$

$$A_3\, N(E) = (2I \;\; 3+I \;\; 4+I \;\; 4+I)$$
$$= (I\ 1\ 1\ 1) = B_3\ (= B_2).$$

Thus the hidden pattern of the system is a fixed point pair given by {(I, 1, 1, 1) (1, 1, 1, 1, 1)}.

We just give one more illustration namely NRMs to analyze the employee and employers relationship model.

Without a congenial relation of the employee and the employer the industrial harmony cannot exist. So the study of employee and employer relationship is an important problem.

For instance the employer expects to achieve consistent production quality product at the optimum production to earn profit. However there may be profit no profit loss in business or heavy loss depending on various factors such as demand, supply, rent, electricity, raw materials, transportations, consumable etc., Since the relationship involves concepts of uncertainty and indeterminacy we are justified in using NRM models.

We have 8 nodes related with the employee which is taken as the nodes of the NRM model. The concepts are self explanatory. We just list them.

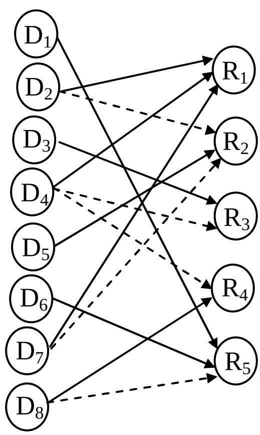
D_1 - Pay with allowances and bonus to the employee
D_2 - Only pay to the employee
D_3 - Pay with allowance to the employee
D_4 - Best performance by the employee
D_5 - Average performance by the employee
D_6 - Poor performance by the employee
D_7 - Employee works for more number of hours
D_8 - Employee work for less number of hours.

These eight nodes are taken as the domain space of the NRM model.

Now we proceed onto describe the nodes of the range space of the NRM model which relates to the employer.

R_1 - Maximum profit to the employer
R_2 - Only profit to the employer
R_3 - Neither profit nor loss to the employer
R_4 - Loss to the employer
R_5 - Heavy loss to the employer.

The neutrosophic directed graph given by the first expert for this problem is as follows:



The associated connection neutrosophic matrix of the neutrosophic directed graph is as follows:

$$N(E_1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & I & I & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & I \end{bmatrix}.$$

Suppose we are interested in studying the on state of the node D_1 - pay with allowances and bonus to the employee and all other nodes of the domain space is in the off state.

To find the effect of $X_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$ on the neutrosophic dynamical system $N(E_1)$.

$X_1 N(E_1) = (0\ 0\ 0\ 0\ 1) = Y_1$ (say)

$$Y_1 (N(E_1))^t \hookrightarrow (1\ 0\ 0\ 0\ 0\ 1\ 0\ I) = X_2 \text{ (say)}$$

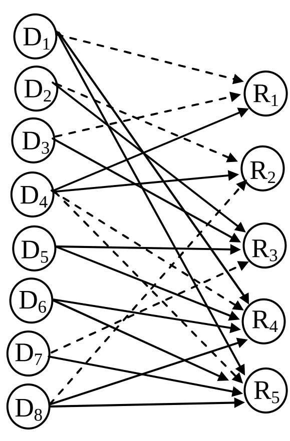
$$X_2 (N(E_1)) \hookrightarrow (0\ 0\ 0\ I\ 1{+}I) = (0\ 0\ 0\ I\ 1) = Y_2 \text{ (say)}$$

$$Y_2 (N(E_1))^t \hookrightarrow (1\ 0\ 0\ 0\ 0\ 1\ 0\ I + I) = (1\ 0\ 0\ 0\ 0\ 1\ 0\ I) = X_3 \text{ (say)}.$$

But $X_3 = X_2$. Thus the hidden pattern of this state vector X_1 is a fixed point pair given by {(1 0 0 0 0 1 0 I), (0 0 0 I 1)}.

We can interpret in the usual way. Now suppose we have yet another expert say, second expert wants to give his opinion for the problem.

Suppose the directed graph given by him is as follows:



The neutrosophic connection matrix associated with this neutrosophic directed graph is as follows:

$$N(E_2) = \begin{bmatrix} I & 0 & 0 & 1 & 1 \\ 0 & I & 1 & 0 & 0 \\ I & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & I & I \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & I & 0 & 1 \\ 0 & I & 0 & 1 & 1 \end{bmatrix}.$$

Suppose let T_1 = (1 0 0 0 0 0 0 0) be the state vector the expert wants to work with. We see all the nodes in the domain space is off except the node D_1.

To find the effect of T_1 on $N(E_2)$.

$T_1N(E_2)$ = (I 0 0 1 1) = S_1 (say)

$$\begin{aligned} S_1 (N(E_2))^t &\hookrightarrow (I + 2, 0, I, 1+2I, I, I, 1, 2) \\ &= (1\ 0\ I\ I\ I\ I\ 1\ 1) \\ &= T_2 \text{ (say)} \end{aligned}$$

$$\begin{aligned} T_2 (N(E_2)) &= (3I, 2I, 4I, 2I+1, 3+2I) \\ &\hookrightarrow (I\ I\ I\ 1\ 1) \\ &= S_2 \text{ (say)} \end{aligned}$$

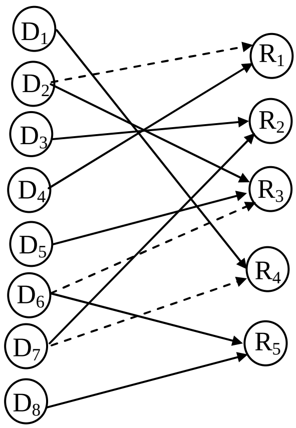
$$\begin{aligned} S_2 (N(E_2))^t &\hookrightarrow (1\ I\ I\ I\ I\ I\ 1\ 1) \\ &= T_3 \text{ (say)} \end{aligned}$$

$$T_3 (N(E_2)) \hookrightarrow (I\ I\ I\ 1\ 1) = S_3 \text{ (say).}$$

Thus we see the hidden pattern is a fixed point pair given by {(1, I, I, I, I, I, 1, 1), (I, I, I, 1, 1)}.

Thus we see the resultant vectors for the same on state of the node happens to be different.

Now a third expert also wants to give his opinion of the employee - employer relations problems with same set of domain space nodes and range space nodes is given by following directed neutrosophic graph;



The associated connection neutrosophic matrix of this neutrosophic directed graph is as follows:

$$N(E_3) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ I & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & I & 0 & 1 \\ 0 & 1 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

This expert also wants to study the effect of the on state of the node D_1 in the domain space.

Let $V_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$. To find the effect of V_1 on the neutrosophic dynamical system $N(E_3)$.

$$V_1 N(E_3) = (0\ 0\ 0\ 1\ 0) = W_1 \text{ (say)}$$

$$W_1 (N(E_3))^t \hookrightarrow (1\ 0\ 0\ 0\ 0\ 0\ I\ 0) = V_2 \text{ (say)}$$

$$V_2 (N(E_3)) = (0\ I\ 0\ 1{+}I\ 0) \hookrightarrow (0\ I\ 0\ 1\ 0) = W_2 \text{ (say)}$$

$$W_2 (N(E_3))^t \hookrightarrow (1\ 0\ I\ 0\ 0\ 0\ I\ 0) = V_3 \text{ (say)}$$

$$V_3 (N(E_3)) \hookrightarrow (0\ I\ 0\ 1\ 0) = W_3 \text{ (say)}.$$

Thus we see hidden pattern of the neutrosophic dynamical system is a fixed point pair given by
$\{(1\ 0\ I\ 0\ 0\ 0\ I\ 0), (0\ I\ 0\ 1\ 0)\}$.

We see all the three experts wants their resultant should be used. Thus in such cases when multi experts wants equal importance to be given; we make use of the combined neutrosophic relational maps model.

The combined neutrosophic relational maps model is described in a line or two in the following.

Finite number of NRMs can be combined together to produce the joints effect of all NRMs.

Suppose $N(E_1), N(E_2), \ldots, N(E_r)$ be the neutrosophic relational matrices given by experts of the NRMs with nodes $R_1, R_2, \ldots, R_m$ and $D_1, D_2, \ldots, D_n$ then the combined NRM is represented by the neutrosophic relational matrix

$$N(E) = N(E_1) + N(E_2) + \ldots + N(E_r).$$

We will now illustrate this situation by the above example we have just worked with the three experts in the employer - employee problem using the NRM model.

The three connection neutrosophic matrices given by them for the range space nodes $R_1, R_2, \ldots, R_5$ and $D_1, D_2, \ldots, D_8$ are $N(E_1)$, $N(E_2)$ and $N(E_3)$.

Let $N(E) = N(E_1) + N(E_2) + N(E_3)$.

$$N(E) = \begin{bmatrix} I & 0 & 0 & 2 & 2 \\ 1 & 2I & 2 & 0 & 0 \\ I & 1 & 2 & 0 & 0 \\ 3 & 1 & I & I & I \\ 0 & 1 & 2 & I & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & I & I & 1 \\ 0 & I & 0 & 2 & 2 \end{bmatrix}.$$

If $T_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$ is the state vector with only the node D_1 in the on state and all other nodes are in the off state to find the effect of T on N(E).

$$T_1\, N(E) = (0\ 0\ 2\ 2) = S_1 \text{ (say)}$$

$$\begin{aligned} S_1(N(E))^t &\hookrightarrow (1\ I\ I\ I\ I\ 1\ 1\ 1) \\ &= T_2 \text{ (say)} \end{aligned}$$

$$\begin{aligned} T_2\,(N(E)) &\hookrightarrow (1\ 1\ 1\ 1\ 1) \\ &= S_2 \text{ (say)} \end{aligned}$$

$$\begin{aligned} S_2(N(E))^t &\hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) \\ &= T_3 \text{ (say).} \end{aligned}$$

We see the hidden pattern is a fixed point pair given by {(1 1 1 1 1 1 1 1), (1 1 1 1 1)}.

Now we proceed onto define a new type of NCM and NRM.

We define the notion of modified NCM and modified NRM.

We see the modified NCM is the same as that of the usual NCM except in the case of modified NCM the expert gives the directed neutrosophic graph whose edge weights are from the bi-interval $[0, I] \cup [0, 1]$.

Secondly we do not use the usual multiplication of matrices but use the min-max or max-min or min-min or max-max operation.

The main advantage of using this Modified Neutrosophic Cognitive Maps (MNCM) model is that we overcome the arbitrariness of the thresholding the resultant vector at each stage.

However we start only with the state vector $A = (a_1, \ldots, a_n)$ where $a_i \in \{0, 1, I\}$ that is a off state in case $a_i = 0$, on state in case $a_i = 1$ and indeterminate state in case $a_i = I$, $1 \leq i \leq n$.

Further we arrive at a fixed point or limit cycle as the hidden pattern as we have the number of entries in the matrix is finite so arrive at the hidden pattern in a finite number of steps.

We will first illustrate this situation by an example.

Example 3.5: Suppose we are interested in studying the strategic planning simulation based on MNCMs knowledge and differential game.

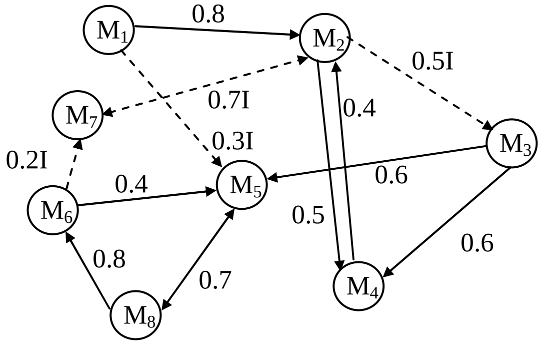
The adjoining edges of the modified NCMs are weighted with elements from $[0, I] \cup [0, 1]$.

For more about this problem please refer [8].

Let

M_1 - Competitiveness
M_2 - Market share
M_3 - Quality control
M_4 - Competitors advertisements
M_5 - Market demand
M_6 - Economic conditions
M_7 - Productivity
M_8 - Sales price

The modified NCM associated with this problem given by an expert is as follows:



Let the associated connection matrix of the neutrosophic directed weighted graph be as follows:

$$MN(E) = \begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} M_1 & M_2 & M_3 & M_4 & M_5 & M_6 & M_7 & M_8 \end{array} \\ \begin{bmatrix} 0 & 0.8 & 0 & 0 & 0.3I & 0 & 0 & 0 \\ 0 & 0 & 0.5I & 0.5 & 0 & 0 & 0.7I & 0 \\ 0 & 0 & 0 & 0.6 & 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0 & 0.4 & 0 & 0.2I & 0 \\ 0 & 0.7I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0.8 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we use the max min operation to study the problem.

Suppose the expert is interested in the on state of the node M_1 that is A_1 = (1 0 0 0 0 0 0 0).

To find max min (A_1, MN(E))

= A_1 o MN(E) = (0 0.8 0 0 0.3I 0 0 0)

→ (1 0.8 0 0 0.3I 0 0 0) = A_2 (say).

We only update as there is no point in thresholding the vector (→ denotes the operation of updating is done).

max min (A_2, MN(E)) = (0 0.8 0.5I 0.5 0.3I 0 0.7I 0.3I)

→ (1, 0.8, 0.5I, 0.5, 0.3I, 0, 0.7I, 0.3I) = A_3 (say).

We find max min (A_3, MN(E))

= A_3 o MN(E) and so on.

We see certainly we arrive at a fixed point or a limit cycle as the entries are from the finite set

{0, 1, 0.8, 0.4, 0.2I, 0.3I, 0.5I, 0.5, 0.6, 0.7, 0.7I}.

The advantage of using the modified NCM is that we do not have the operation of thresholding which in many cases happen to be arbitrary, in the hands of the expert.

Further we do not get the resultant as 0 or 1 or I we get more sensitive values between $0 \leq a_i \leq 1$ and $0 \leq b_i \leq I$.

Thus we can use modified NCMs in the place of NCMs and use the max min (or min max) operation to arrive at a solution.

We can use this new technique to find more reliable solutions.

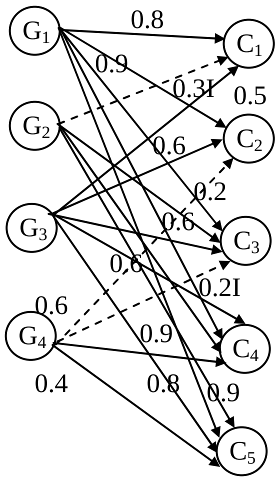
Next we proceed onto describe the modified form of the Neutrosophic Relational Maps (NRMs) model.

If in the NRM directed graph the edge weights are weighted with entries from [0, 1] ∪ [0, I] we call them as the modified NRMs or MNRMs.

We will illustrate this situation by an example or two.

Example 3.6: Suppose we are interested in studying the problem of child labour and the influence of the government on it.

Let the neutrosophic directed graph given by the expert be as follows:



The related connection neutrosophic matrix of the modified neutrosophic directed weighted graph is

$$MN(E) = \begin{array}{c} \\ G_1 \\ G_2 \\ G_3 \\ G_4 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0.8 & 0.9 & 0.6 & 0.8 & 0.4 \\ 0.3I & 0 & 0.2 & 0.7 & 0.9 \\ 0.5 & 0.6 & 0.6 & 0.9 & 0.6 \\ 0 & 0.4I & 0.2I & 0.9 & 0.4 \end{bmatrix} \end{array}.$$

Here

G_1 - no steps is taken by the government to provide alternatives when agriculture has failed.

G_2 - government has not taken any legal remedies to prevent child labour.

G_3 - No education with monthly stipend for school children in villages whose life is miserable.

G_4 - Child labourers are not vote banks so no concern over their welfare.

These nodes of government are taken as the domain space of the MNRM.

Now we give the nodes / attributes associated with the range space of the MNRMs.

C_1 - Acute poverty

C_2 - Failure of agriculture

C_3 - No means to food then how to continue education hence dropout so has become child labourers

C_4 - Tea shops and petty hotels provide job with food and shelter

C_5 - Long hours of work and related health problems.

Now using MN(E) we find the effect of the state vector $X_1 = (1\ 0\ 0\ 0)$ from the MNRMs domain space using max min operation.

max min $\{X_1, MN(E)\} = X_1 \text{ o } MN(E)$

$= (0.8, 0.9, 0.6, 0.8, 0.4) = Y_1$ (say).

max min $\{Y_1, MN(E)\} = Y_1 \text{ o } (MN(E))^t$

$= (0.9 \;\; 0.7 \;\; 0.8 \;\; 0.8)$ and so on.

We work till we arrive at a fixed point pair or a limit cycle.

Once again we keep on record that solution using modified NRM is not only sensitive or accurate but it is free from the personal bias of the expert as thresholding is not done at each stage of the problem.

We give yet another illustration of the modified NRM.

