How Massive Gravitons (and Gravitinos) may Affect and Modify The Fundamental Singularity Theorem
(Irrotational Geodestic Singularities from the Raychaudhuri Equation)

Andrew Beckwith
Chongqing University Department of Physics;
E-mail: abeckwith@uh.edu
Chongqing, PRC, 400044

The author presents the consequences of what if Gravitinos in the Electroweak era, all $10^{8}-10^{12}$ of them have an (almost) invariant energy from the beginning of cosmology. This in turn may lead to massive Gravitons. This invariant energy constitutes an initial energy value at the start of the universe which can be used to obtain, at the onset of inflation Kauffman's lower bound to a non zero initial radius of the universe. We reference Theorem 6.1.2 of the book by Ellis, Maartens, and MacCallum in order to argue that if there is a non zero initial scale factor, that there is a partial breakdown of the Fundamental Singularity theorem which is due to the Raychaudhuri equation. The arguments for why the scale factor may not go to zero are reviewed, with suggestions as to linkage as to how to avoid the Anthropic principle in having continuity of $\hbar$ from a prior to the present universe. We also look at how different models of contributing vacuum energy, initially may affect divergence from the first singularity theorem and its results. This document implicitly uses Salvoy's (1983) results.

Key words: 'heavy' Graviton, SUSY, Raychaudhuri equation, Fundamental Singularity theorem

1. Introduction

We wish to investigate if a nonzero graviton mass would lead to conditions which could lead to a modification of the first singularity theorem, 6.12 of the recent book by Ellis, Maartens, and MacCallum. Note that as given in a prior paper, by the author (Beckwith), **Appendix A** summarizes why we think gravitons should be massive, i.e. having a small rest mass, and this is in turn linked to a paper by Steven Kauffmann as to presumably how a nonzero initial energy could lead to a non zero lower bound non singular initial radius of the universe. The present document is to determine what may contribute to a nonzero initial radius, i.e. not just an initial nonzero energy value, as Kauffman's paper would imply, and how different models of contributing vacuum energy, initially may affect divergence from the first singularity theorem.

2. Looking at the First Singularity theorem and how it could fail

We will look at what is given by Ellis, Maartens, and MacCallum. (2012) as to how to state the fundamental singularity theorem

Theorem 6.1(Irrotational Geodestic singularities) *If* $\Lambda \le 0$, $\rho + 3p \ge 0$, *and* $\rho + p > 0$ *in a fluid flow for which* $\dot{u} = 0$, $\omega = 0$ *and* $H_0 > 0$ *at some time* s_0, *then a spacetime singularity , where either* $\ell(\tau) \to 0$ *or* $\sigma \to \infty$, *occurs at a finite proper time* $\tau_0 \le H_0$ *before* s_0.

As was brought up by Beckwith, (2013a), if there is a non zero initial energy for the universe, a supposition which is counter to ADM theory as seen in Kolb and Turner (1991), then the supposition by Kauffman (2012) is supportable with evidence. I.e. then if there is a non zero initial energy, is this in any way counter to Theorem 6.1 above? We will review this question, keeping in mind that. $\ell(\tau) \to 0$ is in reference to a scale factor, as written by Ellis, Maartens, and MacCallum. (2012), vanishing. What we will be doing will be considering what if $\ell(\tau) \ne 0$, and then see if it is still possible for $\sigma \to \infty$.

I.e. the arguments as to if $\sigma \to \infty$*, which we claim will not happen, if* $\ell(\tau) \neq 0$ will then be tied into a review as to what may be setting the Planck's constant to be what it is in early times, reviewing the recent U. V. S. Seshavatharam, S. Lakshminarayana paper as to if the Planck constant is actually variable. To do so we will first of all give arguments as to why, first, the fine structure constant, α would be likely invariant since the beginning, due to a ratio behavior of M for mass within a Hubble radius, and T for space-time temperature values, which would have profound implications for the fine structure constant, α, which in turn would go to the heart of determining if Planck's constant as defined below could have a constant value. Planck's constant being invariant would in turn say some startling things about if Kauffman's conjecture of being able to avoid having a scale factor $\ell(\tau) \to 0$, *i.e. that instead we would be looking at* $\ell(\tau) \neq 0$, and also avoiding $\sigma \to \infty$. *The tie in with Gravitinos, and the details of such can be observed in Beckwith(2013b)*

3. Looking at how to form the Planck constant and also the fine structure constant. Leading to $\ell(\tau) \neq 0$ for all scale factors.

