Talent's error

1. Introduce

Euclid was a Greek mathematician, often referred to as the "Father of Geometry". He was active in Alexandria during the reign of Ptolemy I (323–283 BC). His Elements is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics(especially geometry) from the time of its publication until the late 19th or early 20th century. In the Elements, Euclid deduced the principles of what is now called Euclidean geometry from a small set of axioms. Euclid also wrote works on perspective, conic sections, spherical geometry, number theory and rigor. His methodology has also influenced many great scientists.

Most conclusions of a mathematical talent are correct. People adore him. But if a talent has made an imperceptible mistake, most people will trust it was correct also.

2. Prime is infinite

One of Euclid's famous proof is prime is infinite.

Suppose prime is finite, P = {p|2, 3, 5 … p_k }. Constructing a number p_{k+1} = 2×3×5×…×p_k + 1. All of 2, 3, 5 ,…, p_k can't divide

p_{k+1}. Either p_{k+1} is a bigger prime or p_{k+1} is a composite number that can resolve a prime being bigger than p_k.

So supposition is false, prime is infinite.

It's a clever proof.

3. Prime is finite

The interesting thing is like below.

Suppose prime is infinite. Constructing a number k= $p_k/(p_k-1)$, p_k is a prime, k is not an integer, otherwise (p_k-1) can divide p_k, then p_k has found a divisor and is a composite number. Because prime is infinite, $k = \lim_{p_k \to \infty} p_k / (p_k - 1) = 1$, k is an integer. (p_k-1) can divide p_k, then p_k has found a divisor and is a composite number.

So supposition is false, prime is finite.

4. Who is wrong

Both prime being finite and prime being infinite are correct? One of they must be wrong. But who is correct? Most people will say prime being infinite is correct. But why, I think the only reason is that it's Euclid's proof, no other more strong reason.

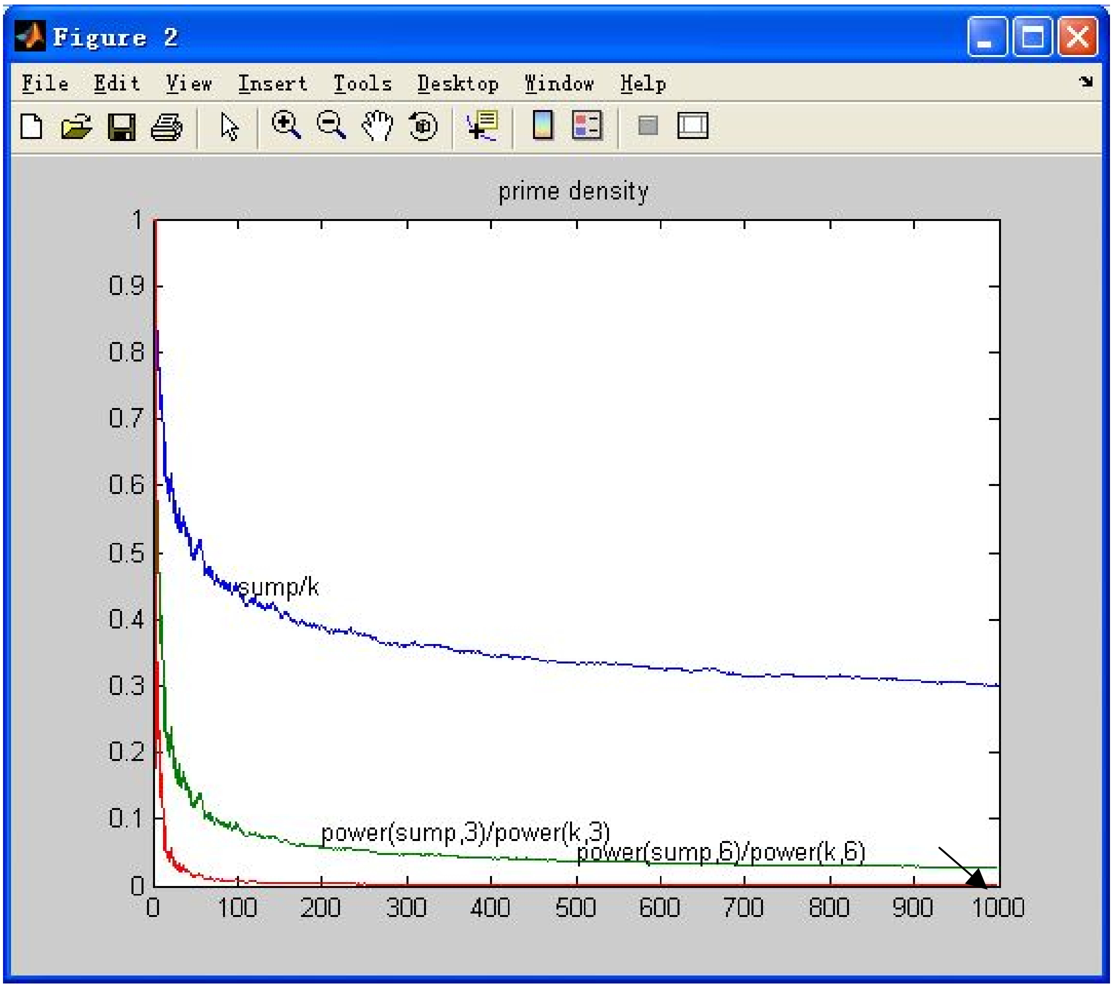
Firstly, let me make an experiment.