Example 3.7: Suppose we are interested in studying the student teacher model using the modified NRM.

The attributes related with the students and teachers are as follows:

The following are the attributes related with the students which are as follows:

S_1 - Private schools charge a very high fee and donation.

S_2 - Private schools do not admit children whose parental income is low or educational qualification is low.

S_3 - Private schools develop a caste among its students.

S_4 - Private schools promote more of English than vernacular so poor students find it difficult to follow as most of them are the first generation learners.

S_5 - In government run schools there are not adequate number of qualified teachers. Even those who are in schools concentrate on private tuitions and above all some side business like LIC agent, etc.

S_6 - Both in government and private schools there are not many teachers who could encourage and motivate the students to their best in every field for the reasons best known to them.

This is taken as the domain space of the modified MNRM.

We now proceed onto define in a line or two the domain nodes / attributes of the MNRM in a line or two.

C_1 - Parents are poor and are not in a position to pay huge amount as fees.

C_2 - Parents are incapable of helping / assisting in their home work etc as these children are the first generation learners.

C_3 - Children get into bad company and become dropouts when they face problems of communication in class room.

C_4 - Poor children are discouraged by teachers and sometimes even ridiculed in front of others often for no fault of theirs.

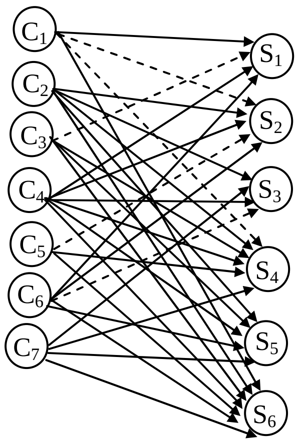
C_5 - Children have to walk long distance to reach school as there is neither a good road facility nor buses are available to suit the school timings.

C_6 - Due to gender difference, female children are stopped from school after a stage and made to work for parents / or as baby sitters or as domestic servants in other houses.

C_7 - Government keeps a indifferent attitude towards children's issues as they are not their vote banks and no punishment is given to those who practice child labour.

We using (C_1, C_2 ,…, C_7) as the domain attributes and {S_1, S_2, …, S_6} as the range attributes obtain the MNRM directed neutrosophic graph for working with the MNRM model.

The neutrosophic directed graph is as follows:



The neutrosophic matrix associated with weighted neutrosophic graph is as follows:

$$MN(E) = \begin{bmatrix} 0.6 & 0.8I & 0 & 0.3I & 0 & 0.6 \\ 0 & 0.7 & 0.6 & 0.7 & 0.5 & 0.7 \\ 0.3I & 0 & 0 & 0.8 & 0.6 & 0.7 \\ 0.8 & 0.8 & 0.7 & 0.6 & 0.5 & 0.6 \\ 0 & 0.2I & 0 & 0.7 & 0 & 0.7 \\ 0.5 & 0.2 & 0.I & 0 & 0.4 & 0.5 \\ 0 & 0 & 0.5 & 0.6 & 0.7 & 0.4 \end{bmatrix}.$$

Suppose the expert wants to work with the on state of the vector C_1 from the domain space and all other nodes are in the off state.

To find the effect of $X_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0)$ on the dynamical system MN(E).

max-min $\{X_1, MN(E)\} = X_1 \text{ o } MN(E)$
$= (0.6\ \ 0.8I\ \ 0\ 0.3I\ \ 0\ 0.6)$
$= Y_1$ (say).

We now find max-min $\{Y_1, MN(E)^t\}$
$= Y_1 \text{ o } (MN(E))^t = (0.8I\ \ 0.7\ \ 0.6\ \ 0.8\ \ 0.6\ \ 0.5\ \ 0.4)$
$\to (1\ \ 0.7\ \ 0.6\ \ 0.8\ \ 0.6\ \ 0.5\ \ 0.4) = X_2$ (say)

('$\to$' denotes the resultant vector has been updated).

We find max-min $\{X_2, MN(E)\}$

$= X_2 \text{ o } MN(E) = (0.8\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6\ \ 0.7) = Y_2$ (say) and so on. Of course after a finite number of times we will arrive at a fixed point pair or a limit cycle as the number of terms we deal is {0, 0.6, 0.8I, 0.3I, 0.7, 0.5, 0.8, 0.2I, 0.I, 0.2 and 0.4}.

The advantage is the bias of thresholding is removed and the resultant vector is not 0, 1 or I but elements of the form a or bI, a, b $\in$ [0, 1].

Now having seen MNCM and MNRM models we now proceed onto give yet another formulation of new operation for the combined neutrosophic cognitive maps and combined neutrosophic relational maps models.

If $N(E) = \sum_{i=1}^{n} N(E_i)$ where N(E) is the combined NCM of n experts opinion. We see the large numbers are n + nI and n and nI.

We can convert N(E) into a neutrosophic element whose entries are from the set $\langle [0, I] \cup [0,1] \rangle = \{a + bI \mid a, b \in [0, 1]\}$.

We do this by dividing

$$\frac{n(E)}{n} = \{\text{every element of N(E) is divided by n}\}.$$

Now we use for the new neutrosophic matrix $\frac{n(E)}{n}$ the max min operation or max-max operation or min max operation or min-min operation.

The operation of thresholding is over come by this method. Thus we get a resultant after a finite stage as the maximum number of elements with which we work is p^2 and (p is finite) (N(E) is a $p \times p$ matrix).

On similar lines we work with combined NRM.

Suppose we have m experts who give opinion on a problem using NRM say with the associated neutrosophic matrices are;

$N(E_1), N(E_2), \ldots, N(E_m)$.

We know $N(E) = \sum_{i=1}^{n} N(E_i)$. We now find $\frac{N(E)}{m}$.

Suppose $\frac{N(E)}{m} = M(E)$ we find the effect of any state vector on M(E) using only max min (or max max or min max or min min) operation we get the hidden pattern as a fixed point pair or a limit cycle pair.

This method also is free from the bias of the thresholding operations.

Finally we define the new notion of linked neutrosophic relational maps. These models are useful when we have two sets of attributes which are not comparable but get a link through another set of attributes.

Suppose we have an NRM model with the set of domain and range space say $(A_1, A_2, \ldots, A_n)$ which is the attributes related with school children the problem which forms the domain space and ($B_1, \ldots, B_t$) be the set of attributes related with the teachers which forms the range space of the NRM model.

We can have a NRM relating to these two disjoint sets of attributes and $N(E_1)$ be a $n \times t$ neutrosophic matrix.

Suppose ($G_1, \ldots, G_m$) be the attributes associated with the government officials and private owners. Clearly there does not exist directly any relation between ($A_1, \ldots, A_n$) and ($G_1, \ldots, G_m$) however we get a link between them by the following method.

Now if

$$N(E_1) = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \overset{\begin{array}{cccc} B_1 & B_2 & \ldots & B_t \end{array}}{\begin{bmatrix} & & & \\ & & (a_{ij}) & \\ & & & \\ & & & \end{bmatrix}}$$

where $a_{ij} \in \langle [0, 1] \cup [0,1] \rangle = \{a + bI \mid a, b \in [0, 1]\}$ be the neutrosophic matrix associated with the NRM model, where student attributes are taken as the domain space and that of the teachers attributes as the range space of the NRM model.

Now certainly we know there is a relation between the school teachers and the government private sectors or quasi government sectors. We find the NRM model associated with ($B_1, B_2, \ldots, B_t$) and ($G_1, G_2, \ldots, G_m$) as follows:

$$N(E_2) = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_t \end{array} \overset{\begin{array}{cccc} G_1 & G_2 & \ldots & G_m \end{array}}{\begin{bmatrix} & & & \\ & & (b_{ij}) & \\ & & & \\ & & & \end{bmatrix}}$$

where $b_{ij} \in \langle [0, 1] \cup [0,I] \rangle = \{a + bI \mid a, b \in [0, 1], 1 \le i \le t$ and $1 \le j \le m$.

We get the linked NRM as follows:

$N(E) = N(E_1) \times N(E_2)$

$$= \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} G_1 \;\; G_2 \;\; \ldots \;\; G_m \\ \left[\begin{array}{ccc} & & \\ & (c_{ij}) & \\ & & \\ & & \end{array} \right] \end{array}$$

where $c_{ij} \in \langle [0, 1] \cup [0, I] \rangle = \{a + bI \mid a, b \in [0, 1]\}, 1 \le i \le n$, $1 \le j \le m$.

N(E) is called the linked NRM of the two NRMs.

By this method we can interlink two attributes which cannot be associated but we have a indirect relation. This shows the interrelation between students attributes and the government attributes.

We will illustrate this by an example.

Example 3.8: Let us consider problem of child labour problem where P_1, P_2, P_3, P_4 and P_5 are the attributes related with the parents which is described in a line or two.

P_1 : Poverty and lack of money to spare for children’s education and hence they migrate to different places in search of lively hood and survival

P_2 : Importance and the value of education not known / not properly understood

P_3 : Family problem

P_4 : Hereditary job

P_5 : Frustration on the existing system of education in rural school.

Now we proceed onto describe the problems related with the children which is described in a line or two in the following.

C_1 - Children not properly motivated

C_2 - No school with accommodating fee structure

C_3 - Parents not educated enough to help the children

C_4 - Looking after the younger children and doing household work specially when the parents move in search of job.

Now we proceed onto describe the nodes associated with the government.

G_1 - Government has not provided good schools with moderate fee structure in the near by vicinity of every village

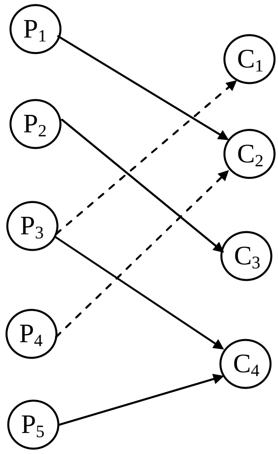
G_2 - No proper road and transport facilities available for the school children to reach the school

G_3 - There are not adequate number of well trained teachers to take care of the children

G_4 - Poor parents who depend on the small income of their children's daily labour are not adequately taken care by the government to motivate them to send the children to school

G_5 - There is no provision in the existing school system for migrating children to continue their studies in other schools.

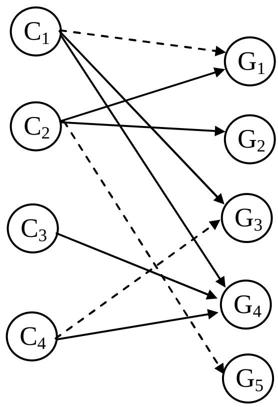
Let the neutrosophic graph given by the expert using $\{P_1, P_2, P_3, P_4, P_5\}$ as the domain space and $\{C_1, C_2, C_3, C_4\}$ as the range space be as follows:



The associated connection neutrosophic matrix of the NRM is as follows:

$$N(E_1) = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ I & 0 & 0 & 1 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

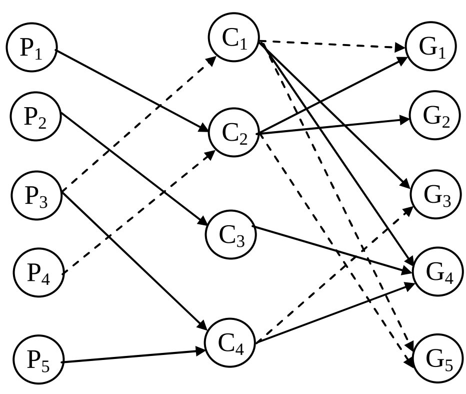
Now the neutrosophic graph given by the expert taking $\{C_1, \ldots, C_4\}$ as the nodes of the domain space and $\{G_1, G_2, \ldots, G_5\}$ as the nodes of the range space as follows:



The neutrosophic matrix associated with the neutrosophic graph is as follows:

$$N(E_2) = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \begin{array}{ccccc} G_1 & G_2 & G_3 & G_4 & G_5 \end{array} \\ \begin{bmatrix} I & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & I \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & I & 1 & 0 \end{bmatrix} \end{array}.$$

Now the linked NRM is given by the following neutrosophic linked graph;



Now the linked NRM associated matrix is as follows:

N(E) = N(E_1) × N(E_2)

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ I & 0 & 0 & 1 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} I & 0 & 1 & 1 & I \\ 1 & 1 & 0 & 0 & I \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & I & I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & I \\ 0 & 0 & 0 & 1 & 0 \\ I & 0 & 2I & 2I & I \\ I & I & 0 & 0 & I \\ 0 & 0 & I & I & 0 \end{bmatrix}$$

$$\cong \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{matrix} G_1 & G_2 & G_3 & G_4 & G_5 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & I \\ 0 & 0 & 0 & 1 & 0 \\ I & 0 & I & I & I \\ I & I & 0 & 0 & I \\ 0 & 0 & I & I & 0 \end{bmatrix} \end{array}.$$

Using this linked NRM matrix N(E) we can work for any resultant vector. This is the use of studying and defining linked NRM.

Finally the authors wish to keep on record that all the illustration are not worked using real data only as examples for the reader to understand these notions.

We now proceed onto describe the Combined Disjoint Block NRM and Combined Overlap Block NRM .

Now we proceed onto describe the definition and working of the combined disjoint block NCM, combined disjoint block NRM and combined overlap block NRM.

For the definition of Combined Disjoint Block Neutrosophic Cognitive Maps (CDBNCMs) model. Please refer [8, 10].

We just illustrate this situation by an example.

Example 3.9: Suppose one is interested in studying the problems related with HIV/AIDS migrant labourers.

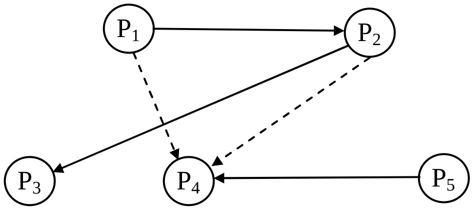
The fifteen attributes $P_1, P_2, \ldots, P_{15}$ related with them are as follows:

P_1 - No binding with family
P_2 - Male chauvinism
P_3 - Women as inferior objects
P_4 - Bad company and bad habits
P_5 - Socially irresponsible
P_6 - Uncontrollable sexual feelings
P_7 - Food habits, gluttony
P_8 - More leisure
P_9 - No other work for the brain
P_{10} - Only physically active
P_{11} - Visits Commercial Sex Workers (CSWs)
P_{12} - Enjoys life - jolly mood
P_{13} - Unreachable by friends or relatives
P_{14} - Pride in visiting countless CSWs
P_{15} - Failure of agriculture.

Suppose we study the problem by dividing it into 3 equal classes;

$C_1 = \{P_1, P_2, P_3, P_4, P_5\}$
$C_2 = \{P_6, P_7, P_8, P_9, P_{10}\}$ and
$C_3 = \{P_{11}, P_{12}, P_{13}, P_{14}, P_{15}\}$.

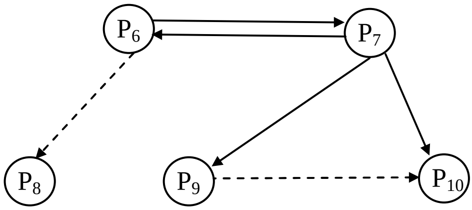
The related neutrosophic graph with C_1 is as follows:



The neutrosophic connection matrix is as follows:

$$= \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & I & 0 \\ 0 & 0 & 1 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

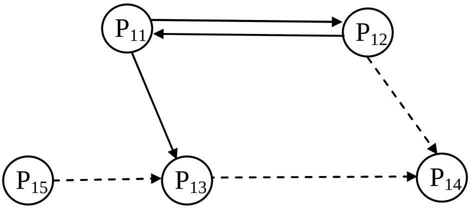
The related neutrosophic graph of the nodes $\{P_6, P_7, P_8, P_9, P_{10}\}$ is as follows:



The related neutrosophic matrix of this neutrosophic graph is as follows:

$$= \begin{array}{c} \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{array} \begin{array}{c} \begin{array}{ccccc} P_6 & P_7 & P_8 & P_9 & P_{10} \end{array} \\ \begin{bmatrix} 0 & 1 & I & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The directed graph associated with the attributes P_{11}, P_{12}, P_{13}, P_{14} and P_{15} is as follows:



The neutrosophic matrix associated with this neutrosophic graph is as follows:

$$= \begin{array}{c} \\ P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \\ P_{15} \end{array} \begin{array}{c} \begin{array}{ccccc} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix} \end{array}.$$

Now we give the related Combined Block Disjoint Neutrosophic Connection Matrix (CDBNCM) given by N(A).

