**Scientia Magna**
**Vol. , No.**

# Smarandache Seminormal Subgroupoids

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**Abstract** In this paper, we define Smarandache seminormal subgroupoids. We have proved some results for finding the Smarandache seminormal subgroupoids in $Z(n)$ when $n$ is even and $n$ is odd.

**Keywords** Smarandache groupoids, Smarandache seminormal subgroupoids.

## §1. Introduction and preliminaries

In [5] and [6], W.B.Kandasamy defined new classes of Smarandache groupoids using $Z_n$. In this paper we define and prove some theorems for construction of Smarandache seminormal subgroupoids according as $n$ is even or odd.

**Definition 1.1.** A non-empty set of elements $G$ is said to form a groupoid if in $G$ is defined a binary operation called the product, denoted by $*$ such that $a * b \in G \;\; \forall a, b \in G$. We denote groupoids by $(G, *)$.

**Definition 1.2.** Let $(G, *)$ be a groupoid. A proper subset $H \subset G$ is a subgroupoid if $(H, *)$ is itself a groupoid.

**Definition 1.3.** Let $S$ be a non-empty set. $S$ is said to be a semigroup if on $S$ is defined a binary operation $*$ such that

1. for all $a$, $b \in S$ we have $a * b \in S$.
2. for all $a$, $b$, $c \in S$ we have $a * (b * c) = (a * b) * c$.

$(S, *)$ is a semi-group.

**Definition 1.4.** A Smarandache groupoid $G$ is a groupoid which has a proper subset $S$ such that $S$ under the operation of $G$ is a semigroup.

**Definition 1.5.** Let $(G, *)$ be a Smarandache groupoid. A non-empty subgroupoid $H$ of $G$ is said to be a Smarandache subgroupoid if $H$ contains a proper subset $K$ such that $K$ is a semigroup under the operation $*$.

**Definition 1.6.** Let $G$ be a Smarandache groupoid. $V$ be a Smarandache subgroupoid of $G$. We say $V$ is a Smmarandache seminormal subgroupoid if $aV = V$ for all $a \in G$ or $Va =$

$V$ for all $a \in G$.
e.g. Let $(G, *)$ be groupoid given by the following table:

| $*$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|---|
| $a_0$ | $a_0$ | $a_3$ | $a_0$ | $a_3$ | $a_0$ | $a_3$ |
| $a_1$ | $a_2$ | $a_5$ | $a_2$ | $a_5$ | $a_2$ | $a_5$ |
| $a_2$ | $a_4$ | $a_1$ | $a_4$ | $a_1$ | $a_4$ | $a_1$ |
| $a_3$ | $a_0$ | $a_3$ | $a_0$ | $a_3$ | $a_0$ | $a_3$ |
| $a_4$ | $a_2$ | $a_5$ | $a_2$ | $a_5$ | $a_2$ | $a_5$ |
| $a_5$ | $a_4$ | $a_1$ | $a_4$ | $a_1$ | $a_4$ | $a_1$ |

It is a Smarandache groupoid as $\{a_3\}$ is a semigroup. $V = \{a_1, a_3, a_5\}$ is a Smarandache subgroupoid,also $aV = V$. Therefore $V$ is Smarandache seminormal subgroupoid in $G$.

**Definition 1.7.** Let $Z_n = \{0, 1, \ldots, n-1\}$, $n \geq 3$ and $a, b \in Z_n \backslash \{0\}$. Define a binary operation $*$ on $Z_n$ as follows:
$a * b = ta + ub \pmod n$ where $t, u$ are two distinct elements in $Z_n \backslash \{0\}$ and $(t, u) = 1$. Here $'+'$ is the usual addition of two integers and $'ta'$ means the product of the two integers $t$ and $a$.

Elements of $Z_n$ form a groupiod with respect to the binary operation $*$. We denote these groupiod by $\{Z_n(t, u), *\}$ or $Z_n(t, u)$ for fixed integer n and varying $t, u \in Z_n \backslash \{0\}$ such that $(t, u) = 1$. Thus we define a collection of groupiods $Z(n)$ as follows
$Z(n) = \{\{Z_n(t, u), *\} |$ for integers $t, u \in Z_n \backslash \{0\}$ such that $(t, u) = 1\}$.

