Energy – momentum Vectors for matter and gravitational field.

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Abstract

This paper is the simple example of the using the First Noether theorem generalized on asymmetric metric tensors in curved spaces and taking into account the second derivatives. It defines the ordinary energy-momentum vector for matter and the energy-momentum vector for gravitational field.

From [1] (item 2.2 - page 36):
For infinitesimal space-time translations

$$x'^k = x^k + \delta x^k$$

we can take δx^k for the transformation parameters $\delta\omega^i$ and, if we take the transformation law $u'(x') = u(x)$ into account and also (3.3) from [2], we obtain

$$X^k{}_n = \delta^k{}_n \qquad \Psi_{in} = 0 \qquad (n = 1, 2, 3, 4)$$

And θ transforms into a tensor of rank 2:
(k, n, ℓ = 1, 2, 3, 4) From [2] (3.1.3) :

$$\theta^k{}_n = \left[\frac{\partial L}{\partial u_{i,k}} - \partial_{\mathbf{l}}\left(\frac{\partial L}{\partial u_{i,k\mathbf{l}}}\right)\right]\cdot u_{i;n} - L\cdot\delta^k{}_n + $$

$$+\frac{\partial L}{\partial u_{i,\mathbf{l}k}}\cdot\partial_{\mathbf{l}} u_{i;n} \qquad (1)$$

From [2] (3.1.2) : $$j_{\mathbf{l}} = \tfrac{1}{2}\cdot g^{\mu\nu}\cdot b_{\mu\nu,\mathbf{l}} \qquad (1.1)$$

Integrals of form (3.3.5) from [2] represent the time-conserved four-vector

$$P_{\mathbf{l}} = \int(\theta^1{}_{\mathbf{l}} - \int dx^1\cdot j_{\mathbf{l}})\cdot dx^2\cdot dx^3\cdot dx^4 \qquad (1.2)$$

$P_{\mathbf{l}}$ is **a vector of energy-momentum for the matter.**

If instead of u_i we take the asymmetric metric tensor $g_{\mu\nu}$, then from (1.35) of [3] we obtain :

$$\delta g_{\mu\nu} = (\delta\vec{e}_\mu, \vec{e}_\nu) + (\vec{e}_\mu, \delta\vec{e}_\nu) = (g_{\sigma\nu} \cdot \Gamma^\sigma{}_{\mu\lambda} + g_{\mu\sigma} \cdot \Gamma^\sigma{}_{\nu\lambda}) \cdot \delta x^\lambda =$$

$$= g_{\mu\nu,\lambda} \cdot \delta x^\lambda = \Psi_{\mu\nu\lambda} \cdot \delta x^\lambda \text{ hence } \Psi_{\mu\nu n} = g_{\mu\nu,n} \qquad (2)$$

Using (2) and $X^k{}_n = \delta^k{}_n$ and $g_{\mu\nu;m} = 0$ [that from [2] (3.1.1)] and (3.1.2) and (3.1.3) of [2] we can obtain **the vector of energy-momentum for the gravitational field**:

$$P_n = \int(\theta^1{}_n - \int dx^1 \cdot j_n) \cdot dx^2 \cdot dx^3 \cdot \mathbf{K} \cdot dx^N =$$

$$= \int(\theta^1{}_n - \int dx^1 \cdot \tfrac{1}{2} \cdot g^{m\nu} \cdot b_{m\nu,\,n}) \cdot dx^2 \cdot dx^3 \cdot \mathbf{K} \cdot dx^N \qquad (3)$$

$$\theta_n{}^k = -[\frac{\partial L}{\partial g_{\mu\nu,k}} - \partial_{\mathbf{l}}(\frac{\partial L}{\partial g_{\mu\nu,k\mathbf{l}}})] \cdot g_{\mu\nu,n} - L \cdot \delta_n{}^k -$$

$$- \frac{\partial L}{\partial g_{\mu\nu,\mathbf{l}k}} \cdot g_{\mu\nu,n\mathbf{l}} \qquad (4)$$

$$j_n = n_k \cdot X^k{}_n = \tfrac{1}{2} \cdot g^{m\nu} \cdot b_{m\nu,k} \cdot \delta^k{}_n = \tfrac{1}{2} \cdot g^{m\nu} \cdot b_{m\nu,\,n} \qquad (5)$$

Literature :

1. Bogoliubov N.N., Shirkov D.V. Introduction to the theory of quantized fields (Wiley, 1980)(ISBN 0471042234)(600dpi)(T)(637s)_PQft_.djvu
2. The generalizations of the First Noether theorem. http://viXra.org/abs/1304.0106
3. Christoffel symbols for asymmetric metric tensors. http://viXra.org/abs/1302.0072