A common underlying cause for quantum phenomena and cosmological data

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Abstract

The science of physics possesses today an adequate amount of knowledge that allows us to search for the first principles that govern physical reality. It is in the spirit of this search that we performed the study presented in this edition. The physical theories of the last century did not have the necessary completeness in order to justify the quantum phenomena and the cosmological data. There is a fundamental physical law that prevails from the microcosm to the observations we perform billions of light years away.

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Introduction

The study we present in the current edition is based on two assumptions that are taken as axioms. The first assumption is that the rest masses m_0 and electric charges q of material particles increase with the passage of time (selfvariations). The second assumption is that the consequences of the selfvariations propagate through four-dimensional spacetime with a zero arc length: $dS^2 = 0$. The set of consequences arising from these two assumptions constitutes the "theory of selfvariations".

An immediate consequence of the statements-axioms we have introduced, is the concept of the generalized photon: a particle carrying energy E, linear momentum $\mathbf{P}$, and moving with velocity $\boldsymbol{\upsilon}$, of magnitude $\|\boldsymbol{\upsilon}\| = c$, in every inertial frame of reference. The generalized photon correlates the material particle with its surrounding spacetime. In its simplest version, the generalized photon is emitted by the material particle into its surrounding spacetime. When the material particle is electrically charged, the generalized photon, apart from energy and momentum, also carries electric charge.

The following figure represents the arbitrary motion of a material point particle moving with velocity $\mathbf{u}$ in an inertial frame of reference $O(x, y, z, t)$.

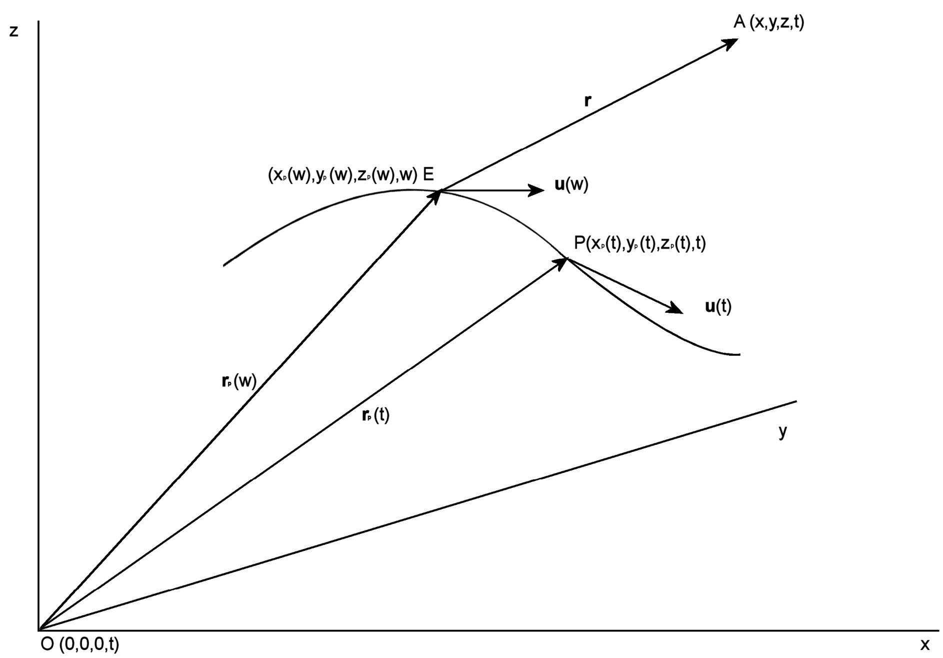

Figure 1 : A material point particle moving arbitrarily. As the material particle moves from point $E(x_p(w), y_p(w), z_p(w), w)$ to point $P(x_p(t), y_p(t), z_p(t), t)$, the generalized photon moves from point $E(x_p(w), y_p(w), z_p(w), w)$ to point $A(x, y, z, t)$.

A generalized photon is emitted by the material particle at time $w = t - \frac{r}{c}$, from point $E(x_p(w), y_p(w), z_p(w), w)$, and arrives at time t at point $A(x, y, z, t)$. The velocity of the generalized photon in Figure 1, is

$$\boldsymbol{\upsilon} = \frac{c}{r}\boldsymbol{r}$$

where $r = \|r\|$. We express the vector $\frac{\boldsymbol{\upsilon}}{c}$ in the trigonometric form

$$\frac{\boldsymbol{\upsilon}}{c} = \begin{bmatrix} \frac{\upsilon_x}{c} \\ \frac{\upsilon_y}{c} \\ \frac{\upsilon_z}{c} \end{bmatrix} = \begin{bmatrix} \cos\delta \\ \sin\delta\cos\omega \\ \sin\delta\sin\omega \end{bmatrix}$$

Furthermore, we define the following two vectors

$$\boldsymbol{\beta} = \begin{bmatrix} -\sin\delta \\ \cos\delta\cos\omega \\ \cos\delta\sin\omega \end{bmatrix}$$

$$\boldsymbol{\gamma} = \begin{bmatrix} 0 \\ -\sin\omega \\ \cos\omega \end{bmatrix}$$

The vectors $\frac{\boldsymbol{\upsilon}}{c}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ constitute a right-handed, orthonormal vector basis that accompanies the generalized photon in its motion. The consequences of the selfvariations are expressed as functions of the parameters $w = t - \frac{r}{c}, r, \delta, \omega$.

The basic study of the selfvariations leads to two fundamental theorems: the "Fundamental Mathematical Theorem", and the "Trajectory Representation Theorem". The first theorem allows us to correlate any change in energy manifested on the material particle at point $E(x_p(w), y_p(w), z_p(w), w)$ with a corresponding change in energy at point $A(x,y,z,t)$ of Figure 1. The second theorem represents the tangent vector, the curvature and the torsion of the trajectory of the material particle onto the geometric characteristics of the generalized photon in the surrounding spacetime. The two theorems allow us to express quantitatively the consequences of the selfvariations on the surrounding spacetime of the material particle. As a consequence of the selfvariations, in the surrounding spacetime of the material particle there is energy of density D

$$D = -c\frac{\partial m_0}{\partial w}\frac{1}{4\pi\gamma^3 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^4}$$

and momentum of density J

$$\boldsymbol{J} = D\frac{\boldsymbol{\upsilon}}{c^2}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$, and $\mathbf{u} = \mathbf{u}(w)$.

If the material particle is electrically charged, then in the surrounding spacetime there is also electric charge of density ρ

$$\rho = -\frac{\partial q}{c\partial w}\frac{1}{4\pi\gamma^2 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}$$

and electric current of density j

$$\boldsymbol{j} = \rho\boldsymbol{\upsilon}$$

The Lienard-Wiechert potentials

$$V = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\mathbf{u}}{c^2}\right)}$$

$$\mathbf{A} = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\mathbf{u}}{c^2}\right)}\mathbf{u}$$

are not compatible with the theory of selfvariations. Therefore, they are replaced by the potentials of the selfvariations

$$V=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}+\frac{q\left(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}\right)}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}$$

$$\boldsymbol{A}=V\frac{\boldsymbol{\upsilon}}{c^2}$$

where $\boldsymbol{\alpha}=\boldsymbol{\alpha}(w)$ is the acceleration of the material particle.

The potentials of the selfvariations are separated into two individual pairs

$$V_u=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}$$

$$\boldsymbol{A}_u=V_u\frac{\boldsymbol{\upsilon}}{c^2}$$

and

$$V_\alpha=\frac{q\left(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}\right)}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}$$

$$\boldsymbol{A}_\alpha=V_\alpha\frac{\boldsymbol{\upsilon}}{c^2}$$

The $\left(V_u, A_u\right)$ pair gives the electromagnetic field $\left(\boldsymbol{\varepsilon}_u, \mathbf{B}_u\right)$ that accompanies the electrically charged material particle

$$\boldsymbol{\varepsilon}_u=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)$$

$$\boldsymbol{B}_u=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}$$

The $\left(V_\alpha, A_\alpha\right)$ pair gives the electromagnetic radiation

$$\boldsymbol{\varepsilon}_\alpha = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)-\boldsymbol{\alpha}\right]$$

$$\boldsymbol{B}_\alpha = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\alpha}\right]$$

The pair (V_α, A_α) of the electromagnetic radiation potentials does not depend on the distance r. For each couple $(\boldsymbol{\varepsilon}, \mathbf{B})$ the following relation holds

$$\boldsymbol{B} = \frac{\boldsymbol{\upsilon}}{c^2}\times\boldsymbol{\varepsilon}$$

The energy-momentum tensor for the generalized photon that results from the selfvariation of the rest mass m_0 of the material particle is given by the matrix Φ^{ij}

$$\Phi^{ij} = \frac{D}{c^2}\begin{bmatrix} c^2 & c\upsilon_x & c\upsilon_y & c\upsilon_z \\ \upsilon_x c & \upsilon_x^2 & \upsilon_x\upsilon_y & \upsilon_x\upsilon_z \\ \upsilon_y c & \upsilon_y\upsilon_x & \upsilon_y^2 & \upsilon_y\upsilon_z \\ \upsilon_z c & \upsilon_z\upsilon_x & \upsilon_z\upsilon_y & \upsilon_z^2 \end{bmatrix}$$

where $\begin{bmatrix} c \\ \upsilon_x \\ \upsilon_y \\ \upsilon_z \end{bmatrix} = \begin{bmatrix} \upsilon^0 \\ \upsilon^1 \\ \upsilon^2 \\ \upsilon^3 \end{bmatrix}$, $i, j = 0,1,2,3$

The energy-momentum tensor for the generalized photon that results from the selfvariation of the electric charge q of the material particle is given by the matrix Φ^{ij}

$$\Phi^{ij} = \begin{bmatrix} w & cS_x & cS_y & cS_z \\ cS_x & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ cS_y & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ cS_z & \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} - \frac{\rho V}{c^2}\begin{bmatrix} c^2 & c\upsilon_x & c\upsilon_y & c\upsilon_z \\ \upsilon_x c & \upsilon_x^2 & \upsilon_x\upsilon_y & \upsilon_x\upsilon_z \\ \upsilon_y c & \upsilon_y\upsilon_x & \upsilon_y^2 & \upsilon_y\upsilon_z \\ \upsilon_z c & \upsilon_z\upsilon_x & \upsilon_z\upsilon_y & \upsilon_z^2 \end{bmatrix}$$

where $(S_x, S_y, S_z) = \mathbf{S} = \varepsilon_0\boldsymbol{\varepsilon}\times\mathbf{B}$ is the Poynting vector, $W = \frac{1}{2}\varepsilon_0\left(\boldsymbol{\varepsilon}^2 + c^2\mathbf{B}^2\right)$ and

$$\sigma_{\alpha\beta} = \varepsilon_0\left(-\varepsilon_\alpha\varepsilon_\beta - c^2 B_\alpha B_\beta + W\delta_{\alpha\beta}\right)$$

$$\delta_{\alpha\beta} = \begin{cases} 1, if\ \alpha = \beta \\ 0, if\ \alpha \neq \beta \end{cases}$$

$$(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (\varepsilon_x, \varepsilon_y, \varepsilon_z) = \boldsymbol{\varepsilon}$$
$$(\mathrm{B}_1, \mathrm{B}_2, \mathrm{B}_3) = (\mathrm{B}_x, \mathrm{B}_y, \mathrm{B}_z) = \mathbf{B}$$

$$\alpha, \beta = 1, 2, 3$$

The energy-momentum tensors Φ^{ij} give us important information about the energy content of the surrounding spacetime of the material particle. Furthermore, they are related with the gravitational and the electromagnetic interaction. As we progress in our study however, it becomes evident that there is information about the energy content and the properties of spacetime, that is not contained within the Φ^{ij} tensors.

