Bounds upon Graviton Mass – using the difference between graviton propagation speed and HFGW transit speed to observe post-Newtonian corrections to gravitational potential fields

Updated to take into account early universe cosmology

Andrew Beckwith
Chongqing University Department of Physics;
E-mail: abeckwith@uh.edu
Chongqing, PRC, 400044

The author presents a post-Newtonian approximation based upon an earlier argument in a paper by Clifford Will as to Yukawa revisions of gravitational potentials, in part initiated by gravitons having explicit mass dependence in their Compton wave length. Prior work with Clifford Will's idea was stymied by the application to binary stars and other such astrophysical objects, with non-useful frequencies topping off near 100 Hertz, thereby rendering Yukawa modifications of Gravity due to gravitons effectively an experimental curiosity which was not testable with any known physics equipment. This work improves on those results. Futhermore we argue in favor of both a non zero initial radius of the universe, using Kenneth Kauthman's work and also try to make the case for a non zero graviton mass. We use Salvoy's document of 1983 to argue in favor of both a non zero initial radius of the universe, and non zero graviton mass (heavy gravitons). We claim that a non-zero initial radius (of the universe) supports the massive graviton hypothesis. Which is the main point of our document. Also Gravitinos in the Electroweak era, all $10^8 - 10^{12}$ of them have an (almost) invariant energy from the beginning of cosmology. This invariant energy constitutes an initial energy value at the start of the universe which can be used to obtain, at the onset of inflation Kauffman's lower bound to a non zero initial radius of the universe.

Key words: Graviton mass, Yukawa potential, Post Newtonian Approximation, SUSY

1. Introduction

Post-Newtonian approximations to General Relativity have given physicists a view as to how and why inflationary dynamics can be measured via deviation from simple gravitational potentials. One of the simplest deviations from the Newtonian inverse power law is a Yukawa potential modification of gravitational potentials. It is apparent that a graviton's mass (assuming it is massive) would factor directly into the Yukawa exponential term modification of gravity. This present paper tries to indicate how a smart experimentalist could use a suitably configured gravitational wave detector as a way to obtain more realistic upper bounds for the mass of a graviton, and explores how to use this idea as a template to investigate modifications of gravity along the lines of a Yukawa potential modification as given by Clifford Will. **Appendix A** summarizes why we think gravitons should be massive, i.e. having a small rest mass. We will show how our findings dovetail with recycled information (from previous cycles) into a new universe. Presumably the information transferred via massive Gravitons will be responsible for setting Planck's constant at a particular value at the onset of a new universe.

Secondly, this paper will address an issue of great import to the development of experimental gravity. Namely, if an upper mass to the graviton mass is identified; can an accelerator physicist use the theoretical construction Eric Davis posited in his book in the section "Producing Gravitons via Quantization of the coupled Maxwell-Einstein fields" for obtaining an experimental bound to the graviton mass, to refine our understanding of graviton Synchrotron radiation. A brief review of Chen, Chen, and Noble's application of the Gersenshtein effect will be made, to potentially improve their statistical estimates of the range of graviton production.

2. Giving an upper bound to the mass of a graviton.

The easiest way to ascertain the mass of a graviton is to investigate if or not there is a slight difference in the speed of graviton 'particle' propagation and that of HFGW in transit from a 'source' to the detector. Visser's (1998) mass of a graviton paper presents a theory which passes the equivalence test, but which has possible problem with depending upon a non-dynamical background metric. Note that gravitons are assumed by both Visser, and also in Clifford Will's write up of experimental GR, to have mass

This document also accepts the view that there is a small graviton mass, which the author has estimated to be on the order of 10^{-60} kilograms. This is small enough so the following approximation is valid. Here, v_g is the speed of graviton 'propagation', λ_g is the Compton wavelength of a graviton with $\lambda_g = h/m_g c$, and $f \approx 10^{10}$ Hertz in line with L. Grischuck's treatment of relic HFGWs. In addition, the high value of relic HFGWs leads to naturally fulfilling $hf >> m_g c^2$ so that

$$v_g / c \approx 1 - \frac{1}{2} \cdot \left(c / \lambda_g \cdot f \right)^2 \tag{1}$$