We argue that inputs into the early Planck expression argue in favor of a constant Planck constant, and that it remains constant due to inputs into the fine structure constant given by U. V. S. Seshavatharam, S. Lakshminarayana as, if characteristic cosmic Hubble mass can be expressed as as given by the expression given below with $H_0 \cong 70.4^{+1.3}_{-1.4}$ Km/sec/Mpc which will lead to, if we look at present values, of $M_0 \cong 8.848 \times 10^{32}$ g, when $M_0 \cong \rho_c V_0 \cong \left(c^3 / 2GH_0\right)$, and $\rho_c \cong 3H_0^2 / 8\pi G$ as a present critical density , with $R_0 \cong \left(c/H_0\right)$ as a defined Hubble radius, and $V_0 \cong (4\pi/3) \cdot R_0^3$ as a defined volume of space of analysis, leading to if, the fine structure α remains invariant

$$\ln\left(\sqrt{\frac{aT_0^4}{\rho_c \cdot c^2}} \cdot \sqrt{\frac{4\pi\varepsilon_o G \cdot M_0^2}{e^2}}\right) \cong \frac{1}{\alpha} \qquad (1)$$

If the above expression for the fine structure constant is fixed from the beginning, this means than then there could be very interesting inter relationships between temperature T and mass M, which will be going to what could happen if Kauffman (2012) is correct. To do so, we should also look at another part of the V. S. Seshavatharam, S. Lakshminarayana article which has , a relationship between the find structure constant, α as given by Eq. (2) below, where m_P, m_e are for proton and electron mass.

$$\alpha \cong \sqrt{\frac{m_e}{M_0}} \frac{e^2}{G \cdot \left(n \cdot m_p\right) \cdot m_e} \qquad (2)$$

The objection to using m_P, m_e for proton and electron mass values is that before the Electroweak era, that there were no protons and electrons. Needless to say, if α remains invariant, in line with Paul Langacker, Gino Segre and Matthew J. Strassler's statements that variations of the fine structure constant may not provide meaningful limits, then it probably is safe to speak of an interactive ration between background cosmological temperature T, and a mass M. I.e. if temperature T does not become infinite, and Mass M is also not infinite, but still proportional, this in itself may eliminate $\sigma \to \infty$. *Given this, we will examine what happens to a defined Plank's value as given by, if n is a quantum number to the following expression from the* V. S. Seshavatharam, S. Lakshminarayana article

$$n \cdot \hbar \cong \sqrt{\frac{M_0}{m_e}} \cdot \frac{G \cdot \left(n \cdot m_p\right) \cdot m_e}{c} \quad (3)$$

We will then re define Eq. (3) to read as (eliminating the quantum number n)

$$\hbar \cong \sqrt{M_0} G m_P \sqrt{m_e} \quad (4)$$

I.e. locking in of the value of Planck's constant initially would be commensurate with

$$\hbar \propto \sqrt{M_0} \quad (5)$$

We would argue that a given amount of mass, M_0 would be fixed in by initial conditions, at the start of the universe and that if energy, is equal to mass (E = M) that in fact locking in a value of initial energy, according to the dimensional argument of $E \sim \hbar \cdot \omega$ that having a fixed initial energy of $E \sim \hbar \cdot \omega$, with Planck's constant fixed would be commensurate with, for very high frequencies, ω of having a non zero initial energy, thereby confirming in part Kauffmann(2012) for conditions for a non zero lower bound to the cosmological initial radius. If so then we always have $\ell\left(\tau\right) \neq 0$. We will then next examine the consequences of $\ell\left(\tau\right) \neq 0$. I.e. what if $\ell\left(\tau\right) = a(\tau)$ for a FLRW cosmology.

4. $\ell\left(\tau\right) \neq 0$ and what to look for in terms of the Raychaudhuri-Elders equation for $\ell\left(\tau\right) = a(\tau)$ at the start of cosmological expansion in FLRW cosmology

We will start off with $\ell\left(\tau\right) = a(\tau) = a_{initial} e^{\tilde{H}\tau}$ with $\tilde{H}$ an initial huge Hubble parameter

$$3\ddot{a}/a = -4\pi G \cdot \left(\rho + 3p\right) + \Lambda \Rightarrow \dot{a}^2 - 8\pi G \rho a^2 - \Lambda a^2 - const = 0 \quad (6)$$

Equation (6) above becomes, with $\ell\left(\tau\right) = a(\tau) = a_{initial} e^{\tilde{H}\tau}$ introduced will lead to

$$a_{initial}^2 = const / \left[\tilde{H}^2 - \left[8\pi G \rho + \Lambda\right]\right] \Rightarrow a_{initial} = \sqrt{const / \left[\tilde{H}^2 - \left[8\pi G \rho + \Lambda\right]\right]} \quad (7)$$

Here, $\tilde{H}$ as the initial Hubble expansion rate would be enormous. If we postulate that we wish for $a_{initial} = \sqrt{const / \left[\tilde{H}^2 - \left[8\pi G \rho + \Lambda\right]\right]} \neq 0$, it would be, if we have $\rho \equiv \rho_{initial} \neq 0$, and maybe a time varying by temperature $\Lambda = \Lambda_{start-value} T^{\beta}$ as given by Park, (2003), where T is the cosmological temperature, a requirement that

$$\tilde{H}^2 >> \left[8\pi G \rho_{initial} + \Lambda_{start-Value} T^{\beta}\right] \quad (8)$$

The above is a serious restriction upon values of initial energy density and also any would be temperature variance of . $\Lambda = \Lambda_{start-value} T^{\beta}$. I.e. it would be better that the $\Lambda = \Lambda_{start-value} T^{\beta}$ have NO temperature variance and that the cosmological constant be a true constant.

