Bounds upon Graviton Mass – using the difference between graviton propagation speed and HFGW transit speed to observe post-Newtonian corrections to Gravitational potential fields: Updated to take into account early universe cosmology and Penrose Cyclic conformal cosmology

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The author presents a post-Newtonian approximation based upon an earlier argument in a paper by Clifford Will as to Yukawa revisions of gravitational potentials, in part initiated by gravitons having explicit mass dependence in their Compton wave length. Prior work with Clifford Will's idea was stymied by the application to binary stars and other such astrophysical objects, with non-useful frequencies topping off near 100 Hertz, thereby rendering Yukawa modifications of Gravity due to gravitons effectively an experimental curiosity which was not testable with any known physics equipment. This work improves on those results.

Key words: Graviton mass, Yukawa potential, Post Newtonian Approximation

1. Introduction

Post-Newtonian approximations to General Relativity have given physicists a view as to how and why inflationary dynamics can be measured via deviation from simple gravitational potentials. One of the simplest deviations from the Newtonian inverse power law is a Yukawa potential modification of gravitational potentials. It is apparent that a graviton's mass (assuming it is massive) would factor directly into the Yukawa exponential term modification of gravity. This present paper tries to indicate how a smart experimentalist could use a suitably configured gravitational wave detector as a way to obtain more realistic upper bounds for the mass of a graviton, and explores how to use this idea as a template to investigate modifications of gravity along the lines of a Yukawa potential modification as given by Clifford Will. **Appendix A** summarizes why we think gravitons should be massive, i.e. with a small rest mass. We will summarize how our findings dovetail with a larger than Planck length radius of initial universe, and its connections with both cyclic conformal cosmology (Penrose) and recycled information into a new universe. Presumably the information transferred via massive Gravitons will be for setting Planck's constant at a given value at the onset of a new universe.

Secondly, this paper will address an issue of great import to the development of experimental gravity. Namely, if an upper mass to the graviton mass is identified; can an accelerator physicist use the theoretical construction Eric Davis raised in his book in the section "Producing Gravitons via Quantization of the coupled Maxwell-Einstein fields" for how an experimental bound to the graviton mass, as considered in this document, to refine our understanding of graviton Synchrotron radiation. A brief review of Chen, Chen, and Noble's application of the Gersenshtein effect will be made, to potentially improve their statistical estimates as to the range of graviton production.

2. Giving an upper bound to the mass of a graviton.

The easiest way to ascertain the mass of a graviton is to investigate if or not there is a slight difference in the speed of graviton 'particle' propagation and of HFGW in transit from a 'source' to the detector. Visser's (1998) mass of a graviton paper presents a theory which passes the equivalence test, but which has possible problem with depending upon a non-dynamical background metric. I.e. gravitons are assumed by both him, and also Clifford Will's write up of experimental G.R. to have mass

This document also accepts the view that there is a small graviton mass, which the author has estimated to be on the order of 10^{-60} kilograms. This is small enough so the following approximation is valid. Here, v_g is the speed of graviton 'propagation', λ_g is the Compton wavelength of a graviton with $\lambda_g = h/m_g c$, and $f \approx 10^{10}$ Hertz in line with L. Grischuck's treatment of relic HFGWs. In addition, the high value of relic HFGWs leads to naturally fulfilling $hf >> m_g c^2$ so that

$$v_g / c \approx 1 - \frac{1}{2} \cdot \left(c / \lambda_g \cdot f \right)^2 \tag{1}$$

But equation (1) above is an approximation of a much more general result which may be rendered as

$$\left(v_g / c \right)^2 \equiv 1 - \left(m_g c^2 / E \right)^2 \tag{2}$$

The terms m_g and E refers to the graviton rest mass and energy, respectively. Now physics researchers can ascertain what E is, with experimental data from a gravitational wave detector, and the next question needs to be addressed, relating to Visser's model. Namely; if D is the distance between a detector and the source of a HFGW/ Graviton emitter source

$$1 - v_g / c = 5 \times 10^{-17} \cdot \left[\frac{200Mpc}{D} \right] \cdot \left(\frac{\Delta t}{1\sec} \right) \tag{3}$$

The above formula depends upon, $\Delta t = \Delta t_a - \left(1 + Z\right) \cdot \Delta t_e$ with where Δt_a and Δt_e are the differences in arrival time and emission time of the two signals (HFGW and Graviton propagation), respectively, and Z is the redshift of the source.

Specifically, the situation for HFGW is that for early universe conditions, that $Z \geq 1100$, in fact for very early universe conditions in the first few milliseconds after the big bang, that $Z \sim 10^{25}$. This is an enormous number.