But equation (1) above is an approximation of a much more general result which may be rendered as

$$\left(v_g / c \right)^2 \equiv 1 - \left(m_g c^2 / E \right)^2 \tag{2}$$

The terms m_g and E refers to the graviton rest mass and energy, respectively. Now Physics researchers can ascertain what E is, with experimental data from a gravitational wave detector, and the next question needs to be addressed, relating to Visser's model. Namely; if D is the distance between a detector and the source of a HFGW/ Graviton emitter source

$$1 - v_g / c = 5 \times 10^{-17} \cdot \left[\frac{200 Mpc}{D} \right] \cdot \left(\frac{\Delta t}{1 \sec} \right) \tag{3}$$

The above formula depends upon, $\Delta t = \Delta t_a - (1+Z)\cdot\Delta t_e$ with where Δt_a and Δt_e are the differences in arrival time and emission time of the two signals (HFGW and Graviton propagation), respectively, and Z is the redshift of the source.

Specifically, the situation for HFGW is that for early universe conditions, that $Z \geq 1100$, in fact for very early universe conditions in the first few milliseconds after the big bang, that $Z \sim 10^{25}$. This is an enormous number.

The first question which needs to be asked is, if the Visser non-dynamical background metric is correct, for early universe conditions so as to avoid the problem of the limit of small graviton mass does not coincide with pure GR, and the predicted perihelion advance, for example, violates experiment . A way forward would be to configure data sets so in the case of early universe conditions that one is examining appropriate $Z >> 1100$ but with extremely small Δt_e times, which would reflect upon generation of HFGW before the electro weak transition, and after the INITIAL onset of inflation.

I.e. a Gravitational wave detector system should be employed as to pin point experimental conditions so to high accuracy, the following is an adequate presentation of the difference in times, Δt. I.e.

$$\Delta t = \Delta t_a - (1+Z)\cdot\Delta t_e \quad \rightarrow \Delta t_a - \varepsilon^{+} \approx \Delta t_a \tag{4}$$

The closer the emission times for production of the HFGW and Gravitons are to the time of the initial nucleation of vacuum energy of the big bang, the closer we can be to experimentally using equation (4) above as to give experimental criteria for stating to very high accuracy the following.

$$1 - v_g / c \cong 5\times 10^{-17}\cdot\left[\frac{200Mpc}{D}\right]\cdot\left(\frac{\Delta t_a}{1\sec}\right) \tag{5}$$

More exactly this will lead to the following relationship which will be used to ascertain a value for the mass of a graviton. By necessity, this will push the speed of graviton propagation very close to the speed of light. In this, we assume an enormous value for D

$$v_g / c \cong 1 - 5\times 10^{-17}\cdot\left[\frac{200Mpc}{D}\right]\cdot\left(\frac{\Delta t_a}{1\sec}\right) \tag{6}$$

This equation (6) relationship should be placed into $\lambda_g = h / m_g c$, with a way to relate this above value of $(v_g/c)^2 \equiv 1 - (m_g c^2/E)^2$, with an estimated value of E as an average value from field theory calculations, as well as to make the following argument rigorous, namely

$$\left[1 - 5\times 10^{-17}\cdot\frac{200Mpc}{D}\cdot\frac{\Delta t_a}{1\sec}\right]^2 \cong 1 - \left(\frac{m_g c^2}{E}\right)^2 \tag{7}$$

A suitable numerical treatment of this above equation, with data sets could lead to a range of bounds for m_g, as a refinement of the result given by Clifford Will for graviton Compton wavelength bounded behavior for a lower bound to the graviton mass, assuming that h is the Planck's constant.

