

Derivation of the Pauli Exclusion Principle

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Abstract: In generally, the Pauli Exclusion Principle follows from the spectroscopy whereas its origin is not good understood. To understand fully this principle, most important is origin of quantization of the azimuthal quantum number i.e. the angular momentum quantum number. Here, on the base of the theory of ellipse and starting from very simple physical condition, I quantized the azimuthal quantum number.

1. Introduction

The Pauli Exclusion Principle says that no two identical half-integer-spin fermions may occupy the same quantum state simultaneously. For example, no two electrons in an atom can have the same four quantum numbers. They are the principal quantum number n that denotes the number of the de Broglie-wave lengths λ in a quantum state, the azimuthal quantum number l (i.e. the angular momentum quantum number), the magnetic quantum number m and the spin s .

On the base of the spectrums of atoms, placed in magnetic field as well, follows that the quantum numbers take the values:

$$n = 1, 2, 3, \dots$$

$$l = 0, 1, 2, \dots, n - 1$$

$$m = -l, \dots, +l$$

$$s = \pm 1/2.$$

The three first quantum numbers n , l , and m are the integer numbers and define a state in which can be maximum two electrons with opposite spins.

The magnetic quantum number m determines the projection of the azimuthal quantum number l on the arbitrary chosen axis. This axis can overlap with a diameter of the circle $l = 0$.

To understand fully the Pauli Exclusion Principle we must answer following questions concerning the azimuthal quantum number l :

1.
What is physical meaning of this quantum number?
2.
Why the l numbers are the natural numbers only?
3.
Why the zero is the lower limit?
4.
Why the $n - 1$ is the upper limit?

To answer these questions we must apply the theory of ellipse, especially the formula for its circumference C and eccentricity e . When we use the complete elliptic integral of the second kind and the Carlson symmetric form [1], we obtain for circumference C of an ellipse following formula

$$C = 2\pi a [1 - (1/2)^2 e^2 / 1 - (1 \cdot 3 / (2 \cdot 4))^2 e^4 / 3 - (1 \cdot 3 \cdot 5 / (2 \cdot 4 \cdot 6))^2 e^6 / 5 - \dots], \quad (1)$$

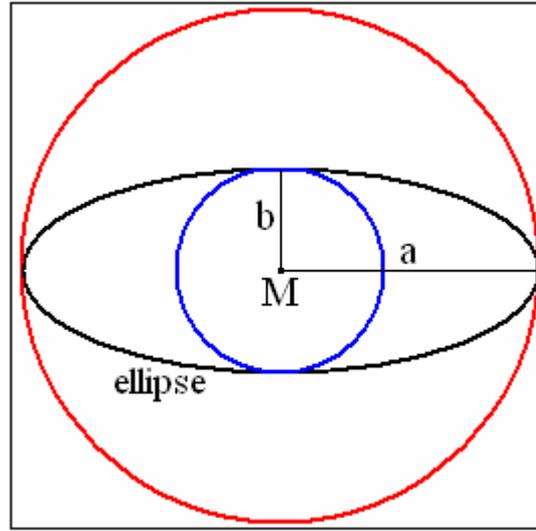
where a is the major radius and e is the eccentricity defined as follows

$$e = [\text{sqrt}(a^2 - b^2)]/a, \quad (2)$$

where b is the minor radius.

2. Calculations

In the figure, the circumference of the ellipse $C_{\text{de-Broglie}}$ is $C_{\text{de-Broglie}} = n\lambda = 2\pi n\lambda$, where the n is the principal quantum number whereas the λ is the reduced de Broglie-wave length. Assume that there are allowed only ellipses that circumference is the arithmetic mean of the circumferences of two circles that radii are equal to the major and minor radii of the ellipse.



Similarly as for the circumference of the ellipse, the circumferences of the circles must be equal to a natural number multiplied by the de Broglie-wave length. This leads to following definitions

$$a = j\lambda \text{ and } b = k\lambda. \quad (3)$$

Notice that $j = k = 0$ has no sense.

Then, we can rewrite formula (2) as follows

$$e = [\text{sqrt}(j^2 - k^2)]/j. \quad (4)$$

It is the natural assumption that the allowed circumferences of the ellipse should be the arithmetic mean of the sum of the circumferences of the two circles. It leads to following conclusion

$$(j + k)/2 = n. \quad (5)$$

Define some number l as follows

$$(j - k)/2 = l. \quad (6)$$

Formulae (5) and (6) lead to following relations

$$j = n + l, \quad (7)$$

$$k = n - l. \quad (8)$$

Since the j , k and n are the integers so the number l must be an integer as well.

On the base of formulae (7) and (8) we can rewrite formula (4) as follows

$$e = 2[\text{sqrt}(nl)]/(n + l). \quad (9)$$

We can see that due to the square root, this formula has no real sense for $l < 0$. Since the l cannot be negative then from formulae (5) and (6) follows that $l < n$.

On the base of formulae (3) and (7), we can rewrite formula (1) as follows

$$C_K = 2\pi(n+l)\lambda[1 - (1/2)^2e^2/1 - (1\cdot3/(2\cdot4))^2e^4/3 - (1\cdot3\cdot5/(2\cdot4\cdot6))^2e^6/5 - \dots]. \quad (10)$$

Notice that for $n = l$ is $e = 1$ and then $C_{\text{de-Broglie}} > C_K$ i.e. l cannot be equal to n . For $l = 0$ is $C_{\text{de-Broglie}} = C_K$ and because l cannot be negative then the $l = 0$ is the lower limit for l .

Some recapitulation is as follows. We proved that the azimuthal quantum number l

- 1) is associated with transitions between the states j and k ,
- 2) is the integer,
- 3) cannot be negative and the lower limit is zero,
- 4) the $n - 1$ is the upper limit.

Some abbreviation of it is as follows

$$l = 0, 1, 2, \dots, n - 1.$$

The Quantum Physics is timeless because a quantum particle disappears in one region of a field or spacetime and appears in another, and so on. There are no trajectories of individual quantum particles. Quantum Physics is about the statistical shapes and their allowed orientations. Such procedure simplifies considerably the Quantum Physics.

3. Summary

In generally, the Pauli Exclusion Principle follows from the spectroscopy whereas its origin is not good understood. To understand fully this principle, most important is origin of quantization of the azimuthal quantum number i.e. the angular momentum quantum number. Here, on the base of the theory of ellipse and starting from very simple physical condition, I quantized the azimuthal quantum number.

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References

- [1] Carlson, B. C. (1995). "Computation of real or complex elliptic integrals".
Numerical Algorithms **10**: 13-26. arXiv:math/9409227 .