The first question which needs to be asked is, if the Visser non-dynamical background metric is correct, for early universe conditions so as to avoid the problem of the limit of small graviton mass does not coincide with pure GR, and the predicted perihelion advance, for example, violates experiment . A way forward would be to configure data sets so in the case of early universe conditions that one is examining appropriate $Z >> 1100$ but with extremely small Δt_e times, which would reflect upon generation of HFGW before the electro weak transition, and after the INITIAL onset of inflation.

I.e. a Gravitational wave detector system should be employed as to pin point experimental conditions so to high accuracy, the following is an adequate presentation of the difference in times, Δt . I.e.

$$\Delta t = \Delta t_a - \left(1 + Z\right) \cdot \Delta t_e \quad \rightarrow \Delta t_a - \varepsilon^+ \approx \Delta t_a \tag{4}$$

The closer the emission times for production of the HFGW and Gravitons are to the time of the initial nucleation of vacuum energy of the big bang, the closer we can be to experimentally using equation (4) above as to give experimental criteria for stating to very high accuracy the following.

$$1 - v_g / c \cong 5 \times 10^{-17} \cdot \left[\frac{200Mpc}{D} \right] \cdot \left(\frac{\Delta t_a}{1\sec} \right) \tag{5}$$

More exactly this will lead to the following relationship which will be used to ascertain a value for the mass of a graviton. By necessity, this will push the speed of graviton propagation very close to the speed of light. In this, we are assuming an enormous value for D

$$v_g / c \cong 1 - 5 \times 10^{-17} \cdot \left[\frac{200 Mpc}{D} \right] \cdot \left(\frac{\Delta t_a}{1 \sec} \right) \tag{6}$$

This equation (6) relationship should be placed into $\lambda_g = h / m_g c$, with a way to relate this above value of $\left(v_g / c \right)^2 \equiv 1 - \left(m_g c^2 / E \right)^2$, with an estimated value of E as an average value from field theory calculations, as well as to make the following argument rigorous, namely

$$\left[1 - 5 \times 10^{-17} \cdot \frac{200 Mpc}{D} \cdot \frac{\Delta t_a}{1 \sec} \right]^2 \cong 1 - \left(\frac{m_g c^2}{E} \right)^2 \tag{7}$$

A suitable numerical treatment of this above equation, with data sets could lead to a range of bounds for m_g, as a refinement of the result given by Clifford Will for graviton Compton wavelength bounded behavior for a lower bound to the graviton mass, assuming that h is the Planck's constant.

$$\begin{aligned} \lambda_g \equiv \frac{h}{m_g c} &> 3 \times 10^{12} km \cdot \left(\frac{D}{200 Mpc} \cdot \frac{100 Hz}{f} \right)^{1/2} \cdot \left(\frac{1}{f \Delta t} \right)^{1/2} \\ &\cong 3 \times 10^{12} km \cdot \left(\frac{D}{200 Mpc} \cdot \frac{100 Hz}{f} \right)^{1/2} \cdot \left(\frac{1}{f \Delta t_a} \right)^{1/2} \end{aligned} \tag{8}$$

The above equation (8) gives an upper bound to the mass m_g as given by

$$m_g < \left(\frac{c}{h} \right) \Big/ 3 \times 10^{12} \, km \cdot \left(\frac{D}{200 Mpc} \cdot \frac{100 Hz}{f} \right)^{1/2} \cdot \left(\frac{1}{f \cdot \Delta t_a} \right)^{1/2} \tag{9}$$

Needless to say, an estimation of the bound for the graviton mass m_g, and the resulting Compton wavelength λ_g would be important to get values of the following formula for experimental validation

$$V(r)_{gravity} \cong \frac{MG}{r} \exp\left(r / \lambda_g \right) \tag{10}$$

Clifford Will gave for values of frequency $f \equiv 100$ Hertz enormous values for the Compton wavelength, i.e. values like $\lambda_g > 6 \times 10^{19} km$. Such enormous values for the

Compton wavelength make experimental tests of equation (10) practically infeasible. Values of $\lambda_g \approx 10^{-5}$ centimeters or less for very high HFGW data makes investigation of equation (10) above far more tractable.