$$\begin{aligned}\lambda_g &\equiv \frac{h}{m_g c} > 3\times 10^{12} km\cdot\left(\frac{D}{200Mpc}\cdot\frac{100Hz}{f}\right)^{1/2}\cdot\left(\frac{1}{f\Delta t}\right)^{1/2} \\ &\cong 3\times 10^{12} km\cdot\left(\frac{D}{200Mpc}\cdot\frac{100Hz}{f}\right)^{1/2}\cdot\left(\frac{1}{f\Delta t_a}\right)^{1/2}\end{aligned} \tag{8}$$

The above equation (8) gives an upper bound to the mass m_g as given by

$$m_g < \left(\frac{c}{h}\right) \Big/ 3\times 10^{12}\, km\cdot\left(\frac{D}{200Mpc}\cdot\frac{100Hz}{f}\right)^{1/2}\cdot\left(\frac{1}{f\cdot\Delta t_a}\right)^{1/2} \tag{9}$$

Needless to say, an estimation of the bound for the graviton mass m_g, and the resulting Compton wavelength λ_g would be important to get values of the following formula for experimental validation

$$V(r)_{gravity} \cong \frac{MG}{r}\exp\left(r/\lambda_g\right) \tag{10}$$

Clifford Will gave for values of frequency $f \equiv 100$ Hertz enormous values for the Compton wavelength, i.e. values like $\lambda_g > 6\times 10^{19} km$. Such enormous values for the Compton wavelength make experimental tests of equation (10) practically infeasible. Values of $\lambda_g \approx 10^{-5}$ centimeters or less for very high HFGW data makes investigation of equation (10) above far more tractable.

3. Application to Gravitational Synchrotron radiation, in accelerator physics

Eric Davis, quoting Pisen Chen's article written in 1994 estimates that a typical storage ring for an accelerator will be able to give approximately $10^{-6} - 10^{3}$ gravitons per second. Eric uses Chen's article about Photon conversion into Gravitons, to suggest a way we can use accelerators as a somewhat focused graviton source. Quoting Pisen Chen's 1994 article, the following for graviton emission values for a circular accelerator system, with m the mass of a graviton, and M_P being the Planck mass. N as mentioned below is the number of 'particles' in a ring for an accelerator system, and n_b is an accelerator physics parameter for bunches of particles, which for the LHC is set by Pisen Chen to the value of 2800, and N for the LHC is about 10^{11}. And, for the LHC Pisen Chen sets γ at 0.88×10^{2}, with $\rho[m] \approx 4300$. Here, $m \sim m_{graviton}$ acts as a mass charge.

$$N_{GSR} \sim 5.6 \cdot n_b^2 \cdot N^2 \cdot \frac{m^2}{M_P^2} \cdot \frac{c \cdot \gamma^4}{\rho} \tag{11}$$

The immediate consequence of the prior discussion would be to obtain a more realistic set of bounds for the graviton mass, which could considerably refine the estimate of 10^{11} gravitons produced per year at the LHC, with realistically 365 x 86400 *seconds* = 31536000 *seconds* in a year, leading to 3.171×10^3 gravitons produced per second. Refining an actual permitted value of bounds for the accepted graviton mass, m, as given above, while keeping $M_P \sim 1.2209 \times 10^{19}$ *GeV*/c^2 would allow for a more precise value for gravitons per second which significantly enhances the chance of actual detection, since right now for the LHC there is too much general uncertainty about where to place a detector for actually capturing / detecting a graviton. Note that Eric Davis explicitly uses Pisen Chen's calculations of $10^{-6} - 10^3$ gravitons produced to make a feasibility argument as to non zero graviton mass.

4. Conclusion, falsifiable tests for the Graviton are closer than the physics community thinks

The physics community now has an opportunity to experimentally infer the existence of gravitons as a knowable and verifiable experimental datum with the onset of the LHC as an operating system. Even if the LHC is not used, Pisen Chen's parameterization of inputs from the table right after his equation (8) as inputs into equation (11) above will permit the physics community to make progress toward the detection of Gravitons for, say the Brookhaven laboratory site circular ring accelerator system. See **Appendix B** for that table. Tony Rothman's statement about needing a detector the size of Jupiter to obtain a single experimentally falsifiable set of procedures is defensible only if the wave-particle duality induces so much uncertainty as to the mass of the purported graviton, that worst case model building and extraordinarily robust parameters for a Rothman style graviton detector have to be put in place.

