Special Relativity Predictions Using The Derivative

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Abstract

A peculiar result will be demonstrated with a spherical light wave (SLW) under special relativity (SR) when the SLW is intersecting the line $y = y_g$ for some $y_g > 0$ and in between two origins that are in relative motion. It will be shown SR predicts in the measurements of an unprimed frame that the SLW moves closer to the origin of a primed frame when measured from the line $y = y_g$. It will also be shown SR predicts in the measurements of the primed frame that the same SLW never moves closer to the primed origin when measured from the line $y = y_g$.

Keywords – Special Relativity, Light Sphere, Light Cone, Spherical light wave

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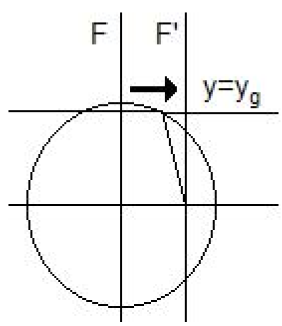
Introduction

Assume two coordinate systems F and F' are in relative motion in the standard configuration and a light pulse is emitted from the origins when they are co-located. Based on the light postulate, it is fundamental to SR that the SLW expands spherically from the origin of each frame F and F'. Otherwise, there will be light rays that do not measure a constant c. Assume some fixed $y_g > 0$. In the coordinates of the unprimed frame, once the SLW acquires the unprimed coordinate $(0, y_g, 0)$, it moves two different directions along the line $y = y_g$, one being in a direction closer to the primed origin when measured from the line $y = y_g$ and the other being in a direction away from both origins. In the coordinates of the primed frame, once the SLW acquires the primed coordinate $(0, y_g, 0)$, it moves two different directions along the line $y = y_g$, one in a direction closer to the unprimed origin, but further from the primed origin, and the other in a direction further from both origins. It will be proven that SR predicts both of the results above.

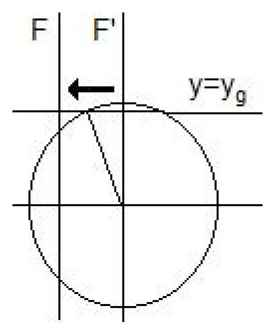
Method

Assume two coordinate systems F and F' are in relative motion in the standard configuration and a light pulse is emitted from the origins when they are co-located. The following two figures represent the predictions of SR for the SLW given the unprimed frame values of $x > 0$, $y_g > 0$ with $y = y_g$, $z = 0$ and $y_g / c < t < y_g \gamma / c$.



In the view of the unprimed frame, the SLW must expand spherically from the origin of the frame by the light postulate. When the SLW is in between the two origins and intersecting the line $y=y_g$, the expanding SLW moves closer to the primed origin when measured from the line $y=y_g$. The arrow represents the direction of intersection of the SLW with the line $y=y_g$.

Figure 1



In the view of the primed frame, the SLW must expand spherically from the origin of the frame by the light postulate. When the SLW is in between the two origins and intersecting the line $y=y_g$, the expanding SLW moves further from the primed origin when measured from the line $y=y_g$. The arrow represents the direction of intersection of the SLW with the line $y=y_g$.

Figure 2

It will be proven that SR predicts the results in both figure 1 and figure 2.

To prove the results of figure 1 in the unprimed system, the equation for the SLW is $c^2t^2 = x^2 + y^2 + z^2$. Next, assume $x > 0$ and some fixed $y_g > 0$ with $y = y_g$ and $z = 0$ then we have, $c^2t^2 = x^2 + {y_g}^2$. So, $x = \sqrt{c^2t^2 - {y_g}^2}$. Also, the primed origin is located at

vt for any time t in the unprimed frame. So, in the measurements of the unprimed frame, the distance to the primed origin from the location of the SLW intersecting the line $y = y_g$ given $y_g/c < t < y_g\gamma/c$ is $d' = \sqrt{(x - vt)^2 + y_g^{\,2}}$ with $x = \sqrt{c^2t^2 - y_g^{\,2}}$ or

$$d' = \sqrt{\left(\sqrt{c^2t^2 - y_g^{\,2}} - vt\right)^2 + y_g^{\,2}}\,.$$

To show the distance to the primed origin d' decreases as time increases between $y_g/c < t < y_g\gamma/c$, the partial derivative of d' with respect to time must be calculated and shown to be negative on that interval of time. So, calculate the partial derivative below.