The study we presented up to this point has been conducted without a quantitative determination of the selfvariations. We made the assumption of the selfvariations in order to undertake the relevant calculations, but we have not determined quantitatively the rate at which they evolve, i.e. the $\frac{\partial m_0}{\partial w}$ and $\frac{\partial q}{\partial w}$. In order to study the consequences of the selfvariations, we have to quantitatively determine these rates.

The quantitative determination of the selfvariations is made on the basis of the total energy E_s and the total momentum $\mathbf{P}_s$ emitted simultaneously in all directions, by the material particle. The rest mass m_0 and the electric charge q of the material particle vary according to the operators

$$\frac{\partial}{\partial t} \to -\frac{i}{\hbar} E_s$$
$$\nabla \to \frac{i}{\hbar} \mathbf{P}_s$$

where h is Planck's constant, and $\hbar = \frac{h}{2\pi}$. The law of selfvariations expresses a continuous interaction between the material particle and the generalized photons.

The partial contribution of an individual generalized photon to the law of selfvariations is determined by the percentage-function Φ. Due to this, function Φ has a fundamental role in the energy content of the generalized photon.

The energy E and momentum $\mathbf{P}$ of the generalized photon that is related to the selfvariation of the rest mass m_0 of the material particle, are given by the equations

$$E = \Phi \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + i\hbar \frac{\partial \Phi}{\partial t}$$

$$\mathbf{P} = \Phi \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \nabla \Phi$$

The equations that give the energy and momentum of the generalized photon that is related to the selfvariation of the electric charge of the material particle, are of similar form.

The energy E and the momentum $\mathbf{P}$ of the generalized photon do not obey the simple relation

$$\mathbf{P} = E \frac{\boldsymbol{\upsilon}}{c^2}$$

That relation is a special case of the general relation

$$\mathbf{P} = E \frac{\boldsymbol{\upsilon}}{c^2} - \frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \boldsymbol{\beta} - \frac{i\hbar}{r \sin\delta} \frac{\partial \Phi}{\partial \omega} \boldsymbol{\gamma}$$

The generalized photon determines the relation of the material particle with the surrounding spacetime. Furthermore, it is related with the energy content of spacetime and, hence, with the very properties of spacetime. Because of this, a large part of the study we present in the present edition concerns the generalized photon and its properties. The resulting equations contain an exceptionally large body of data and information. Thus, we shall confine ourselves to a brief report for the structure and the properties of the generalized photon.

The generalized photon carries four energy-momentum pairs, each of which transforms autonomously, independently of the rest, according to Lorentz-Einstein. Two of these pairs do not possess rest energy, do not depend on the distance r from the material particle, are defined both on the material particle and on the surrounding spacetime, while they do not possess intrinsic angular momentum (spin). The other two energy-momentum pairs have, respectively, rest energy

$$\pm \frac{c\hbar}{r} \frac{\partial \Phi}{\partial \delta}$$

$$\pm \frac{c\hbar}{r \sin\delta} \frac{\partial \Phi}{\partial \omega}$$

Their energy and momentum are inversely proportional to the distance r from the material particle, they are not defined on the material particle but only on the surrounding spacetime, while they possess intrinsic angular momentum (spin), given respectively by

$$-i\hbar\frac{\partial\Phi}{\partial\delta}\boldsymbol{\gamma}$$

$$\frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\beta}$$

The total intrinsic angular momentum $\mathbf{S}$ of the generalized photon is given by relation

$$\mathbf{S}=\frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\beta}-i\hbar\frac{\partial\Phi}{\partial\delta}\boldsymbol{\gamma}$$

The intrinsic angular momentum of the generalized photon exhibits some remarkable properties. The first is that it does not depend on the distance r from the material particle, while it is also defined on the material particle itself. Furthermore, the component

$$S_u=i\hbar\frac{\partial\Phi}{\partial\omega}$$

in the direction of the velocity of the material particle, remains invariant under the action of the Lorentz-Einstein transformations and is, therefore, constant in all inertial reference frames. Another property of the intrinsic angular momentum of the generalized photon is that it does not vanish even if we consider that the material particle is motionless. In other words, the generalized photon carries intrinsic angular momentum even in the inertial reference frame in which the material particle is at rest. In that sense, we can characterize the intrinsic angular momentum of the generalized photon as “rest angular momentum”. One final property, which is not included in the present edition is the following: during the interaction of the generalized photon with a material particle, the variation $\Delta\mathbf{S}$ of the angular momentum of the generalized photon manifests a component along the direction of the vector $\frac{\boldsymbol{\upsilon}}{c}$.

Of particular interest is the fact that the generalized photon, in its general version, implies the existence of rest energy in the surrounding spacetime of the material particle. The existence of this energy results as a general consequence of the equations of the theory of selfvariations.

We remind that the law of the selfvariations has been stated on the basis of the total energy E_s and the total momentum $\mathbf{P}_s$ of the generalized photons emitted simultaneously and in all directions by the material particle. We can easily prove that between the energy E_s and the momentum $\mathbf{P}_s$ the following relation holds

$$\mathbf{P}_s=E_s\frac{\mathbf{u}}{c^2}$$

where $\mathbf{u}=\mathbf{u}(w)$ is the velocity of the material particle at the moment of emission of the generalized photons. The energy E_s is always correlated with a rest energy

$E_0 \neq 0$ through equation $E_s = \gamma E_0$, where $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$. Therefore, in the energy E_s, which results from the aggregation of the generalized photons, a rest mass of $\frac{E_0}{c^2} \neq 0$ is implicit. The law of selfvariations expresses exactly the interaction between the rest mass m_0 of the material particle, and the rest mass $\frac{E_0}{c^2}$ that results from the aggregation of the generalized photons.

The physical object that results from the aggregation of the generalized photons, always accompanies the material particle. Because of this, we named it "accompanying particle". The accompanying particle has rest mass $\frac{E_0}{c^2}$, while in the part of spacetime it occupies it holds that $dS^2 = 0$. The combination $\frac{E_0}{c^2} \neq 0$ and $dS^2 = 0$, leads to the conclusion that the accompanying particle corresponds to an intermediate state between "matter" ($\frac{E_0}{c^2} \neq 0$) and the "photon" ($dS^2 = 0$). This intermediate state of matter is the cause of quantum phenomena, and its prediction constitutes one of the most important results of the theory of selfvariations.

In Nature, the system material particle-accompanying particle exists and behaves as a "generalized particle" which extends in a part of spacetime. The part of space occupied by the generalized particle can be the point where the material particle is located, or it can extend up to an infinite distance away from the material particle. In the part of spacetime where the generalized particle extends, the trajectories and velocities of the generalized photons are altered with respect to the strictly defined trajectories and velocities presented in Figure 1. There is an extreme case where the concepts of trajectory and velocity of the generalized photon become meaningless; they are not defined. The same is true for the trajectory and velocity of the material particle in case it is located in the part of spacetime occupied by the generalized particle. This prediction provides us with the basic idea about the method we have to develop in order to study the generalized particle.

One way in which to study the internal structure and physical properties of the generalized particle, is to eliminate the velocity, which also represents the trajectory, from the equations of the theory of selfvariations. This elimination of the velocity can be accomplished in several ways. One is to introduce into the equations of the theory of selfvariations the potential energy U of the material particle. The resulting equation is the time-independent wave equation of Schrödinger

$$\nabla^2\Psi = -\frac{2m_0\left(\varepsilon - U\right)}{\hbar^2}\Psi$$

The differential equations of the theory of selfvariations are of first order. When we convert them to second order equations, we can eliminate the velocity without having to introduce potential energy, or any other physical quantity, into the equations. The elimination of velocity leads to the Klein-Gordon equation. As a special case of the Klein-Gordon for $m_0 = 0$, we get the wave equation

$$\nabla^2\Psi - \frac{\partial^2\Psi}{c^2\partial t^2} = 0$$

which appears in Maxwell's theory of electromagnetism.

Observing the way in which we use Schrödinger's operators in quantum mechanics, we realize that, what we are primarily doing, is to eliminate the kinematic characteristics of the material particle from the resulting differential equations. Dirac does the same thing in the method he develops, in combination, of course, with his additional assumptions, in order to derive his eponymous equation.

In order to study the internal structure of the generalized particle we have to answer specific questions. These questions, and more generally all the issues concerning the generalized particle, are completely different from the ones we usually have to answer when we study physical reality.

The material particle can be located at any position in the part of spacetime it occupies. Judging by the success of quantum mechanics and by the high accuracy calculations it permits, we conclude that statistical interpretation is one way of studying the internal structure of the generalized particle. However, the theory of selfvariations poses a question, the answer to which, leads us to an unknown territory of physical reality.

In order to study the internal structure of the generalized particle we have to answer the question, how is the total rest mass of the generalized particle distributed between the material particle (m_0) and the accompanying particle ($\frac{E_0}{c^2}$). During the quantitative determination of this particular distribution, the Schrödinger and Klein-Gordon equations show up, together with the wave equation of Maxwell's electromagnetic theory. In the part of spacetime occupied by the generalized particle, an external cause suffices to shift the rest mass towards either the material particle or the accompanying particle. In the first case, the generalized particle behaves as a material particle, which moves on a defined trajectory, with defined velocity, energy, etc. In the second case, the generalized particle spreads in spacetime, while the consequences of the aggregation of the generalized photons are intensified. This is the phenomenon we observe in the double-slit experiment.

The law of selfvariations results in the differential equation

$$\left(m_0 c^2 + i\hbar \frac{\dot{m}_0}{m_0} \right)^{\bullet} = 0$$

the only unknown being the rest mass m_0 of the material particles. This simple equation contains as information and rationalizes, the totality of the cosmological data within a Universe that is flat and static, with the exception of a very slight variation of the fine structure constant predicted by the equations of the theory of selfvariations for observations at distances greater than $6\times10^9 ly$.

The redshift z of a distant astronomical object located at distance r is given by equation

$$z=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}-1$$

where k is a constant and A is a scalar parameter that obeys the inequality

$$\frac{z}{1+z}<A<1$$

for every value of the redshift z. Therefore, the value of parameter A is close to 1, with $A<1$.

The distance $r=r(z)$ of a distant astronomical object as a function of the redshift z, is given by equation

$$r=\frac{c}{k}\ln\left(\frac{A}{A-z(1-A)}\right)$$

In Diagram 1 we present the plot of the function $r=r(z)$ for $A=0.900, A=0.950, A=0.990, A=0.999$ up to $z=5$. We observe that, as we increase the value of parameter A, the curve tends to become a straight line. This result is not accidental. It is proven that, for $A\to1^-$, function $r=r(z)$ gives Hubble's law.

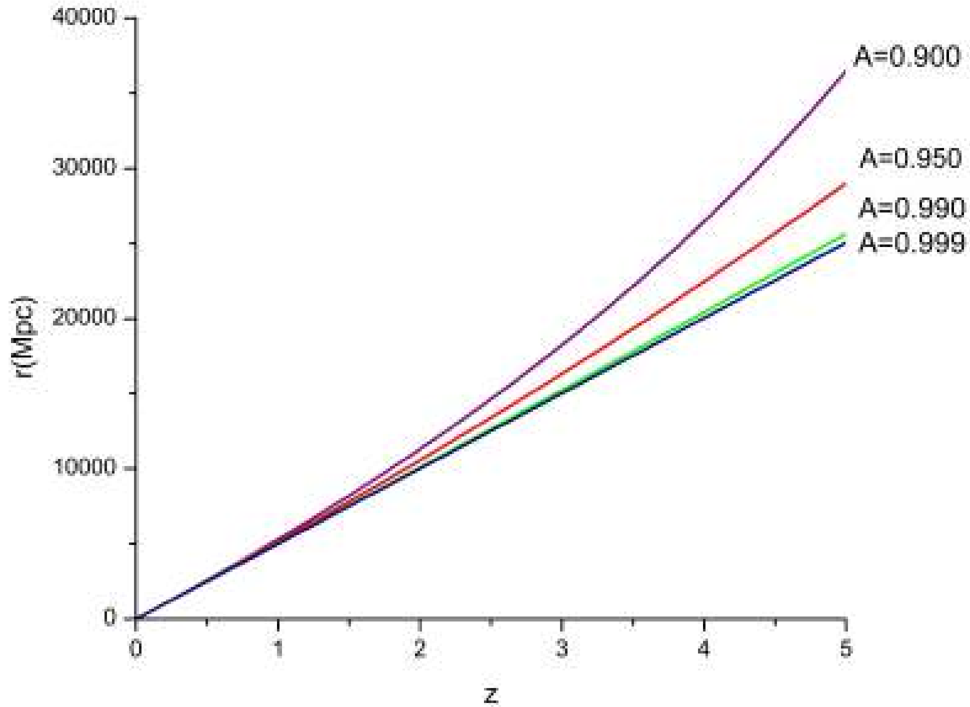

Diagram 1: The plot $r = r(z)$ of the distance of an astronomical object as a function of redshift z, for $A = 0.900, A = 0.950, A = 0.990, A = 0.999$. As the value of the parameter A is increased, the curve $r = r(z)$ tends to a straight line.