A suitably configured detector can help with bracketing a range of masses for the graviton, as a physical entity subject to measurements, without needing to be so massive. Such an effort requires obtaining rigorous verification of the approximation used to the effect that $\Delta t = \Delta t_a - (1+Z) \cdot \Delta t_e \quad \rightarrow \Delta t_a - \varepsilon^+ \approx \Delta t_a$ is a defensible approximation. Furthermore, having realistic estimates for distance D as inputs into equation (9) above is essential.

The expected pay offs of making such an investment would be to determine the range of validity of equation (10) , i.e. to what degree is gravitation as a force is amendable to post Newtonian approximations.

The author asserts that equation (10) can only be realistically be tested and vetted for sub atomic systems, and that with the massive Compton wavelength specified by Clifford Will cannot be done with low frequency gravitational waves.

Furthermore, a realistic bounding of the graviton mass would permit a far more precise calibration of equation (11) as given by Pisen Chen in his 1994 article. We refer the reader to **Appendix C** for how we expect that Eq. (11) and **Appendix A**, Eq. (A4) may be combined to yield experimentally falsifiable tests for a massive graviton. We state that a non-zero initial radius (of the universe) supports the massive graviton hypothesis.

Note that **Appendix D** gives the details of how Yurov links both initial inflation and the speed up of acceleration of the universe a billion years ago. In addition a brief mention of Padmanabhan's contruction of the present era inflaton is mentioned. Notice that the inflaton for today is given by Padmanabhan , which Beckwith views as important to understand what Yurov meant by linking initial inflation with the expansion of the universe speeding up today, i.e. Yurov's second inflation. Also, if gravitons are linkable to DE, as stated by Beckwith's Journal of cosmology publication (2011), in the present era, and if gravitons are super partnered with Gravitinos as stated by Beckwith (2013), Kauffman's non zero initial radius becomes essential, and we also have a description of how the fine structure constant over time, as given by Appendix C, Eq. (C2) , could be invariant. That in turn would lead to Planck's constant being invariant from cosmological cycle to cycle without invoking the Anthropic principle. Appendix A gives a crucial lower bound for the initial radius of the universe. Appendix C makes the case for a non zero Graviton mass super partered to (via SUSY) to Gravitinos.

Appendix A: Indirect support for a massive graviton

We follow the recent work of Steven Kenneth Kauffmann, which sets an upper bound to concentrations of energy, in terms of how he formulated the following equation put in below as Eq. (A1). Equation (A1) specifies an inter-relationship between an initial radius R for an expanding universe, and a "gravitationally based energy" expression we will call $T_G(r)$ which lead to a lower bound to the radius of the universe at the start of the Universe's initial expansion, with manipulations. The term $T_G(r)$ is defined via Eq.(A2) afterwards. We start off with Kauffmann's

$$R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G(r + r'') d^3 r'' \qquad \text{(A1)}$$

Kauffmann calls $\left(\frac{c^4}{G}\right)$ a "Planck force" which is relevant due to the fact we will employ Eq. (A1) at the initial instant of the universe, in the Planckian regime of space-time. Also, we make full use of setting for small r, the following:

$$T_G(r + r'') \approx T_{G=0}(r) \cdot const \sim V(r) \sim m_{Graviton} \cdot n_{Initial-entropy} \cdot c^2 \qquad \text{(A2)}$$

I.e. what we are doing is to make the expression in the integrand proportional to information leaked by a past universe into our present universe, with Ng style quantum infinite statistics use of

$$n_{Initial-entropy} \sim S_{Graviton-count-entropy} \qquad \text{(A3)}$$

Then Eq. (A1) will lead to

$$R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G\left(r + r''\right) d^3 r'' \approx const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right]$$

$$\Rightarrow R \cdot \left(\frac{c^4}{G}\right) \geq const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right] \quad \text{(A4)}$$

$$\Rightarrow R \geq \left(\frac{c^4}{G}\right)^{-1} \cdot \left[const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right]\right]$$