I.e. look at if $\Lambda = \Lambda_{start-value} T^{\beta} \leq 0$ and that we have fidelity with the above theorem ? Only true if

I . $\Lambda_{start-value} \leq 0$ if $\beta = 0$

II. $\Lambda = \Lambda_{start-value} T^{\beta} \leq 0$ if $\beta > 0$, meaning that we would have, potentially a negative cosmological parameter which would initially be strongly negative.. I.e. this is not a good idea. Either that or we would have zero temperature initially

III. $\Lambda = -\left|\Lambda_{start-value}\right| T^{-|\beta|} \leq 0$, which we assert is true for higher than four dimensional Brane theory models. Even so, the restriction given by Eq. (8) would still have to hold and would be then re written as

$$\tilde{H}^2 >> \left[8\pi G \rho_{initial} - \left|\Lambda_{start-Value}\right| T^{-|\beta|}\right] \tag{9}$$

Note that at a minimum, the $\rho_{initial}$ terms in Eq. (8) and Eq.(9) are due to what Beckwith outlined in Beckwith(2013a) as well as judicious use of Salvoy (1983).

5. What if $\ell(\tau) \neq 0$ is always true, and then see if it is still possible for $\sigma \to \infty$?

We will then have to go back to the Raychaudhuri-Ehlers equation in terms of the scale factor, in general, if what if $\ell(\tau) \neq 0$, and then see if it is still possible for $\sigma \to \infty$. *To do so, look at*

$$\ddot{\ell} = \left(\frac{1}{3}\right) \cdot \left[-2\left(\sigma^2 - \omega^2\right) + \bar{\nabla}_a \dot{u}^a + \dot{u}_a \dot{u}^a - 4\pi G \cdot \left(\rho + 3 \cdot p\right) + \Lambda\right] \cdot \ell \tag{10}$$

Now, set $\sigma \to \infty$. *Then one has infinite negative acceleration. Now look at it again. How can one have infinite negative acceleration if* $\ell(\tau) \neq 0$ for all values of τ ? Not possible.

One can have a high rate of negative acceleration, but to say it is infinite, while having $\ell(\tau) \neq 0$ **always true** while having Eq.(10) hold is false by inspection.

6. Conclusion: Non singular solutions to cosmological evolution require new thinking.

The classical equations referred to and referenced here will need to be re done, and there is much to be gained by an appropriate rendering of Eq. 8 and Eq. 9 above, in the future. If there is quintessence, i.e. the situation will become immensely more complicated for the reasons stated above. $\rho_{initial}$ due to Gravitons and Gravitinos plus perhaps cyclic cosmology contributions as stated by Penrose (2012) will provide at least a modicum of a density factor which will be followed up in fine tuning Eq. (8) and Eq. (9) above.

Appendix A: Indirect support for a massive graviton

We follow the recent work of Steven Kenneth Kauffmann, which sets an upper bound to concentrations of energy, in terms of how he formulated the following equation put in below as Eq. (A1). Equation (A1) specifies an inter-relationship between an initial radius R for an expanding universe, and a "gravitationally based energy" expression we will call $T_G(r)$ which lead to a lower bound to the radius of the universe at the start of the Universe's initial expansion, with manipulations. The term $T_G(r)$ is defined via Eq.(A2) afterwards. We start off with Kauffmann's

$$R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G\left(r + r''\right) d^3 r'' \tag{A1}$$

Kauffmann calls $\left(\frac{c^4}{G}\right)$ a "Planck force" which is relevant due to the fact we will employ Eq. (A1) at the initial instant of the universe, in the Planckian regime of space-time. Also, we make full use of setting for small r, the following:

$$T_G\left(r + r''\right) \approx T_{G=0}\left(r\right) \cdot const \sim V(r) \sim m_{Graviton} \cdot n_{Initial-entropy} \cdot c^2 \tag{A2}$$

I.e. what we are doing is to make the expression in the integrand proportional to information leaked by a past universe into our present universe, with Ng style quantum infinite statistics use of

$$n_{Initial-entropy} \sim S_{Graviton-count-entropy} \tag{A3}$$

Then Eq. (A1) will lead to

$$R\cdot\left(\frac{c^4}{G}\right) \geq \int_{|r''|\leq R} T_G\left(r+r''\right)d^3r'' \approx const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right]$$

$$\Rightarrow R\cdot\left(\frac{c^4}{G}\right) \geq const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right] \tag{A4}$$

$$\Rightarrow R \geq \left(\frac{c^4}{G}\right)^{-1}\cdot\left[const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right]\right]$$

Here, $\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right] \sim 10^5$, $m_{Graviton} \sim 10^{-62}\, grams$, and

1 Planck length = l_{Planck} **= 1.616199 × 10^{-35} meters**

where we set $l_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$ with $R \sim l_{Planck}\cdot 10^{\alpha}$, and $\alpha > 0$. Typically $R \sim l_{Planck}\cdot 10^{\alpha}$ is about $10^3\cdot l_{Planck}$ at the outset, when the universe is the most compact. The value of *const* is chosen based on common assumptions about contributions from all sources of early universe entropy, and will be more rigorously defined in a later paper.

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