The energy $E(z)$ which fuels the radiance of astronomical objects, and which originates from the process of fusion, and generally from the conversion of mass into energy, is smaller than the corresponding energy E in our galaxy, according to equation

$$E(z) = \frac{E}{1+z}$$

Therefore, the intrinsic luminosity of the astronomical object is lower than the standard luminosity we use. As a consequence, the luminosity distance R we measure is in fact greater than the real distance r of distant astronomical objects. The relevant calculations lead to equation

$$R = r\sqrt{1+z}$$

Consideration the arithmetic values of the parameters that factor into function $R = R(z)$, we obtain equation

$R = 5000z\sqrt{1+z}$

where the luminosity distance R is given in Mpc. In Diagram 2 we present the plot of function $R = R(z)$ up to $z = 1.5$.

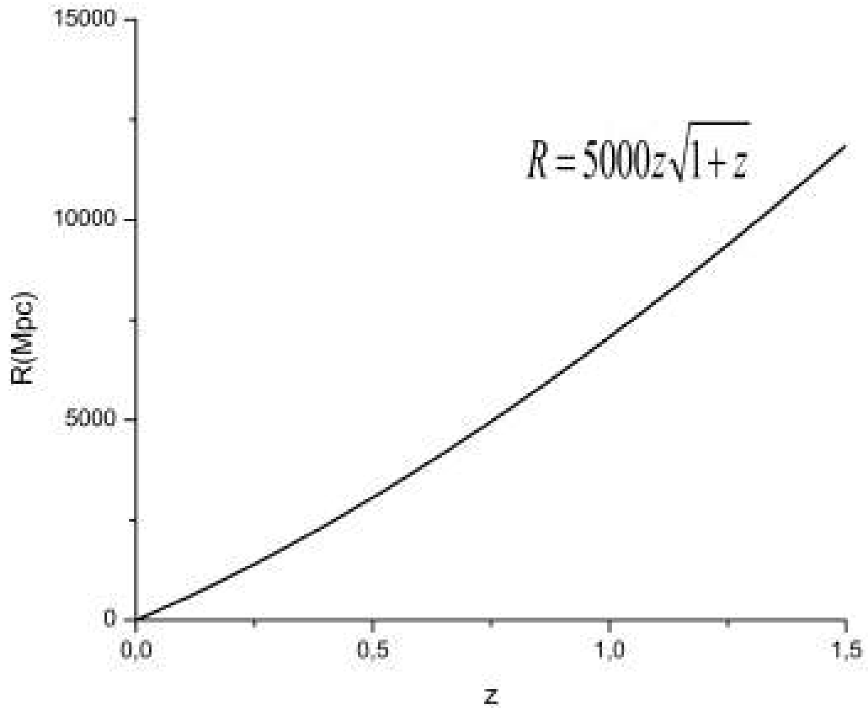

Diagram 2.: The plot of the luminosity distance R of astronomical objects as a function of the redshift z. The measurement of the luminosity distances of type I_a supernova, confirms the theoretical prediction of the law of selfvariations.

Type I_a supernovae are cosmological objects for which we can measure the luminosity distance at great distances. At the end of the last century, these measurements were performed by the independent scientific groups of Adam J. Riess and Saul Perlmutter. The graph that results from those measurements, exactly matches Diagram 2, which is theoretically predicted by the law of selfvariations. The concept of dark energy was invented in order to justify the inconsistency between the Standard Cosmological Model and Diagram 2.

At cosmological scales, the rest mass $m_0(r)$ with which an astronomical object exerts gravitational action at distance r from itself, is given by equation

$$m_0(r) = m_0 \frac{0.001}{1 - 0.999e^{-2\times10^{-7}r}}$$

where m_0 is the laboratory value of the rest mass. The distance r is measured in Mpc.

For values of r of the order of kpc, it turns out that $m_0 = m_0(r)$. For $r = 100kpc$ we get $m_0(r) = 0.99999 m_0$. Consequently, the strength of the gravitational interaction is not affected on the scale of galactic distances. The selfvariations do not affect the stability of the solar system and of galaxies.

On the contrary, at distances of the order of magnitude of Mpc, a clearly smaller value of mass $m_0(r)$ compared to m_0, is predicted. For $r = 100Mpc$ we get $m_0(r) = 0.98 m_0$. For even larger distances, the ratio $\frac{m_0(r)}{m_0}$ becomes even smaller. For an astronomical object located at a distance corresponding to redshift $z = 9$, it is $\frac{m_0(r)}{m_0} = 0.1$. The strength of the gravitational interaction exerted by an astronomical object with $z = 9$ on our galaxy is just 10% of the expected. For still greater distances, the gravitational interaction practically vanishes. This is why gravity cannot play the role attributed to it by the Standard Cosmological Model.

The Thomson scattering coefficient

$$\sigma_\tau = \frac{8\pi}{3} \frac{q^4}{m_0^2 c^2}$$

as well as the Klein-Nishina scattering coefficient

$$\sigma = \frac{3}{8} \sigma_\tau \frac{m_0 c^2}{E} \left[\ln\left(\frac{2E}{m_0 c^2} \right) + \frac{1}{2} \right]$$

obtain different values, namely

$$\sigma_\tau(r) = \frac{8\pi}{3} \frac{q^4(r)}{m_0^2(r) c^2}$$

and

$$\sigma(r) = \frac{3}{8} \sigma_\tau(r) \frac{m_0(r) c^2}{E(r)} \left[\ln\left(\frac{2E(r)}{m_0(r) c^2} \right) + \frac{1}{2} \right]$$

respectively, at distant astronomical objects. The mathematical calculations give

$$\frac{\sigma_\tau(r)}{\sigma_\tau} = \frac{\sigma(r)}{\sigma} = \left(\frac{1 - A e^{-\frac{kr}{c}}}{1 - A} \right)^2$$

At very large distances ($r \to \infty$), and equivalently for the very early Universe, we get

$$\frac{\sigma_\tau(r \to \infty)}{\sigma_\tau} = \frac{\sigma(r \to \infty)}{\sigma} = \left(\frac{1}{1-A}\right)^2$$

Because of the inequality $\frac{z}{1+z} < A < 1$ we see that $A \to 1^-$ and, therefore, the Thomson and Klein-Nishina scattering coefficients obtain enormous values in the very early Universe. Consequently, in its very early stages, the Universe went through a phase during which it was opaque to electromagnetic radiation. The cosmic microwave background radiation originates from that period. The theory of selfvariations predicts that, in that phase, the temperature of the Universe was slightly above $0K$. Furthermore, it predicts that the cosmic microwave background radiation originates from the whole extent of the space occupied by the Universe.

The ionization and excitation energy $X_n(r) = X_n(z)$ of the atoms of distant astronomical objects differs from the laboratory value X_n according to equation

$$X_n(z) = \frac{X_n}{1+z}$$

This equation has consequences regarding the degree of ionization of distant astronomical objects. In other words, the redshift z affects the degree of ionization of atoms in distant astronomical objects. Boltzmann's formula

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\frac{X_n}{KT}}$$

gives the number of excited atoms N_n, that occupy the energy level n on a stellar surface which is in thermodynamic equilibrium. With X_n we denote the excitation energy from the ground energy level 1 to the energy level n, T denotes the temperature of the stellar surface in Kelvins $K = 1.38 \times 10^{-23} \frac{J}{K}$ is Boltzmann's constant, and g_n is the degree of degeneracy of energy level n (that is, the number of energy levels in which the energy level n splits in a magnetic field). At distant astronomical objects Boltzmann's formula becomes

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\frac{X_n}{KT(1+z)}}$$

From this equation it follows that the degree of ionization at distant astronomical objects is greater than expected. The mathematical calculations lead to the conclusion that the Universe went through a phase of ionization. The dependence of the degree of ionization, as well as of the Thomson and Klein-Nishina scattering coefficients, on the redshift z, demands an overall re-evaluation of the electromagnetic spectra we receive from distant astronomical objects.

The law of selfvariations correctly predicts the structures in the Universe. It predicts the monstrous webs of matter in between vast expanses of empty space which we observe with current observational instruments. At smaller scales, it predicts galaxies and galactic clusters.

The theory of selfvariations also solves a fundamental problem concerning physical reality, which the physical theories of the last century were unable to solve: the arrow of time is included within the equations of the theory of selfvariations. The Universe comes from the vacuum and evolves towards a particular direction defined by the selfvariations. As mentioned earlier, at cosmological scales, all the equations resulting from the law of selfvariations give at the limit, for $r \to \infty$, that the initial form of the Universe only slightly differs from the vacuum at a temperature of $0K$. The origin of matter from the vacuum, in combination with the principles of conservation, with which the law of selfvariations agrees, necessitate that the energy content of the Universe remains zero. The selfvariations continually "remove" the Universe from the state of the vacuum, while at the same time the Universe remains consistent with its origin.

In contrast to what happens at the macrocosm, the equations predict that in the laboratory the arrow of time does not exist. This prediction definitively solves the problem with the arrow of time.

A measure of the future evolution of the Universe is the rate of increase of the redshift z predicted by the law of selfvariations. Substituting the arithmetic values of the parameters into the corresponding equation, we get

$$\dot{z} = z \cdot 6.3 \times 10^{-11} year^{-1}$$

It is very characteristic the fact that one simple differential equation, having as a unique unknown the rest mass, contains as information, and at the same time justifies, the totality of the cosmological data, as we observe and record them, from the time of Hubble up to the present. Generally, the equations of the theory of Selfvariations contain an extremely large amount of data and information.

CHAPTER 2

The study of the selfvariations for an arbitrarily moving point particle

2.1 Introduction

In this chapter we present the fundamental study for the mathematical background of the theory of selfvariations. We prove a set of equations which permits us the following: We can represent in the surrounding spacetime of a material particle any kinematic characteristic which concerns the material particle. At every point of spacetime, the velocity, the acceleration, the tangent vector, the curvature and the torsion of the trajectory of the material particle can be mapped in a one-to-one correspondence. This mapping allows us to take the next step: we exactly determine the contribution of the material particle to the energy content of the surrounding spacetime. What emerges is a continuous interaction of every material particle with the surrounding spacetime.
The equations are proven for a material point particle in arbitrary motion. We present a more general statement of the equations in the Appendix at the end of the book.