## §2. Smarandache seminormal subgroupoids when $n$ is even

When $n$ is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

**Theorem 2.1.** Let $Z_n(t, t+1) \in Z(n)$, n is even, $n > 3$ and $t = 1, \ldots, n-2$. Then $Z_n(t, t+1)$ is Smarandache groupoid.

**Proof.** Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= 2xt + x \\ &= (2t+1)x \equiv x \bmod n \end{aligned}$$

$\therefore \{x\}$ is a semigroup in $Z_n(t, t+1)$.
$\therefore Z_n(t, t+1)$ is a Smarandache groupoid when $n$ is even.

**Remark:** In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t, t+1) \in Z(n)$.

Proof: When $t$ is even

$0 * t + \frac{n}{2} * (t+1) \equiv \frac{n}{2} \bmod n$.
$\frac{n}{2} * t + 0 * (t+1) \equiv 0 \bmod n$.
$\frac{n}{2} * t + \frac{n}{2} * (t+1) \equiv \frac{n}{2} \bmod n$.

$0 * t + 0 * (t + 1) \equiv 0 \bmod n.$

Therefore,$\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t, t + 1)$.

When $t$ is odd

$0 * t + \frac{n}{2} * (t + 1) \equiv 0 \bmod n.$

$\frac{n}{2} * t + 0 * (t + 1) \equiv \frac{n}{2} \bmod n.$

$\frac{n}{2} * t + \frac{n}{2} * (t + 1) \equiv \frac{n}{2} \bmod n.$

$0 * t + 0 * (t + 1) \equiv 0 \bmod n.$

Therefore,$\{0, \frac{n}{2}\}$ is a semigroup in $Z_n(t, t + 1)$.

**Theorem 2.2.** Let $n > 3$ be even and $t = 1, \ldots, n - 2$,

1. If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \ldots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.

2. If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \ldots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid in $Z_n(t, t+1) \in Z(n)$.

**Proof.**

1. Let $\frac{n}{2}$ is even.
   $\Rightarrow \frac{n}{2} \in A_0$
   We will show that $A_0$ is subgroupoid
   . Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t + 1) \\ &= (x_i + x_j)t + x_j \equiv x_k \bmod n \end{aligned}$$

   for some $x_k \in A_0$ as $(x_i + x_j)t + x_j$ is even.
   $\therefore x_i * x_j \in A_0$
   Thus $A_0$ is subgroupoid in $Z_n(t, t + 1)$.
   Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t + 1) \\ &= (2t + 1)x \equiv x \bmod n \end{aligned}$$

   $\therefore \{x\}$ is a semigroup in $A_0$.
   Thus $A_0$ is a subgroupoid in $Z_n(t, t + 1)$.

2. Let $\frac{n}{2}$ is odd.
   $\Rightarrow \frac{n}{2} \in A_1$
   We will show that $A_1$ is subgroupoid.
   Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$\begin{aligned} x_i * x_j &= x_i t + x_j(t + 1) \\ &= (x_i + x_j)t + x_j \equiv x_k \bmod n \end{aligned}$$

   for some $x_k \in A_1$ as $(x_i + x_j)t + x_j$ is odd.
   $\therefore x_i * x_j \in A_1$

Thus $A_1$ is subgroupoid in $Z_n(t, t+1)$.
Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + x(t+1) \\ &= (2t+1)x \equiv x \bmod n \end{aligned}$$

$\therefore \{x\}$ is a semigroup in $A_1$.
Thus $A_1$ is a Smardandache subgroupoid in $Z_n(t, t+1)$.

**Theorem 2.3.** Let $n > 3$ be even and $t = 1, \ldots, n-2$,

1. If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \ldots, n-2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.
2. If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \ldots, n-1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, t+1) \in Z(n)$.

