The Simplest Proof of Fermat Last Theorem(1)

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Abstract

In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $4P$ and P, where P is an odd prime. We rediscover the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

Theorem:The simplest proof of Fermat last theorem.We have Fermat equation $x^{4P} - y^{4P} = z^{4P}$, where P is odd prime. We prove that if y and z are integer numbers then x, x^4 and x^P are irrational numbers.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \qquad (1)$$

where J denotes a $4m$ th root of unity, $J^{4m} = 1$, m=1,2,3,…, t_i are the real numbers.

S_i is called the automorphic functions(complex hyperbolic functions) of order $4m$ with $4m-1$ variables [2,5,7].

$$S_i = \frac{1}{4m}\left[e^{A_1} + 2e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + 2\sum_{j=1}^{m-1} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{2m}\right)\right]$$

$$+\frac{(-1)^{(i-1)}}{4m}\left[e^{A_2}+2\sum_{j=1}^{m-1} e^{D_j}\cos\left(\phi_j-\frac{(i-1)j\pi}{2m}\right)\right] \quad (2)$$

where $i=1,...,4m$;

$$A_1=\sum_{\alpha=1}^{4m-1} t_\alpha, \quad A_2=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha, \qquad H=\sum_{\alpha=1}^{2m-1} t_{2\alpha}(-1)^\alpha, \quad \beta=\sum_{\alpha=1}^{2m} t_{2\alpha-1}(-1)^\alpha,$$

$$B_j=\sum_{\alpha=1}^{4m-1} t_\alpha\cos\frac{\alpha j\pi}{2m}, \quad \theta_j=-\sum_{\alpha=1}^{4m-1} t_\alpha\sin\frac{\alpha j\pi}{2m},$$

$$D_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\cos\frac{\alpha j\pi}{2m}, \quad \phi_j=\sum_{\alpha=1}^{4m-1} t_\alpha(-1)^\alpha\sin\frac{\alpha j\pi}{2m},$$

$$A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)=0. \quad (3)$$

From (2) we have its inverse transformation[5,7]

$$e^{A_1}=\sum_{i=1}^{4m} S_i, \quad e^{A_2}=\sum_{i=1}^{4m} S_i(-1)^{1+i}$$

$$e^H\cos\beta=\sum_{i=1}^{2m} S_{2i-1}(-1)^{1+i}, \quad e^H\sin\beta=\sum_{i=1}^{2m} S_{2i}(-1)^i,$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}\cos\frac{ij\pi}{2m}, \quad e^{B_j}\sin\theta_j=-\sum_{i=1}^{4m-1} S_{1+i}\sin\frac{ij\pi}{2m},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\cos\frac{ij\pi}{2m}, \quad e^{D_j}\sin\phi_j=\sum_{i=1}^{4m-1} S_{1+i}(-1)^i\sin\frac{ij\pi}{2m}. \quad (4)$$

(3) and (4) have the same form.

From (3) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=1 \quad (5)$$

From (4) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix}$$

$$=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1} \end{vmatrix} \qquad (6)$$

where

$$(S_i)_j = \frac{\partial S_i}{\partial t_j} \text{[7]}$$

From (5) and (6) we have circulant determinant

$$\exp\left[A_1 + A_2 + 2H + 2\sum_{j=1}^{m-1} (B_j + D_j)\right] = \begin{vmatrix} S_1 & S_{4m} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{4m} & S_{4m-1} & \cdots & S_1 \end{vmatrix} = 1 \qquad (7)$$

Assume $S_1 \neq 0, S_2 \neq 0, S_i = 0$, where $i = 3,...,4m.$ $S_i = 0$ are $(4m-2)$ indeterminate equations with $(4m-1)$ variables. From (4) we have

$$e^{A_1} = S_1 + S_2, \quad e^{A_2} = S_1 - S_2, \quad e^{2H} = S_1^2 + S_2^2$$

$$e^{2B_j} = S_1^2 + S_2^2 + 2S_1 S_2 \cos\frac{j\pi}{2m}, \quad e^{2D_j} = S_1^2 + S_2^2 - 2S_1 S_2 \cos\frac{j\pi}{2m} \qquad (8)$$