2.2 Arbitrarily moving material point particle

The theory of selfvariations is based upon two hypotheses which are taken as axioms.

a) The rest mass and the electric charge of the material particles increase slightly with the passage of time. We shall call this increase "selfvariations".
b) The consequences of the selfvariations propagate within the four-dimensional spacetime with a vanishing four-dimensional arc length:
$dS^2 = 0$

In an inertial frame of reference $S(0,x,y,z,t)$, according to the second postulate, the velocity of propagation of the selfvariations $\boldsymbol{\upsilon}$ remains constant as a vector

$$\upsilon = \begin{bmatrix} \upsilon_x \\ \upsilon_y \\ \upsilon_z \end{bmatrix} = \text{constant} \qquad (2.2.1)$$

This vector has magnitude

$$\|\upsilon\| = \sqrt{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2} = c \qquad (2.2.2)$$

The selfvariations cause energy changes to every material particle and, as a consequence, energy, linear momentum and angular momentum propagate into the surrounding spacetime.
We shall later call the carrier of this energy, *"generalized photon"*. Initially, we will refer to the generalized photon as a signal emitted by the material particle, moving with velocity $\boldsymbol{\upsilon}$, and, as our study advances, its properties as a real physical object will be revealed.
We consider an inertial frame of reference $S(0,x,y,z,t)$ and a material point particle moving with velocity $\boldsymbol{u}$ as depicted in figure 2.2.1.

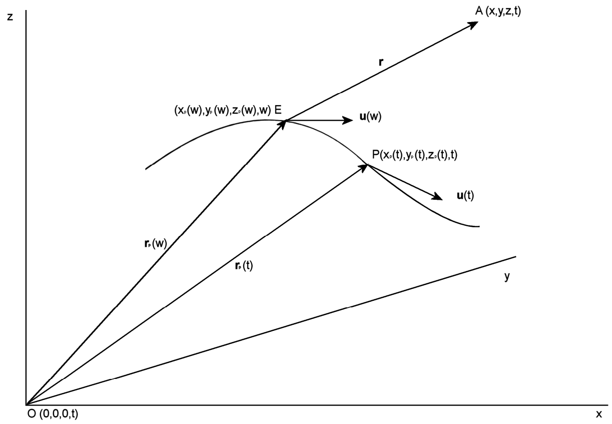

Figure 2.2.1 Material point particle in arbitrary motion. As the material particle moves from point $E(x_p(w), y_p(w), z_p(w), w)$ to point $P(x_p(t), y_p(t), z_p(t), t)$, a generalized photon moves from point $E(x_p(w), y_p(w), z_p(w), w)$ to point $A(x, y, z, t)$.

At moment t, when the particle is located at point $P(x_p(t), y_p(t), z_p(t), t)$, the rest mass m_o and the electric charge q of the particle act at point $A(x, y, z, t)$ with the value they had at time $\Delta t = \frac{\|\boldsymbol{r}\|}{c} = \frac{r}{c}$, when the material particle was located at $E(x_p(t - \frac{r}{c}), y_p(t - \frac{r}{c}), z_p(t - \frac{r}{c}), t - \frac{r}{c})$.

During the time interval $\Delta t = \frac{r}{c}$ the material particle moved from point E to point P, while the generalized photon moved from point E to point A.

We now denote

$$W = t - \frac{r}{c} \tag{2.2.3}$$

Hence, the coordinates of E are

$$E(x_p(w), (y_p(w), (z_p(w), w) \tag{2.2.4}$$

The vector $\boldsymbol{r} = \overrightarrow{EA}$ of figure 2.2.1 is given by

$$\boldsymbol{r} = \overrightarrow{EA} = \begin{bmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{bmatrix} \tag{2.2.5}$$

The velocity of propagation of the selfvariations $\boldsymbol{\upsilon}$ is given by

$$\boldsymbol{\upsilon} = \frac{c}{r}\boldsymbol{r} = \frac{c}{r}\begin{bmatrix} x - x_p(w) \\ y - y_p(w) \\ z - z_p(w) \end{bmatrix} \tag{2.2.6}$$

Here,

$$r = \|\boldsymbol{r}\| = \sqrt{\left(x - x_p(w)\right)^2 + \left(y - y_p(w)\right)^2 + \left(z - z_p(w)\right)^2} \tag{2.2.7}$$

The velocity $\boldsymbol{u}=\boldsymbol{u}(w)$ of the material particle at point E, where it emitted the generalized photon, is

$$\boldsymbol{u}=\boldsymbol{u}(w)=\begin{bmatrix}\dfrac{dx_p(w)}{dw}\\ \dfrac{dy_p(w)}{dw}\\ \dfrac{dz_p(w)}{dw}\end{bmatrix} \qquad (2.2.8)$$

From equation (2.2.7) we have

$$\frac{\partial r}{\partial t}=\frac{1}{2r}\left[2\left(\left(x-x_p(w)\right)\left(-\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t}\right)\right)+2\left(\left(y-y_p(w)\right)\left(-\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t}\right)\right)\right.$$
$$\left.+2\left(\left(z-z_p(w)\right)\left(-\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t}\right)\right)\right]$$

Taking into account equations (2.2.5) and (2.2.6) we have

$$\frac{\partial r}{\partial t}=-\frac{1}{r}(\boldsymbol{r}\cdot\boldsymbol{u})\frac{\partial w}{\partial t}$$

And with equation (2.2.3) we get

$$\frac{\partial r}{\partial t}=-\frac{1}{r}(\boldsymbol{r}\cdot\boldsymbol{u})\left(1-\frac{\partial r}{c\partial t}\right)$$

Taking into consideration that $\dfrac{\boldsymbol{r}}{r}=\dfrac{\boldsymbol{\upsilon}}{c}$, as deduced by equation (2.6.6) we obtain

$$\frac{\partial r}{\partial t}=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c}\left(1-\frac{\partial r}{c\partial t}\right)$$

and finally

$$\frac{\partial r}{\partial t}=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c\left(1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \qquad (2.2.9)$$

where $\boldsymbol{u}=\boldsymbol{u}(w)$ and $\boldsymbol{\upsilon}\cdot\boldsymbol{u}=u_x u_x+u_y u_y+u_z u_z$.

Similarly, starting from equation (2.2.7) and differentiating with respect to x,y,z we get

$$\nabla r=\begin{bmatrix}\dfrac{\partial r}{\partial x}\\ \dfrac{\partial r}{\partial y}\\ \dfrac{\partial r}{\partial z}\end{bmatrix}=\frac{1}{1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c} \qquad (2.2.10)$$

From equation (2.2.3) we obtain initially

$$\frac{\partial w}{\partial t}=\frac{1}{1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}} \qquad (2.2.11)$$

Similarly, from equation (2.2.3) we have $\nabla w = \nabla\left(t - \frac{r}{c}\right) = -\frac{1}{c}\nabla r$ and, in combination with equation (2.2.10), we get

$$\nabla w = -\frac{1}{c^2\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon} \qquad (2.2.12)$$

From equation (2.2.7) and after differentiating with respect to x, we get

$$\frac{\partial r}{\partial x} = \frac{1}{2r}\left[\left(x - x_p(w)\right)\left(1 - \frac{\partial x_p(w)}{\partial x}\right) - \left(y - y_p(w)\right)\frac{\partial y_p(w)}{\partial x} - \left(z - z_p(w)\right)\frac{\partial z_p(w)}{\partial x}\right]$$

Equivalently,

$$\frac{\partial r}{\partial x} = \frac{1}{r}\left[\left(x - x_p(w)\right)\left(1 - \frac{dx_p(w)}{dw}\frac{\partial w}{\partial x}\right) - \left(y - y_p(w)\right)\left(1 - \frac{dy_p(w)}{dw}\frac{\partial w}{\partial x}\right)\right.$$

$$\left. - \left(z - z_p(w)\right)\left(1 - \frac{dz_p(w)}{dw}\frac{\partial w}{\partial x}\right)\right]$$

and also,

$$\frac{\partial r}{\partial x} = \frac{x - x_p(w)}{r} - \frac{1}{r}\left[\left(x - x_p(w)\right)\left(\frac{dx_p(w)}{dw}\right) + \left(y - y_p(w)\right)\left(\frac{dy_p(w)}{dw}\right)\right.$$

$$\left. + \left(z - z_p(w)\right)\left(\frac{dz_p(w)}{dw}\right)\right]\frac{\partial w}{\partial x}$$

Taking into account equations (2.2.8) and (2.2.6) we arrive at

$$\frac{\partial r}{\partial x} = \frac{\upsilon_x}{c} - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c}\frac{\partial w}{\partial x}$$

and substituting $\frac{\partial w}{\partial x} = -\frac{\upsilon_x}{c^2\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}$

as inferred from equation (2.2.12), we finally obtain

$$\frac{\partial r}{\partial x} = -\frac{1}{c\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\upsilon_x \qquad (2.2.13)$$

Following the same procedure differentiating with respect to y and z, we finally have

$$\nabla r = \frac{1}{\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c} \qquad (2.2.14)$$

Differentiating with respect to time t, we obtain from equation (2.2.5)

$$\frac{\partial \boldsymbol{r}}{\partial t} = \begin{bmatrix} -\frac{\partial x_p(w)}{\partial t} \\ -\frac{\partial y_p(w)}{\partial t} \\ -\frac{\partial z_p(w)}{\partial t} \end{bmatrix} = \begin{bmatrix} -\frac{dx_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dy_p(w)}{dw}\frac{\partial w}{\partial t} \\ -\frac{dz_p(w)}{dw}\frac{\partial w}{\partial t} \end{bmatrix}$$

Taking into consideration equation (2.2.8) $\frac{\partial \boldsymbol{r}}{\partial t} = -\boldsymbol{u}\frac{\partial w}{\partial t}$, and in combination with equation (2.2.11), we finally get

$$\frac{\partial \boldsymbol{r}}{\partial t} = -\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\boldsymbol{u} \tag{2.2.14}$$

From equation (2.2.6) we successively obtain

$$\boldsymbol{\upsilon} = \frac{c}{r}\boldsymbol{r}$$

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = -\frac{c}{r^2}\frac{\partial r}{\partial t}\boldsymbol{r} + \frac{c}{r}\frac{\partial \boldsymbol{r}}{\partial t}$$

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = -\frac{1}{r}\frac{\partial r}{\partial t}\boldsymbol{\upsilon} + \frac{c}{r}\frac{\partial \boldsymbol{r}}{\partial t} \tag{2.2.15}$$

taking into account $\frac{c}{r}\boldsymbol{r} = \boldsymbol{\upsilon}$. Substituting into equation (2.2.15) the quantity $\frac{\partial r}{\partial t}$, from equation (2.2.9), and $\frac{\partial \boldsymbol{r}}{\partial t}$, from (2.2.14), we finally obtain relation

$$\frac{\partial \boldsymbol{\upsilon}}{\partial t} = \frac{c}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{\left(\boldsymbol{\upsilon}\cdot\boldsymbol{u}\right)}{c^2}\boldsymbol{\upsilon} - \boldsymbol{u}\right] \tag{2.2.16}$$

Starting from equation (2.2.6) we get $\upsilon_x = \frac{c}{r}\left(x - x_p(w)\right)$, and differentiating with respect to x we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p(w)\right) + \frac{c}{r}\left(1 - \frac{\partial x_p(w)}{\partial x}\right)$$

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p(w)\right) + \frac{c}{r}\left(1 - \frac{dx_p(w)}{dw}\frac{\partial w}{\partial x}\right)$$

Since $\frac{dx_p(w)}{dw} = u_x$, as arises from equation (2.2.8), we have that

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{c}{r^2}\frac{\partial r}{\partial x}\left(x - x_p(w)\right) + \frac{c}{r}\left(1 - u_x\frac{\partial w}{\partial x}\right)$$

and considering that $\frac{\partial r}{\partial x} = \frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\upsilon_x$ from equation (2.2.13), and that $\frac{\partial w}{\partial x} = -\frac{1}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\upsilon_x$ from equation (2.2.12), we get

$$\frac{\partial \upsilon_x}{\partial x} = -\frac{\upsilon_x^2}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} + \frac{c}{r}\left(1 + \frac{\upsilon_x u_x}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\right)$$

and finally

$$\frac{\partial \upsilon_x}{\partial x} = \frac{c}{r} + \frac{\upsilon_x (u_x - \upsilon_x)}{cr\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \tag{2.2.17}$$

Working similarly, we finally obtain

$$\frac{\partial \upsilon_i}{\partial x_j} = \begin{cases} \dfrac{c}{r} + \dfrac{\upsilon_i (u_i - \upsilon_i)}{cr\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} & \textit{for} \quad i = j \\ \dfrac{\upsilon_j (u_i - \upsilon_i)}{cr\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} & \textit{for} \quad i \neq j \end{cases} \tag{2.2.18}$$

where, $i, j = 1,2,3 \quad (x_1, x_2, x_3) = (x, y, z)$.