**Proof.** By Theorem 2.1, $Z_n(t, t+1)$ is a Smarandache groupoid.

1. Let $\frac{n}{2}$ is even. Then by Theorem 2.2, $A_0 = \{0, 2, \ldots, n-2\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$.
   Now we show that either $aA_0 = A_0$ or $A_0 a = A_0$ $\forall$ $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$.
   Case 1: $t$ is even.
   Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

   $$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &\equiv a_j \bmod n \end{aligned}$$

   for some $a_j \in A_0$ as $at + a_i(t+1)$ is even.
   $\therefore a * a_i \in A_0$ $\forall$ $a_i \in A_0$.
   $\therefore aA_0 = A_0$.
   Thus, $A_0$ is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.
   Case 2: $t$ is odd.
   Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

   $$\begin{aligned} a_i * a &= a_i t + a(t+1) \\ &\equiv a_j \text{ mod n} \end{aligned}$$

   for some $a_j \in A_0$ as $a_i t + a(t+1)$ is even.
   $\therefore a_i * a \in A_0$ $\forall$ $a_i \in A_0$.
   $\therefore A_0 a = A_0$.
   Thus $A_0$ is a Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

2. Let $\frac{n}{2}$ is odd. Then by Theorem 2.2, $A_1 = \{1, 3, 5, \ldots, n-1\}$ is Smarandache subgroupoid of $Z_n(t, t+1)$.
   Now we show that either $aA_1 = A_1$ or $A_1 a = A_1$ $\forall$ $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$ .

Case 1: $t$ is even.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i(t+1) \\ &= (a + a_i)t + a_i \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_1$ as $(a + a_i)t + a_i$ is odd.

$\therefore a * a_i \in A_1 \ \forall\ a_i \in A_1$.

$\therefore aA_1 = A_1$.

Thus $A_1$ is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

Case 2: $t$ is odd.

Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + a(t+1) \\ &\equiv a_j \text{ mod n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + a(t+1)$ is odd.

$\therefore a_i * a \in A_1 \ \forall\ \ a_i \in A_1$.

$\therefore A_1 a = A_1$.

Thus $A_1$ is Smarandache seminormal subgroupoid in $Z_n(t, t+1)$.

By the above theorem we can determine the Smarandaache seminormal subgroupoid in $Z_n(t, t+1)$ of $Z(n)$ when $n$ is even and $n > 3$.

| **n** | $n/2$ | $t$ | $Z_n(t, t+1)$ | **Smarandache seminormal subgroupoid in** $Z_n(t, t+1)$ |
|---|---|---|---|---|
| 4 | 2 | 1 | $Z_4(1, 2)$ | $\{0, 2\}$ |
| | | 2 | $Z_4(2, 3)$ | |
| 6 | 3 | 1 | $Z_6(1, 2)$ | $\{1, 3, 5\}$ |
| | | 2 | $Z_6(2, 3)$ | |
| | | 3 | $Z_6(3, 4)$ | |
| | | 4 | $Z_6(4, 5)$ | |
| 8 | 4 | 1 | $Z_8(1, 2)$ | $\{0, 2, 4, 6\}$ |
| | | 2 | $Z_8(2, 3)$ | |
| | | 3 | $Z_8(3, 4)$ | |
| | | 4 | $Z_8(4, 5)$ | |
| | | 5 | $Z_8(5, 6)$ | |
| | | 6 | $Z_8(6, 7)$ | |
| 10 | 5 | 1 | $Z_{10}(1, 2)$ | $\{1, 3, 5, 7, 9\}$ |
| | | 2 | $Z_{10}(2, 3)$ | |
| | | 3 | $Z_{10}(3, 4)$ | |
| | | 4 | $Z_{10}(4, 5)$ | |
| | | 5 | $Z_{10}(5, 6)$ | |
| | | 6 | $Z_{10}(6, 7)$ | |
| | | 7 | $Z_{10}(7, 8)$ | |
| | | 8 | $Z_{10}(8, 9)$ | |
| 12 | 6 | 1 | $Z_{12}(1, 2)$ | $\{0, 2, 4, 6, 8\}$ |
| | | 2 | $Z_{12}(2, 3)$ | |
| | | 3 | $Z_{12}(3, 4)$ | |
| | | 4 | $Z_{12}(4, 5)$ | |
| | | 5 | $Z_{12}(5, 6)$ | |
| | | 6 | $Z_{12}(6, 7)$ | |
| | | 7 | $Z_{12}(7, 8)$ | |
| | | 8 | $Z_{12}(8, 9)$ | |
| | | 9 | $Z_{12}(9, 10)$ | |
| | | 10 | $Z_{12}(10, 11)$ | |