Here, $\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy}\right] \sim 10^5$, $m_{Graviton} \sim 10^{-62}\, grams$, and

1 Planck length = l_{Planck} **= 1.616199 × 10^{-35} meters**

where we set $l_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$ with $R \sim l_{Planck} \cdot 10^{\alpha}$, and $\alpha > 0$. Typically $R \sim l_{Planck} \cdot 10^{\alpha}$ is about $10^3 \cdot l_{Planck}$ at the outset, when the universe is the most compact. The value of *const* is chosen based on common assumptions about contributions from all sources of early universe entropy, and will be more rigorously defined in a later paper.

Appendix B: Graviton paper table from Pisen Chen

Storage Rings	**PEP-11**	**LEP-1**	**LEP-II**	**HERA**	**LHC**
$\varepsilon\left[GeV\right]$	**9**	**50**	**100**	**880**	**7000**
$\gamma\left[10^3\right]$	**18**	**100**	**100**	**7.5**	**.88**
$N\left[10^{10}\right]$	**3.8**	**45**	**45**	**10**	**10**
n_b	**1700**	**4**	**4**	**210**	**2800**
$l\left[cm\right]$	**3.46**	**6.24**	**6.24**	**27.7**	**18.4**
$\rho\left[m\right]$	**500**	**4300**	**4300**	**1035**	**4300**
Gravitational SR					
$\omega_0\left[kHz\right]$	**600**	**70**	**70**	**290**	**70**
$N_{GSR}\left[10^{-7}\sec^{-1}\right]$	$\mathbf{1.3}\times 10^3$	**38**	**150**	$\mathbf{6}\times 10^6$	$\mathbf{18}\times 10^8$
Resonant conversion					
$\omega_c\left[10^9 GHz\right]$	**3.5**	**70**	**560**	**.12**	$\mathbf{4.8}\times 10^{-5}$
$N_{\mathrm{Re}s}\left[10^{-7}\sec^{-1}\right]$	**.1**	**.1**	**,3**	10^3	$\mathbf{2}\times 10^5$

Appendix C: How to get stricter bounds to Eq. (A4) above

We will give justification for the recent work of Steven Kenneth Kauffmann, which sets an upper bound to an initial concentrations of energy in the early universe, and afterwards gives a minimum non zero lower bound to the permitted radius of the early universe. Gravitinos in the very early universe are converted into Gravitons once Electro-Weak symmetry breaking has commenced, and that this conversion results in some (most?) of the mass of $10^8 - 10^{12}$ Gravitons being conferred to 10^{50} Gravitons as a result. This in turn leads to a lower bound to initial cosmological radius, R, well before the Electroweak era being non zero, especially if each Gravitino can have up to 1 TeV in energy.

The idea was to mix results in Salvoy's 1983 document with Ng's re-statement of entropy to obtain up to $10^8 - 10^{12}$ Gravitinos in the Electroweak era, as leading to SUSY super partnered (to the Gravitons) 10^{50} Gravitons in the Electroweak era. If the Gravitons are massive, this will imply each Gravitino can have up to 1 TeV energy. If Gravitinos have such large energies , then if the total energy of the initial universe formed is $10^8 - 10^{12}$ times 1 TeV , we will have by Steven Kenneth Kaufmann, a non zero minimum initial radius R, to the universe. I.e. a non zero singularity.

The upper bound in initial energy of the early universe is used in Eq. (A4) to specify a crucial term in an integral on the right hand side off Eq. (A4) which when performed in Eq.(A4) will give a lower bound to the initial radius of the universe. Kauffman specifies a specific energy, initially in the early universe, which we assume is the same as the energy content of Gravitinos in the Electro weak era.The governing assumption is that if matter is energy, initially, that what is the energy content of $10^8 - 10^{12}$ Gravitinos in the Electroweak era is the same as the initial energy initially introduced in the beginning of the expansion of the universe. I.e. the energy content of Gravitinos in the Electroweak era, all $10^8 - 10^{12}$ of them, constitutes the upper bound in initial energy of the universe, at the onset of inflation.