Equations (2.2.18) can be summarized in equation

$$grad\boldsymbol{\upsilon} = \frac{c}{r} I + \frac{1}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c} \otimes (\boldsymbol{u} - \boldsymbol{\upsilon}) \tag{2.2.19}$$

where,

$$grad\boldsymbol{\upsilon} = \begin{bmatrix} \frac{\partial \upsilon_x}{\partial x} & \frac{\partial \upsilon_x}{\partial y} & \frac{\partial \upsilon_x}{\partial z} \\ \frac{\partial \upsilon_y}{\partial x} & \frac{\partial \upsilon_y}{\partial y} & \frac{\partial \upsilon_y}{\partial z} \\ \frac{\partial \upsilon_z}{\partial x} & \frac{\partial \upsilon_z}{\partial y} & \frac{\partial \upsilon_z}{\partial z} \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\alpha} \otimes \boldsymbol{b} = \begin{bmatrix} a_1 b_1 & a_2 b_1 & a_3 b_1 \\ a_1 b_2 & a_2 b_2 & a_3 b_2 \\ a_1 b_3 & a_2 b_3 & a_3 b_3 \end{bmatrix} \tag{2.2.20}$$

This holds for any two arbitrary vectors

$\boldsymbol{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

We now have $\nabla \cdot \boldsymbol{\upsilon} = \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z}$, and from equations (2.2.18) we get

$$\nabla \cdot \boldsymbol{\upsilon} = \frac{3c}{r} + \frac{\upsilon_x (u_x - \upsilon_x) + \upsilon_y (u_y - \upsilon_y) + \upsilon_z (u_z - \upsilon_z)}{cr\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)}$$

$$\nabla\cdot\boldsymbol{\upsilon}=\frac{3c}{r}+\frac{\upsilon_x u_x+\upsilon_y u_y+\upsilon_z u_z-\upsilon_x^2+\upsilon_y^2+\upsilon_z^2}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}$$

and since $\upsilon_x^2+\upsilon_y^2+\upsilon_z^2=c^2$ and $\upsilon_x u_x+\upsilon_x u_y+\upsilon_z u_z=\boldsymbol{\upsilon}\cdot\boldsymbol{u}$, we see that

$$\nabla\cdot\boldsymbol{\upsilon}=\frac{3c}{r}+\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}-c^2}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}$$

Finally, we arrive at relation

$$\nabla\cdot\boldsymbol{\upsilon}=\frac{2c}{r} \tag{2.2.21}$$

Now, we consider the curl of vector $\boldsymbol{\upsilon}$

$$\nabla\times\boldsymbol{\upsilon}=curl\boldsymbol{\upsilon}=\begin{bmatrix}\frac{\partial\upsilon_z}{\partial x}-\frac{\partial\upsilon_y}{\partial z}\\ \frac{\partial\upsilon_x}{\partial z}-\frac{\partial\upsilon_z}{\partial x}\\ \frac{\partial\upsilon_y}{\partial x}-\frac{\partial\upsilon_x}{\partial y}\end{bmatrix} \tag{2.2.22}$$

Taking into account equations (2.2.18) we obtain

$$\nabla\times\boldsymbol{\upsilon}=curl\boldsymbol{\upsilon}=\frac{1}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}(\boldsymbol{\upsilon}\times\boldsymbol{u}) \tag{2.2.23}$$

where,

$$\boldsymbol{\upsilon}\times\boldsymbol{u}=\begin{bmatrix}\upsilon_y u_z-\upsilon_z u_y\\ \upsilon_z u_x-\upsilon_x u_z\\ \upsilon_x u_y-\upsilon_y u_x\end{bmatrix}$$

We now consider the acceleration vector

$$\boldsymbol{\alpha}=\boldsymbol{\alpha}(w)=\frac{d\boldsymbol{u}(w)}{dw}=\begin{bmatrix}\frac{du_x(w)}{dw}\\ \frac{du_y(w)}{dw}\\ \frac{du_z(w)}{dw}\end{bmatrix} \tag{2.2.24}$$

of the material particle at the moment w, located at point E of figure 2.2.1. We have that

$$\frac{\partial u_x}{\partial t}=\frac{\partial u_x(w)}{\partial t}=\frac{du_x(w)}{dw}\frac{\partial w}{\partial t}=\alpha_x\frac{\partial w}{\partial t}$$

and since, from equation (2.2.11), it is $\frac{\partial w}{\partial t}=\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}$, we get $\frac{\partial u_x}{\partial t}=\frac{\alpha_x}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}$.

Working similarly for the differentials $\frac{\partial u_x}{\partial t}$ and $\frac{\partial u_z}{\partial t}$, we get

$$\frac{\partial \boldsymbol{u}}{\partial t} = \frac{1}{1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}} \boldsymbol{\alpha} \qquad (2.2.25)$$

For the differentiation of the velocity $\boldsymbol{u} = \boldsymbol{u}(w)$ with respect to x, y, z we initially get

$$\frac{\partial u_x}{\partial x} = \frac{\partial u_x(w)}{\partial x} = \frac{du_x(w)}{dw} \frac{\partial w}{\partial x} = \alpha_x \frac{\partial w}{\partial x}.$$

Similarly, from equation (2.2.12) we have that

$$\frac{\partial w}{\partial x} = -\frac{u_x}{c^2\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)}, \text{ hence } \frac{\partial u_x}{\partial x} = -\frac{\upsilon_x \alpha_x}{c^2\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)}.$$

Working similarly we finally obtain

$$\frac{\partial u_i}{\partial x_j} = -\frac{\upsilon_j \alpha_i}{c^2\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \qquad i, j = 1,2,3 \qquad (2.2.26)$$

Here we use the notation $(x_1, x_2, x_3) = (x, y, z)$

From equation (2.2.26) we obtain

$$grad\boldsymbol{u} = -\frac{1}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c} \otimes \boldsymbol{u} \qquad (2.2.27)$$

We now consider the vector

$$\boldsymbol{b} = \boldsymbol{b}(w) = \frac{d\boldsymbol{\alpha}(w)}{dw} \qquad (2.2.28)$$

Working as we did in order to prove equations (2.2.16), (2.2.25) and (2.2.26), we arrive at relations

$$\frac{\partial \boldsymbol{\alpha}}{\partial t} = \frac{1}{1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}} \boldsymbol{b} \qquad (2.2.29)$$

$$\frac{\partial \alpha_i}{\partial x_j} = -\frac{\upsilon_j b_i}{c^2\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \qquad i, j = 1,2,3 \qquad (2.2.30)$$

where $(x_1, x_2, x_3) = (x, y, z)$, and

$$grad\boldsymbol{\alpha} = -\frac{1}{c\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c} \otimes \boldsymbol{b} \qquad (2.2.31)$$

The equations of this paragraph express the fact that in every inertial reference frame the velocity $\boldsymbol{\upsilon}$ of the selfvariations remains constant as a vector with magnitude $\|\boldsymbol{\upsilon}\| = c$. It can easily be proven that all the equations are consistent with the Lorentz-Einstein transformations, as we pass from one inertial reference frame to another. The equations we have proven are fundamental for the theory of selfvariations. As we advance our study, we will find that they allow us to correlate any physical quantity defined on the material particle, with any physical quantity defined on the surrounding spacetime. Using the concept of information, we can correlate any information concerning the material particle with any information concerning the surrounding spacetime. Part of this information are the potential fields, while the quantum phenomena arise spontaneously.

2.3 The trigonometric form of the velocity of selfvariations

Starting from equation (2.2.2) we get $\left\|\frac{\boldsymbol{\upsilon}}{c}\right\| = 1$ for every inertial reference frame. We express the unit vector $\frac{\boldsymbol{\upsilon}}{c}$ into the trigonometric form

$$\frac{\boldsymbol{\upsilon}}{c} = \begin{bmatrix} \frac{\upsilon_x}{c} \\ \frac{\upsilon_y}{c} \\ \frac{\upsilon_z}{c} \end{bmatrix} = \begin{bmatrix} \cos\delta \\ \sin\delta\cos\omega \\ \sin\delta\sin\omega \end{bmatrix} \qquad (2.3.1)$$

where $\delta = \delta(x, y, z, t)$ and $\omega = \omega(x, y, z, t)$ are functions of the coordinates x, y, z, t in an inertial frame of reference $S(0, x, y, z, t)$.

From equation (2.3.1) we see that

$$\frac{\upsilon_x}{c} = \cos\delta = \frac{\boldsymbol{\upsilon}}{c}\boldsymbol{e}_1 \qquad \text{(a)}$$

$$\frac{\upsilon_y}{c} = \sin\delta\cos\omega = \frac{\boldsymbol{\upsilon}}{c}\boldsymbol{e}_2 \qquad \text{(b)} \qquad (2.3.2)$$

$$\frac{\upsilon_z}{c} = \sin\delta\sin\omega = \frac{\boldsymbol{\upsilon}}{c}\boldsymbol{e}_3 \qquad \text{(c)}$$

where $\boldsymbol{e}_1 = \hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{e}_2 = \hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\boldsymbol{e}_3 = \hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

We now consider the vectors

$$\boldsymbol{\beta} = \begin{bmatrix} -\sin\delta \\ \cos\delta\cos\omega \\ \cos\delta\sin\omega \end{bmatrix} \qquad (2.3.3)$$

and

$$\boldsymbol{\gamma} = \begin{bmatrix} 0 \\ -\sin\omega \\ \cos\omega \end{bmatrix} \qquad (2.3.4)$$

It is easily proven that the set of vectors $\{\frac{\boldsymbol{\upsilon}}{c}, \boldsymbol{\beta}, \boldsymbol{\gamma}\}$ form a right-handed orthonormal vector basis which is defined at every point A of figure 2.2.1. Furthermore, the following relations hold:

$$\frac{\partial}{\partial \delta}\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\boldsymbol{\beta}$$
$$\frac{\partial}{\partial \omega}\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\sin\delta\boldsymbol{\gamma}$$
$$\frac{\partial \boldsymbol{\beta}}{\partial \delta}=-\frac{\boldsymbol{\upsilon}}{c} \tag{2.3.5}$$
$$\frac{\partial \boldsymbol{\beta}}{\partial \omega}=\cos\delta\boldsymbol{\gamma}$$
$$\frac{\partial \boldsymbol{\gamma}}{\partial \delta}=0$$
$$\frac{\partial \boldsymbol{\gamma}}{\partial \omega}=-\sin\delta\frac{\boldsymbol{\upsilon}}{c}-\cos\delta\boldsymbol{\beta}$$