## §3. Smarandache seminormal subgroupoids depending on $t$ and $u$ when $n$ is even

When $n$ is even we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$ when $t$ is even and $u$ is odd or when $t$ is odd and $u$ is even.

**Theorem 3.1.** Let $Z_n(t,u) \in Z(n)$,if n is even,$n > 3$ and for each $t, u \in Z_n$,if one is even and other is odd then $Z_n(t,u)$ is Smarandache groupoid.

**Proof.** Let $x = \frac{n}{2}$

Then

$$\begin{aligned} x * x &= xt + xu \\ &= (t+u)x \equiv x \bmod n \end{aligned}$$

$\therefore \{x\}$ is a semigroup in $Z_n(t,u)$.

$\therefore Z_n(t,u)$ is a Smarandache groupoid when $n$ is even.

**Remark:** In the above theorem we can also show that beside $\{n/2\}$ the other semigroup is $\{0, n/2\}$ in $Z_n(t,u) \in Z(n)$.

Proof:

1. When $t$ is even and $u$ is odd,
   $0 * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$.
   $\frac{n}{2} * t + 0 * u \equiv 0 \bmod n$.
   $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$.
   $0 * t + 0 * u \equiv 0 \bmod n$.
   Therefore,$\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t,u)$.

2. When $t$ is odd and $u$ is even,
   $0 * t + \frac{n}{2} * u \equiv 0 \bmod n$.
   $\frac{n}{2} * t + 0 * u \equiv \frac{n}{2} \bmod n$.
   $\frac{n}{2} * t + \frac{n}{2} * u \equiv \frac{n}{2} \bmod n$.
   $0 * t + 0 * u \equiv 0 \bmod n$.
   Therefore,$\{0, \frac{n}{2}\}$ is semigroup in $Z_n(t,u)$.

**Theorem 3.2.** Let $n > 3$ be even and $t, u \in Z_n$

1. If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \ldots, n-2\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t,u) \in Z(n)$ when one of $t$ and $u$ is odd and other is even.

2. If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \ldots, n-1\} \subseteq Z_n$ is Smarandache subgroupoid of $Z_n(t,u) \in Z(n)$ when one of $t$ and $u$ is odd and other is even.

**Proof.**

1. Let $\frac{n}{2}$ is even.
   $\Rightarrow \frac{n}{2} \in A_0$

We will show that $A_0$ is subgroupoid

. Let $x_i, x_j \in A_0$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \bmod n$$

for some $x_k \in A_0$ as $x_i t + x_j u$ is even.

$\therefore x_i * x_j \in A_0$

$\therefore A_0$ is a subgroupoid in $Z_n(t, u)$.

Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t + u) \equiv x \bmod n \end{aligned}$$

$\therefore \{x\}$ is a semigroup in $A_0$.

Thus, $A_0$ is a Smarandache subgroupoid in $Z_n(t, u)$

2. Let $\frac{n}{2}$ is odd.

   $\Rightarrow \frac{n}{2} \in A_1$

   We will show that $A_1$ is subgroupoid.

   Let $x_i, x_j \in A_1$ and $x_i \neq x_j$. Then

$$x_i * x_j = x_i t + x_j u \equiv x_k \bmod n$$

   for some $x_k \in A_1$ as $x_i + x_j u$ is odd.

   $\therefore x_i * x_j \in A_1$

   $\therefore A_1$ is subgroupoid in $Z_n(t, u)$.

   Let $x = \frac{n}{2}$. Then

$$\begin{aligned} x * x &= xt + xu \\ &= x(t + u) \equiv x \bmod n \end{aligned}$$

   $\therefore \{x\}$ is a semigroup in $A_1$.

