The spherical solution of the cosmology and the revised gravity field equation

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ABSTRACT

In the general relativity theory, using Einstein's revised gravity field equation (add the cosmological term), discover the spherical solution of the cosmology. In this time, the cosmology constant is concerned about the Hubble's constant. In this case, the general relativity theory only treats the present universe because the Hubble's constant is the inverse of the present universe's age.

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I.Introduction

This theory is that it discovers the spherical solution of the cosmology using the revised gravity field equation(add the cosmological term).

The spherical solution (The Schwarzshild solution) of the general relativity is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \tag{1}$$

II. Additional chapter-I

In this theory, the general relativity theory's revised field equation (add the cosmological term) is written completely.

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4}T_{\mu\upsilon} \tag{2}$$

Eq (2) multiply $g^{\mu\upsilon}$ and does contraction,

$$g^{\mu\upsilon}R_{\mu\upsilon} - \frac{1}{2}g^{\mu\upsilon}g_{\mu\upsilon}R - \Lambda g^{\mu\upsilon}g_{\mu\upsilon}$$

$$= -R - 4\Lambda = -\frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda} \tag{3}$$

Therefore, Eq (2) is

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}(-4\Lambda + \frac{8\pi G}{c^4}T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4}T_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} \tag{4}$$

In this time, the spherical coordinate system's vacuum solution is by $T_{\mu\upsilon} = 0$

$$R_{\mu\upsilon} = -\Lambda g_{\mu\upsilon} \tag{5}$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \tag{6}$$

Using Eq(6)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = \Lambda A \tag{7}$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = -\Lambda B \tag{8}$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -\Lambda r^2 \tag{9}$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \tag{10}$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0$$

$$R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \tag{11}$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

By Eq(11),

$$\dot{B} = 0 \tag{12}$$

By Eq(7) and Eq(8),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}\left(\frac{A'}{A} + \frac{B'}{B}\right) = -\frac{(AB)'}{rAB^2} = 0 \tag{13}$$

Therefore,

$$A = \frac{1}{B} \tag{14}$$

If Eq(9) is inserted by Eq(14),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left(\frac{r}{B}\right)' = -\Lambda r^2 \tag{15}$$

If solve Eq(15)

$$\frac{r}{B} = r + C - \frac{1}{3}\Lambda r^3 \rightarrow \frac{1}{B} = 1 + \frac{C}{r} - \frac{1}{3}\Lambda r^2 \tag{16}$$

In this time, be able to think following the formula.

$$C = -\frac{2GM}{c^2} \quad , \quad \Lambda = 3H_0^{\ 2}/c^2$$

The Hubble's constant H_0. (17)

$$\frac{1}{B} = 1 - \frac{2GM}{rc^2} - H_0^{\ 2} r^2 / c^2 \tag{18}$$

Therefore, Eq(18) is

$$A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} - H_0^{\ 2} r^2 / c^2 \tag{19}$$

To know Eq(19)'s third term, does Newton's limitation

$$\frac{d^2 x^\lambda}{d\tau^2} = -\Gamma^\lambda{}_{\mu\upsilon} \frac{dx^\mu}{d\tau} \frac{dx^\upsilon}{d\tau}$$

$$\frac{d^2 r}{dt^2} \approx \frac{1}{2} c^2 \frac{\partial(-A)}{\partial r} = -\frac{GM}{r^2} + H_0{}^2 r \qquad (20)$$

To know Eq(20)'s second term, use galaxies.

$$V_{galaxxy} = \frac{r_{galaxy}}{T_0} = H_0 r_{galaxy} \,,$$

$$a_{galaxy} = -\frac{GM_{galaxy}}{r_{galaxy}{}^2} + \frac{V_{galaxy}}{T_0} = -\frac{GM_{galaxy}}{r_{galaxy}{}^2} + H_0 \frac{r_{galaxy}}{T_0} = -\frac{GM_{galaxy}}{r_{galaxy}{}^2} + H_0{}^2 r_{galaxy}$$

M_{galaxy} is the galaxy's mass, V_{galaxy} is the velocity of the galaxy and the other galaxy, r_{galaxy} is the distant of the galaxy and the other galaxy, a_{galaxy} is the acceleration of the galaxy and the other galaxy, T_0 is the present universe's age.