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
P_1	0	1	0	I	0	0	0	0	0	0	0	0	0	0	0
P_2	0	0	1	I	0	0	0	0	0	0	0	0	0	0	0
P_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
P_6	0	0	0	0	0	0	1	I	0	0	0	0	0	0	0
P_7	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0
P_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_9	0	0	0	0	0	0	0	0	0	I	0	0	0	0	0
P_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{11}	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
P_{12}	0	0	0	0	0	0	0	0	0	0	1	0	0	I	0
P_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	I	0
P_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{15}	0	0	0	0	0	0	0	0	0	0	0	0	I	0	0

Using N(A) the dynamical system of the given model we study the effect of state vector.

Suppose S = (0 1 0 0 0 0 0 0 0 0 0 0 0 0 0)

that is only the attribute male ego is in the on state and all other nodes are in the off state.

To find the effect of S on the dynamical system N(A).

SN (A) $\hookrightarrow$ (0 1 1 I 0 0 0 0 0 0 0 0 0 0 0) = S_1 (say)

S_1N (A) $\hookrightarrow$ (0 1 1 I 0 0 0 0 0 0 0 0 0 0 0) = S_2 (= S_1)

Thus the hidden pattern is a fixed point. That is a person with male ego treats women as inferior object and one cannot say that they had bad company or bad habits.

Now we study the on state of the nodes A_1, A_7 and A_1. Let Y = (1 0 0 0 0 0 1 0 0 0 1 0 0 0 0). To find the effect of Y on the dynamical system N(A).

$$YN(A) \hookrightarrow (1\ 1\ 0\ I\ 0\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0) = Y_1 \text{ (say)}$$

$$Y_1N(A) \hookrightarrow (1\ 1\ 1\ \ I\ 0\ 1\ 1\ I\ 1\ 0\ 1\ 1\ 1\ 0\ 0) = Y_2 \text{ (say)}$$

$$Y_2N(A) \hookrightarrow (1\ 1\ 1\ I\ 0\ 1\ 1\ I\ 1\ 0\ 1\ 1\ 1\ I\ 0) = Y_3 \text{ (say)}$$

$$Y_3N(A) \hookrightarrow (1\ 1\ 1\ I\ 0\ 1\ 1\ I\ 1\ 0\ 1\ 1\ 1\ I\ 0) = Y_4 \text{ (say)}.$$

Clearly $Y_4 = Y_3$. Thus the hidden pattern associated with the state vector X is a fixed point. One can as in case of usual NCMs interpret the resultant state vector.

Now we can also have the elements in each of the C_i; to be different ($1 \leq i \leq 3$). Here we have worked with each C_i having just only 5 elements. For working with different number of elements please refer [8, 10].

Now for the same problem we describe how the combined block overlap NCM works.

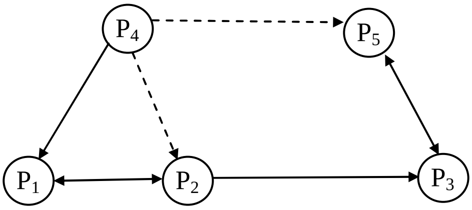
We work with the attributes $\{P_1, P_2, \ldots, P_{15}\}$ described earlier.

Let us take $C_1 = (P_1, P_2, P_3, P_4, P_5)$, $C_2 = (P_4, P_5, P_6, P_7, P_8, P_9, P_{10})$, $C_3 = (P_9, P_{10}, P_{11}, P_{12})$, $C_4 = (P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15})$ and $C_5 = (P_{15}, P_1, P_2, P_3)$.

We have 5 classes certainly the classes are not disjoint they over lap.

Now using these attributes in these 5 classes we give the neutrosophic directed graphs and its related neutrosophic connection matrices.

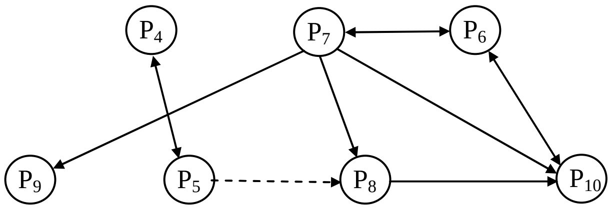
The directed neutrosophic graph associated with $\{P_1, P_2, P_3, P_4, P_5\}$ is as follows:



The related matrix is as follows:

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & I \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

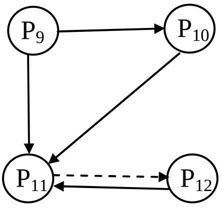
The directed graph given by the expert related with the attributes $C_2 = \{P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$ are as follows:



The related connection neutrosophic matrix is as follows:

	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
P_4	0	1	0	0	0	0	0
P_5	1	0	0	0	I	0	0
P_6	0	0	0	1	0	0	1
P_7	0	0	1	0	1	1	1
P_8	0	0	0	0	0	0	1
P_9	0	0	0	0	0	0	0
P_{10}	0	0	1	0	0	0	0

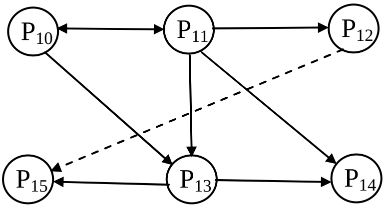
The directed neutrosophic given related with $C_3 = \{P_9, P_{10}, P_{11}, P_{12}\}$ is as follows:



The related connection matrix of C_3 is as follows:

	P_9	P_{10}	P_{11}	P_{12}
P_9	0	1	1	0
P_{10}	0	0	1	0
P_{11}	0	0	0	1
P_{12}	0	0	I	0

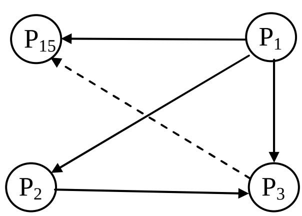
The directed graph related to the attributes in $C_4 = \{P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}\}$ is as follows:



The related connection matrix;

	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
P_{10}	0	1	0	1	0	0
P_{11}	1	0	1	1	1	0
P_{12}	0	0	0	0	0	I
P_{13}	0	0	0	0	1	1
P_{14}	0	0	0	0	0	0
P_{15}	0	0	0	0	0	0

The directed graph related with the final class of attributes $C_5 = \{P_{15}, P_1, P_2, P_3\}$ is as follows:



The related connection matrix is as follows:

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
P_1	0	2	1	0	0	0	0	0	0	0	0	0	0	0	1
P_2	I	0	2	0	0	0	0	0	0	0	0	0	0	0	0
P_3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	I
P_4	1	I	0	0	I	0	0	0	0	0	0	0	0	0	0
P_5	0	0	1	1	0	0	0	I	0	0	0	0	0	0	0
P_6	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
P_7	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0
P_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_9	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
P_{10}	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0
P_{11}	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
P_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	I
P_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
P_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{15}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Now we study the effect of any state vector on the dynamical system W where W is 15 × 15 neutrosophic matrix. For consider the state vector A_1, A_5, A_9, A_{11} and A_{15} in the on state and all other vectors are in the off state.

That is let

$$X = (1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 1).$$

To find the effect of X on the neutrosophic dynamical system W.

$$XW \hookrightarrow (1\ 1\ 1\ 1\ 1\ 0\ 0\ I\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = X_1 \text{ (say)}$$

$$X_1W \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 0\ I\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = X_2 \text{ (say)}$$

$$X_2W \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ I\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = X_3 \text{ (say)}$$

$$X_3W \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ I\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = X_4 \text{ (say)}$$

$$X_4 = X_3 .$$

Hence hidden pattern is a fixed point and one can interpret the on state of the nodes.

Combined Disjoint Block NRM and Combined Overlap Block NRM.

When the problem under investigation has several attributes it becomes very difficult to handle them as a whole and all the more it becomes difficult to get the expert opinion and the related neutrosophic directed graph so we face the problem of the management of all the attributes together so in this chapter we give new methods by which the problem can be managed section by section or part by part without sacrificing any thing. We define here these four methods.

DEFINITION 3.2: *Consider a NRM model with n elements say* $\{D_1, D_2,\ldots, D_n\}$ *in the domain space and* $\{R_1, R_2,\ldots, R_m\}$

elements in the range space. Suppose we imagine that both m and n are fairly large numbers. Now we divide the n elements (i.e., attributes) of the domain space into t blocks each (assuming n is not a prime) such that no element finds its place in two blocks and all the elements are necessarily present in one and only one block such that each block has the same number of elements.

Similarly we divide the m attributes of the range space in r blocks so that no element is found in more than one block ie every element is in one and only one block. Thus each block has equal number of elements in them (As m is also a assurance to be a large number which is not a prime. Now if we take one block from the t block and one block from the r-block and form the neutrosophic directed graph using an expert's opinion. It is important to see no block repeats itself either from the r blocks or from the t blocks i.e., each block from the range space as well as the domain space occurs only once.

Now we take the directed graph of each pair of blocks (one block taken from the domain space and one block taken from the range space) and their related neutrosophic connection matrix. We use all these connection matrices and form the $n \times m$ *neutrosophic matrix, which is called the connection matrix of the Combined Disjoint Block of the NRM of equal sizes.*

Now instead of dividing them into equal size blocks we can also divide them into different size blocks so that still they continue to be disjoint.

Thus if we use equal size block we mention so other wise we say the Combined Disjoint Block of the NRM of varying sizes.

Now we give the definition of combined over lap blocks NRM.

DEFINITION 3.3: *Let us consider a NRM where the attributes related with the domain space say* $D = \{D_1,\ldots, D_n\}$ *is such that n is very large and the attributes connected with the range* *space say R = R_1,…, R_m} is such that m is very large. Now we want to get the neutrosophic directed graph, it is very difficult to manage the directed graph got using the expert opinion for the number of nodes is large equally, large is the number of edges also it may happen n and m may be primes and further it may be the case where some of the attributes may be common.*

In such a case we adopt a new technique called the combined over lap block of equal length. Take the n attributes of the domain space divide them into blocks having same number of element also see that the number elements overlap between any two blocks is either empty the same number of element; carryout the similar procedure in case of the range space elements also.

Now pair the blocks by taking a block from the range elements and a block from the domain elements draw using the experts opinion the neutrosophic directed graph. See that each block in the pairs occur only once. Now using the neutrosophic directed graphs we obtain for every pair neutrosophic relational matrices using these matrices we form the n $\times$ m matrices which is the matrix related with the combined overlap block of the NRM of equal size if we vary the number of elements in each of these blocks we call such model as the combined overlap block of the NRM with varying sizes of blocks.

It has become pertinent to mention here that we can by all means divide a NCM, if the attributes so formed into disjoint blocks of all them as combined disjoint block of the NCM. On similar lines we can divide NCM into overlap blocks of equal size and using this form the combined overlap block NCM. Already this concept for FCM have been used by Bart Kosko [1-3].

Here we only define and study it in case of NCMs. As our main motivation in this book is to give examples we describe the problem of HIV/AIDS affected migrant labour from rural areas of Tamilnadu we illustrate all these models only by

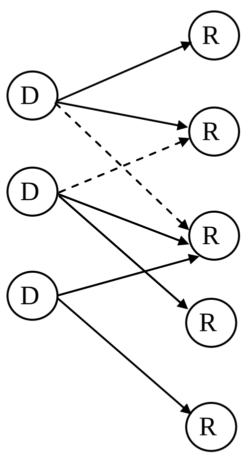
HIV/AIDS affected migrant laboures and the problems faced by them.

Now using the NRM model given in [8-9] using the attributes of the domain space $\{D_1\ D_2, \ldots, D_6\}$ and that of the range space $\{R_1, \ldots, R_{10}\}$.

We give he disjoint block decomposition of them and give the related directed neutrosophic graphs and their associated neutrosophic connection matrices.

$$C_1 = \{D_1\ D_2\ D_3), (R_1\ R_2\ R_3 \ldots R_5)\} \text{ and}$$
$$C_2 = \{(D_4\ D_5\ D_6), (R_6\ R_7\ R_8\ R_9\ R_{10})\}.$$

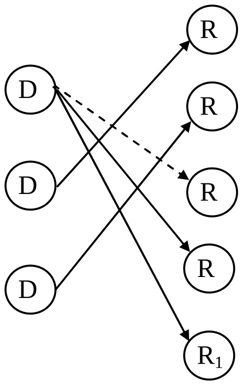
The neutrosophic directed graph for the class C_1 given by the expert is as follows:



The related neutrosophic connection matrix is

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 1 & 1 & I & 0 & 0 \\ 0 & I & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

The directed graph for the class
$C_2 = \{(D_4\ D_5\ D_6), (R_6\ R_7\ R_8\ R_9\ R_{10})\}$ given by the expert is as follows.



The related neutrosophic connection matrix is

$$\begin{array}{c} \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{matrix} R_6 & R_7 & R_8 & R_9 & R_{10} \end{matrix} \\ \begin{bmatrix} 0 & 0 & I & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Using these two connection matrices we obtain the 6 × 10 matrix associated with the combined disjoint block NRM which we denote by C(N).

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{matrix} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \end{matrix} \\ \begin{bmatrix} 1 & 1 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we analyze the effect of the state vector X = (0 0 1 0 0 0) i.e., only the attribute D_3 alone is in the on state and all other nodes are in the off state.

$$\begin{aligned} XC(N) &\hookrightarrow (0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0) = Y \\ YC(N)^T &\hookrightarrow (I\ 1\ 1\ 0\ 0\ 0) = X_1 \\ X_1\ C(N) &\hookrightarrow (I\ I\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0) = Y_1 \\ Y_1\ C(N)^T &\hookrightarrow (I\ 1\ 1\ 0\ 0\ 0) = X_2\ (= X_1). \end{aligned}$$

The hidden pattern is a fixed point given by the binary pair {(I 1 1 0 0 0), (I I 1 1 1 00 0 0 0)}, which has influence in the system which is evident by the resultant vector.

Let us now consider the state vector

Y = (0 0 0 0 0 0 1 0 0 0) i.e., only the node R_7 is in the on state and all other nodes are in the off state. Effect on Y on the neutrosophic dynamical system C(N) is given by

$$\begin{aligned} YC(N)^T &\hookrightarrow (0\ 0\ 0\ 0\ 0\ 1) = X \\ XC(N) &\hookrightarrow (0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0). \end{aligned}$$

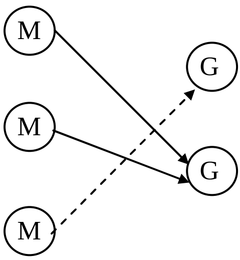
Thus we see the binary pair {(0 0 0 0 0 0 1 0 0 0). Thus pair {(0 0 0 0 0 0 1 0 0 0), (0 0 0 0 0 1) is a fixed point of the dynamical system. This pair has no influence on the dynamical system.

Now let us consider the model given in page with the domain space {M_1 M_2,…, M_9} where this expert wishes to combine the attributes M_9 and M_{10}.

The attributes of the range space are taken as {G_1, …, G_6} where G_1 and G_2 are combined together to form the attribute G_2 so G_3 is labeled as G_2, G_4 as G_3 and so on G_6 as G_5. Now we use the method of disjoint block decomposite and divide these sets into 3 classes

$C_1 = \{(M_1\ M_2\ M_3), (G_1\ G_2)\}$,
$C_2 = \{(M_4\ M_5\ M_6), (G_3\ (G_4)\}$ and
$C_3 = \{(M_7\ M_8\ M_9), (G_5\ (G_6)\}$.

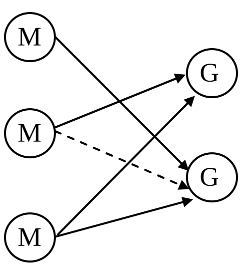
Now using the expert's opinion we give the related directed neutrosophic graphs for the classes C_1, C_2 and C_3 respectively.



The associated neutrosophic connection matrix is

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \end{array} \begin{array}{c} \begin{array}{cc} G_1 & G_2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ I & 0 \end{bmatrix} \end{array}$$

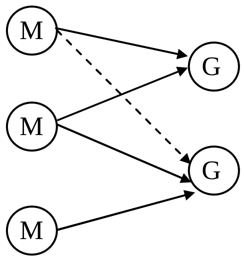
The directed neutrosophic graph given by the expert related with the class C_2.



The related connection matrix.

$$\begin{array}{c} \\ M_4 \\ M_5 \\ M_6 \end{array} \begin{array}{c} \begin{array}{cc} G_3 & G_4 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 1 & I \\ 1 & 1 \end{bmatrix} \end{array}$$

Now using the experts opinion give the related neutrosophic directed graph for the class C_3.



The related connection matrix

$$\begin{array}{c} \\ M_7 \\ M_8 \\ M_9 \end{array} \begin{array}{c} \begin{array}{cc} G_5 & G_6 \end{array} \\ \begin{bmatrix} 1 & I \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

Now using these three connection matrices we get the related relational matrix of the combined disjoint block NRM which is denoted by C(M) and C(M) is a 9 × 6 matrix.