   Thus $A_1$ is a Smarandache subgroupoid in $Z_n(t, u)$.

**Theorem 3.3.** Let $n > 3$ be even and $t = 1, \ldots, n - 2$.

1. If $\frac{n}{2}$ is even then $A_0 = \{0, 2, \ldots, n - 2\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, u) \in Z(n)$ when one of $t$ and $u$ is odd and other is even.

2. If $\frac{n}{2}$ is odd then $A_1 = \{1, 3, \ldots, n - 1\} \subseteq Z_n$ is Smarandache seminormal subgroupoid of $Z_n(t, u) \in Z(n)$ when one of $t$ and $u$ is odd and other is even.

**Proof.** By Theorem 3.1, $Z_n(t, u)$ is a Smarandache groupoid.

1. Let $\frac{n}{2}$ is even. Then by Theorem 3.2, $A_0 = \{0, 2, \ldots, n - 2\}$ is Smarandache subgroupoid of $Z_n(t, u)$.

   Now we show that either $aA_0 = A_0$ or $A_0 a = A_0$ $\forall$ $a \in Z_n = \{0, 1, 2, \ldots, n - 1\}$.

Case 1: $t$ is even and $u$ is odd.
Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_0$ as $at + a_i u$ is even.
$\therefore a * a_i \in A_0 \ \forall \ a_i \in A_0$.
$\therefore aA_0 = A_0$.
Thus, $A_0$ is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.
Case 2: $t$ is odd and $u$ is even.
Let $a_i \in A_0$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \text{ mod n} \end{aligned}$$

for some $a_j \in A_0$ as $a_i t + au$ is even.
$\therefore a_i * a \in A_0 \ \forall \ a_i \in A_0$.
$\therefore A_0 a = A_0$.
Thus, $A_0$ is Smarandache seminormal subgroupoid in $Z_n(t, u)$.

2. Let $\frac{n}{2}$ is odd then by Theorem is $A_1 = \{1, 3, 5, \ldots, n-1\}$ is Smarandache subgroupoid of $Z_n(t, u)$.
Now we show that either $aA_1 = A_1$ or $A_1 a = A_1 \ \forall \ a \in Z_n = \{0, 1, 2, \ldots, n-1\}$ .
Case 1: $t$ is even and $u$ is odd.
Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} a * a_i &= at + a_i u \\ &\equiv a_j \bmod n \end{aligned}$$

for some $a_j \in A_1$ as $at + a_i u$ is odd.
$\therefore a * a_i \in A_1 \ \forall \ a_i \in A_1$.
$\therefore aA_1 = A_1$.
Thus, $A_1$ is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.
Case 2: $t$ is odd and $u$ is even.
Let $a_i \in A_1$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$

$$\begin{aligned} a_i * a &= a_i t + au \\ &\equiv a_j \text{ mod n} \end{aligned}$$

for some $a_j \in A_1$ as $a_i t + au$ is odd.
$\therefore a_i * a \in A_1 \ \forall \ a_i \in A_1$.
$\therefore A_1 a = A_1$.
Thus, $A_1$ is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

By the above theorem we can determine Smarandaache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$ for $n > 3$, when $n$ is even and when one of $t$ and $u$ is odd and other is even.