3. Application to Gravitational Synchrotron radiation, in accelerator physics

Eric Davis, quoting Pisen Chen's article written in 1994 estimates that a typical storage ring for an accelerator will be able to give approximately $10^{-6} - 10^{3}$ gravitons per second. Quoting Pisen Chen's 1994 article, the following for graviton emission values for a circular accelerator system, with m the mass of a graviton, and M_P being the Planck mass. N as mentioned below is the number of 'particles' in a ring for an accelerator system, and n_b is an accelerator physics parameter for bunches of particles, which for the LHC is set by Pisen Chen to the value of 2800, and N for the LHC is about 10^{11}. And, for the LHC Pisen Chen sets γ at $.88 \times 10^{2}$, with $\rho[m] \approx 4300$. Here, $m \sim m_{graviton}$ acts as a mass charge.

$$N_{GSR} \sim 5.6 \cdot n_b^2 \cdot N^2 \cdot \frac{m^2}{M_P^2} \cdot \frac{c \cdot \gamma^4}{\rho} \tag{11}$$

The immediate consequence of the prior discussion would be to obtain a more realistic set of bounds for the graviton mass, which could considerably refine the estimate of 10^{11} gravitons produced per year at the LHC, with realistically 365 x 86400 *seconds* = 31536000 *seconds* in a year, leading to 3.171×10^{3} gravitons produced per second. Refining an actual permitted value of bounds for the accepted graviton mass, m, as given above, while keeping $M_P \sim 1.2209 \times 10^{19}$ *GeV*/c^2 would allow for a more precise value for gravitons per second which significantly enhances the chance of actual detection, since right now for the LHC there is too much general uncertainty about where to place a detector for actually capturing / detecting a graviton.

4. Conclusion, falsifiable tests for the Graviton are closer than the physics community thinks

The physics community now has an opportunity to experimentally infer the existence of gravitons as a knowable and verifiable experimental datum with the onset of the LHC as an operating system. Even if the LHC is not used, Pisen Chen's parameterization of inputs from the table right after his equation (8) as inputs into equation (11) above will permit the physics community to make progress toward the detection of Gravitons for, say the Brookhaven laboratory site circular ring accelerator system. See **Appendis B** for that table. Tony Rothman's statement about needing a detector the size of Jupiter to obtain a single experimentally falsifiable set of procedures is defensible only if the wave-particle duality induces so much uncertainty as to the mass of the purported graviton, that worst case model building and extraordinarily robust parameters for a Rothman style graviton detector have to be put in place.

A suitably configured detector can help with bracketing a range of masses for the graviton, as a physical entity subject to measurements. Such an effort requires obtaining rigorous verification of the approximation used to the effect that $\Delta t = \Delta t_a - \left(1+Z\right)\cdot \Delta t_e$ $\to \Delta t_a - \varepsilon^{+} \approx \Delta t_a$ is a defensible approximation. Furthermore, having realistic estimates for distance D as inputs into equation (9) above is essential.

The expected pay offs of making such an investment would be to determine the range of validity of equation (10) , i.e. to what degree is gravitation as a force is amendable to post Newtonian approximations.

The author asserts that equation (10) can only be realistically be tested and vetted for sub atomic systems, and that with the massive Compton wavelength specified by Clifford Will cannot be done with low frequency gravitational waves.

Furthermore, a realistic bounding of the graviton mass would permit a far more precise calibration of equation (11) as given by Pisen Chen in his 1994 article. We refer the reader to **Appendix C** for how we expect that Eq. (11) and **Appendis A**, Eq (A4) may be combined to yield experimentally falsifiable tests for a massive graviton.

Appendix A: Indirect support for a massive graviton

We follow the work of Seven Kenneth Kauffmann in terms of how he formulated the following equation for specifying an inter relationship between an initial radius we will call R for an expanding universe, and a "gravitationally based energy" expression we will call $T_G\left(r\right)$ which lead to, with manipulations, We start off with Kauffman's

$$R\cdot\left(\frac{c^4}{G}\right) \geq \int\limits_{|r''|\leq R} T_G\left(r+r''\right) d^3 r'' \qquad \text{(A1)}$$

Kauffmann calls $\left(\frac{c^4}{G}\right)$ a "Planck force" which is relevant due to the fact we will emply Eq. (A1) at the initial time step of the universe, in the Planckian regime of space-time.Also, we make full use of setting for small r, the following:

$$T_G\left(r+r''\right) \approx T_{G=0}\left(r\right)\cdot const \sim V(r) \sim m_{Braviton}\cdot n_{Initial-entropy}\cdot c^2 \qquad \text{(A2)}$$

I.e. what we are doing is to make the expressin in the integrand proportinal to information leaked by a past universe into our present universe, with Ng style quantum infinite statistics use of

$$n_{Initial-entropy} \sim S_{Graviton-count-entropy} \qquad \text{(A3)}$$

Then Eq. (A1) will lead to

$$R\cdot\left(\frac{c^4}{G}\right)\geq\int_{|r''|\leq R}T_G\left(r+r''\right)d^3r''\approx const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy}\sim S_{Graviton-count-entropy}\right]$$

$$\Rightarrow R\cdot\left(\frac{c^4}{G}\right)\geq const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy}\sim S_{Graviton-count-entropy}\right] \qquad \text{(A4)}$$

$$\Rightarrow R\geq\left(\frac{c^4}{G}\right)^{-1}\cdot\left[const\cdot m_{Graviton}\cdot\left[n_{Initial-entropy}\sim S_{Graviton-count-entropy}\right]\right]$$