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \end{array} \begin{array}{c} \begin{array}{cccccc} G_1 & G_2 & G_3 & G_4 & G_5 & G_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & I & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & I \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Let us consider the effect of the state vector $X = (0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1)$ of the dynamical system where only the attributes M_4 and M_9 are in the on state and all other nodes are in the off state. The effect of X on C(M) is given by

$$
\begin{aligned}
XC(M) &\hookrightarrow (0\ 0\ 0\ 1\ 0\ 1) &=& \ Y \\
Y\,C(M)^T &\hookrightarrow (0\ 0\ 0\ 1\ I\ 1\ I\ 1\ 1) &=& \ X_1 \\
X_1\,C(M) &\hookrightarrow (0\ 0\ I\ 1\ I\ 1) &=& \ Y_1 \\
Y_1\,C(M) &\hookrightarrow (0\ 0\ 0\ 1\ I\ I\ I\ I\ 1) &=& \ X_2 \\
X_2\,C(M) &\hookrightarrow (0\ 0\ I\ I\ I\ 1) &=& \ Y_2 \\
Y_2\,C(M) &\hookrightarrow (0\ 0\ 0\ 1\ I\ I\ I\ I\ I\ I) &=& \ X_3 \\
X_3\,C(M) &\hookrightarrow (0\ 0\ I\ I\ I\ I) &=& \ Y_3 = Y_2 \\
Y_2\,C(M) &\hookrightarrow X_3.
\end{aligned}
$$

Thus the binary pair is a fixed point given by {(0 0 I I I I), (0 0 0 1 I I I I 1)} which shows a very strong influence on the system.

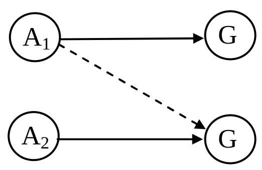
Now we illustrate the model in which the blocks are disjoint but the number of blocks are not the same.

We consider the model with the sets of attributes given by $\{A_1, A_2,\ldots,A_6\}$ and $\{G_1\ G_2\ldots\ G_5\}$.

Let us divide into equivalence classes.

$$C_1 = \{(A_1\ A_2)\ (G_1\ G_4)\} \text{ and}$$
$$C_2 = \{(A_3\ A_4\ A_5\ A_6),\ (G_2\ G_3\ G_5)\}$$

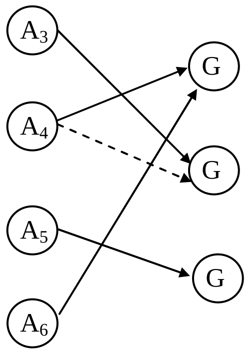
The directed neutrosophic graph related with the class C_1.



The connection neutrosophic matrix associated with the above graph is

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} \begin{array}{cc} G_1 & G_4 \end{array} \\ \begin{bmatrix} 1 & I \\ 0 & 1 \end{bmatrix} \end{array}$$

The related neutrosophic direct graph given the expert associated with the class C_2 is given by



The related connection matrix is as follows:

$$\begin{array}{c} \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}\begin{array}{c} \begin{array}{ccc} G_2 & G_3 & G_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & I & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we give the associated connection 6 × 5 matrix C(S) given by

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array}\begin{array}{c} \begin{array}{ccccc} G_1 & G_2 & G_3 & G_4 & G_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

C(S) is the neutrosophic matrix of the combined disjoint block NRM of unequal sizes, consider the state vector X = (1 0 0 1 0 0) in which the attributes A_1 and A_4 are in the on state and all other nodes are in the off state.

Effect of X on the dynamical system C(S) is given by

X C(S)	↪	(1 1 I I 0)	=	Y
$Y\ C(S)^T$	↪	(1 I 0 1 0 1)	=	X_1
X_1 C (S)	↪	(1 1 I I 0)	=	Y_1 (= Y).

Thus the hidden pattern of the dynamical system is the fixed point given by the binary pair {(1 1 I I 0) (1 I 0 1 0 1)}.

The nodes A_1 and A_4 together have a very strong impact on C(S). Now consider the state vector Y = (0 0 0 1 0) i.e., only the attribute G_4 is in the on state and all other vectors are in off state.

The effect of Y on the dynamical system C(S) is given by

$YC(S)^T$	↪	(I 1 0 0 0 0)	=	X
X (C (S))	↪	(I 0 0 I 0)	=	Y_1
$Y_1\ C(S)^T$	↪	(I 1 0 0 0 0)	=	X_1 = X.

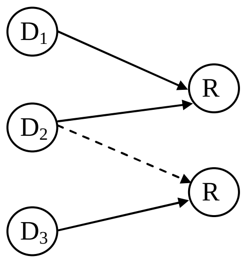
Thus the hidden point is the fixed point given by the binary pair {(I 1 0 0 0 0), (I 0 0 1 0)}.

The effect of the node G_4 on the dynamical system C(S) is not very strong.

Now let us consider yet a new model.

Let the domain space of the NRM be taken as {(D_1 D_2,…, D_8) } and that of the range space be {R_1 R_2…, R_5)} consider the classes C_1 and C_2 where C_1 = {(D_1 D_2 D_3), (R_1 R_2)} and C_2 = {(D_4 D_5 D_6 D_7 D_8), (R_3 R_4 R_5)}.

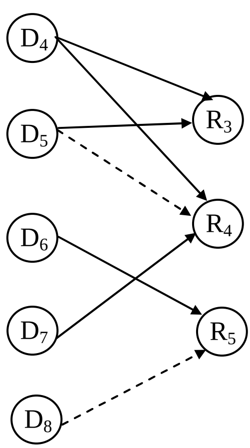
Now we obtain the experts opinion and give the directed graph related to them.



The related neutrosophic connection matrix is as follows:

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \end{array}\begin{array}{c} \begin{array}{cc} R_1 & R_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & I \\ 0 & 1 \end{bmatrix} \end{array}.$$

The directed graph related to the class C_2 is as follows:



The related connection matrix is given as follows:

$$\begin{array}{c} \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array} \begin{array}{c} \begin{array}{ccc} R_3 & R_4 & R_5 \end{array} \\ \left[\begin{array}{ccc} 1 & 1 & 0 \\ 1 & I & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{array}\right] \end{array}$$

Now we obtain the related neutrosophic connection matrix of the combined disjoint block NRM of different sizes given by the 8 × 5 matrix and is denoted by C(G)

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 1 & I & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & I & 0 \\ 0 & 0 & 1 & I & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I \end{array}\right] \end{array}$$

Now we obtain the effect of a state vector X = (0 0 0 0 1 0 0 0) where only the node D_5 is in the on state and all other nodes are in the off state.

The resultant vector of X using the dynamical system C(G)

$$\begin{array}{llll} XC(G) & \hookrightarrow & (0\ 0\ 1\ I\ 0) = & Y \\ Y\ C(G)^T & \hookrightarrow & (0\ 0\ 0\ I\ 1\ 0\ 0\ 0) = & X_1 \\ X_1\ C(G) & \hookrightarrow & (0\ 0\ 1\ I\ 0) = & Y_1\ (=Y). \end{array}$$

Thus the fixed point is given by the binary pair {(0 0 1 *I* 0), (0 0 0 1 10 0 0)}, which has a least effect on the system.

Let us consider the state vector Y = (1 0 0 0 0) that is R_1 is in the on state and all nodes in the rage space are in the off state

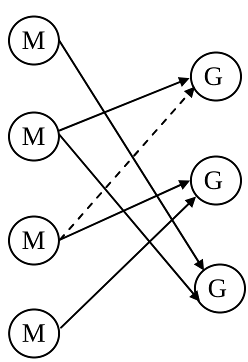
$$\begin{array}{llll} YC(G)^T & \hookrightarrow & (1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0) & = & X \\ XC(G) & \hookrightarrow & (1\ I\ 0\ 0\ 0) & = & Y_1 \\ YC(G)^T & \hookrightarrow & (1\ I\ I\ 0\ 0\ 0\ 0\ 0\ 0) & = & X_1 \\ X_1\, C(G) & \hookrightarrow & (1\ I\ 0\ 0\ 0) & = & Y_2\ (=Y_1). \end{array}$$

Thus the hidden pattern of the dynamical system is a fixed point given by the binary pair {(1 I 0 0 0), (1 I I 0 0 0 0 0)} according to this expert the effect of the node R_1 is least on the system.

Consider the NRM given in the attributes are taken as $\{M_1\ M_2,\ldots M_{10}\}$ and $\{G_1\ G_2,\ldots.G_7\}$ the combined block disjoint NRM of varying sizes is calculated using the classes

$$C_1 = \{M_1\ M_2\ M_3\ M_4), (G_1\ G_2\ )\},$$
$$C_2 = \{\ M_5\ M_6\ M_7\ M_8), (G_4\ G_5\ )\} \text{ and}$$
$$C_3 = \{(M_9\ M_{10}), (G_6, G_7)\}.$$

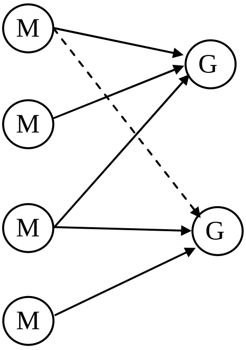
The neutrosophic directed graph of class C_1 given by the expert is as follows:



The related neutrosophic connection matrix

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \begin{array}{c} \begin{array}{ccc} G_1 & G_2 & G_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ I & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

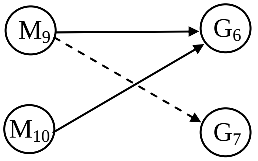
Now using the expert opinion we give directed graph for class C_2.



The related connection matrix is as follows:

$$\begin{array}{c} \\ M_5 \\ M_6 \\ M_7 \\ M_8 \end{array} \begin{array}{c} \begin{array}{cc} G_4 & G_5 \end{array} \\ \begin{bmatrix} 1 & I \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

Now the directed graph given by the expert is



The related neutrosophic matrix is as follows:

$$\begin{array}{c} \\ M_9 \\ M_{10} \end{array} \begin{array}{c} \begin{array}{cc} G_6 & G_7 \end{array} \\ \begin{bmatrix} 1 & I \\ 1 & 0 \end{bmatrix} \end{array}$$

Now we give the relational connection matrix of the combined disjoint block NRM of varying sizes.

We denote this 10 × 7 matrix by N(W).

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \\ M_{10} \end{array} \begin{array}{c} \begin{array}{ccccccc} G_1 & G_2 & G_3 & G_4 & G_5 & G_6 & G_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ I & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & I & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & I \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Let us consider the state vector X = (0 0 0 0 0 0 0 1 0 0) where only node M_8 alone is in the on state and all other nodes are in the off state.

The effect of X on the dynamical system C(W) is given by

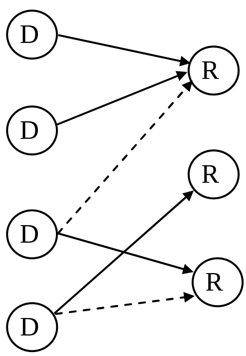
$$\begin{array}{llll} XC(W) & \hookrightarrow & (0\ 0\ 0\ 0\ 1\ 0\ 0) & = Y \\ YC(W)^T & \hookrightarrow & (0\ 0\ 0\ 0\ I\ 0\ 1\ 1\ 0\ 0) & = X_1 \\ X_1\ X(W) & \hookrightarrow & (0\ 0\ 0\ I\ 1\ 0\ 0) & = Y_1 \\ Y_1\ (C(W))^T & \hookrightarrow & (0\ 0\ 0\ 0\ I\ I\ I\ 1\ 0\ 0) & \\ X_2\ C(W) & \hookrightarrow & (0\ 0\ 0\ I\ I\ 0\ 0) & = Y_2 \\ Y_2\ C(W)^T & \hookrightarrow & (0\ 0\ 0\ 0\ I\ I\ I\ 1\ 0\ 0) & = X_3\ (=X_2). \end{array}$$

Thus the fixed point is given by the binary pair {(0 0 0 I I 0 0), (0 0 0 0 I I I 1 0 0)}. The reader is expected to give the effect of the node M_8 on the system.

Next we consider the combined overlap block NRM used in modeling the HIV/AIDS migrant labourers problem. The classes of attributes used are $\{D_1, D_2, \ldots, D_8\}$ and $\{R_1, R_2, \ldots, R_5\}$.

We divide these into over lap blocks C_1, C_2 C_3 and C_4 where $C_1 = \{(D_1\ D_2\ D_3\ D_4), (R_1\ R_2\ R_3)\}$, $C_2 = \{(D_3\ D_4\ D_5\ D_6), (R_2\ R_3\ R_4)\}$, $C_3 = \{(D_5\ D_6, D_7, D_8), (R_3\ R_4\ R_5)\}$ and $C_4 = \{(D_7\ D_8\ D_1, D_2), (R_4\ R_5\ R_1)\}$.

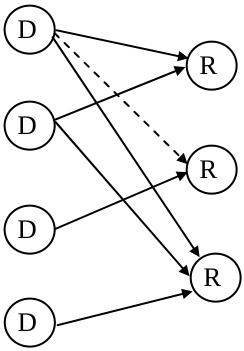
Now we obtain using the experts opinion the directed graph of these NRMs. The directed graph related to the class C_1.



The related connection matrix is as follows:

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ I & 0 & 1 \\ 0 & 1 & I \end{bmatrix} \end{array}$$

The directed graph given by the expert related to the class C_2.

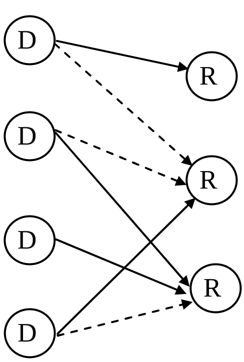


The related connection matrix is given as follows :

$$\begin{array}{c} \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array} \begin{array}{c} \begin{array}{ccc} R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 1 & I & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Directed graph given by the expert for the class C_3 is as follows:

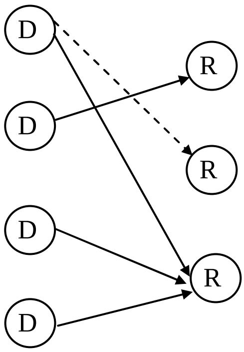
The related connection matrix of the above directed graph.



The related neutrosophic matrix is as follows:

$$\begin{array}{c} \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array} \begin{array}{c} \begin{array}{ccc} R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 1 & I & 0 \\ 0 & I & 1 \\ 0 & 0 & 1 \\ 0 & 1 & I \end{bmatrix} \end{array}$$

The directed graph given by the expert for the last class C_4.



The related neutrosophic matrix is as follows:

$$\begin{array}{c} \\ D_7 \\ D_8 \\ D_1 \\ D_2 \end{array} \begin{array}{c} \begin{array}{ccc} R_4 & R_5 & R_6 \end{array} \\ \begin{bmatrix} 0 & I & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Now using these four connection matrices we obtain the relational neutrosophic matrix of the combined block overlap NRM denoted by N(V).

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{array}\begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \left[\begin{array}{ccccc} 2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ I & 1 & I & 1 & 0 \\ 0 & 2 & I & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & I & 1 \\ 1 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 2 & I \end{array}\right] \end{array}$$

Let X = (0 0 0 0 1 0 0 0) be the state vector with the node D_5 to be in the on state and all other nodes are in the off state the effect of X on the dynamical system N(V).

$$\begin{array}{llll} XN(V) & \hookrightarrow & (0\ 0\ 1\ 1\ 0) & = Y \\ YN(V)^T & \hookrightarrow & (0\ 0\ I\ I\ 1\ I\ 0\ 1) & = X_1 \\ X_1 N(V) & \hookrightarrow & (I\ I\ I\ 1\ I) & = Y_1 \\ Y_1 N(V)^T & \hookrightarrow & (I\ I\ I\ I\ 1\ I\ I\ 1) & = X_2 \\ X_2 (N(V)) & \hookrightarrow & (I\ I\ I\ 1\ I) & = Y_2\ (= Y_1). \end{array}$$

Thus the fixed point is a binary pair given by
{(I I I 1 I), (I I I I 1 I I 1)}which has a very strong influence on the dynamical system as D_5 makes all nodes to on state or in an indeterminate state.

Next we consider the state vector Y = (1 0 0 0) that is only the node R_1 is in the on state and all other nodes are in the off state. The effect of Y on the dynamical system N(V) is given by

$$\begin{array}{llll} Y(N(V))^T & \hookrightarrow & (1\ 1\ I\ 0\ 0\ 0\ 1\ 0) & = X \\ XN(V) & \hookrightarrow & (1\ I\ I\ I\ I) & = Y_1 \\ Y_1 (N(V)) & \hookrightarrow & (1\ 1\ I\ I\ I\ I\ I\ I) & = X_1 \\ X_1 N(V) & \hookrightarrow & (1\ I\ I\ I\ I) & = Y_2 = Y_1. \end{array}$$

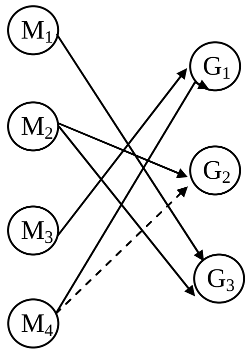
Thus the hidden pattern of the dynamical system is a fixed point given by the binary pair {(1 I I I I) (1 1 I I I I I I)}as before this node has very strong impact on the system.