| **n** | $n/2$ | $t$ | $Z_n(t,u)$ | **Smarandache seminormal subgroupoid in** $Z_n(t,u)$ |
|---|---|---|---|---|
| 4 | 2 | 1 | $Z_4(1,2)$ | $\{0,2\}$ |
| | | 2 | $Z_4(2,3)$ | |
| 6 | 3 | 1 | $Z_6(1,2), Z_6(1,4)$ | $\{1,3,5\}$ |
| | | 2 | $Z_6(2,1), Z_6(2,3), Z_6(2,5)$ | |
| | | 3 | $Z_6(3,2), Z_6(3,4)$ | |
| | | 4 | $Z_6(4,1), Z_6(4,3), Z_6(4,5)$ | |
| | | 5 | $Z_6(5,2), Z_6(5,4)$ | |
| 8 | 4 | 1 | $Z_8(1,2), Z_8(1,4), Z_8(1,6)$ | $\{0,2,4,6\}$ |
| | | 2 | $Z_8(2,1), Z_8(2,3), Z_8(2,5), Z_8(2,7)$ | |
| | | 3 | $Z_8(3,2), Z_8(3,4)$ | |
| | | 4 | $Z_8(4,1), Z_8(4,3), Z_8(4,5), Z_8(4,7)$ | |
| | | 5 | $Z_8(5,2), Z_8(5,4), Z_8(5,6)$ | |
| | | 6 | $Z_8(6,1), Z_8(6,5), Z_8(6,7),$ | |
| | | 7 | $Z_8(7,2), Z_8(7,4), Z_8(7,6),$ | |
| 10 | 5 | 1 | $Z_{10}(1,2), Z_{10}(1,4), Z_{10}(1,6), Z_{10}(1,8)$ | $\{1,3,5,7,9\}$ |
| | | 2 | $Z_{10}(2,1), Z_{10}(2,3), Z_{10}(2,5), Z_{10}(2,7), Z_{10}(2,9)$ | |
| | | 3 | $Z_{10}(3,2), Z_{10}(3,4), Z_{10}(3,8),$ | |
| | | 4 | $Z_{10}(4,1), Z_{10}(4,3), Z_{10}(4,5), Z_{10}(4,7), Z_{10}(4,9)$ | |
| | | 5 | $Z_{10}(5,2), Z_{10}(5,4), Z_{10}(5,6), Z_{10}(5,8)$ | |
| | | 6 | $Z_{10}(6,1), Z_{10}(6,5), Z_{10}(6,7),$ | |
| | | 7 | $Z_{10}(7,2), Z_{10}(7,4), Z_{10}(7,6), Z_{10}(7,8)$ | |
| | | 8 | $Z_{10}(8,1), Z_{10}(8,3), Z_{10}(8,5), Z_{10}(8,7), Z_{10}(8,9)$ | |
| | | 9 | $Z_{10}(9,2), Z_{10}(9,4), Z_{10}(9,8)$ | |

| **n** | ***n*/2** | ***t*** | $\boldsymbol{Z_n(t,u)}$ | **Smarandache seminormal subgroupoid in** $Z_n(t,u)$ |
|---|---|---|---|---|
| 12 | 6 | 1 | $Z_{12}(1,2), Z_{12}(1,4), Z_{12}(1,6), Z_{12}(1,8), Z_{12}(1,10)$ | $\{0,2,4,6,8,10\}$ |
| | | 2 | $Z_{12}(2,1), Z_{12}(2,3), Z_{12}(2,5), Z_{12}(2,7), Z_{12}(2,9), Z_{12}(2,11)$ | |
| | | 3 | $Z_{12}(3,2), Z_{12}(3,4), Z_{12}(3,8), Z_{12}(3,10)$ | |
| | | 4 | $Z_{12}(4,1), Z_{12}(4,3), Z_{12}(4,5), Z_{12}(4,7), Z_{12}(4,9), Z_{12}(4,11)$ | |
| | | 5 | $Z_{12}(5,2), Z_{12}(5,4), Z_{12}(5,6), Z_{12}(5,8)$ | |
| | | 6 | $Z_{12}(6,1), Z_{12}(6,3), Z_{12}(6,5), Z_{12}(6,7), Z_{12}(6,11)$ | |
| | | 7 | $Z_{12}(7,2), Z_{12}(7,4), Z_{12}(7,6), Z_{12}(7,8), Z_{12}(7,10)$ | |
| | | 8 | $Z_{12}(8,1), Z_{12}(8,3), Z_{12}(8,5), Z_{12}(8,7), Z_{12}(8,9), Z_{12}(8,11)$ | |
| | | 9 | $Z_{12}(9,2), Z_{12}(9,4), Z_{12}(9,8), Z_{12}(9,10)$ | |
| | | 10 | $Z_{12}(10,1), Z_{12}(10,3), Z_{12}(10,7), Z_{12}(10,9), Z_{12}(10,11)$ | |
| | | 11 | $Z_{12}(11,2), Z_{12}(11,4), Z_{12}(11,6), Z_{12}(11,8), Z_{12}(11,10)$ | |

## §4. Smarandache seminormal subgroupoids when $n$ is odd

When $n$ is odd we are interested in finding Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$.We have proved the similiar result in [4].