$$\frac{\partial d'}{\partial t} = \frac{\left(\dfrac{c^2t}{\sqrt{c^2t^2 - y_g^{\,2}}} - v\right)\left(\sqrt{c^2t^2 - y_g^{\,2}} - vt\right)}{\sqrt{\left(\sqrt{c^2t^2 - y_g^{\,2}} - vt\right)^2 + y_g^{\,2}}}\,.$$

Based on $\sqrt{c^2t^2 - y_g^{\,2}}$ above, $y_g/c < t$ is immediate. Also since $c > v$, then $\left(\dfrac{c^2t}{\sqrt{c^2t^2 - y_g^{\,2}}} - v\right) > 0$. Finally, if $t < y_g\gamma/c$ then $\left(\sqrt{c^2t^2 - y_g^{\,2}} - vt\right) < 0$. So, if $y_g/c < t < y_g\gamma/c$ then $\dfrac{\partial d'}{\partial t} < 0$. Therefore, as the SLW expands away from the unprimed origin, time proceeds forward. As time proceeds forward on the interval $y_g/c < t < y_g\gamma/c$ with $x > 0$ in the measurements of the unprimed frame, the distance of the SLW to the primed origin as measured from the line $y = y_g$ decreases since $\dfrac{\partial d'}{\partial t} < 0$. Hence, the SLW moves in a direction closer to the primed origin when measured from the line $y = y_g$. So, the above SR calculations predict the results of figure 1.

Finally, it is shown that SR also predicts the conditions of figure 2, which represent the view of the primed frame. In that figure, as the SLW expands away from the origin of the primed frame and is located in between the two origins while intersecting the line $y = y_g$, the SLW only moves further from the primed origin when measured from the line $y = y_g$.

The equation for the SLW in the primed frame is $c^2 t'^2 = x'^2 + y'^2 + z'^2$. In the standard configuration, $y' = y$ and $z' = z$. Next, assume the same conditions $y = y_g$ and $z = 0$. Then, below is the equation for x'.

$$x' = \pm\sqrt{c^2 t'^2 - y_g{}^2} \; .$$

For this case, $x' < 0$ is assumed and so the plus is not applicable, thus,

$$x' = -\sqrt{c^2 t'^2 - y_g{}^2} \; .$$

Now, calculate the partial derivative below,

$$\frac{\partial x'}{\partial t'} = \frac{-c^2 t'}{\sqrt{c^2 t'^2 - y_g{}^2}} \, .$$

Since $t' > 0$ then $\frac{\partial x'}{\partial t'} < 0$. Hence, given time only proceeds forward, as t' increases, x' decreases since $\frac{\partial x'}{\partial t'} < 0$. But with $x' < 0$, then a decreasing x' becomes more negative, so x'^2 increases and thus $d' = \sqrt{x'^2 + y_g{}^2}$ increases. Therefore, the expanding SLW only moves further from the primed origin as it intersects the line $y' = y_g$ and is measured from the primed origin to the line $y' = y_g$ in the view of the primed frame. Thus, calculations of SR also predict the conditions of figure 2.

Therefore, SR predicts the one expanding SLW moves closer to the primed origin and does not move closer to the primed origin when it is in between the two origins and measured from the same line $y = y' = y_g$.

Experiment

If the mathematics above is correct, then it should be possible to force SR into a physical contradiction using an experiment that exploits the two frames' disagreement on the direction of motion of the SLW along the line $y = y' = y_g$. So, consider a primed frame car with a light detector, which is located at the bottom center of the car. If the detector is struck twice by light, then the car instantly acquires the same speed and same direction of motion as the unprimed frame. Hence, the car enters the unprimed frame if the detector is struck twice by light. Also, the experiment requires a flat piece of glass that has a reflective mirror surface on one side and a light-absorbing surface on the opposite side of the glass. Because of the standard configuration, the unprimed frame moves in the negative x direction in the view of the primed frame.

Next, assume some $y_g > 0$ and $v = \frac{1}{2}c$. The car is placed at $(0, 2y_g)$ in the primed frame with the front of the car pointing in the negative x direction and the bottom light detector directly facing the primed origin. Then, the glass is placed parallel to the primed y-axis centered at the primed coordinate $\left(-y_g \big/ \sqrt{4c^2/v^2 - 1}, y_g\right)$ with the reflective side perpendicular to and facing the primed y-axis. Finally, a SLW is emitted from the origin of the primed frame. See figure 3.