Next we consider the combined overlap block NRM to study the given model. Take the sets of attributes given by {M_1 M_2…, M_{10}} and (G_1 G_2… G_7}.

We divide them into over lapping blocks given by the classes C_1, C_2 and C_3 where

$$C_1 = \{(M_1\ M_2\ M_3\ M_4), (G_1\ G_2\ G_3)\}$$
$$C_2 = \{(M_4\ M_5\ M_6\ M_7), (G_3\ G_4\ G_5)\} \text{ and}$$
$$C_3 = \{(M_7\ M_8\ M_9\ M_{10}), (G_5\ G_6\ G_7)\}.$$

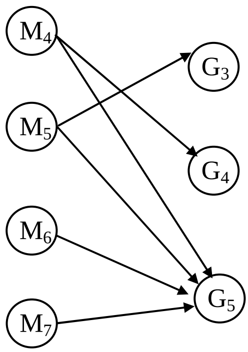
The directed graph for the class C_1 as given by the expert as follows:



The related connection matrix of the NRM is as follows:

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \begin{array}{c} \begin{array}{ccc} G_1 & G_2 & G_3 \end{array} \\ \left[\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & I & 0 \end{array}\right] \end{array}$$

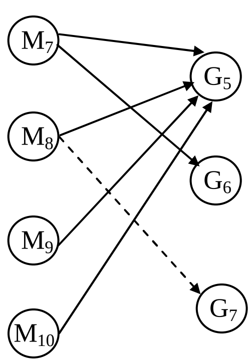
The neutrosophic directed graph given by the expert for the class C_2 is as follows:



The related connection matrix of the above directed graph is as follows:

$$\begin{array}{c} \\ M_4 \\ M_5 \\ M_6 \\ M_7 \end{array}\begin{array}{c} \begin{array}{ccc} G_3 & G_4 & G_5 \end{array} \\ \left[\begin{array}{ccc} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{array}\right] \end{array}.$$

Now we using the experts opinion give the directed graph of the last class.



The related neutrosophic connection matrix is given below:

$$\begin{array}{c} \\ M_7 \\ M_8 \\ M_9 \\ M_{10} \end{array} \begin{array}{c} \begin{array}{ccc} G_5 & G_6 & G_7 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ I & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Now using there three connection matrices we obtain the 10 × 7 connection neutrosophic matrix associated with the combined block overlap NRM where the blocks are of equal size.

We denote this matrix by N(Q).

$$\begin{array}{c} \\ M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ M_8 \\ M_9 \\ M_{10} \end{array} \begin{array}{c} \begin{array}{ccccccc} G_1 & G_2 & G_3 & G_4 & G_5 & G_6 & G_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & I & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & I & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & I & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

Let X = (0 0 0 0 0 0 0 1 0 0) be the state vector in which only the node M_8 is in the on state and all other nodes are in the off state. The effect of X on the dynamical system N(Q).

$$\begin{aligned} XN(Q) &\hookrightarrow (0\ 0\ 0\ 0\ 1\ 0\ 1) = Y \\ YN(Q)^T &\hookrightarrow (0\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 1\ 1) = X_1 \\ X_1N(Q) &\hookrightarrow (1\ I\ 0\ 1\ 1\ 1\ 1) = Y_1 \end{aligned}$$

$$Y_1\, N(Q)^T \hookrightarrow (0\ I\ 1\ I\ 1\ 1\ 1\ 1\ 1\ 1) = X_2$$
$$X_2\, N(Q) \hookrightarrow (I\ I\ I\ I\ I\ 1\ 1) = Y_2.$$

It is left as exercise to reader to find the fixed point.

Let us consider the state vector Y = (0 0 1 0 0 0 0) i.e., only the node G_3 is on the on state and all other nodes are in the off state.

The effect of Y on N(Q)

$$YN(Q)^T \hookrightarrow (1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0) = X$$
$$XN(Q) \hookrightarrow (1\ 0\ 1\ 1\ 1\ \ 0\ 0) = Y_1$$
$$Y_1\, (NQ)^T \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ ) = X_1$$
$$Y_1\, N(Q) \hookrightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1).$$

Thus the hidden pattern of the dynamical system is a fixed point given by the binary pair
{(1 1 1 1 1 1 1 1), (1 1 1 1 1 1 1 1 1 1)}.

The node G_3 has a very strong influence on the system so all nodes come to on state.

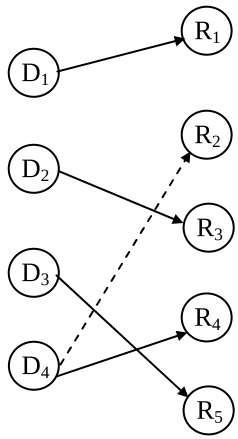
We see the combined overlap block NRM is every sensitive to the situation.

Next we consider the combined overlap block FRM of different sizes.

Let us consider the attributes given in page 201-202 with set of attributes {(D_1 D_2 D_3 D_4), (R_1 R_2 R_3 R_4 R_5)}, C_2 = {(D_3 D_4 D_5), (R_3 R_4 R_5 R_6)}, C_3 = {(D_4 D_5 D_6 D_1, D_2), (R_6 R_7 R_8 R_{10})}.

Now we analyze the model.

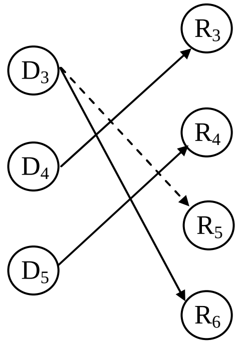
The directed given by the expert for the class C_1 of the NRM.



The related relation matrix is as follows:

$$\begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} \begin{array}{c} \begin{array}{ccccc} R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & I & 0 & 1 & 0 \end{bmatrix} \end{array}$$

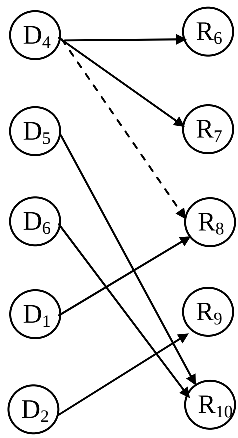
Directed graph given by the expert relative to the class C_2 is



The related matrix of the directed graph is as follows:

$$\begin{array}{c} \\ D_3 \\ D_4 \\ D_5 \end{array}\begin{array}{c} \begin{array}{cccc} R_3 & R_4 & R_5 & R_6 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & I & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \end{array}.$$

Now the expert opinion in the form of direct graph for the class C_3. The related connection matrix for this neutrosophic directed bigraph is as follows:



$$\begin{array}{c} \\ D_4 \\ D_5 \\ D_6 \\ D_1 \\ D_2 \end{array}\begin{array}{c} \begin{array}{ccccc} R_6 & R_7 & R_8 & R_9 & R_{10} \end{array} \\ \left[\begin{array}{ccccc} 1 & 1 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}.$$

Now using this matrices we give the connection matrix N((T) which gives the related combined overlap block NRM. This is a 6 × 10 matrix which is as follows:

$$
\begin{array}{c}
 \\ D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6
\end{array}
\begin{array}{c}
\begin{array}{cccccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \end{array} \\
\left[\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & I & 1 & 1 & 0 & 1 & 1 & I & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

Now we consider the effect of the state vector X = (0 0 0 0 1 0) on the dynamical system where only the node D_5 is in the on state and all nodes are in the off state.

$$
\begin{array}{llll}
XN(T) & \hookrightarrow & (0\ 0\ 0\ 1 0\ 0\ 0\ 0\ 0\ 1) & = \ Y \\
YN(T)^T & \hookrightarrow & (0\ 0\ 0\ 1\ 1\ 1) & = \ X_1 \\
X_1\, N(T) & \hookrightarrow & (0\ I\ 1\ 1\ 0\ 1\ 1\ I\ 0\ 1) & = \ Y_1 \\
Y_1\, N(T)^T & \hookrightarrow & (I\ 1\ 1\ 1\ 1\ 1) & = \ X_2 \\
X_2\, N(T) & \hookrightarrow & (I\ I\ 1\ 1 0\ 1\ 1\ 0\ 1\ 1) & = \ Y_2 \\
Y_2\, (N(T)^T & \hookrightarrow & (I\ 1\ 1\ 1\ 1\ 1) & = \ X_3 \\
 & & & = \ (=X_2).
\end{array}
$$

Thus the hidden pattern of the dynamical system is a fixed point given by the binary pair

{(I 1 1 1 1 1), (I I 1 1 0 1 1 0 1 1)}.

The influence of D_5 on the dynamical system is very strong so it affects most of the nodes.

Let us consider the state vector Y = (0 0 1 0 0 0) where the attribute D_3 alone is in the on state and all other nodes are the off state.

The effect of Y on the dynamical system (N(T)) is given by

$$
YN(T) \hookrightarrow (0\ 0\ 0\ 0\ 0\ 1 0\ 0\ 0\ 0) = X
$$

$$
\begin{array}{llll}
XN(T)^T & \hookrightarrow & (0\ 0\ 1\ 1\ 0\ 0) & = Y_1 \\
Y_1 N(T) & \hookrightarrow & (0\ I\ 1\ 1\ 0\ 1\ 1\ I\ 0\ 0) & = X_1 \\
X_1 N(T)^T & \hookrightarrow & (I\ 1\ 1\ 1\ 1\ 0) & = Y_2 \\
Y_2 N(T) & \hookrightarrow & (I\ I\ 1\ 1\ 0\ 11\ I\ 1\ 1) & = X_2 \\
X_2 N(T)^T & \hookrightarrow & (I\ 1\ 1\ 1\ 1\ 1) & = Y_3 \\
Y_3 N(T) & \hookrightarrow & (I\ I\ 1\ 1\ 0\ 1\ 1\ I\ 1\ 1) & = X_3 \\
 & & & (=X_2).
\end{array}
$$

Thus the hidden pattern of the dynamical system is the fixed point given by the binary pair.

{(I 1 1 1 1 1), (I I 1 1 0 1 1 I 1 1)}.The D_3 also has a strong effect on the dynamical system for it influences all nodes expect R_5.

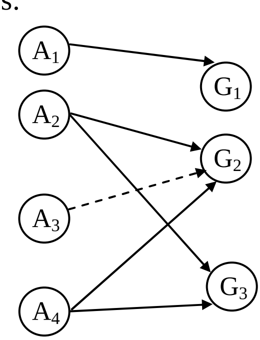
We now study another set of attributes related with the HIV/AIDS affected migrant labours.

Let us consider the sets attributes $\{(A_1\ A_2\ ,\ldots.\ A_6), (G_1\ G_2, \ldots, G_5)\}$ of the NRM.

This is divided into two overlapping classes C_1 and C_2

$$C_1 = \{(A_1\ A_2\ A_3\ A_4), (G_1\ G_2\ G_3)\} \text{ and}$$
$$C_2 = \{(A_4\ A_5\ A_6)\ (G_3\ G_4\ G_5)\}.$$

The directed graph as given by the expert for the attributes in class C_1 is as follows:

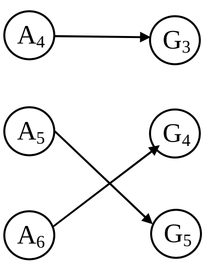


The related relational neutrosophic matrix is as follows:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{ccc} G_1 & G_2 & G_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & I & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}.$$

The directed graph given by the expert for the class

$C_2 = \{(A_4\ A_5\ A_6)\ (G_3\ G_4\ G_5)\}$ is as follows:



The related connection matrix is as follows:

$$\begin{array}{c} \\ A_4 \\ A_5 \\ A_6 \end{array}\begin{array}{c} \begin{array}{ccc} G_3 & G_4 & G_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}.$$

The matrix related to the combined block overlap NRM denoted by N(V).

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{ccccc} G_1 & G_2 & G_3 & G_4 & G_5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Consider the state vector X = (0 0 0 10 0) in which only the node A_4 is in the on state and all other nodes are in the off state. The effect of X on N(V)

$$\begin{array}{llll} XN(U) & \hookrightarrow & (0\ 1\ 1\ 0\ 0) & = Y \\ YN(U)^T & \hookrightarrow & (0\ 1\ I\ 1\ 0\ 0) & = X_1 \\ X_1\ N(U) & \hookrightarrow & (0\ 1\ 1\ 0\ 0) & = Y_1 = Y. \end{array}$$

Thus the hidden pattern of the dynamical system is a fixed point given by the binary pair {(0 1 1 0 0), (0 1 I 1 0 0)}.

It has meager effect on the system.

Next we consider the state vector Y = (1 0 0 0 0) i.e., only the node G_1 is in the on state all other nodes are in the off state. The effect of Y on the dynamical system N(U) is given by

$$\begin{array}{llll} G_1\ N(U)^T & \hookrightarrow & (1\ 0\ 0\ 0\ 0\ 0) = & X \\ XN(U) & \hookrightarrow & (1\ 0\ 0\ 0\ 0) \ = & Y_1\ (= Y). \end{array}$$

Thus the hidden pattern of the dynamical system is a binary pair given by {(1 0 0 0 0) (1 0 0 0 0 0 0)}.

The effect is least on the system.

It is important to mention here that once the connection matrix of the combined block disjoint NRM, or combined block overlap NRM is formed it is a matter of routine to determine the fixed point or limit cycle.

Once again we leave it for the reader to modify the FRM-program to the case of NRM only in it is matter of an addition I also in the coordinate where the indeterminate I behaves as I.I = I but 1 + I = 0 r + I if r > 1 is 1 and 1 + rI if r > 1 is I. Thus the interested reader is requested to construct a C program.

Chapter Four

NEUTROSOPHIC RELATIONAL EQUATIONS MODEL AND THEIR PROPERTIES

In this chapter we describe define and develop the neutrosophic relational maps model and describe them by some illustrations.

NREs are built analogous to FREs (Fuzzy Relational Equations) model.

We denote by N $= \langle [0, I] \cup [0, 1] \rangle$
$= \langle [0, 1] \cup I \rangle$
$= \{a + bI \mid a, b \in [0, 1]\}$

the collection of all fuzzy neutrosophic numbers.

Any matrix which takes its entries from N is called as fuzzy neutrosophic matrix or just neutrosophic matrix. As from the very observation one can easily understand the nature of the neutrosophic matrix to be fuzzy or otherwise.

A binary neutrosophic relation $R_N(x, y)$ may assign to each element of X two or more elements of Y or the indeterminate I.

Some basic operations on functions such as the inverse and composition are applicable to binary relations as well.

Given a neutrosophic relation $R_N(X, Y)$ its domain is a neutrosophic set on $\langle X \cup I \rangle$, domain R whose membership function is defined by dom (R(x)) = max $R_N(x, y)$ for each $x \in \langle X \cup I \rangle$.

That is each element of set $\langle X \cup I \rangle$ belongs to the domain of R to the degree equal to the strength of its strongest relation to any member of set $\langle Y \cup I \rangle$.

The degree may be an indeterminate I also.

Thus this is one of the marked difference between the binary fuzzy relation and the binary neutrosophic relation (However it is pertinent to mention here that I is used as neutrosophic or indeterminacy so one does not (at times depending on the expert) distinguish 0.5I or 0.3I or 0.9I it is replaced by I. Thus in general one does not welcome terms like a + bI where a, b $\in$ [0, 1].

The range of $R_N(X, Y)$ is a neutrosophic relation on Y, ran R whose membership is defined by

$$\text{ran } R(y) = \max_{x \in X} R_N(x, y) \text{ for each } y \in Y,$$

that is the strength of the strongest relation that each element of Y has to an element of X is equal to the degree of that elements membership in the range of R or it can be indeterminate I.

The height of a neutrosophic relation $R_N(x, y)$ is a number; h(R) or an indeterminate I defined by

$$h_N(R) = \max_{y \in \langle Y \cup I \rangle} \max_{x \in \langle X \cup I \rangle} R_N(x, y).$$

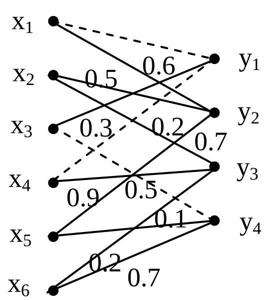
That is $h_N(R)$ is the largest membership grade attained by any pair (x, y) in R or the indeterminate I.

A convenient representation of the neutrosophic binary relation $R_N(X,Y)$ are membership matrices.

$$R = [\gamma_{xy}] \text{ where } \gamma_{xy} \in R_N(x,y).$$

Another useful representation of a binary neutrosophic relation is a neutrosophic Sagittal diagram.