The density of prime ((count of prime)/(count of odd number)) is oscillating to trend to 0 when odd number is increasing.

Prime density data table like below, which displays the regularity very clearly.

2K+1	*sum(prime)/K*	$sum(p)^3/K^3$	$sum(p)^6/K^6$
00000003	1.0000	1.0000	1.0000
00000005	1.0000	1.0000	1.0000
00000007	1.0000	1.0000	1.0000
00000011	0.8000	0.5120	0.2621
00000013	0.8333	0.5787	0.3349
…	…	…	…
00000673	0.3601	0.0467	0.0022
00000677	0.3609	0.0470	0.0022
00000683	0.3607	0.0469	0.0022
00000691	0.3594	0.0464	0.0022
00000701	0.3571	0.0456	0.0021
…	…	…	…
00098807	0.1920	0.0071	0.0001
00098809	0.1920	0.0071	0.0001
00098837	0.1919	0.0071	0.0000
00098849	0.1919	0.0071	0.0000
…	…	…	…

The function plot like below.



From the experiment, it seems the evidence supports prime being finite.

5. Talent's error

To find out the root cause, I have checked both proofs carefully. Please read the proof again. "Constructing a number p_{k+1} = $2 \times 3 \times 5 \times \ldots \times p_k$ + 1. All of 2, 3, 5 ,…, p_k can't divide p_{k+1}. **Either p_{k+1} is a bigger prime or p_{k+1} is a composite number that can resolve a prime being bigger than p_k**." The bold section has included 3 prerequisites.

1. A number is either a prime, or a composite number.
2. Nature number can never reach infinite.
3. A composite number can always resolve a prime.

For prerequisite 1, it's easy to find an exception. Neither 1 is prime, nor composite number. Is infinite a composite number? Many people don't think infinite is a number. Then what's infinite?

For prerequisite 2, if nature number can never reach infinite, does the distance from infinite become more and more big or small?

If the distance from infinite becomes more and more big, it means that infinite is increasing also. It means that $\infty + 1 > \infty$, it's unreasonable.

If the distance from infinite becomes more and more small, when $(\infty - 2 \times 3 \times 5 \times \ldots \times p_k) < \varepsilon$, ε is smaller than any number, it means it can be smaller than 1. If $\varepsilon < 1$, then $(\infty - 2 \times 3 \times 5 \times \ldots \times p_k) < \varepsilon < 1$, $\Rightarrow$ p_{k+1} =

$2 \times 3 \times 5 \times \ldots \times p_k + 1 > \infty$, $\Rightarrow$ p_{k+1} has exceeded infinite.

For prerequisite 3, When $p_{k+1} > \infty$, neither it is a prime, nor can resolve a bigger prime.

So Euclid's proof is wrong.

6. Conclusion

A great mathematician can hardly make a mistake, but if it's an imperceptible mistake, which will mislead mathematician for a long time because of people's adoration.

Though Euclid is a great mathematician, he is a human being also. Both the experiment and logic analyze have demonstrated the famous proof is wrong.

Whatever, Euclid is a mathematician with my full respects for his great achievements.

References

[1] John Friedlander and Henryk Iwaniec, The polynomial X2+ Y4 captures its primes, 148 (1998), 945-1040
[2] John Friedlander and Henryk Iwaniec, Asymptotic sieve for primes, 148 (1998), 1041-1065
[3] University of TongJi ,Higher mathematics, 465(1991)
[4] E. Bombieri, The asymptotic sieve, Mem. Acad. Naz. dei XL, 1/2 (1976), 243-269.
[5] W. Duke, J.B. Friedlander, and H. Iwaniec, Equidistribution of roots of a quadratic congruence to prime moduli, Ann. of Math. 141 (1995), 423-441.
[6] C.L. Stewart and J. Top, On ranks of twists of elliptic curves and power-free values of binary forms, J. Amer. Math. Soc. 8 (1995), 943-973.
[7] E. Fouvry and H. Iwaniec, Gaussian primes, Acta Arith. 79 (1997), 249-287.
[8] J. Friedlander and H. Iwaniec, Bombieri's sieve, in Analytic Number Theory, Proc. Halberstam Conf., Allerton Park Illnois, June 1995, ed. B. C. Berndt et al., pp. 411-430, BirkhÄauser (Boston), 1996.
[9] , The polynomial X2 + Y 4 captures its primes, Ann. of Math. 148 (1998), 945-1040.
[10] G. Harman, On the distribution of ®p modulo one, J. London Math. Soc. 27 (1983),9-18.