(21)

III. Additional chapter-II

In this time, proves that is $\Lambda = 3H_0{}^2 / c^2$ in the cosmology.

Therefore, the general relativity theory's revised field equation (add the cosmological term) is written completely.

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R - \Lambda g_{\mu\upsilon} = R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R - 3H_0{}^2 / c^2 g_{\mu\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon} \,, \Lambda = 3H_0{}^2 / c^2 \qquad (22)$$

$$\tilde{T}_{\mu\upsilon} = T_{\mu\upsilon} - \frac{\Lambda c^4}{8\pi G} g_{\mu\upsilon} = T_{\mu\upsilon} - \frac{3H_0{}^2 c^2}{8\pi G} g_{\mu\upsilon} \qquad (23)$$

$$\tilde{T}_{\mu\upsilon} = \tilde{p} g_{\mu\upsilon} + (\tilde{p} / c^2 + \tilde{\rho}) U_\mu U_\upsilon \qquad (24)$$

$$\tilde{p} = p - \frac{\Lambda c^4}{8\pi G} = p - \frac{3H_0{}^2 c^2}{8\pi G} \,, \qquad (25)$$

$$\tilde{\rho} = \rho + \frac{\Lambda c^2}{8\pi G} = \rho + \frac{3H_0{}^2}{8\pi G} = \rho + \rho_c \,, \quad \rho_c = \frac{3H_0{}^2}{8\pi G} \qquad (26)$$

Invariant time $d\tau$ of the cosmology is

$$d\tau^2 = dt^2 - \Omega^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

(27)

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^\lambda{}_\lambda) - \Lambda g_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^\lambda{}_\lambda) - \frac{3H_0{}^2}{c^2} g_{\mu\upsilon} \,, \qquad (28)$$

$$T_{\mu\upsilon} = pg_{\mu\upsilon} + (p/c^2 + \rho)U_\mu U_\upsilon \,, \quad U_\mu = (c,0,0,0)$$

$$T_{00} = \rho(t)c^2 \,,\ T_{0i} = 0 \,,\ T_{ij} = p(t)g_{ij} \,, \quad T^\lambda{}_\lambda = -\rho(t)c^2 + 3p(t) \tag{29}$$

$$R_{00} = 3\frac{\ddot{\Omega}}{\Omega} = -\frac{4\pi G}{c^4}(\rho c^2 + 3p) + \Lambda \tag{30}$$

$$R_{ij} = -(\Omega\ddot{\Omega} + 2\dot{\Omega}^2 + 2k)\frac{g_{ij}}{\Omega^2} = -\frac{4\pi G}{c^4}(\rho c^2 - p)g_{ij} - \Lambda g_{ij} \tag{31}$$

$$(\Omega\ddot{\Omega} + 2\dot{\Omega}^2 + 2k)\frac{1}{\Omega^2} = \frac{4\pi G}{c^4}(\rho c^2 - p) + \Lambda \tag{32}$$

Therefore, Eq(30) - $3\times$ Eq(32) is

$$-(6\dot{\Omega}^2 + 6k)\frac{1}{\Omega^2} = -\frac{16\pi G}{c^2}\rho - 2\Lambda \quad , \ \Lambda = 3{H_0}^2/c^2$$

$$(\dot{\Omega}^2 + k)\frac{1}{\Omega^2} = \frac{8\pi G}{3c^2}\rho + \Lambda/3 = \frac{8\pi G}{3c^2}\rho + \frac{{H_0}^2}{c^2} \tag{33}$$

In this time,

$$\Omega(t) < \dot{\Omega}(t_0)(t - t_0) + \Omega(t_0) \tag{34}$$

$$\text{If} \quad t = 0 \ , \ \Omega(0) = 0$$

$$0 < -\dot{\Omega}(t_0)t_0 + \Omega(t_0) \quad , \quad t_0 < \frac{\Omega(t_0)}{c\dot{\Omega}(t_0)} = \frac{1}{H_0} \tag{35}$$