Each of the sets X, Y represented by a set of nodes in the diagram, nodes corresponding to one set are clearly distinguished from nodes representing the other set. Elements of $X' \times Y'$ with non zero membership grades in $R_N(X,Y)$ are represented in the diagram by lines connecting the respective nodes. These lines are labeled with the values of the membership grades.



An example of the neutrosophic Sagittal diagram is a binary neutrosophic relation $R_N(X, Y)$ together with the membership neutrosophic matrix which is given below:

$$= \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} I & 0.5 & 0 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.6 & 0 & 0 & I \\ I & 0.9 & 0 & 0 \\ 0 & 0.7 & 0.5 & 0.1 \\ 0 & 0 & 0.2 & 0.7 \end{bmatrix} \end{array}$$

The inverse of a neutrosophic relation $R_N(X, Y)$ denoted by $R_N^{-1}(X, Y)$ for all $x \in X$ and $y \in Y$.

A neutrosophic membership matrix $R^{-1} = \left[\gamma_{xy}^{-1}\right]$ representing $R_N^{-1}(Y, X)$ is the transpose of the matrix R for R_N (X,Y) which means that the rows of R^{-1} equal the columns of R and the columns of R^{-1} equal rows of R.

Clearly $(R^{-1})^{-1} = R$ for any binary neutrosophic relation.

Consider any two binary neutrosophic relation $P_N(X, Y)$ and Q_N (Y, Z) with a common set Y. The standard composition of these relations which is denoted by $P_N(X, Y)$ o Q(Y, Z) produces a binary neutrosophic relation

$R_N(X, Z)$ on $X \times Z$ defined by

$$R_N(x, z) = [P \text{ o } Q]_N(x, z) = \max_{y \in Y} \min\left[P_N(x, y), Q_N(y, z)\right]$$

for all $x \in X$ and for all $z \in Z$.

This composition satisfies the following relations.

$$[P_N(X, Y) \text{ o } Q_N(Y, Z)]^{-1} = Q_N^{-1}(Z, Y) \text{ o } P_N^{-1}(Y, X).$$

Further $[P_N(X, Y) \text{ o } Q_N(Y, Z)] \text{ o } R_N(Z, W)$

$$= [P_N(X, Y) \text{ o } [Q_N(Y, Z) \text{ o } R_N(Z, W)];$$

that is the standard max min composition is associative and its inverse equals to reverse composition of the inverse relation.

However standard composition is not commutative because $Q_N(Y, Z)$ o $P_N(X, Y)$ is not well defined when $X \neq Z$. Even if $X = Z$ and Q_N (Y, Z) o $P_N(X, Y)$ are well defined still we can have $P_N(X, Y) \text{ o } Q_N(Y, Z) \neq Q_N(Y, Z) \text{ o } P_N(X, Y)$.

Composition of binary neutrosophic relation can be performed conveniently in terms of membership matrices of the relations. Let P = $[p_{ik}]$, Q = $[q_{kj}]$ and R = $[r_{ij}]$ be membership matrices of binary relations such that R = P o Q.

We write using matrix notation

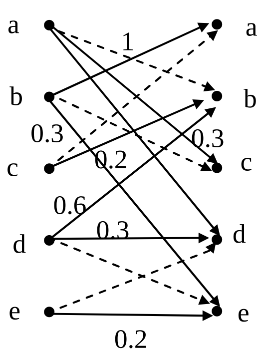
$$[r_{ij}] = [p_{ik}] \text{ o } [q_{kj}] \text{ where } r_{ij} = \max_k \min\left(p_{ik}, q_{kj}\right).$$

The neutrosophic membership matrix and the neutrosophic Sagittal diagram is as follows for any set X = {a, b, c, d, e}.

The neutrosophic matrix of X is

$$N = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} 0 & I & 0.3 & 0.2 & 0 \\ 1 & 0 & I & 0 & 0.3 \\ I & 0.2 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0.3 & I \\ 0 & 0 & 0 & I & 0.2 \end{array}\right] \end{array}.$$

Neutrosophic membership matrix for X is given above and the neutrosophic Sagittal diagram of X is as follows:



Neutrosophic diagram or graph is left for the reader as an exercise. Several properties about fuzzy neutrosophic relational maps is discussed in NFRM.

We do not deal in this book in this direction for we need only models to tackle social problems for socio scientist.

Keeping this in mind we introduce some examples to show how the Neutrosophic Relational Equations (NRE) is used in our study.

Example 4.1: We study the bonded labour problem using NRE. We describe the main attributes associated with the bonded labour problem.

B_1 - No knowledge of any other work has made them not only bonded but live in penury.

B_2 - Advent of power looms and globalization has made them still poorer.

B_3 - Salary they earn in a month.

B_4 - No saving so they become more and more bonded by borrowing from the owners they live in perennial debts.

B_5 - Government interferes and frees them but they do not have any work and government does not give them any alternative job.

B_6 - Hours / days of work.

We have taken these six heads B_1, B_2, …, B_6 related to the bonded labourers as the rows of the fuzzy relational matrix. The main attributes / heads O_1, O_2, O_3, O_4 related to the owners of the bonded labourers are;

O_1 - Globalization / introduction of modern textiles machine

O_2 - Profit or no loss

O_3 - Availability of raw goods

O_4 - Demand for finished goods.

Using the heads related to owners along columns the fuzzy relational equations are formed using experts opinions.

The following are the limit sets.

$B_1 \geq 0.5$ Means no knowledge of other work hence live in poverty.

$B_2 \geq 0.5$ Power looms / other modern textile machinery had made their condition from bad to worst.

$B_3 \geq 0.5$ Earning is mediocre ($B_3 < 0.5$ implies the earning does not help them to meet both ends).

$B_4 \geq 0.4$ No savings no debt ($B_4 < 0.4$ implies they are in debt).

$B_5 \geq 0.5$ Government interference has not helped ($B_5 < 0.5$ implies government interference have helped).

$B_6 \geq 0.4$ 10 hours of work with no holidays ($B_6 < 0.4$ implies less than 10 hours of work).

$O_1 \geq 0.5$ The globalizations / government has affected the owners of the bonded labourers drastically ($O_1 < 0.5$ implies; has no impact on owners).

$O_2 \geq 0.5$ Profit or no loss ($O_2 < 5$ implies total loss).

$O_3 \geq 0.6$ Availability of raw materials ($O_3 < 0.6$ implies shortage of raw materials).

$O_4 \geq 0.5$ Just they can meet both ends i.e., demand for finished goods are produced goods balance ($O_4 < 0.5$ implies no demand for the finished product i.e., demand and supply do not balance).

We use the NRE; P_N o Q_N = R_N where P_N, Q_N and R_N are Neutrosophic matrices.

Now we determine the neutrosophic matrix associated with the attributes relating the bonded labours and the owners using NRE.

$$P_N = \begin{array}{c} \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{array} \begin{array}{c} \begin{array}{cccc} O_1 & O_2 & O_3 & O_4 \end{array} \\ \begin{bmatrix} 0.6 & 0 & 0.3I & 0 \\ 0.7 & 0.4 & 0.3 & 0.I \\ 0.3 & 0.4 & 0.3 & 0.3 \\ 0.3I & 0 & 0.3 & 0.4I \\ 0.8 & 0.4I & 0.2 & 0.4 \\ 0 & 0.4 & 0.5 & 0.9 \end{bmatrix} \end{array}$$

Suppose Q_N^T = [0.6 0.5 0.7 0.9].

Now P_N and Q_N are know in the neutrosophic relational equation P_N o Q_N = R_N.

Using the max-min principle in the equation P_N o Q_N = R_N we get R_N^T = {0.6 0.6 0.4 0.4I, 0.6, 0.9}.

In the neutrosophic relational equation P_N o Q_N = R_N, P_N corresponds to the weightages of the expert, Q_N is the profit of the owner given by the experts and R_N is the calculated or the resultant giving the status of the bonded labourers.

Using these we obtain the neutrosophic resultant vector R_N^T = (0.6, 0.6, 0.4, 0.4I, 0.6, 0.9) where hours of days work is the highest and the advent of power looms and globalization has made them still poorer followed by no knowledge of any other work has made them only bonded but live in penury and government does not give them any alternative job remains next maximum.

Example 4.2: Here we study eight symptoms and 10 patients $P_1, P_2, P_3, \ldots, P_{10}$. The 8 symptoms which are taken are $S_1, \ldots, S_8$ are given as follows:

S_1 - Disabled
S_2 - Difficult to cope with
S_3 - Dependent on others
S_4 - Apathetic and unconcerned
S_5 - Blaming oneself
S_6 - Very ill
S_7 - Depressed
S_8 - Anxious and worried.

Now using the experts opinion who is the ward doctor we give the related neutrosophic matrix P_N with weights w_{ij}.

P_N is a 8×10 matrix; P_N

$$= \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{array} \begin{array}{c} \begin{array}{cccccccccc} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} \end{array} \\ \left[\begin{array}{cccccccccc} 0 & 0 & 0.2I & 0.5 & 0 & 0 & 0.6 & 0.7 & 0 & 0.5I \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 1 & 0 & 0.9 & 0.6 \\ 0.5I & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0.8I & 0 & 0.3 & 0 & 0.8 & 0 & 0 \\ 0 & 0.8I & 0.3 & 0 & 0.7 & 1 & 0 & 0.3 & 0.7I & 0.7 \\ 0.3 & 0.7 & 0 & 0.3 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0.9 & 0.4 & 0 & 0 & 0.8I & 0.9 & 0 & 0 & 0 & 0.4 \\ 0.2I & 0 & 0 & 0 & 0 & 0 & 0.7I & 0 & 0.2 & 0.3 \end{array}\right] \end{array}.$$

Let Q_N denote the set of 8 symptoms / diseases, i.e., Q the neutrosophic vector is a 1×8 neutrosophic matrix.

Consider the neutrosophic equation Q_N o $P_N = R_N$; clearly when Q_N and P_N are known we can easily solve the neutrosophic relational equation and obtain the neutrosophic vector R_N.

Let Q_N = (0.3, 0.7, 0.5I, 0.3, 0, 0.3, 0.2, 0.3I) be the neutrosophic vector given by the expert he feels the dependence is an indeterminate concept to some extent and difficult to cope with is present in most patients and in fact all patients suffer from depression.

Q_N o $P_N = R_N$ and we find

R_N = (0.5I, 0.3, 0.3, 0.3, 0.5I, 0.3, 0.7, 0.3, 0.7, 0.6) which shows for the given input the relations with the patients.

Example 4.3: Let us study the children dropping out of school and its relations with parents, private and government schools using NRE.

Attributes related with the educational institutions are

S_1 - Entry into school by poor students is impossible / meager due to fee structure and donations.

S_2 - Private schools shun to admit first generation learners. So parents who are illiterates cannot get admission in these school.

S_3 - Caste and socio economic conditions in private schools makes it impossible for the poor students to thrive - so dropout of it even if by chance get admitted.

S_4 - Poor students suffer communication problems so find it difficult to study and understand in class.

S_5 - Number of teachers in government school is inadequate and those teachers are also busy with side business so do not teach children. In case of private schools most teachers are busy taking tuitions so do not teach properly in class.

S_6 - The school teachers be it private or government do not properly motivate the children.

The attributes related with the school dropouts.

C_1 - Parents are poor and are not in a position to pay huge amount as fees.

C_2 - Parents are not educated so are not in a position to help their children in their home work or revise the daily class lessons.

C_3 - Because children have communication problems; so disinterested in studies seek company of gangs and develop bad habits.

C_4 - Poor children are discriminated and insulted in the classroom for no fault of theirs.

C_5 - Due to gender bias female children are stopped from studies.

C_6 - Government is indifferent to the sufferings of these children. Least punishment is given to persons who practice child labour.

Even though one can mention many more attributes, we select a few for this analysis.

The limit set associated with $S_1, \ldots, S_6$ and $C_1, C_2, \ldots, C_6$ are as follows:

$S_1 \geq 5$ Fee is too high to pay by poor ($S_1 < 0.5$ it is not that high for the poor to pay).

$S_2 \geq 0.5$ Educational qualifications of their parents is poor so no admission for them ($S_2 < 0.5$ means they get admission in spite of their parents poor educational qualification).

$S_3 \geq 0.5$ Poor students feel out of place in these private schools ($S_3 < 0.5$ poor students do not feel out of place in these private schools).

$S_4 \geq 0.5$ Private schools promote English so poor find it difficult to cope up with studies. $S_4 < 0.5$ poor do not find it difficult to study as English is not so dominant in the classroom.

$S_5 \geq 0.4$ Schools have adequate number of qualified devoted teachers ($S_5 < 0.4$ school lack in the number of qualified devoted teachers).

$S_6 \geq 0.5$ Schools do not have motivated teachers to encourage poor ($S_6 < 0.5$ school has motivated encouraging teachers who motivate poor students in particular).

The limit sets related with $C_1, C_2, \ldots, C_7$ are as follows:

$C_1 \geq 0.5$ in ability for the parents to pay fees of their children ($C_1 < 0.5$ poor parents can pay the fees of their children).

$C_2 \geq 0.6$ income and educational level of parents is extremely inadequate ($C_2 < 0.6$ in spite of income and parents educational level; children cope up well in the class).

$C_3 \geq 0.4$ children get into bad company of friends and dropout of school ($C_3 < 0.4$ implies that the children do not dropout of school and seek bad company).

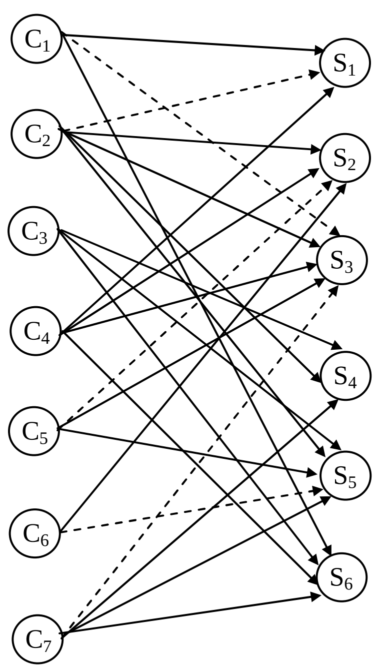
$C_4 \geq 0.4$ poor children are ridiculed in school / classroom by teachers ($C_4 < 0.5$ poor children are not ridiculed by teachers but teachers are helpful in class).

$C_5 \geq 0.4$ Children have to walk a long distance to reach school. No good road or transport facilities ($C_5 < 0.4$ the poor children can reach safely and the roads are good and transport facilities are up to expectation).

$C_6 \geq 0.4$ Children are discriminated by gender so female children are stopped from attending school and are made to do domestic work; as a training to become house wife after marriage ($C_6 < 0.4$ no such discrimination).

$C_7 \geq 0.5$ Indifferent attitude on the part of the government towards providing education to the poor and first generation learners, so these children become school dropout and land ultimately as child labourers ($C_7 < 0.5$ no such indifference).

Now we give the directed graph (neutrosophic Sagittal diagram) associated with this problem.



The associated matrix is as follows:

$$P = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \left[\begin{array}{cccccc} 0.7 & 0 & 0.3I & 0 & 0 & 0.8 \\ 0.2I & 0.5 & 0.6 & 0.7 & 0.4 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0.5 \\ 0.9 & 0.6 & 0.8 & 0 & 0 & 0.7 \\ 0 & 0.5I & 0.7 & 0 & 0.6 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0.5I \\ 0 & 0 & 0.I & 0.6 & 0.8 & 0.7 \end{array}\right] \end{array}$$

Suppose P o Q = R we would give two of the matrices to find the third one.

Now Q = (0.6, 0.2, 0.3, 0.4, 0.5, 0.6) be the supplied matrix to solve the equation P o Q^t = R.

We use the max min operation and find the expected value of R. We see R^t = (0.6, 0.4, 0.5, 0.6, 0.5, 0.5I, 0.6).

In this way we obtain the expected value and can interpret the expected value. Also if R is given we can find Q.

Interested reader can refer [13].

Now we proceed onto define a new type of Neutrosophic Relational Equations. Suppose we have a set of n experts all of them want to work with the same set of attributes and also they use the same limit set. So with each expert is associated a s × t matrix say P_i, $1 \leq i \leq m$.

Now we can define the new average (mean) neutrosophic relational equations model. Given P_1, P_2, …, P_n are n fuzzy neutrosophic s × t matrices.

All these n experts work with the same limit set.

$$\text{We define } P = \frac{1}{n}\sum_{i=1}^{n} P_i \ .$$

Clearly P is again a fuzzy neutrosophic matrix.