**Theorem 4.1.** Let $Z_n(t,u) \in Z(n)$. If $n$ is odd, $n > 4$ and for each $t = 2, \ldots, \frac{n-1}{2}$ and $u = n - (t-1)(t,u) = 1$, then $Z_n(t,u)$ is a Smarandache groupoid.

**Proof.** Let $x \in \{0, \ldots, n-1\}$. Then

$$x * x = xt + xu = (n+1)x \equiv x \bmod n.$$

$\therefore \{x\}$ is semigroup in $Z_n$.

$\therefore Z_n(t,u)$ is a Smarandanche groupoid in $Z(n)$.

**Remark:** We note that all $\{x\}$ where $x \in \{1, \ldots, n-1\}$ are proper subsets which are semigroups in $Z_n(t,u)$.

**Theorem 4.2.** Let $n > 4$ be odd and $t = 2, \ldots, \frac{n-1}{2}$ and $u = n-(t-1)$ such that $(t,u) = 1$ if $s = (n,t)$ or $s = (n,u)$ then $A_k = \{k, k+s, \ldots, k+(r-1)s\}$ for $k = 0, 1, \ldots, s-1$ where $r = \frac{n}{s}$ is a Smarandache subgroupoid in $Z_n(t,u) \in Z(n)$.

**Proof.** Let $x_p, x_q \in A_k$. Then

$$x_p \neq x_q \Rightarrow \left.\begin{array}{l} x_p = k+ps \\ x_q = k+qs \end{array}\right\} p,\ q \in \{0,1,\ldots,r-1\}.$$

Also,

$$\begin{aligned} x_p * x_q &= x_p t + x_q u \\ &= (k+ps)t + (k+qs)(n-(t-1)) \\ &= k(n+1) + ((p-q)t + q(n+1))s \\ &\equiv (k+ls) \bmod n \\ &\equiv x_l \bmod n \end{aligned}$$

$x_l \in A_k$ as $x_l = k+ls$ for some $l \in \{0,1,\ldots,r-1\}$.
$\therefore x_p * x_q \in A_k$
$\therefore A_k$ is a subgroupoid in $Z_n(t,u)$.
By the above remark all singleton sets are semigroup.
Thus,$A_k$ is a Smarandache subgroupoid.

**Theorem 4.3.** Let $n > 4$ be odd and $t = 2, \ldots, \frac{n-1}{2}$ and $u = n-(t-1)$ such that $(t,u) = 1$ if $s = (n,t)$ or $s = (n,u)$ then $A_k = \{k, k+s, \ldots, k+(r-1)s\}$ for $k = 0, 1, \ldots, s-1$ where $r = \frac{n}{s}$ is a Smarandache seminormal subgroupoid in $Z_n(t,u) \in Z(n)$.

**Proof.** By Theorem 4.1, $Z_n(t,u)$ is a Smarandache groupoid Also by Theorem 4.2, $A_k = \{k, k+s, \ldots, k+(r-1)s\}$ for $k = 0, 1, \ldots, s-1$ is Smarandache subgroupoid of $Z_n(t,u)$.

1. If $s = (n,t)$
   Let $x_p \in A_k$ and $a \in Z_n = \{0,1,2,\ldots,n-1\}$. Then

$$\begin{aligned} a * x_p &= at + x_p u \\ &= at + (k+ps)(n-t+1) \\ &= k(n+1) + [(a-k)v_1 + (pn-pt+p)]s \text{ where } t = v_1 s \\ &\equiv k + ls \bmod n \end{aligned}$$

   $x_l \in A_k$ as $x_l = k+ls$ for some $l \in \{0,1,\ldots,r-1\}$
   $\therefore a * x_p \in A_k$
   $\therefore a * A_k = A_k$
   Thus,$A_k$ is a Smarandache seminormal subgroupoid in $Z_n(t,u)$.