Differentiating the vectors $\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}$ with respect to x,y,z,t we obtain the following equations:

$$\nabla\cdot\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\boldsymbol{\beta}\cdot\nabla\delta+\sin\delta\boldsymbol{\gamma}\cdot\nabla\omega \qquad \text{(a)}$$
$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\frac{\partial\delta}{\partial t}\boldsymbol{\beta}+\sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma} \qquad \text{(b)} \tag{2.3.6}$$
$$\nabla\times\frac{\boldsymbol{\upsilon}}{c}=\nabla\delta\times\boldsymbol{\beta}+\sin\delta\nabla\omega\otimes\boldsymbol{\gamma} \qquad \text{(c)}$$
$$grad\frac{\boldsymbol{\upsilon}}{c}=\nabla\delta\otimes\boldsymbol{\beta}+\sin\delta\nabla\omega\otimes\boldsymbol{\gamma} \qquad \text{(d)}$$

$$\nabla\cdot\boldsymbol{\beta}=-\frac{\boldsymbol{\upsilon}}{c}\nabla\delta+\cos\delta\boldsymbol{\gamma}\cdot\nabla\omega \qquad \text{(a)}$$
$$\frac{\partial\boldsymbol{\beta}}{\partial t}=-\frac{\partial\delta}{\partial t}\frac{\boldsymbol{\upsilon}}{c}+\cos\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma} \qquad \text{(b)} \tag{2.3.7}$$
$$\nabla\times\boldsymbol{\beta}=\frac{\boldsymbol{\upsilon}}{c}\times\nabla\delta-\cos\delta\boldsymbol{\gamma}\times\nabla\omega \qquad \text{(c)}$$
$$grad\boldsymbol{\beta}=-\nabla\delta\otimes\frac{\boldsymbol{\upsilon}}{c}+\cos\delta\nabla\omega\otimes\boldsymbol{\gamma} \qquad \text{(d)}$$

$$\nabla\cdot\boldsymbol{\gamma}=-\sin\delta\frac{\boldsymbol{\upsilon}}{c}\nabla\omega-\cos\delta\boldsymbol{\beta}\cdot\nabla\omega \qquad \text{(a)}$$
$$\frac{\partial\boldsymbol{\gamma}}{\partial t}\nabla\cdot=-\sin\delta\frac{\partial\omega}{\partial t}\frac{\boldsymbol{\upsilon}}{c}-\cos\delta\frac{\partial\omega}{\partial t}\boldsymbol{\beta} \qquad \text{(b)} \tag{2.3.8}$$
$$\nabla\times\boldsymbol{\gamma}=\sin\delta\frac{\boldsymbol{\upsilon}}{c}\times\nabla\omega+\cos\delta\boldsymbol{\beta}\times\nabla\omega \qquad \text{(c)}$$
$$grad\boldsymbol{\gamma}=-\sin\delta\nabla\omega\otimes\frac{\boldsymbol{\upsilon}}{c}-\cos\delta\nabla\omega\otimes\boldsymbol{\beta} \qquad \text{(d)}$$

We prove indicatively equation (2.3.6)(a). The rest of the equations are proven along similar lines. Taking into account equation (2.3.1) we get

$$\nabla\cdot\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\frac{\partial}{\partial x}(\cos\delta)+\frac{\partial}{\partial y}(\sin\delta\cos\omega)+\frac{\partial}{\partial z}(\sin\delta\sin\omega)=$$

$$-\sin\delta\frac{\partial\delta}{\partial x}+\cos\delta\frac{\partial\delta}{\partial y}\cos\omega+\cos\delta\frac{\partial\delta}{\partial z}\sin\omega$$

$$+0-\sin\delta\sin\omega\frac{\partial\omega}{\partial y}+\sin\delta\cos\omega\frac{\partial\omega}{\partial z}$$

and considering equations (2.3.3) and (2.3.4), as well as relations

$$\nabla\delta=\begin{bmatrix}\frac{\partial\delta}{\partial x}\\\frac{\partial\delta}{\partial y}\\\frac{\partial\delta}{\partial z}\end{bmatrix},\ \nabla\omega=\begin{bmatrix}\frac{\partial\omega}{\partial x}\\\frac{\partial\omega}{\partial y}\\\frac{\partial\omega}{\partial z}\end{bmatrix}$$

we finally obtain

$$\nabla\cdot\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\boldsymbol{\beta}\cdot\nabla\delta+\sin\delta\boldsymbol{\gamma}\cdot\omega.$$

We now expand the vector of velocity $\boldsymbol{u}=\boldsymbol{u}(w)$ with respect to the vector basis $\{\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}\}$ as

$$\boldsymbol{u}=\boldsymbol{u}(w)=u_1\frac{\boldsymbol{\upsilon}}{c}+u_2\boldsymbol{\beta}+u_3\boldsymbol{\gamma}=\left(\boldsymbol{u}\cdot\frac{\boldsymbol{\upsilon}}{c}\right)\frac{\boldsymbol{\upsilon}}{c}+(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}+(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}$$

and combining with equations (2.2.16) we get

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{c}\frac{\boldsymbol{\upsilon}}{c}-\left(\boldsymbol{u}\frac{\boldsymbol{\upsilon}}{c}\right)\frac{\boldsymbol{\upsilon}}{c}-(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}-(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)=\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}+(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\right]$$

Considering equations (2.3.6)(b) we get

$$\frac{\partial\delta}{\partial t}\boldsymbol{\beta}+\sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma}=-\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}+(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\right]$$

and finally

$$\frac{\partial\delta}{\partial t}=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{2.3.9}$$

$$\sin\delta\frac{\partial\omega}{\partial t}=-\frac{\boldsymbol{u}\cdot\boldsymbol{\gamma}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{2.3.10}$$

because of the linear independence of the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

We now write vectors $\nabla\delta$ and $\nabla\omega$ as a linear combination of vectors $\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}$.

$$\nabla\delta = \lambda_1 \frac{\boldsymbol{\upsilon}}{c} + K\boldsymbol{\beta} + L\boldsymbol{\gamma} \tag{2.3.11}$$

$$\nabla\omega = \lambda_2 \frac{\boldsymbol{\upsilon}}{c} + M\boldsymbol{\beta} + N\boldsymbol{\gamma} \tag{2.3.12}$$

We combine equations (2.2.16) and (2.2.19), and get relation

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right) + \left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{\upsilon} = \frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\boldsymbol{\upsilon} - \boldsymbol{u}\right] + \left[\frac{1}{r}I + \frac{1}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}\otimes(\boldsymbol{u}-\boldsymbol{\upsilon})\right]\boldsymbol{\upsilon} =$$

$$\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\boldsymbol{\upsilon} - \boldsymbol{u}\right] + \frac{1}{r}\boldsymbol{\upsilon} + \frac{1}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(\frac{\boldsymbol{\upsilon}}{c}\otimes(\boldsymbol{u}-\boldsymbol{\upsilon})\right)\boldsymbol{\upsilon}$$

Using the identity

$$(\boldsymbol{\alpha}\otimes\boldsymbol{\beta})\boldsymbol{c} = (\boldsymbol{\alpha}\cdot\boldsymbol{c})\boldsymbol{\beta} \tag{2.3.13}$$

which holds for every set of vectors $\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{c}$, we see that

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right) + \left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{\upsilon} = \frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\boldsymbol{\upsilon} - \boldsymbol{u}\right] + \frac{1}{r}\boldsymbol{\upsilon} + \frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}(\boldsymbol{\upsilon}-\boldsymbol{u}) =$$

$$\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\boldsymbol{\upsilon} + \frac{1}{r}\boldsymbol{\upsilon} - \frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon} = \frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right) + \left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right) - 1\right]\boldsymbol{\upsilon} = \boldsymbol{0}$$

That is,

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right) + \left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{\upsilon} = \boldsymbol{0} \tag{2.3.14}$$

Into equation (2.3.13) we replace $\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)$ from equation (2.3.6)(b), and $grad\frac{\boldsymbol{\upsilon}}{c}$ from equation (2.3.6)(d), and obtain

$$\frac{\partial\delta}{\partial t}\boldsymbol{\beta} + \sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma} + \left(\nabla\delta\otimes\boldsymbol{\beta} + \sin\delta\nabla\omega\otimes\boldsymbol{\gamma}\right)\boldsymbol{\upsilon} = \boldsymbol{0}$$

Using the identity (2.3.13) we get

$$\frac{\partial\delta}{\partial t}\boldsymbol{\beta} + \sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma} + (\boldsymbol{u}\cdot\nabla\delta)\boldsymbol{\beta} + \sin\delta(\boldsymbol{\upsilon}\cdot\nabla\omega)\boldsymbol{\gamma} = 0$$

and due to the linear independence of the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ we see that

$$\frac{\partial\delta}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla\delta = 0 \tag{2.3.15}$$

$$\frac{\partial\omega}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla\omega = 0 \tag{2.3.16}$$

Combining equations (2.3.15) and (2.3.11) we obtain

$$\frac{\partial \delta}{\partial t} + \lambda_1 = 0$$

$$\lambda_1 = -\frac{\partial \delta}{\partial t}$$

Through equation (2.3.9) we have that

$$\lambda_1 = \frac{\boldsymbol{u} \cdot \boldsymbol{\beta}}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)}$$

and replacing into equation (2.3.11) we get

$$\nabla \delta = \frac{\boldsymbol{u} \cdot \boldsymbol{\beta}}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c^2} + K\boldsymbol{\beta} + L\boldsymbol{\gamma} \tag{2.3.17}$$

Performing the corresponding combinations, we arrive at equation

$$\nabla \omega = \frac{\boldsymbol{u} \cdot \boldsymbol{\gamma}}{\sin \delta r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c} + M\boldsymbol{\beta} + N\boldsymbol{\gamma} \tag{2.3.18}$$

We shall now prove that $K = \frac{1}{r}$, $L = 0$, $M = 0$, $N = \frac{1}{r \sin \delta}$, hence equations (2.3.17) and (2.3.18) obtain their final form

$$\nabla \delta = \frac{\boldsymbol{u} \cdot \boldsymbol{\beta}}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c^2} + \frac{1}{r}\boldsymbol{\beta} \tag{2.3.19}$$

$$\nabla \omega = \frac{\boldsymbol{u} \cdot \boldsymbol{\gamma}}{\sin \delta r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c^2} + \frac{1}{r \sin \delta}\boldsymbol{\gamma} \tag{2.3.20}$$

We will prove that $K = \frac{1}{r}$, $L = 0$. In a similar manner we can also calculate the factors M, N. From equation (2.3.2)(a) we successively obtain

$$\cos \delta = \frac{\upsilon_x}{c}$$

$$-\sin \delta \nabla \delta = \nabla\left(\frac{\upsilon_x}{c}\right)$$

We calculate $\nabla\left(\frac{\upsilon_x}{c}\right)$ from equations (2.2.18), hence we have

$$-\sin \delta \nabla \delta = \frac{1}{r}\boldsymbol{e}_1 - \frac{\upsilon_x - u_x}{r\left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2}\right)} \frac{\boldsymbol{\upsilon}}{c} \tag{2.3.21}$$

where $\boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

We take the inner product of equation (2.3.21) with vector $\boldsymbol{\beta}$ and obtain

$$-\sin \delta \boldsymbol{\beta} \cdot \nabla \delta = \frac{1}{r}\boldsymbol{e}_1 \cdot \boldsymbol{\beta}$$

From equation (2.3.17) we have $\boldsymbol{\beta}\cdot\nabla\delta = K$, hence we have

$$-\sin\delta K = \frac{1}{r}\boldsymbol{e}_1\cdot\boldsymbol{\beta}$$

From equation (2.3.3) we obtain

$$\boldsymbol{e}_1\cdot\boldsymbol{\beta} = -\sin\delta$$

Therefore,

$$-\sin\delta K = \frac{1}{r}\left(-\sin\delta\right)$$

Finally, we obtain

$$K = \frac{1}{r}$$

We take the inner product of equation (2.3.21) with vector $\boldsymbol{\gamma}$ and obtain

$$-\sin\delta\boldsymbol{\gamma}\cdot\nabla\delta = \frac{1}{r}\boldsymbol{e}_1\cdot\boldsymbol{\gamma}$$

From equation (2.3.17) it holds that $\boldsymbol{\gamma}\cdot\nabla\delta = L$, hence

$$-\sin\delta L = \frac{1}{r}\boldsymbol{e}_1\cdot\boldsymbol{\gamma}$$

From equation (2.3.4) we see that $\boldsymbol{e}_1\cdot\boldsymbol{\gamma} = 0$, therefore $-\sin\delta L = 0$, and finally $L = 0$.