Example [2]. Let $4m = 12$. From (3) we have

$$A_1 = (t_1 + t_{11}) + (t_2 + t_{10}) + (t_3 + t_9) + (t_4 + t_8) + (t_5 + t_7) + t_6,$$

$$A_2 = -(t_1 + t_{11}) + (t_2 + t_{10}) - (t_3 + t_9) + (t_4 + t_8) - (t_5 + t_7) + t_6,$$

$$H = -(t_2 + t_{10}) + (t_4 + t_8) - t_6,$$

$$B_1 = (t_1 + t_{11})\cos\frac{\pi}{6} + (t_2 + t_{10})\cos\frac{2\pi}{6} + (t_3 + t_9)\cos\frac{3\pi}{6} + (t_4 + t_8)\cos\frac{4\pi}{6} + (t_5 + t_7)\cos\frac{5\pi}{6} - t_6,$$

$$B_2 = (t_1 + t_{11})\cos\frac{2\pi}{6} + (t_2 + t_{10})\cos\frac{4\pi}{6} + (t_3 + t_9)\cos\frac{6\pi}{6} + (t_4 + t_8)\cos\frac{8\pi}{6} + (t_5 + t_7)\cos\frac{10\pi}{6} + t_6,$$

$$D_1 = -(t_1 + t_{11})\cos\frac{\pi}{6} + (t_2 + t_{10})\cos\frac{2\pi}{6} - (t_3 + t_9)\cos\frac{3\pi}{6} + (t_4 + t_8)\cos\frac{4\pi}{6} - (t_5 + t_7)\cos\frac{5\pi}{6} - t_6,$$

$$D_2 = -(t_1 + t_{11})\cos\frac{2\pi}{6} + (t_2 + t_{10})\cos\frac{4\pi}{6} - (t_3 + t_9)\cos\frac{6\pi}{6} + (t_4 + t_8)\cos\frac{8\pi}{6} - (t_5 + t_7)\cos\frac{10\pi}{6} + t_6,$$

$$A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2) = 0, \quad A_2 + 2B_2 = 3(-t_3 + t_6 - t_9). \qquad (9)$$

From (8) and (9) we have

$$\exp[A_1 + A_2 + 2(H + B_1 + B_2 + D_1 + D_2)] = S_1^{12} - S_2^{12} = (S_1^3)^4 - (S_2^3)^4 = 1. \qquad (10)$$

From (9) we have

$$\exp(A_2 + 2B_2) = [\exp(-t_3 + t_6 - t_9)]^3 . \quad (11)$$

From (8) we have

$$\exp(A_2 + 2B_2) = (S_1 - S_2)(S_1^2 + S_2^2 + S_1 S_2) = S_1^3 - S_2^3 . \quad (12)$$

From (11) and (12) we have Fermat's equation

$$\exp(A_2 + 2B_2) = S_1^3 - S_2^3 = [\exp(-t_3 + t_6 - t_9)]^3 . \quad (13)$$

Fermat proved that (10) has no rational solutions for exponent 4 [8].

Therefore we prove we prove that (13) has no rational solutions for exponent 3. [2]

Theorem 1 . Let $4m = 4P$, where P is an odd prime, $(P-1)/2$ is an even number.

From (3) and (8) we have

$$\exp[A_1 + A_2 + 2H + 2\sum_{j=1}^{P-1} (B_j + D_j)] = S_1^{4P} - S_2^{4P} = (S_1^P)^4 - (S_2^P)^4 = 1 . \quad (14)$$

From (3) we have

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = [\exp(-t_P + t_{2P} - t_{3P})]^P . \quad (15)$$

From (8) we have

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P . \quad (16)$$

From (15) and (16) we have Fermat's equation

$$\exp[A_2 + 2\sum_{j=1}^{\frac{P-1}{4}} (B_{4j-2} + D_{4j})] = S_1^P - S_2^P = [\exp(-t_P + t_{2P} - t_{3P})]^P . \quad (17)$$

Fermat proved that (14) has no rational solutions for exponent 4 [8]. Therefor we prove that (17) has no rational solutions for prime exponent P .

Theorem 2.The simplest proof of Fermat last theorem.

We have the Fermat equation

$$x^{4P} - y^{4P} = z^{4P} \quad (18)$$

where P is the odd prime.

We rewrite (18)

$$(x^P)^4 - (y^P)^4 = (z^P)^4 . \quad (19)$$

Fermat proved that (19) has no integer solutions for exponent 4 . We assume that y and z are integer numbers, y^P and z^P also are integer numbers. We have x and x^P are irrational numbers. Therefore we prove (18) has no integer solutions

We rewrite (18)

$$(x^4)^P - (y^4)^P = (z^4)^P \qquad (20)$$

We assume that y and z are integer numbers, y^4 and z^4 also are integer numbers. From theorem 1 we prove x^4 is irrational number, x also is irrational number. Therefore we prove (20) has no integer solutions for prime exponent *P*.

Conclusion.In(18) we prove that if y and z are integer numbers then x, x^4 and x^P are irrational numbers. .

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