Here, $\left[n_{Initial-entropy}\sim S_{Graviton-count-entropy}\right]\sim 10^5$, $m_{Graviton}\sim 10^{-62}\, grams$, and

1 planck length = 1.616199 × 10^{-35} meters

where we set $l_{Planck}=\sqrt{\frac{\hbar G}{c^3}}$ with $R\sim l_{Planck}\cdot 10^{\alpha}$, and $\alpha>0$. Typically $R\sim l_{Planck}\cdot 10^{\alpha}$ is about $10^3\cdot l_{Planck}$

Appendix B: Graviton paper table from Pisen Chen

Storage Rings	PEP-11	LEP-1	LEP-II	HERA	LHC
$\varepsilon[GeV]$	**9**	**50**	**100**	**880**	**7000**
$\gamma[10^3]$	**18**	**100**	**100**	**7.5**	**.88**
$N[10^{10}]$	**3.8**	**45**	**45**	**10**	**10**
n_b	**1700**	**4**	**4**	**210**	**2800**
$l[cm]$	**3.46**	**6.24**	**6.24**	**27.7**	**18.4**
$\rho[m]$	**500**	**4300**	**4300**	**1035**	**4300**
Gravitational SR					
$\omega_0[kHz]$	**600**	**70**	**70**	**290**	**70**
$N_{GSR}[10^{-7}\sec^{-1}]$	**1.3**$\times 10^3$	**38**	**150**	**6**$\times 10^6$	**18**$\times 10^8$
Resonant conversion					
$\omega_c[10^9 GHz]$	**3.5**	**70**	**560**	**.12**	**4.8**$\times 10^{-5}$
$N_{Res}[10^{-7}\sec^{-1}]$	**.1**	**.1**	**,3**	10^3	**2**$\times 10^5$

Appendix C. Summarizing how to get stricter bounds to Eq. (A4) above

To start, we can first of all review briefly what was done by Beckwith, in 2011 in the Journal of cosmology. In this publication,Beckwith outlined how there may be a contribution via a small massive graviton as to re acceleration of the universe. Here the value of $m_{Graviton} \sim 10^{-62}\, grams$ to get a speed up of acceleration of cosmological expansion a billion years ago. This though, does not cleave to the essence of the problem, though, as seen in growing neutrino mass theories and cosmology, there conceivably could be growing mass for early universe gravitons. The author thinks not and appeals to what was done in another publication, i.e. a talk in Italy 2011, see pages 36-43 where a variant of Penrose cyclic conformal cosmology is appealed to as a recycling of prior universe information in order to have a uniform Planck's contant, per cycle. The idea is also stated by R. Penrose in his recent book. Needless to state that the author views that a minimum entropy of 10^5 as a transfer mechanism from prior to present space time is important and should be reviewed, partly to give constant values to the present value of Planck's constant from cycle to cycle. Cyclic conformal cosmology, or a test confirming its initial CMBR pattern would in itself do much to add to at least a plausable explanation as to how consistancy in Planck's constant from cycle to cycle could be maintained. Also, the author appeals to a calculation given by Giovanni, as to the effect that if the Penrose supposition of

$$
\begin{aligned}
&10^{10^{90}} \approx 10^{Final-entropy-Gravitons=S_{FINAL-GRAVITONS}} \\
&\sim Phase \;\; space \;\; due \;\; to \;\; Gravitons
\end{aligned}
\tag{C1}
$$

is true (given to us by Penrose) then the relationship given by Giovanni in his book for entropy that the present entropy of the universe, i.e. about $10^{88} - 10^{90}$ is obtained by integrating from 10-^ 11 Hertz down to 10^-19 Hertz would do much to argue for a constant graviton mass, as given above. Furthermore, this could be with work also connected with what the author referred to as first and second inflation, as given by his work in 2010. This would be a project warranting serious investigation. First inflation is the typical inflation given in cosmology, whereas the 2^{nd} inflation is the speed up of the universe, as referenced in Beckwith's 2011 publications. Linking the two analytically, partly due to Yurov's suggestion would in the mind of the author, clinch a constant graviton mass. The Yurov suggestion will be worked upon seriously by the author in future publications and is crucial to making initial and final graviton mass, about the same order of magnitude.

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