THE ENERGY OF THE GRAVITATIONAL FIELD FOR THE SCHWARZSCHILD METRIC

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Abstract

This paper is the application of [1] to the Schwarzschild metric. And it gives the energy of the Earth gravitational field.

Content

1). The vector of energy-momentum for the gravitational field.

Let us take from [1] (4):

$$\theta_n{}^k = -L\cdot\delta_n{}^k - \left[\frac{\partial L}{\partial g_{\mu\nu,k}} - \partial_{\mathbf{l}}\left(\frac{\partial L}{\partial g_{\mu\nu,k\mathbf{l}}}\right)\right]\cdot g_{\mu\nu,n} -$$

$$-\frac{\partial L}{\partial g_{\mu\nu,\mathbf{l}k}}\cdot g_{\mu\nu,n\mathbf{l}} \qquad (1.1)$$

and from [1] (5):

$$j_n = n_k\cdot X^k{}_n = \tfrac{1}{2}\cdot g^{m\nu}\cdot b_{m\nu,k}\cdot\delta^k{}_n = \tfrac{1}{2}\cdot g^{m\nu}\cdot b_{m\nu,\,n} \qquad (1.2)$$

Let us take the Lagrangian in this form:

$$L = \eta\cdot R\cdot\sqrt{-g} \qquad \eta = \frac{c^3}{16\cdot\pi\cdot\gamma} \qquad R = R^\rho{}_{\nu\rho\lambda}\cdot g^{\lambda\nu}$$

here $R^\rho{}_{\nu\mu\lambda}$ - the Riemann tensor of curvature.
From [1] (3) we have:

$$P_n = \int (\theta^1{}_n - \int dx^1 \cdot \tfrac{1}{2} \cdot g^{m\nu} \cdot b_{m\nu,\,n}) \cdot dx^2 \cdot dx^3 \cdot dx^4 \qquad (1.3)$$

2). The application of item 1 to the Schwarzschild metric.

The components of this metric tensor which are not equal zero are:

$$g_{11} = 1 - \frac{2 \cdot m}{r} \qquad g_{22} = -\frac{1}{1 - \frac{2 \cdot m}{r}} \qquad g_{33} = -r^2 \qquad g_{44} = -r^2 \cdot \sin^2\theta$$

This tensor is symmetric and so $b_{m\nu} = 0 \quad (2.1)$.
Formula (1.1) became such (at κ = 1 and n = 1) :

$$\theta_1{}^1 = -\eta \cdot R \cdot \sqrt{-g} \cdot \delta_1{}^1$$

The energy of the gravitational field is:

$$E = c \cdot P_1 = c \cdot \int \theta^1{}_1 \cdot dx^2 \cdot dx^3 \cdot dx^4$$

The curvature of this space is this:

$$R = -\frac{2 \cdot m^2}{r^4 \cdot (1 - \frac{2 \cdot m}{r})}$$

And for the energy we get:

$$E = c \cdot \eta \cdot \int_{R_1}^{R_2} dr \cdot \int_0^{\pi} d\theta \cdot \int_0^{2\pi} d\varphi \cdot r^2 \cdot \sin\theta \cdot \frac{2 \cdot m^2}{r^4 \cdot (1 - \frac{2 \cdot m}{r})} =$$

$$= 4 \cdot \pi \cdot m \cdot c \cdot \eta \cdot \ln \left| \frac{(1 - \frac{2 \cdot m}{R_2})}{(1 - \frac{2 \cdot m}{R_1})} \right|$$

3). The energy of the Earth gravitational field.

For the Earth m = 0.45 см, $R_1 = 6380\ km,\ \ R_2 = \infty$; $\eta \approx 0.8 \cdot 10^{37}\ \frac{g}{\sec}$

$E = 1.9 \cdot 10^{40}\ erg \approx m_{equ} \cdot c^2 \qquad m_{equ} = 2.1 \cdot 10^{19}\ g = 3.5 \cdot 10^{-9} \cdot M_{Earth}$

That means that the equivalent mass of the Earth gravitational field is approximately in billion times less than the Earth mass.

Literature :

1 Energy – momentum Vectors for matter and gravitational field.
http://viXra.org/abs/1304.0130