The idea is that the early universe had a certain amount of information transferred to it via recycling, and that this information, initially, may have been encoded in $10^8 - 10^{12}$ Gravitinos before the Electroweak era.

Furthermore, after $10^8 - 10^{12}$ Gravitinos are formed, the Gravitinos are linked by SUSY to Gravitons in the Electroweak era. The $10^8 - 10^{12}$ Gravitinos are linked to 10^{50} Gravitons due to the Electroweak phase transition.

We will reference a paper cited by Beckwith in Beckwith's (2013) Rencontres De Moriond contribution , which was written by C.A. Salvoy (1983) and take results to show that the total mass of gravitinos in the early universe is the same as the total mass of gravitions during the Electroweak era. In Beckwith's (2013) document for Rencontres De Moriond (2013) the heading for relationships we seek is given by the following header.

2. Forming $m_{3/2}$ for a Gravitino and linking it to Massive Graviton Contributions in electro weak era: For the Machian relationship

First, Primordial Gravitinos might each have a mass up to 1 TeV, and there are up to $10^8 - 10^{12}$ Gravitinos in the Electroweak Era.

Secondly, we assume the number of Gravitons, i.e. about 10^{50} in the Electroweak era is due to the presence of $10^{8}-10^{12}$ Gravitinos in the Electroweak era. Each Gravitino initially is is up to 1 TeV in mass, and if mass is initially the same as energy, the presence of non zero energy for Gravitinos will show up in the formulation of a nonzero energy expression $T_{G}\left(r\right)$ of Eq. (A2) in Appendix A . Since $T_{G}\left(r\right)$ has a non zero value, this 1 TeV times $10^{8}-10^{12}$ initial energy placed into $T_{G}\left(r\right)$ insures that there is a lower non zero bound for the initial radii, R .

The idea was to mix results in Salvoy's 1983 documet with having $N_{electro-weak(gravitinos)}$ as the number of Gravitinos in the Electroweak era ($10^{8}-10^{12}$) , and $N_{electro-weak(gravitons)}$ as the number of Gravitons in the Electroweak era (10^{50}) and $M_{total-initial-mass}$ as the initial mass of the universe before the Electroweak era. If M(mass)= Energy, then Eq.(C1) below is a statement that there is a finite upper bound to energy as is used by Kauffman to obtain a finite lower bound to the initial radius of the universe he calls R.

$$M_{total-initial-mass} = N_{electro-weak(gravitinos)} \cdot m_{3/2} = N_{electro-weak(gravitons)} \times 10^{38} \cdot m_{graviton} \quad \text{(C1)}$$

Then the electroweak regime would have an entropy value of S ~ 10^{50} . If this entropy, S, value is treated by the Ng quantum infinite statistics S ~ N, where S is entropy, and N is a numerical particle count, this substitution S~ N by Ng will lead to entropy being represented by a graviton count $N_{electro-weak} \sim 10^{50}$ for Gravitons which would be compared with a Gravitino count $10^{8}-10^{12}$ during the Electroweak era., with the mass of an individual Gravitino, which is called $m_{3/2}$ being 10^{38} times larger in mass than the graviton . Eq. (C2) is the initial entropy value in electro weak regime due to a huge number of Gravitons. The relationship of S (entropy) ~ N (particle count) by Ng permits a statement of Graviton entropy in the Electroweak era as

$$N_{electro-weak} \sim 10^{50} \quad \text{(C2)}$$

The first and second eqn. above form our relationship for making a linkage of massive gravitons linked to SUSY gravitinos This would lead to , USING Salvoy's (1983) result of

$$\sum m_{BOSONS} - \sum m_{FERMIONS} = 0 \quad \text{(C3)}$$

If Gravitons as Bosons, are super partnered to Gravitinos as Fermions, and we use Eq. (C3) above we obtain as given by Beckwith (2013)