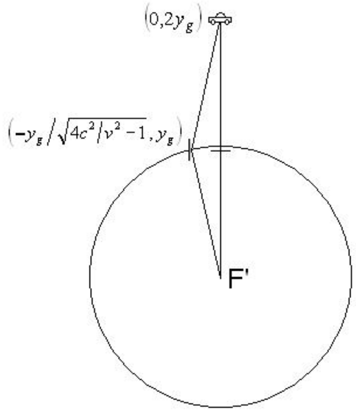


Figure 3

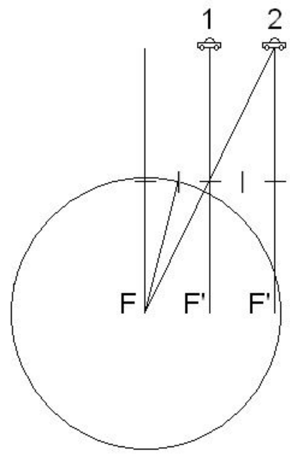


Figure 4

From figure 3, the SLW emitted from the origin of F' will strike the reflective side of the glass located at the location $\left(-y_g/\sqrt{4c^2/v^2-1},y_g\right)$. That causes a light beam to be reflected from $\left(-y_g/\sqrt{4c^2/v^2-1},y_g\right)$ to the coordinate $(0,2y_g)$. As the reflected beam travels toward $(0,2y_g)$, the expanding SLW travels up the y-axis of the primed frame and strikes the car's detector at time $t_1'=2y_g/c$. Then, the reflected beam will strike the car's detector at $t_2'=2y_g/\sqrt{c^2-v^2/4}$. Therefore, SR predicts the car will be struck twice by light. So, the car leaves the primed frame and enters the unprimed frame based on the primed frame's light postulate.

Next, assume that the unprimed origin was co-located with the primed origin when the SLW was emitted. The unprimed frame sees a completely different picture. See figure 4. In the unprimed frame, the SLW strikes the light absorbing side of the glass when the primed frame y-axis is located at position 1. Since the glass is light absorbing on that side, the light beam is absorbed. Afterwards, the bottom of the car is met by the expanding SLW when the primed frame's y-axis is at location 2. However, as one can see, there is no logic using the light postulate in the unprimed frame that will cause light from the SLW to strike the bottom detector of the car twice. More specifically, there is no light-reflecting surface that is exposed to the unprimed frame's SLW that would change the direction of a light beam such that it reflects in the correct direction to meet the bottom center of the primed frame's moving car. Therefore, assuming the truth of the light postulate in the unprimed frame, the car will not be struck twice by light. Hence, SR predicts that the car will remain in the primed fame.

Consequently, SR predicts that the car will move from the primed frame to the unprimed frame. Also, SR predicts that the car will not move from the primed frame, which is a physical contradiction.

Conclusion

It was proven in the coordinates of the unprimed frame, if $x > 0$, $y = y_g > 0$, $z = 0$ and $y_g / c < t < y_g \gamma / c$, then as the SLW expands in the context of the unprimed frame, the SLW moves in a direction closer to the primed origin when measured from the line $y = y_g$. On the other hand, it was proven in the coordinates of the primed frame that the same expanding SLW always moves in a direction further from the primed origin when measured from the line $y = y_g$.

Therefore, while the SLW is in between the two origins of F and F' and also intersecting the line $y = y_g$, SR predicts that the one SLW physically moves two different directions along the same line $y = y_g$. Additionally, under these same conditions, SR also predicts the one expanding SLW moves closer to the F' origin and does not move closer to the F' origin when measured from the line $y = y_g$.

In addition, a simple experiment was proposed in which SR predicts a car moves from the primed frame to the unprimed frame based on light postulate in the primed frame. But also, SR predicts the car does not move to the unprimed frame based on the truth of the light postulate in the unprimed frame. Therefore, SR predicts one SLW causes a car to changes frames and also, SR predicts the same SLW does not cause that same car to change frames.

References

[1] Einstein A., in *The Principle of Relativity* (Dover, New York) 1952, p. 37.