By Eq(33),Eq(35)

$$\rho_0 = \frac{3c^2}{8\pi G}\{\{\frac{\dot{\Omega}(t_0)}{\Omega(t_0)}\}^2 + \frac{k}{\Omega^2(t_0)} - \frac{{H_0}^2}{c^2})\} = \frac{3c^2}{8\pi G}\{\frac{{H_0}^2}{c^2} + \frac{k}{\Omega^2(t_0)} - \frac{{H_0}^2}{c^2}\} = \frac{3c^2}{8\pi G}\frac{k}{\Omega^2(t_0)}$$

$$\rho_0 = \rho(t_0) \tag{36}$$

By Eq(26),

$$\tilde{\rho}_0 = \rho_0 + \frac{\Lambda c^2}{8\pi G} = \rho_0 + \frac{3{H_0}^2}{8\pi G} = \frac{3c^2}{8\pi G}(\frac{k}{\Omega^2(t_0)} + {H_0}^2/c^2), \tag{37}$$

Therefore, the real present universe density $\tilde{\rho}_0 = \tilde{\rho}(t_0)$ is

$$\tilde{\rho}_0 = \frac{3c^2}{8\pi G}(\frac{k}{{\Omega_0}^2} + {H_0}^2/c^2) \,,\ \Omega_0 = \Omega(t_0) \,,\ \rho_c = \frac{3{H_0}^2}{8\pi G} \tag{38}$$

IV. Conclusion

Therefore, the spherical solution of the cosmology is

$$d\tau^2 = (1 - \frac{2GM}{rc^2} - H_0{}^2 r^2 / c^2) dt^2 - \frac{1}{c^2} [\frac{dr^2}{(1 - \frac{2GM}{rc^2} - H_0{}^2 r^2 / c^2)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2]$$

(39)

It found the spherical solution of the cosmology. And the cosmology constant is concerned about the Hubble's constant. In this case, the general relativity theory only treats the present universe because the Hubble's constant is the inverse of the present universe's age.

The general relativity theory's revised field equation (add the cosmological term) is written completely.

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} (T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) - \Lambda g_{\mu\upsilon} \quad (39)$$

*The co-moving system is

$$d\tau^2 = dt^2 - \frac{1}{c^2} [U(t,r) dr^2 + V(t,r)(d\theta^2 + \sin^2 \theta d\phi^2)]$$

$$T_{00} = \rho c^2 \text{ ,otherwise } T^{ti} = T^{ij} = 0 \quad (40)$$

$$\frac{1}{U} R_{rr} = \frac{1}{U} (\frac{V''}{V} - \frac{V'^2}{2V^2} - \frac{U'V'}{2UV}) - \frac{\ddot{U}}{2U} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{V}}{2UV} = -\frac{4\pi G}{c^2} \rho - \Lambda \quad (41)$$

$$\frac{1}{V} R_{\theta\theta} = -\frac{1}{V} + \frac{1}{U} (\frac{V''}{2V} - \frac{U'V'}{4UV}) - \frac{\ddot{V}}{2V} - \frac{\dot{V}\dot{U}}{4VU} = -\frac{4\pi G}{c^2} \rho - \Lambda \quad (42)$$

$$U = R^2(t) f(r) \quad , \quad V = R^2(t) r^2$$

Eq(41),Eq(42) are

$$\frac{f'(r)}{rf^2(r)} + [\ddot{R}(t)R(t) + 2\dot{R}^2(t)] = \frac{4\pi G}{c^2} R^2(t)\rho(t) + \Lambda R^2(t) \quad (43)$$

$$[\frac{1}{r^2} - \frac{1}{r^2 f(r)} + \frac{f'(r)}{2rf^2(r)}] + [\ddot{R}(t)R(t) + 2\dot{R}^2(t)] = \frac{4\pi G}{c^2} R^2(t)\rho(t) + \Lambda R^2(t) \quad (44)$$

$$\frac{f'(r)}{rf^2(r)} = \frac{1}{r^2} - \frac{1}{r^2 f(r)} + \frac{f'(r)}{2rf^2(r)} = -2k \quad (45)$$

$$f(r) = \frac{1}{1 - kr^2}$$

Therefore, in the revised gravity field equation (add the cosmological term), co-moving system's invariant time is

$$d\tau^2 = dt^2 - R^2(t) [\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (46)$$

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