We use P as the only model to work with the problem. The advantages of using this model are:

(i) In the first place no expert feels that his opinion was not given enough representation.
(ii) The theory of large numbers.
(iii) We make no compromise as all the experts use the same limit set.

Thus the new mean neutrosophic relation equation is the best suited model to be used under these stipulated conditions.

Finally we mention about the linked Neutrosophic Relational Equations or linked NRE. We can link several NREs.

We will describe this in the following:

Suppose we have say 3 set of attributes say $S_1, \ldots, S_n$, $P_1, P_2, \ldots, P_m$ and $T_1, \ldots, T_p$ where $\{S_1, \ldots, S_n\}$ and $\{P_1, P_2, \ldots, P_m\}$ are connected by a Sagittal diagram and $\{P_1, \ldots, P_m\}$ and $\{T_1, T_2, \ldots, T_p\}$ are also related by a Sagittal diagram. Both have an associated limit set. That is we have B_1 to be the $n \times m$ neutrosophic matrix.

$$B_1 = \begin{array}{c} \\ S_1 \\ S_2 \\ \vdots \\ S_n \end{array} \begin{array}{c} \begin{array}{cccc} P_1 & P_2 & \cdots & P_m \end{array} \\ \left[\begin{array}{c} \\ (p_{ij}) \\ \\ \\ \end{array} \right] \end{array}$$

where $p_{ij} \in \{[0, 1] \cup nI \mid n \in [0, 1]\}$; $1 \leq i \leq n$ and $1 \leq j \leq m$.

Let B_2 be the m × p neutrosophic matrix of the NRE.

$$B_2 = \begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array} \overset{\begin{array}{cccc} T_1 & T_2 & \ldots & T_p \end{array}}{\left[\begin{array}{c} \\ (s_{ij}) \\ \\ \end{array} \right]}$$

where $s_{ij} \in \{[0, 1] \cup nI \mid n \in [0, 1]\}$, $1 \le i \le m$ and $1 \le j \le p$.

We see $B = B_1 \times B_2$ where × is the max min operation.

We get

$$B = \begin{array}{c} \\ S_1 \\ S_2 \\ \vdots \\ S_n \end{array} \overset{\begin{array}{cccc} T_1 & T_2 & \ldots & T_p \end{array}}{\left[\begin{array}{c} \\ (t_{ij}) \\ \\ \end{array} \right]}$$

where $t_{ij} \in \{[0, 1] \cup nI \mid n \in [0, 1]\}$ $1 \le i \le n$ and $1 \le j \le p$.

Suppose we have say some r number of NREs of different attributes but they are all linked consecutively and we cannot directly connect them; then we can use the linked NRE to interconnect those compatible ones and get a NRE associated with it.

Using the NREs we can construct 2-adaptive fuzzy neutrosophic models for they can be realized as the modified NREs (MNRMs) described in chapter three. The same NRE matrix can be used as the MNRMs dynamical system, instead of the input vectors as $(a_1, \ldots, a_n)$ $a_i \in [0, 1] \cup nI$; $1 \le i \le n$.

We use only the on or off state of the state vector but the final hidden pattern will have the resultant as a pair of vectors of the form

$\{(x_1, \ldots, x_n), (y_1, y_2, \ldots, y_t) \mid x_i, y_j \in [0, 1] \cup mI; m \in [0, 1]; 1 \leq i \leq n; 1 \leq j \leq t\}$.

However it is pertinent here to keep on record that these NREs can also be the 2-adaptive fuzzy neutrosophic model of the NAM's which will be described and defined in chapter VI of this book.

We will illustrate this simple situation by an example.

Consider the mixed NRM matrix in page 71 of this book.

$$MN(E) = \begin{array}{c} \\ G_1 \\ G_2 \\ G_3 \\ G_4 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \left[\begin{array}{ccccc} 0.8 & 0.9 & 0.6 & 0.8 & 0.4 \\ 0.3I & 0 & 0.2 & 0.7 & 0.9 \\ 0.5 & 0.6 & 0.6 & 0.9 & 0.6 \\ 0 & 0.4I & 0.2I & 0.9 & 0.4 \end{array}\right] \end{array}.$$

We will first stipulate the limit system so that the neutrosophic matrix MN(E) can serve as the NRE model for this problem.

$G_1 \geq 0.6$ no steps taken by government to provide alternatives for agriculture failure ($G_1 < 0.6$ implies government provides some alternative for agriculture failure).

$G_2 \geq 0.4$ Government has not taken any legal remedies to prevent child labours. ($G_2 < 0.4$ government has taken legal remedies to prevent child labour).

$G_3 \geq 0.5$ government does not give education with monthly stipend for poor school children in villages whose life is miserable due to poverty ($G_3 < 0.5$ government gives free education with monthly stipend for those in the villages who suffer due to poverty).

$G_4 \geq 0.4$ child labourer are not vote banks so no concern over their welfare ($G_4 < 0.4$ government is concerned over the child labourer and their welfare).

Now we give the limit set associated with the attributes $\{C_1, C_2, C_3, C_4, C_5\}$.

$C_1 \geq 0.4$ these children do not suffer from acute poverty ($C_1 < 0.4$; children do not suffer from acute poverty).

$C_2 \geq 0.5$ the parents of these children suffer due to failure of agriculture ($C_2 < 0.5$ these children's parents do not suffer from failure of agriculture).

$C_3 \geq 0.4$ these children suffer from acute poverty so starvation, hence no food so in this state they cannot think of education so they dropout of school and work as child labourers ($C_3 < 0.4$ implies the family does not suffer from acute poverty so not in the precarious state of sending their children to work by stopping them from school).

$C_4 \geq 0.5$; Tea shops employ them as child labourers and give them free food and shelter ($C_4 < 0.5$ tea shops do not employ them and give them free food and shelter).

$C_5 \geq 0.4$; Long hours of work so they suffer from health related problem ($C_5 < 0.4$ they are not made to work for long hours).

Now using this as the limit set of the NRE we proceed onto work with this model using MN(E) as the matrix for working.

Let Q = (0.8, 0.9, 0.6, 0.8, 0.4) be the given matrix.

To find max min $\{Q, M_N(E)^t\} = Q \text{ o } (M_N(E))^t$
$= (0.9, 0.7, 0.8, 0.8)$.

We see from this resultant no steps is taken by the government to provide alternatives when agriculture has failed, government has not taken action against those who practice child labour as $G_2 = 0.7$.

Since $G_3 = 0.8$ government is not providing any financial assistance to the students family whose life is miserable due to poverty.

Finally $G_4 = 0.8$ clearly shows child labourers are not vote banks so no concern over their welfare.

Now we proceed onto use a NRE as a MNRM by the property of 2-adaptive fuzzy neutrosophic models. Consider the neutrosophic relational equation matrix P associated with the child labour problem in page 135.

$$P = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} 0.7 & 0 & 0.3I & 0 & 0 & 0.8 \\ 0.2I & 0.5 & 0.6 & 0.7 & 0.4 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0.5 \\ 0.9 & 0.6 & 0.8 & 0 & 0 & 0.7 \\ 0 & 0.5I & 0.7 & 0 & 0.6 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0.5I \\ 0 & 0 & 0.I & 0.6 & 0.8 & 0.7 \end{bmatrix} \end{array}$$

Let P be taken as the Modified Neutrosophic Relational Maps (MNRM) model.

To find the effect of the state vector X = (1 0 0 0 0 0 0) on the dynamical system.

That is only the node parents are poor so they cannot afford to pay the fee in private school is in the 'ON' state and all other nodes are in the 'OFF' state.

To find the effect of X on P.

$$X \text{ o } P = \max \min \{X, P\}$$
$$= (0.7, 0, 0.3I, 0, 0, 0.8) = Y \text{ (say)}$$

$$Y \text{ o } P^t = \max \min (Y, P^t)$$
$$= (0.8, 0.3I, 0.5, 0.7, 0.3I, 0.5I, 0.7)$$
$$\rightarrow (1, 0.3I, 0.5, 0.7, 0.3I, 0.5I, 0.7)$$

($\rightarrow$ denotes the vector has been updated)

$$= X_1 \text{ (say)}$$

$$X_1 \text{ o } P = \max \min \{X_1, P\}$$
$$= (0.7, 0.6, 0.7, 0.6, 0.7, 0.8) = Y_1 \text{ (say)}$$

$$Y_1 \text{ o } P^t = \max \min \{Y_1, P^t\} = (0.8, 0.6, 0.7, 0.7, 0.6, 0.7)$$
$$\rightarrow (1, 0.6, 0.6, 0.7, 0.7, 0.6, 0.7) = X_2 \text{ (say)}$$

$$X_2 \text{ o } P = \max \min \{X_2, P\}$$
$$= (0.7, 0.6, 0.7, 0.6, 0.7, 0/8)$$
$$= Y_2 \text{ (say).}$$

We see $Y_2 = Y_1$ which implies the neutrosophic hidden pattern of the system is a fixed point pair given by

$$\{(0.7, 0.6, 0.7, 0.6, 0.7, 0.8), (1, 0.6, 0.6, 0.7, 0.7, 0.6, 0.7)\}$$

All the values in every node is greater than or equal to 0.6.

In this way we can study using 2-adaptive fuzzy neutrosophic model the various properties of problem using these models as they are interrelated we can compare the solutions.

Chapter Five

NEUTROSOPHIC BIDIRECTIONAL ASSOCIATIVE MEMORIES

We in this chapter describe the Neutrosophic Bidirectional Associative Memories model. This model can be applied in place of Bidirectional Associative Memories (BAM) model provided the experts are convinced of the presence of an element of indeterminacy in the problem under investigation.

Unless one is sure of the presence of indeterminacy one need not use this model.

We just define and describe the functioning of this model.

Let N = $[-n, n] \cup [-nI, nI]$ ($n \geq 1$) denote the neutrosophic interval i.e., if n = 1 we call N = $[-1, 1] \cup [-I, I]$ to be fuzzy neutrosophic interval by default of notation and convention.

Any element $x \in N$ is of the form $x + yI$ where $x, y \in [-n, n]$. It is possible that $x = 0$ or $y = 0$ can also occur.

One should be well versed with the notion of Bidirectional Associative Memories (BAM). We know problems where artificial neural networks have the most promise and are those with a real world flavour; sociological problems may involve the concept of indeterminacy or may involve the concept of indeterminacy.

Even in the analysis of a criminal case or a civil case we see the concept of indeterminacy is very high, in India it is highly dependent on caste, economy, power they wield in society and in political circle. In such cases the models which can involve indeterminacy can be used.

We accept that $\frac{dI}{dt} = kI$ i.e., the derivative of the indeterminate relative to time factor change; k a real quantity. For time factor can change an indeterminate value may be other factors can make it change to a real value.

That neuronal activations change the indeterminate with time.

Thus these new models can recognize ill defined problems.

For instance in the criminal case the person say X accused of some criminal act might be an indeterminate. But in due course of time the investigation can prove the involvement is a lesser indeterminate or a greater indeterminate or false or real. So with time the indeterminate I would have become kI; $k > 1$ (or $k < 1$) or just k.

One cannot even after complete investigation relative to time (in due course of time) say the I remains as I.

Might be other factors can make the same I a real value if proved to be criminal or proved to be an innocent.

Thus we can say mind cannot always think conceptually but in certain cases one's mind can also remain indeterminate over a fact. When we wish to go forth to describe the mathematical structure for it we have to describe it as an indeterminate.

It is important to mention that it may so happen given an attribute the indeterminate value I at a time period k may in due course of $(k + n)^{th}$ time period; $n \geq 1$ may become zero or a real number also vice versa can occur.

An event at time t having a real coordinate at the $(t + m)^{th}$ time period may become I or zero. It is also probable that 0 becoming I or a real value.

Here F_X^I will denote a neuronal field which contains n-neurons (simple processing units some of the processing units at any stipulated period may be an indeterminate) which includes also indeterminate neurons.

Let F_Y^I denote a neuron field which contains p neurons which also includes the indeterminate neuron.

These neuron fields will be known as neuron neutrosophic fields.

The neuronal neutrosophic dynamical system is described by a first order differential equation that govern the time evolution of the neuronal activations.

$$\dot{x}_i = g_i (X, Y, \ldots)$$

$$\dot{y}_j = h_j (X, Y, \ldots),$$

where x_i and y_j denote respectively the activation time function of the i^{th} neuron in F_X^I and the j^{th} neuron in F_Y^I.

The over dot denotes time differentiation, g_i and h_j are some functions of X, Y, …, where $X(t) = (x_1(t), \ldots, x_n(t))$ and $Y(t) = (y_1(t), \ldots, y_p(t))$ define the state of the neuronal dynamical neutrosophic system at time t.

We can process as in case of BAM's described in [ ] with the only change the indeterminate I changes with time as $\frac{dI}{dt} = kI$ ($k \in R$ (R - reals)) .

Let us suppose that the field F_X^I with n neurons is synaptically connected to the field F_Y^I of p-neurons.

Let m_{ij} be the synapse where the axon from the i^{th} neuron in F_X^I terminates, m_{ij} can be positive, negative, zero or an indeterminate. The synaptic matrix M is a $n \times p$ neutrosophic matrix whose entries are synaptic efficacies m_{ij}.

$M = (m_{ij})$ describes the forward projections from the neuronal neutrosophic field F_X^I to the neuronal neutrosophic field F_Y^I.

Unidirectional neutrosophic networks occurs when a neuron field synaptically intraconnects to itself. The neutrosophic matrix M be a n by n square neutrosophic matrix.

A bidirectional neutrosophic network (neutrosophic bidirectional network) occur if $M = N^T$ and $N = M^T$.

Keeping in mind the BAM structure we modify the signal functions in case of indeterminacy S_i and S_j as follows:

$$S_i(x_i^k) = \begin{cases} 1 & \text{if } x_i^k > U_i \quad U_i \text{ real} \\ I & \text{if } x_i^k > U_i \quad U_i \text{ an indeterminate} \\ 0 & \text{if } x_i^k < U_i \\ 1 \text{ or } I & \text{if } x_i^k \text{ is real and } U_i \text{ indeterminate or} \\ & \quad \text{vice versa according to the experts needs} \\ S_i(x_i^{k-1}) & \text{if } x_i^k = U_i \ (U_i \text{ can be indeterminate or real}) \end{cases} \tag{1}$$

where $U = (U_1, \ldots, U_n)$ for F_X^I neurons.

If $V = (V_1, \ldots, V_p)$ for F_Y^I neurons the threshold signal functions corresponds to

$$S_j(y_j^k) = \begin{cases} 1 & \text{if } y_j^k > V_j \\ S_j(y_j^{k-1}) & \text{if } y_j^k = V_j \\ I & \text{if } y_j^k > V_j \text{ (both } y_j^k \text{ and } V_j \text{ are} \\ & \text{indeterminates)} \\ 0 & \text{if } y_j^k < V_j \\ 1 \text{ or } I & \text{if one of } y_j^k \text{ or } V_j \text{ is real and the other} \\ & \text{is an indeterminate} \end{cases} \quad (2)$$

Thus as in case of BAM work for the solution. The only change is we instead of working with an interval $[–n, n]$ $(n \geq 1)$ work with the neutrosophic interval $N = [–n, n] \cup [–nI, nI]$.

Also the associated synaptic connection matrix M gets its entries from N. The input vector X_k also gets its entries only from $[–n, n]$.

The binary signal vector after being acted on by the synaptic connection matrix M gets its value in $N = [–n, n] \cup [–nI, nI]$. Thus if in the BAM we replace the interval $[–n, n]$ by N and the matrix (m_{ij}) to take its entries from N and F_X^I and F_Y^I are neuronal fields having the indeterminate.

When the threshold functions are taken as those defined in (1) and (2) then we call the BAM as Neutrosophic Bidirectional Associative Memories (NBAM).

We illustrate this by the following example.

Example 5.1: We give here the NBAM model which analyses the cause for female infanticide in India and also try to find out which factor dominates the most by taking into account an experts opinion.

The expert who is an NGO wishes to work in the scale N = [–5, 5] ∪ [–5I, 5I]. The synaptic connection neutrosophic matrix M is given below by the following table.

X/Y	Rich	Upper middle class	Middle class	Lower middle class	Poor	Very poor
Poverty	–1	–2	–4	2	4	3
Property division	2I	I	I	0	0	0
Torture of inlaws	3	4	4	4	–3	0
Economic loss or burden	1	5	5	3	0	I
Social stigma	5	4	4	5	3	4
No.of children	0	2	3	1	I	–1
Finding suitable good groom	4	3	3	I	0	.1

This 7 × 6 neutrosophic matrix M represents the forward synaptic projection from the neuronal field F_X^I to the neuronal field F_Y^I. The 7 × 6 neutrosophic matrix M takes its entries from N = [–5, 5] ∪ [–5I, 5I].

