Four Prime-Generating Recurrences

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Abstract : Prime number generating recurrences are introduced .
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1 Introduction

Let $a_1 = 1$, for $n > 1$, $a_n = a_{n-1} + lcm(a_{n-1}, n)$ [1] . This sequence has properties related to primes. For instance $a_{n+1}/a_n - 1$ consists of 1's or primes only .

2 Main result

Prime Number Generator I
Let $b_n = b_{n-1} + lcm(n-1, b_{n-2})$ *with* $b_1 = 2$ *and* $b_2 = 2$
then $a_n = b_{n+2}/b_n - 1$ *is either* 1 *or prime* [2] .

Conjecture 1
1. Every term of this sequence is either prime or 1 .
2. Every odd prime number is member of this sequence .

Maxima implementation

```
n;
ans:0;
n1:2;
n2:2;
list:[1,1];
```

```
(for k from 3 thru n do
(ans:n1+lcm(k-1,n1) ,
list:append(list,[ans/n1-1]) ,
n1:n2 ,
n2:ans))$
print(list);
```

Prime Number Generator II

Let $b_n = b_{n-1} + lcm(\lfloor\sqrt{2}\cdot n\rfloor, b_{n-1})$ *with* $b_1 = 2$
then $a_n = b_{n+1}/b_n - 1$ *is either* 1 *or prime* .

Conjecture 2

1. Every term of this sequence is either prime of the form $\lfloor\sqrt{2}\cdot n\rfloor$ or 1 .
2. Every prime of the form $\lfloor\sqrt{2}\cdot n\rfloor$ is member of this sequence .

Maxima implementation

```
n;
ans:0;
n1:2;
list:[1];
(for k from 2 thru n do
(ans:n1+lcm(floor(sqrt(2)*k),n1) ,
list:append(list,[ans/n1-1]) ,
n1:ans))$
print(list);
```

Prime Number Generator III

Let $b_n = b_{n-1} + lcm(\lfloor\sqrt{3}\cdot n\rfloor, b_{n-1})$ *with* $b_1 = 3$
then $a_n = b_{n+1}/b_n - 1$ *is either* 1 *or prime* .

Conjecture 3

1. Every term of this sequence is either prime of the form $\lfloor\sqrt{3}\cdot n\rfloor$ or 1 .
2. Every prime of the form $\lfloor\sqrt{3}\cdot n\rfloor$ is member of this sequence .

Maxima implementation

```
n;
ans:0;
n1:3;
list:[1];
(for k from 2 thru n do
(ans:n1+lcm(floor(sqrt(3)*k),n1) ,
list:append(list,[ans/n1-1]) ,
n1:ans))$
print(list);
```

Prime Number Generator IV

Let $b_n = b_{n-1} + lcm(\lfloor\sqrt{n^3}\rfloor, b_{n-1})$ *with* $b_1 = 2$
then $a_n = b_{n+1}/b_n - 1$ *is either* 1 *or prime* .

Conjecture 4

1. Every term of this sequence is either prime of the form $\lfloor\sqrt{n^3}\rfloor$ or 1 .
2. Every prime of the form $\lfloor\sqrt{n^3}\rfloor$ is member of this sequence .

Maxima implementation

```
n;
ans:0;
n1:2;
list:[1];
(for k from 2 thru n do
(ans:n1+lcm(floor(sqrt(k^3)),n1) ,
list:append(list,[ans/n1-1]) ,
n1:ans))$
print(list);
```

References

[1] OEIS Foundation Inc. (2011), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A135504 .
[2] OEIS Foundation Inc. (2011), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A217663 .