The equations of this paragraph promote the theory of selfvariations considerably, and their fundamental character will become obvious as our study continues. One first fundamental conclusion emerges from equations (2.3.15) and (2.3.16). The functions $\delta = \delta\left(x,y,z,t\right)$ and $\omega = \omega\left(x,y,z,t\right)$ remain invariable on the trajectory of the generalized photon. Through equations (2.3.1), (2.3.3) and (2.3.4) we conclude that the vector basis $\{\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}\}$ accompanies without change, that is remaining constant, the motion of the generalized photon. We can, of course, straightforwardly prove that

$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right) + \left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{\upsilon} = 0$$

$$\frac{\partial\boldsymbol{\beta}}{\partial t} + \left(grad\,\boldsymbol{\beta}\right)\frac{\boldsymbol{\upsilon}}{c} = 0 \qquad (2.3.22)$$

$$\frac{\partial\boldsymbol{\gamma}}{\partial t} + \left(grad\,\boldsymbol{\gamma}\right)\frac{\boldsymbol{\upsilon}}{c} = 0$$

by combining equations (2.3.6), (2.3.7) and (2.3.8) with equations (2.3.19) and (2.3.20).

2.4 The generalized photon as a geometric object. Representation of the trajectory of a material point particle

In the present paragraph we shall look for points A_i in the neighborhood of point $A\left(x,y,z,t\right)$ of figure 2.2.1, for which the velocity of the generalized photon is the same with the velocity at point $A\left(x,y,z,t\right)$ at the same moment t. We use the notation

$$\overrightarrow{AA_i} = d\boldsymbol{R} \qquad (2.4.1)$$

and we search for points A_i, i.e. vector $d\boldsymbol{R}$, such that

$$\boldsymbol{\upsilon}(\boldsymbol{R}+d\boldsymbol{R},t)=\boldsymbol{\upsilon}(\boldsymbol{R},t) \tag{2.4.2}$$

According to equations (2.3.1), equation (2.4.2) is equivalent to the relations

$$\delta(\boldsymbol{R}+d\boldsymbol{R},t)=\delta(\boldsymbol{R},t) \tag{2.4.3}$$

and

$$\omega(\boldsymbol{R}+d\boldsymbol{R},t)=\omega(\boldsymbol{R},t) \tag{2.4.4}$$

After expanding the functions $\delta(\boldsymbol{R},t)$ and $\omega(\boldsymbol{R},t)$ in Taylor series up to the first order terms, we obtain

$$\delta(\boldsymbol{R}+d\boldsymbol{R},t)=\delta(\boldsymbol{R},t)+d\boldsymbol{R}\cdot\nabla\delta$$

$$\omega(\boldsymbol{R}+d\boldsymbol{R},t)=\omega(\boldsymbol{R},t)+d\boldsymbol{R}\cdot\nabla\omega$$

Through equations (2.4.3) and (2.4.4) we have that

$$d\boldsymbol{R}\cdot\nabla\delta=0 \tag{2.4.5}$$

$$d\boldsymbol{R}\cdot\nabla\omega=0 \tag{2.4.6}$$

Combining equations (2.3.19) and (2.3.20) we obtain

$$\boldsymbol{t}=\nabla\delta\times\sin\delta\omega=$$

$$\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\gamma}+\frac{\boldsymbol{u}\cdot\boldsymbol{\gamma}}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\beta}\times\frac{\boldsymbol{\upsilon}}{c}+\frac{1}{r^2}\boldsymbol{\beta}\times\boldsymbol{\gamma}$$

$$=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\beta}-\frac{\boldsymbol{u}\cdot\boldsymbol{\gamma}}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\gamma}+\frac{1}{r^2}\frac{\boldsymbol{\upsilon}}{c}$$

taking into account that the set of the vectors $\{\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}\}$ form a right-handed orthonormal vector basis. We now have

$$\boldsymbol{t}=\frac{1}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}-(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}-(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\right]$$

$$\boldsymbol{t}=\frac{1}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{\boldsymbol{\upsilon}}{c}-\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\frac{\boldsymbol{\upsilon}}{c}-(\boldsymbol{u}\cdot\boldsymbol{\beta})\boldsymbol{\beta}-(\boldsymbol{u}\cdot\boldsymbol{\gamma})\boldsymbol{\gamma}\right]$$

and from equation (2.3.9) we get

$$\boldsymbol{t}=\frac{1}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c}\right)}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)\neq\boldsymbol{0} \tag{2.4.7}$$

According to equations (2.4.5) and (2.4.6) the vector $d\boldsymbol{R}$ is parallel to the vector $\boldsymbol{t}\neq\boldsymbol{0}$, hence we finally arrive at relation

$$d\boldsymbol{R}\,\|\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right) \tag{2.4.8}$$

Thus, we conclude that points A and A_i, at which the generalized photon moves with the same velocity $\boldsymbol{\upsilon}$, are arranged parallel to the vector $\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}$. This conclusion is the result of a more general theorem, which we present in the Appendix. For the case of a material point particle the theorem gives relation (2.4.8).

In figure 2.2.1 and for the time interval from $t-\frac{r}{c}$ to t, i.e. for $t-\frac{r}{c}\le w\le t$, the generalized photons emitted by the material point particle reside within a sphere with center $E\left(x_p\left(t-\frac{r}{c}\right),y_p\left(t-\frac{r}{c}\right),z_p\left(t-\frac{r}{c}\right),t-\frac{r}{c}\right)$ and radius $r=\|\boldsymbol{r}\|$. During the same time interval the material particle moved from point E to point $P\left(x_p\left(t\right),y_p\left(t\right),z_p\left(t\right),t\right)$.

We now consider a point E_i in the neighborhood of point E and on the trajectory C_p of the material particle as it moves from point E to point P, from which point E_i was emitted the generalized photon which at moment t is located at point A_i, as depicted in figure 2.4.1.

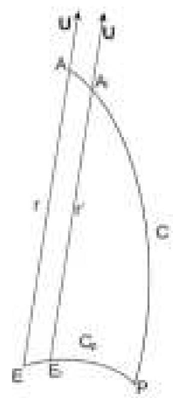

Figure 2.4.1 A material point particle moves from point E to point P on the curved trajectory C_p in the time interval from $w=t-\frac{r}{c}$ to t. The generalized photons emitted by the material particle with the same velocity $\boldsymbol{\upsilon}$, in the time interval $\Delta t=t-w=\frac{r}{c}$, are on curve C at moment t.

Point E_i has coordinates $E_i\left(x_p\left(t-\frac{r'}{c}\right),y_p\left(t-\frac{r'}{c}\right),z_p\left(t-\frac{r'}{c}\right),t-\frac{r'}{c}\right)$, where $\boldsymbol{\upsilon}=\frac{c}{r}\boldsymbol{r}=\frac{c}{r'}\boldsymbol{r}'$.

The points E,P,A appear in figure 2.2.1 as well as in figure 2.4.1, while the points E_i and A_i are shown in figure 2.4.1.

For the vector $\overrightarrow{AA_i}=d\boldsymbol{r}$ we have, according to figure 2.4.1

$$d\boldsymbol{r} = -\boldsymbol{r} + \overrightarrow{EE_i} + \boldsymbol{r}'$$

$$d\boldsymbol{r} = -\boldsymbol{r} + \boldsymbol{u}\left(\frac{r}{c} - \frac{r'}{c}\right) + \boldsymbol{r}'$$

$$d\boldsymbol{r} = -\frac{\boldsymbol{\upsilon}}{c}\left(r - r'\right) + \boldsymbol{u}\left(\frac{r}{c} - \frac{r'}{c}\right)$$

$$d\boldsymbol{r} = -\left(r - r'\right)\left(\frac{\boldsymbol{\upsilon}}{c} - \frac{\boldsymbol{u}}{c}\right) \qquad (2.4.9)$$

For the time interval dw, during which the material particle moved from point E to point E_i, it is $dw = \left(t - \frac{r'}{c}\right) - \left(t - \frac{r}{c}\right) = \frac{r}{c} - \frac{r'}{c}$, therefore from equation (2.4.9) we obtain

$$\overrightarrow{AA_i} = d\boldsymbol{r} = -c\,dw\left(\frac{\boldsymbol{\upsilon}}{c} - \frac{\boldsymbol{u}}{c}\right) \qquad (2.4.10)$$

In figure (2.4.1) we consider curve C which includes all the generalized photons emitted by the material particle during the time interval from $w = t - \frac{r}{c}$ to t towards a particular direction $\frac{\boldsymbol{\upsilon}}{c}$, that is, with the same velocity $\boldsymbol{\upsilon}$.

We now consider the tangent vector $\boldsymbol{t}$ of the curve C at point A

$$\boldsymbol{t} = \frac{d\boldsymbol{r}}{\left\|d\boldsymbol{r}\right\|} = \frac{\frac{\boldsymbol{u}}{c} - \frac{\boldsymbol{\upsilon}}{c}}{\left\|\frac{\boldsymbol{u}}{c} - \frac{\boldsymbol{\upsilon}}{c}\right\|} = \frac{\boldsymbol{u} - \boldsymbol{\upsilon}}{\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\|} \qquad (2.4.11)$$

as follows from equation (2.4.10). For the three-dimensional arc length dS of curve C at point A we obtain from equation (2.4.10)

$$dS = \left\|d\boldsymbol{r}\right\| = dw\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\| \qquad (2.4.12)$$

Now, we calculate the curvature k and the torsion τ of curve C at point A. First, we calculate the curvature vector $\boldsymbol{k}$.

$$\boldsymbol{k} = \frac{d\boldsymbol{t}}{ds} = \frac{d\boldsymbol{t}}{dw\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\|} = \frac{1}{\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\|}\frac{d}{dw}\left(\frac{\boldsymbol{u} - \boldsymbol{\upsilon}}{\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\|}\right) \qquad (2.4.13)$$

Taking into account that $\frac{d\boldsymbol{\upsilon}}{dw} = 0$, $\frac{d\boldsymbol{u}}{dw} = \boldsymbol{\alpha}$ and $\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\| = \sqrt{c^2 + \boldsymbol{u}^2 - 2\left(\boldsymbol{\upsilon} \cdot \boldsymbol{u}\right)}$, we calculate the vector

$$\boldsymbol{n} = \frac{\boldsymbol{k}}{\left\|\boldsymbol{k}\right\|} = \frac{\left(\boldsymbol{u} - \boldsymbol{\upsilon}\right) \times \left[\boldsymbol{\alpha} \times \left(\boldsymbol{u} - \boldsymbol{\upsilon}\right)\right]}{\left\|\boldsymbol{u} - \boldsymbol{\upsilon}\right\|\left\|\left(\boldsymbol{u} - \boldsymbol{\upsilon}\right) \times \boldsymbol{\alpha}\right\|} \qquad (2.4.14)$$

Combining equations (2.4.11) and (2.4.14), we calculate vector $\boldsymbol{b} = \boldsymbol{t} \times \boldsymbol{n}$ appearing in the Frenet formulas:

$$\boldsymbol{b} = \frac{\left(\boldsymbol{u} - \boldsymbol{\upsilon}\right) \times \boldsymbol{\alpha}}{\left\|\left(\boldsymbol{u} - \boldsymbol{\upsilon}\right) \times \boldsymbol{\alpha}\right\|} \qquad (2.4.15)$$