2. If $s = (n, u)$
Let $x_p \in A_k$ and $a \in Z_n = \{0, 1, 2, \ldots, n-1\}$. Then

$$\begin{aligned} x_p * a &= x_p t + au \\ &= (k + ps)(n - u + 1) + au \\ &= k(n+1) + [(a-k)v_2 + (pn - pu + p)]s \text{ where } t = v_2 s \\ &\equiv (k + ls) \bmod n \end{aligned}$$

$x_l \in A_k$ as $x_l = k + ls$ for some $l \in \{0, 1, \ldots, r-1\}$.
$\therefore a * x_p \in A_k$
$\therefore a * A_k = A_k$
Thus $A_k$ is a Smarandache seminormal subgroupoid in $Z_n(t, u)$.

By the above theorem we can determine Smarandache seminormal subgroupoid in $Z_n(t, u)$ when $n$ is odd and $n > 4$.

| **n** | $t$ | $u$ | $Z_n(t,u)$ | $s=(n,u)$ or $s=(n,t)$ | $r=n/s$ | **Smarandache seminormal subgroupoid in $Z_n(t,u)$** |
|---|---|---|---|---|---|---|
| 9 | 3 | 7 | $Z_9(3,7)$ | 3 = (9,3) | 3 | $A_0=\{0,3,6\}$ |
| | | | | | | $A_1=\{1,4,7\}$ |
| | | | | | | $A_2=\{2,5,8\}$ |
| 15 | 3 | 13 | $Z_{15}(3,13)$ | 3 = (15,3) | 5 | $A_0=\{0,3,6,9,12\}$ |
| | | | | | | $A_1=\{1,4,7,10,13\}$ |
| | | | | | | $A_2=\{2,5,8,11,14\}$ |
| | 5 | 11 | $Z_{15}(5,11)$ | $5=(15,5)$ | 3 | $A_0=\{0,5,10\}$ |
| | | | | | | $A_1=\{1,6,11\}$ |
| | | | | | | $A_2=\{2,7,12\}$ |
| | | | | | | $A_3=\{3,8,13\}$ |
| | | | | | | $A_4=\{4,9,14\}$ |
| | 7 | 9 | $Z_{15}(7,9)$ | 3 = (15,9) | 5 | $A_0=\{0,3,6,9,12\}$ |
| | | | | | | $A_1=\{1,4,7,10,13\}$ |
| | | | | | | $A_2=\{2,5,8,11,14\}$ |
| 21 | 3 | 19 | $Z_{21}(3,19)$ | 3 = (21,3) | 7 | $A_0=\{0,3,6,9,12,15,18\}$ |
| | | | | | | $A_1=\{1,4,7,10,13,16,19\}$ |
| | | | | | | $A_2=\{2,5,8,11,14,17,20\}$ |
| | 7 | 15 | $Z_{21}(7,15)$ | 7 = (21,7) | 3 | $A_0=\{0,7,14\}$ |
| | | | | | | $A_1=\{1,8,15\}$ |
| | | | | | | $A_2=\{2,9,16\}$ |
| | | | | | | $A_3=\{3,10,17\}$ |
| | | | | | | $A_4=\{4,11,18\}$ |
| | | | | | | $A_5=\{5,12,19\}$ |
| | | | | | | $A_6=\{6,13,14\}$ |
| | | | | 3 = (21,15) | 7 | $A_0=\{0,3,6,9,12,15,18\}$ |
| | | | | | | $A_1=\{1,4,7,10,13,16,19\}$ |
| | | | | | | $A_2=\{2,5,8,11,14,17,20\}$ |
| | 9 | 13 | $Z_{21}(9,13)$ | 3 = (21,9) | 7 | $A_0=\{0,3,6,9,12,15,18\}$ |
| | | | | | | $A_1=\{1,4,7,10,13,16,19\}$ |
| | | | | | | $A_2=\{2,5,8,11,14,17,20\}$ |

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