Suppose X_k = (2, –4, 3I, 0, –2, –1, –3) be the initial input neutrosophic vector. The initial input is given such that the economic loss or burden is indeterminate, social stigma is absent, finding suitable bride groom is non existence at the time; property division at that time period is taken by the expert; $S(X_k) = (1\ 0\ I\ 0\ 0\ 0\ 0)$.

From the activation equation

$$S(X_k)\,M = (1\ 0\ I\ 0\ 0\ 0\ 0) \circ \begin{bmatrix} -1 & -2 & -4 & 2 & 4 & 3 \\ 2I & I & I & 0 & 0 & 0 \\ 3 & 4 & 4 & 4 & -3 & 0 \\ 1 & 5 & 5 & 3 & 0 & I \\ 5 & 4 & 4 & 5 & 3 & 4 \\ 0 & 2 & 3 & 1 & I & -1 \\ 4 & 3 & 3 & I & 0 & 1 \end{bmatrix}$$

$$= (-1 + 3I,\ -2+4I, -4 + 4I, 2 + 4I, 4-3I, 3)$$

$$= Y_{k+1}$$

$$S(Y_{k+1}) = (1\ I\ I\ I\ 1\ 1)$$

$$S(Y_{k+1})M^t = (6 - 4I, 4I, 12I, 14I + 1, 13I+12, 7I-1, 5+7I)$$

$$= X_{k+2}$$

$$S(X_{k+2}) = (1, I\ I\ I\ I\ I\ I)$$

$$S(X_{k+2})M = (15I - 1, 19I - 2, 20I - 4, 14I - 2, 4+I, 3+5I)$$

$$= Y_{k+3}$$

$$S(Y_{k+3}) = (I\ I\ I\ I\ 1\ I)$$

$$S(Y_{k+3})\ M^+ = X_{k+4}$$

$$= (1, I\ I\ I\ I\ I\ I)$$

is a fixed point.

Thus we see the poverty alone is the cause of female infanticide and all other factors are just indeterminate when the input vector say poverty is the cause of infanticide and torture by inlaws in an indeterminate.

Further all the nodes of social status become indeterminates for this input vector which says poverty is in on state and torture by in laws in the indeterminate state.

Thus if k was the time the female baby was born in 4 to 5 years time only this result can be expected, keeping in mind all other situation are normal i.e., the normal feeling towards a baby, otherwise it would be different.

Next we proceed onto give an example of a vulnerability to HIV/AIDS and factors of migration.

Example 5.2: Causes of migrant labourers vulnerability to HIV/AIDS are discussed under the these heads.

Causes / vulnerability of HIV/AIDS to migrant labourers.

A_1 - No awareness / no education
A_2 - Social status
A_3 - No social responsibility and social freedom
A_4 - Bad company and addictive habits
A_5 - Types of profession
A_6 - Cheap availability of CSWs.

Factors forcing people for migrations.

F_1 - Lack of labour opportunities in their home town.

F_2 - Poverty / seeking better status of life.

F_3 - Mobilization of labour contracts.

F_4 - Infertile land due to implementation of wrong research methodologies / failure of monsoon.

Role of government

G_1 - Alternate job in case of harvest failure, there by stopping migration.

G_2 - Awareness clubs in rural areas about HIV/AIDS.

G_3 - Construction of hospitals in rural areas with HIV/AIDS counseling cell/compulsory HIV/AIDS test before marriage.

G_4 - Failed to stop the misled agricultural techniques followed recently by farmers.

G_5 - No foresight for the government and no precautionary actions taken from the past occurrences.

We have seen that migration is due to lack of job opportunities mainly among the uneducated poor rural. So when they to go other states, they easily become victims of bad habits and including visiting CSWs which makes them HIV/AIDS patients.

We now give the NBAM model M_1 in the scale $[-5, 5] \cup [-5I, 5I]$ taking the attributes of the vulnerability of HIV/AIDS as the neuronal field F_X.

The factors forcing people for migration is taken as the neuronal field F_Y.

$$M_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{cccc} F_1 & F_2 & F_3 & F_4 \end{array} \\ \left[\begin{array}{cccc} 5 & 2I & 4 & 4 \\ 4 & 3 & 5 & 3 \\ I & -2 & 4 & 0 \\ 0 & 4 & 2 & 0 \\ 2 & 4 & 3 & 3I \\ 0 & 4I & 2 & 0 \end{array}\right] \end{array}.$$

Let X_k be the input vector given as (3, 4, –1, –3, –2, 1) at the k[th] period.

$S(X_k) = (1\ 1\ 0\ 0\ 0\ 1)$

From the activation equation

$$S(X_K)M_1 = (9\ \ 6I+3\ \ 11\ \ 7) = Y_{K+1}$$

$$S(Y_{K+1}) = (1\ \ I\ \ 1\ \ 1)$$

Now

$$S(Y_{K+1})\ M_1^t = (13 + 2I, 12+3I, -I+4, 4I+2, 7I+5, 4I+2) = Y_{K+2}$$

$$S(X_{K+2}) = (1\ 1\ 1\ I\ I\ I)$$

We find

$$S(X_{K+2})\ M_1 = (4 + 2I\ \ \ 3+9I\ \ 13+7I\ \ \ 7+3I) = Y_{K+3}$$

$$S(Y_{K+3}) = (1, I, 1\ 1)$$

$$S(Y_{K+3})\ M_1^t = X_{K+4}$$

$$S(X_{K+4}) = (1\ 1\ 1\ I\ I\ I)$$

and we see the resultant binary pair is
{(1, 1, 1, I, I, I), (1, I, 1, 1)}.

This is the way the NBAM model works.

Now we take G_1, G_2, G_3, G_4 and G_5 to be the rows and A_1, A_2, A_3, A_4, A_5 along the columns we get the synaptic connection matrix M_2 in the scale $[-5, 5] \cup [-5I, 5I]$ which is as follows:

$$M_2 = \begin{array}{c} \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \end{array} \begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \end{array} \\ \begin{bmatrix} 3 & 4 & -2 & 3I & -1 & 5 \\ 5 & 4 & 3 & -I & 4I & 4 \\ 1 & 3 & 0 & 1 & 4 & 2 \\ 2 & 3 & -2 & -3 & 0 & 3 \\ 3 & 2 & 4I & 3 & 1 & 4 \end{bmatrix} \end{array}$$

is the neutrosophic bidirectional associative memories matrix associated with NAM is M_2.

Let X_K be the input vector (–3, 4, –2, –1, 3) at the kth instant.

The binary signal neutrosophic vector

$S(X_K) = (0\ 1\ 0\ 0\ 1)$.

From the activation equation

$$S(X_K)M_2 = (8, 6, 3+4I, 3-I, 4I+1, 8\} = Y_{K+1} \text{ (say)}$$

$$S(Y_{K+1}) = (1\ 1\ I\ \ 1\ I\ 8)$$

$$S(Y_{K+1})\ M_2^t = (47,\ 41{=}6I,\ 21+4I, 26-2I, 40+5I) = X_{K+2}$$

$$S(X_{K+2}) = (1\ 1\ 1\ 1\ 1)$$

$$S(Y_{K+2})\ M_2 = (14, 16, 3+4I, 1+2I, 4+4I, 18) = Y_{K+3}$$

$$S(Y_{K+3}) = (1\ 1\ I\ I\ I\ 1)$$

$$S(Y_{K+3})\ M_2^t = (12, 13+6I, 6+5I, 8-5I, 9+8I)$$

$$= X_{K+4}$$

$S(X_{K+4}) \quad = \quad (1\ 1\ 1\ 1\ 1)$

We see the binary pair is as follows:

{(1, 1, 1, 1, 1), (1, 1, I, I, I, 1)}

represents a fixed point of the NBAM dynamical system.

Thus we have seen examples of NBAM dynamical system.

We wish to state none of these examples are researched they are only taken as examples / illustrations of NBAM models.

Finally we wish to state that all neutrosophic combined NRMs can be transformed using 2 adaptive fuzzy models.

For instance if say n experts are working on a problem using NRM model.

We can use the combined NRM,

$$N(E) = \sum_{i=1}^{n} N_i(E)$$

and use this as the NBAM model in the interval

$[-n, n] \cup [-nI, nI]$.

We will illustrate this situation by a simple example.

Example 5.3: Use the combined NRM model given in page 61 studying the employee - employer relationship.

$$N(E) = N(E_1) + N(E_2) + N(E_3)$$

given in page 66-67 of this book.

$$N(E) = \begin{bmatrix} I & 0 & 0 & 2 & 2 \\ 1 & 2I & 2 & 0 & 0 \\ I & 1 & 2 & 0 & 0 \\ 3 & 1 & I & I & I \\ 0 & 1 & 2 & I & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & I & I & 1 \\ 0 & I & 0 & 2 & 2 \end{bmatrix}.$$

We model this matrix N(E) as the NBAM matrix with the interval [–3, 3] ∪ [–3I, 3I].

We will just indicate how the working goes.

Let X_K = (3, 0, –2, 0, 1, –1, 0, 2)

$S(X_K)$ = (1, 0, 0, 0, 1, 0, 0, 1).

We find $S(X_K)$ N(E)

= (I, 1+I 2 4+I 4)

= Y_{K+1},

We now find

$S(Y_{K+1})$ = (I 1 1 1 1)

$S(Y_{K+1})\ N(E)^t$ = (4+I, 3I+2, 3+I, 6I+1, 3+I, 2, 3I+3, 4+I)

= X_{K+2}

$S(X_{K+2})$ = (1, I, 1, I, 1, 1, 1, 1)

$S(X_{K+2})\,N(E) = (6I+1, 4I+3, 4I+5, 4+4I, 6+I)$

$= Y_{K+3}$

$S(Y_{K+3}) = (I, I, 1, 1, 1)$

$S(Y_{K+3})\,N(E)^t = (I+4, 3I+2, 2I+2, 7I, 2I+2, 2, 4I+1, 4+I)$

$= X_{K+4}$

$S(X_{K+4}) = (1, I, 1, I, 1, 1, I, 1)$

We work until we arrive at a fixed pair. Thus using combined NRM we can go to NBAM and vice versa.

Chapter Six

FUZZY NEUTROSOPHIC ASSOCIATIVE MEMORIES MODELS

Now we proceed onto describe the Fuzzy Neutrosophic Associative Memories (FNAM) model.

One is well aware of the Fuzzy Associative Memories (FAM) and the working of it. For the concept of FAM refer [1, 3, 11].

We now proceed to define Neutrosophic Associative Memories (NAM). We note $\langle Z \cup I \rangle$, $\langle Q \cup I \rangle$ and $(\langle R \cup I \rangle)$ denote the ring generated by Z and I or Q and I or R and I where we use the property. I.I = I but I + I = 2I and so sum of I taken n times gives nI.

We define min or max function as follows:

min {x, I} = x where x is real but min {mI, nI} = mI if m < n.

The max function is defined by max {x, I} = I where x is real and max {mI, nI} = nI if m , n. With this definition of max and min in mind we can define the notion of Neutrosophic Associative Memories (NAM).

The simplest NAM encodes the NAM rule or association which associates a p-dimension neutrosophic set A_j (Here by a n-dimensional neutrosophic set we mean the unit neutrosophic hyper cube $[0, 1]^n \cup [0, I]^n$).

These minimal NAMs essentially map one ball in $[0, 1]^n \cup [0, I]^n$. They are comparable to simple neural networks. As FAMs the NAMs need not adaptively be trained. Apart from this the NAM model function on the same rules as the FAM models.

However the fit vector as in case of FAMs are

$B_N = (a_1, \ldots, a_n)$ where

$$a_i = \begin{cases} 0 & \text{if the node is in the off state} \\ 1 & \text{if the node is in the on state} \\ I & \text{if it is an indeterminate.} \end{cases}$$

Here every resultant fit vector $A_N = (a_1, \ldots, a_n)$ can take values in $[0, 1] \cup I$ as in case of the FAMs.

Now we obtain an opinion from the expert which includes the indeterminate values also, so the related matrix of NAM will be called as a neutrosophic vector matrix and will be denoted by N(M).

For $W_1, \ldots, W_7$ and $R_1, R_2, \ldots, R_{10}$ described below we obtain N(M) the neutrosophic vector matrix.

The attributes related with the rural uneducated women are;

W_1 - No basic education

W_2 - No wealth / property

W_3 - No empowerment

W_4 - No recognition of their labour

W_5 - No protection from their spouse

W_6 - No proper health care center to get proper guidance

W_7 - No nutritious food

W_8 - Women are not decision makers for women

W_9 - No voice basically in the traditional set up.

More in rural India the nation gives least importance to the education of women. The literacy of women in India is very low, consequently they are not empowered.

It is pity to say no decision can be made by her. Even in the selection of her life partner or time to get married or her education is not in her hands. Every thing is imposed on her, this is more in rural India.

So many a times very young girls below 10 years are married to men four times their age. So these small girls are sexually exploited and consequently they become victims of HIV/AIDS. Also it is a custom in India food should be served to men, then children; so hardly a poor women in rural India get any thing as nutrition or balanced diet for she eats the remaining food if it remains otherwise starves for Indian women are taught to this sort of life. This is also one of the causes of becoming victims of HIV/AIDS. No stamina so easily infected by their husband.

For a married women has no right to say no to sex to her husband in India be it rural or urban. Thus poor rural women are easily infected by their husbands even if they know that their husbands are infected.

Now we proceed onto give the attributes related with men society and economic status.

R_1 - Male dominated society

R_2 - Traditional and cultural values of the nation

R_3 - Poverty among villagers

R_4 - No proper counseling by doctors to HIV/AIDS infected men

R_5 - Media, TV cinema (includes male ego)

R_6 - Model women

R_7 - No proper spiritual leader to channelize the youth

R_8 - Men has no responsibility or binding with the family.

R_9 - Women in Indian society

R_{10} - Role of women association in the context of these HIV/AIDS affected rural poor women.

Now

$$N(M) = \begin{array}{c} \\ W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \\ W_9 \end{array} \begin{array}{c} \begin{array}{cccccccccc} R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \end{array} \\ \left[\begin{array}{cccccccccc} 0.6 & 0.8 & 0.5 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0.7 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.6 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{array}\right] \end{array}$$

be the neutrosophic vector matrix with entries from [0, 1] $\cup$ I.

Now we study the effect of the fit vector B_N = (0 0 1 0 I 0 0 0 0 0) on N(M).

$$A_N = N(M) \text{ o } B_N.$$

$$= (I, 0.8, 0, 0, 0.9, 0.7, 0, 0, 0) \text{ is the resultant.}$$

The resultant shows R_3 - when poverty in the villages is in the on state, R_5 - influence of media is in an indeterminate state. We get the resultant fit vector.

W_1 - no basic education comes to an indeterminate state, W_5 - takes the highest value 0.9 i.e., No protection from their spouse followed by W_2.

No wealth or property is in their possession with W_6 taking the value 0.7; that is no proper health care center to get proper guidance.

However W_3, W_4, W_7, W_8 and W_9 remain in off state as they take value zero i.e., the fit vector A_N.

Now we find the effect of the fit vector. A_N on N(M) defined by A_N N(M) = max min {(a_1, m_{ij})} = B_N = (0.8, 0.8, 0.9, 0, I, 0, 0, I, 0, 0).

R_5 and R_8 becomes indeterminate i.e., impact of media and men have no responsibility, …, R_4, R_6, R_7, R_9 and R_{10} are only zero for they are not influenced by the fit vector, B_N = (0, 0, 1, 0, I, 0, 0, 0, 0, 0) by its resultant A_N = (I, 0.8, 0, 0, 0.9, 0.7, 0, 0, 0).

R_3 takes the highest value no women empowerment followed by W_1 and W_2 holding the next higher value, no basic education and no wealth / property in women's possession.

We can build NAMs in this manner.

At this juncture we keep on record that we can build 3-adaptive fuzzy model.

For a NRM can be converted into a NAM and a NRE can also be converted into a NAM, that is $C_3 = \{M_1, M_2, M_3\}$ be 3 distinct fuzzy neutrosophic models.

We see C_3 is a new class of 3-adaptive fuzzy neutrosophic model if for any model M_j, we can go to another fuzzy neutrosophic model M_i ($i \neq j$) by some simple transformation function or at time even the identity transformation. (Only operations in models M_i and M_j will be different).

This is true for $1 \leq j \leq 3$. Thus one can go from any M_r to M_s, $r \neq s$; $1 \leq r, s \leq 3$. These three models are such that only operations on them are different.

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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Use of fuzzy neutrosophic models in the study of any social problem or technical problem has become a demand of the day due to the involvement of indeterminacy. For always, one is not in a position to say true or false or assign a fuzzy value to any problem. At times, one is not in a position to commit to anyof the three statements. In that situation, neutrosophic models play a vital role. In this book we introduce several fuzzy neutrosophic models to study social problems.

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