Table 1, mass of different particles and cosmological parameters (rounded off)

M_{Planck}	$M_{TEV} \sim M_{DM} \sim M_{Gravitino}$	M_{DE}	$M_{Graviton}$
$10^{-8}\,kg \sim 10^{16}\,TeV$	$10^{-24}\,kg = 10^{12}\,eV$	$10^{-16}\,M_{DM}$	$10^{-65}\,kg$

The net up shot of the super partner pairing of gravitons and gravitinos is to obtain a non zero $T_G\left(r\right)$. Now to the problem How to insure that the graviton has non zero mass.

To start, we can first review briefly what was done by Beckwith in 2011, in the Journal of Cosmology. In this publication, Beckwith outlined how there may be a contribution via a minimally massive graviton as to re acceleration of the universe. Here the value of $m_{Graviton} \sim 10^{-62}\, grams$ to get a speed up of acceleration of cosmological expansion a billion years ago.

Furthermore, as stated in FXQI, in a discussion by Tom Ray, *No singularity at the Schwarzchild radius not only confirms the quantum nature of the cosmological initial condition, it implies non-quantization of classical space-time. For if the quantum field does not collapse, the universal wavefunction, which is continuous (Kauffmann concludes, " ... only the universe itself, with its cosmological redshift, is actually capable of 'containing' the arbitrarily high frequencies of a quantum field") is physically real and dark energy isn't".*

This end result leads to a finite initial radius of space-time, not zero, and regime massive Gravitons obeying the equation of state , as given by Maggiore

$$\left(\partial_{\mu}\partial^{\varpi} - m_{graviton}\right)\cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^{+}\right]\cdot\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T^{\mu}_{\mu} + \frac{\partial_{\mu}\partial_{\nu}T^{\mu}_{\mu}}{3m_{graviton}}\right) \tag{C4}$$

Giving proof as to the constant value of the graviton mass would allow for further development of Yurov's first and second hypothesis. First inflation is typical inflation whereas second inflation is the speedup of the universe as modeled by Beckwith, in the Journal of Cosmology in 2010.

Appendix D What if an inflaton partly re-emerges in space-time dynamics? At z ~ . 423?

In this section, the author will give further elaboration of a suggestion by Yurov as to linking of initial inflation with the speed up of expansion of the universe, commencing up to today. This section will be focused upon saying something about the inflaton which may be basic as to why the universe has a speed up of inflation. We review, in doing so, the work by Padmanabhan.

Padmanabhan has written up how the 2nd Friedman equation which for **z ~ . 423** may be simplified to read as

$$\dot{H}^2 \cong \left[-2\frac{m}{a^4}\right] \tag{D1}$$

would lead to an inflaton value of , when put in, for scale factor behavior as given by $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$, of, for the inflaton and inflation of (Padmanabhan)

$$\phi(t) = \int dt \cdot \sqrt{-\frac{\dot{H}}{4\pi G}} \qquad \text{(D2)}$$

Assuming a decline of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$, Eq. (D3) yields

$$\phi(t) \sim \sqrt{\frac{2m}{4\pi G}} \cdot \left[2\varepsilon^{+}\right] \cdot t^{2\cdot\varepsilon^{+}} \qquad \text{(D3)}$$

As the scale factor of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ had time of the value of roughly $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ have a power law relationship drop below $a(t) \propto t^{1/2}$, the inflaton took Eq. (D3) 's value which may have been a factor as to the increase in the rate of acceleration, as noted by the q (z) factor , given in Beckwith's Journal of cosmology publication . Note that the way to relate an energy state to the inflaton is , if $a(t) = a_0 t^{\lambda}$, then in the early universe, one has a potential energy term of (Padmanabhan)

$$V(\phi) = V_0 \cdot \exp\left[-\sqrt{\frac{16\pi G}{\lambda}} \cdot \phi(t)\right] \qquad \text{(D4)}$$

A situation where both $\lambda = (1/2) - \varepsilon^{+}$ grows smaller, and, temporarily, $\phi(t)$ takes on Eq. (D3)'s value, even if the time value gets large, and also, if acceleration of the cosmic expansion is taken into account, then there is infusion of energy by an amount dV. The entropy dS ≃dV/T, will lead, if there is an increase in V, as given by Eq. (D4) a situation where there is an effective increase in entropy. Linking the two analytically, partly due to Yurov's suggestion about an explicit linkage between initial and final inflation (initial inflation being what happens right after Planck scale time , and final inflation being the speed up of acceleration seen as of the present era) would in the mind of the author, clinch the case for a non zero graviton mass.

Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No110752. The author thanks Jonathan Dickau for his review and suggestions for improvement given over several months in 2012 and 2013.

Bibliography

A. Beckwith, Applications of Euclidian Snyder Geometry to the Foundations of Space Time Physics",http://vixra.org/abs/0912.0012, v 6 (newest version) [February 2010a]

Andrew Beckwith, "Applications of Euclidian Snyder Geometry to Space Time Physics & Deceleration Parameter (DE Replacement?) Analysis of Linkage Between 1st, 2nd Inflation?", http://vixra.org/abs/1003.0194 [2010b]

A.W. Beckwith, http://journalofcosmology.com/BeckwithGraviton.pdf, Identifying a Kaluza Klein Treatment of a Graviton Permitting a Deceleration Parameter Q(z) as an Alternative to Standard DE, November, 2011a

Andrew Beckwith," If a Machian Relationship Between Gravitons and Gravitinos Exists, What Does Such a Relationship Imply as to Scale Factor and Quinessence Evolution and the Evolution of DM?", http://vixra.org/abs/1303.0224, March 2013

Pisen Chen, "Resonant Photon-Graviton Conversion in E M fields: From Earth to Heaven", SLAC Pub 6666, September (1994)

Eric Davis, "Producing Gravitons via Quantization of the Coupled Maxwell Fields," in Frontiers of Propulsion Science, Progress in Astronautics and Aeronautics Series, Vol. 227, eds. M. G. Millis and E. W. Davis, AIAA Press, Reston, VA, (2009)

Steven Kauffmann, " A Self Gravitational Upper bound On Localized Energy Including that of Virtual Particles and Quantum Fields, which Yield a Passable Dark Energy Density Estimate",
PUT IN WHICH VERSION OF THE DOCUMENT. I CITE THE EQN ON PAGE 12

M. Maggiore, *Gravitational Waves , Volume 1 : Theory and Experiment*, Oxford Univ. Press(2008)

Y.J. Ng, Entropy **10**(4), pp441-461 (2008); Y.J. Ng and H. van Dam, Found Phys **30**, pp795–805 (2000)

T. Padmanabhan, *An Invitation to Astrophysics*, World Scientific series in Astronomy and Astrophysics, Vol. 8

R. Penrose, *Cycles of Time*, The Bodley Head, 2010, London, UK

C.A. Salvoy, "Super symmetric Particle physics: A panorama, pp 299-328; from 18 Rencontres De Moriond, Volume 2 , "Beyond the Standard Model" , edited by J. Tran Thanh Van, Editions Frontieres, B. P. 44 91190 GIF Sur Yvette – France 1983

Matt Visser," Mass of the Graviton, arXiv: gr-qc/ 9705051 v2 Feb 26, 1998

Clifford Will, "Bounding the Mass of the Graviton using Gravitational-Wave observations of inspiralling compact Binaries", arXiv: gr-qc/ 9709011 v1 Sept 4, 1997

Clifford Will, "The confrontation between General Relativity and Experiment", Living Rev. of Relativity, 9, (2006),3, http://www.livingreviews.org/Irr-2006-3

A. Yurov; arXiv: hep-th/028129 v1, 19 Aug, 2002