We remind that the Frenet equations

$$\frac{d\boldsymbol{t}}{ds}=k\boldsymbol{n}$$
$$\frac{d\boldsymbol{n}}{ds}=-k\boldsymbol{t}+\tau\boldsymbol{b} \tag{2.4.16}$$
$$\frac{d\boldsymbol{b}}{ds}=-\tau\boldsymbol{n}$$

uniquely determine the curve C. Having calculated vectors $\boldsymbol{t},\boldsymbol{n},\boldsymbol{b}$ we now determine the curvature k and the torsion τ of curve C from equations (2.4.16). After the necessary calculations, we obtain

$$k=\frac{\sqrt{\|\boldsymbol{u}-\boldsymbol{\upsilon}\|^2\|\boldsymbol{\alpha}\|^2-\left[\boldsymbol{\alpha}\cdot(\boldsymbol{u}-\boldsymbol{\upsilon})\right]^2}}{\|\boldsymbol{u}-\boldsymbol{\upsilon}\|^3} \tag{2.4.17}$$

$$\tau=\frac{\boldsymbol{\alpha}\left[(\boldsymbol{u}-\boldsymbol{\upsilon})\times\dfrac{d\boldsymbol{\alpha}}{dw}\right]}{\|\boldsymbol{\alpha}\|^2\|\boldsymbol{u}-\boldsymbol{\upsilon}\|^2-\left[(\boldsymbol{u}-\boldsymbol{\upsilon})\cdot\boldsymbol{\alpha}\right]^2}\|\boldsymbol{u}-\boldsymbol{\upsilon}\|^2 \tag{2.4.18}$$

We repeat the same procedure deriving vectors $\boldsymbol{t}_p,\boldsymbol{k}_p$ και $\boldsymbol{b}_p$ at point E of the curve C_p of the material particle. For $\|\boldsymbol{u}\|\neq 0$ it is

$$\boldsymbol{t}_p=\frac{\boldsymbol{u}}{\|\boldsymbol{u}\|} \tag{2.4.19}$$

while the three-dimensional arc length is

$$dS_p=\|\boldsymbol{u}\|dw \tag{2.4.20}$$

The curvature vector $\boldsymbol{k}_p$ is given by

$$\boldsymbol{k}_p=\frac{d\boldsymbol{t}_p}{dS_p}=\frac{1}{\|\boldsymbol{u}\|}\frac{d}{dw}\left(\frac{\boldsymbol{u}}{\|\boldsymbol{u}\|}\right)=\frac{\boldsymbol{\alpha}}{\|\boldsymbol{u}\|^2}-\frac{(\boldsymbol{u}\cdot\boldsymbol{\alpha})}{\|\boldsymbol{u}\|^4}\boldsymbol{u}$$

and finally,

$$\boldsymbol{k}_p=\frac{\boldsymbol{u}\times(\boldsymbol{\alpha}\times\mathbf{u})}{\|\mathbf{u}\|^4} \tag{2.4.21}$$

From equation (2.4.21) we get for vector $\boldsymbol{n}_p$

$$\boldsymbol{n}_p=\frac{\boldsymbol{k}_p}{\|\boldsymbol{k}_p\|}=\frac{\boldsymbol{u}\times(\boldsymbol{\alpha}\times\mathbf{u})}{\|\boldsymbol{u}\|\|\boldsymbol{\alpha}\times\mathbf{u}\|} \tag{2.4.22}$$

From equations (2.4.19) and (2.4.22) we get vector $\boldsymbol{b}_p=\boldsymbol{t}_p\times\boldsymbol{n}_p$

$$\boldsymbol{b}_p=\frac{\boldsymbol{u}\times\boldsymbol{\alpha}}{\|\boldsymbol{u}\times\boldsymbol{\alpha}\|} \tag{2.4.23}$$

From the Frenet formulas (2.4.16) for curve C_p, we get for the curvature k_p and the torsion τ_p:

$$k_p=\frac{\sqrt{\|\boldsymbol{u}\|^2\|\boldsymbol{\alpha}\|^2-(\boldsymbol{u}\cdot\boldsymbol{\alpha})^2}}{\|\boldsymbol{u}\|^3} \tag{2.4.24}$$

$$\tau_p = \frac{\boldsymbol{\alpha} \cdot \left(\boldsymbol{u} \times \dfrac{d\boldsymbol{\alpha}}{dw} \right)}{\|\boldsymbol{\alpha}\|^2 \|\boldsymbol{u}\|^2 - (\boldsymbol{u} \cdot \boldsymbol{\alpha})^2} \|\boldsymbol{u}\|^2 \qquad (2.4.25)$$

Comparing equations (2.4.11), (2.4.14), (2.4.15), (2.4.17) and (2.4.18) for curve C, with equations (2.4.19), (2.4.22), (2.4.23), (2.4.24) and (2.4.25) for curve C_p we arrive at the following theorem:
Trajectory representation theorem

"For every direction $\frac{\boldsymbol{\upsilon}}{c}$ the following hold:

a) The map $f : \boldsymbol{u} \to \boldsymbol{u} - \boldsymbol{\upsilon}$ maps the trajectory C_p of the material particle to the curve C of the generalized photons moving with velocity $\boldsymbol{\upsilon}$: $f : \left(\boldsymbol{t}_p, \boldsymbol{n}_p, \boldsymbol{b}_p, k_p, \tau_p \right) \to \left(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}, k, \tau \right)$

b) The map $f^{-1} : \boldsymbol{u} - \boldsymbol{\upsilon} \to \boldsymbol{u}$ maps the curve C of the generalized photons moving with velocity $\boldsymbol{\upsilon}$ to the curve C_p of the material particle: $f^{-1} : \left(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b}, k, \tau \right) \to \left(\boldsymbol{t}_p, \boldsymbol{n}_p, \boldsymbol{b}_p, k_p, \tau_p \right)$ "

According to the "trajectory representation" theorem, if we know the position $P(x, y, z, t)$ of the material particle at moment t and the trajectory C_p at some past time, we can determine the distribution of the generalized photons the material particle has emitted in this specific past time. We know exactly how each kinematic characteristic of the material particle maps to its surrounding spacetime.

2.5 The fundamental mathematical theorem

The interaction of the material point particle with the surrounding spacetime depends on the following four parameters:

- The moment $w = t - \frac{r}{c}$ of emission of the generalized photon by the material particle. All the physical quantities, such as the rest mass, the electric charge, the velocity $\boldsymbol{u} = \boldsymbol{u}(w)$ and the acceleration $\boldsymbol{\alpha} = \boldsymbol{\alpha}(w)$ of the material particle depend upon the moment w of the emission of the generalized photon.
- The distance $r = \|\boldsymbol{r}\|$ of the arbitrary point $A(x, y, z, t)$, as depicted in figure 2.2.1, from the point of emission $E\left(x_p(w), y_p(w), z_p(w), w\right)$ of the generalized photon.
- The direction in space, i.e. the functions $\delta = \delta(x, y, z, t)$ and $\omega = \omega(x, y, z, t)$

In this paragraph we will prove the fundamental equations concerning these four parameters.
Initially we prove that the vectors ∇w, $\nabla \delta$ και $\nabla \omega$ are linearly independent. Let us suppose that $\lambda_1 \nabla w + \lambda_2 \nabla w + \lambda_3 \nabla w = 0$, $\lambda_1, \lambda_2, \lambda_2 \in \mathbb{R}$
Taking into account equations (2.2.12), (2.3.19) and (2.3.20), we obtain

$$-\lambda_1 \frac{1}{c\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}+\lambda_2\left(\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}+\frac{1}{r}\boldsymbol{\beta}\right)+\lambda_3\left(\frac{\boldsymbol{u}\cdot\boldsymbol{\gamma}}{\sin\delta r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}+\frac{1}{r\sin\delta}\boldsymbol{\gamma}\right)=0$$

From the linear independence of the vectors $\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}$ we see that

$$\frac{-\lambda_1}{c}+\lambda_2\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r}+\lambda_3\frac{\boldsymbol{u}\cdot\boldsymbol{\gamma}}{r\sin\delta}=0$$

$$\frac{\lambda_2}{r}=0$$

$$\frac{\lambda_3}{r\sin\delta}=0$$

Finally, we have $\lambda_1=\lambda_2=\lambda_3=0$.

Therefore the vectors $\nabla w,\nabla\delta,\nabla\omega$ are linearly independent.

We now focus our attention on the variation of the quantities w,δ,ω and r on the trajectory of the material particle and on the trajectory of the generalized photon. The following two theorems hold:

Theorem I

$$\frac{\partial w}{\partial t}+\boldsymbol{u}\cdot\nabla w=1 \qquad \text{(a)}$$

$$\frac{\partial \delta}{\partial t}+\boldsymbol{u}\cdot\nabla \delta=0 \qquad \text{(b)} \qquad (2.5.1)$$

$$\frac{\partial \omega}{\partial t}+\boldsymbol{u}\cdot\nabla \omega=0 \qquad \text{(c)}$$

$$\frac{\partial r}{\partial t}+\boldsymbol{u}\nabla r=0 \qquad \text{(d)}$$

Theorem II

$$\frac{\partial w}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla w=0 \qquad \text{(a)}$$

$$\frac{\partial \delta}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla \delta=0 \qquad \text{(b)}$$

$$\frac{\partial \omega}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla \omega=0 \qquad \text{(c)} \qquad (2.5.2)$$

$$\frac{\partial r}{c\partial t}+\frac{\boldsymbol{\upsilon}}{c}\cdot\nabla r=1 \qquad \text{(d)}$$

From equations (2.2.11) and (2.2.12) we have

$$\frac{\partial w}{\partial t}+\boldsymbol{u}\cdot\nabla w=\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}=\frac{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}=1$$

$$\frac{\partial w}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla w=\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}-\frac{\|\boldsymbol{\upsilon}\|^2}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}=0$$

From equations (2.3.9) and (2.3.19) we have

$$\frac{\partial\delta}{\partial t}+\boldsymbol{u}\nabla\delta=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\boldsymbol{u}\left(\frac{(\boldsymbol{u}\cdot\boldsymbol{\beta})}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c^2}+\frac{1}{r}\boldsymbol{\beta}\right)$$

$$=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{(\boldsymbol{u}\cdot\boldsymbol{\beta})\left(\boldsymbol{u}\frac{\boldsymbol{\upsilon}}{c^2}\right)}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r}=$$

$$=\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[-1+\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}+1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right]=0$$

$$\frac{\partial\delta}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\delta=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\boldsymbol{\upsilon}\left(\frac{(\boldsymbol{u}\cdot\boldsymbol{\beta})}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c^2}+\frac{1}{r}\boldsymbol{\beta}\right)$$

$$=-\frac{\boldsymbol{u}\cdot\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{(\boldsymbol{u}\cdot\boldsymbol{\beta})}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\|\boldsymbol{\upsilon}\|^2}{c^2}+\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\beta}}{r}=0$$

since $\|\boldsymbol{\upsilon}\|^2=c^2$ and $\boldsymbol{\upsilon}\cdot\boldsymbol{\beta}=0$.

Similarly, starting from equations (2.3.10) and (2.3.20) we arrive at equations (2.5.1)(c) and (2.5.2)(c).

From equations (2.2.9) and (2.2.10) we get

$$\frac{\partial r}{c\partial t}+\frac{\boldsymbol{u}}{c}\nabla r=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{\boldsymbol{u}}{c}\left(\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c}\right)=0$$

$$\frac{\partial r}{c\partial t}+\frac{\boldsymbol{\upsilon}}{c}\nabla r=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{\boldsymbol{\upsilon}}{c}\left(\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c}\right)=$$

$$=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}+\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}=$$

$$\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)=1$$

With the aid of the above theorems we can prove the following fundamental theorem:

The Fundamental Mathematical Theorem

For every function $f=f(w,\delta,\omega,r)$ the following hold:

A)

$$\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\nabla f=\frac{\partial f}{\partial w} \tag{2.5.3}$$

$$\frac{\partial}{\partial t}\left(f\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\left(f\boldsymbol{\beta}\right)\right)\boldsymbol{u}=\frac{\boldsymbol{\upsilon}}{c}\frac{\partial f}{\partial w} \tag{2.5.4}$$

$$\frac{\partial}{\partial t}\left(f\boldsymbol{\beta}\right)+\left(grad\left(f\boldsymbol{\beta}\right)\right)\boldsymbol{u}=\boldsymbol{\beta}\frac{\partial f}{\partial w} \tag{2.5.5}$$

$$\frac{\partial}{\partial t}\left(f\boldsymbol{\gamma}\right)+\left(grad\left(f\boldsymbol{\gamma}\right)\right)\boldsymbol{u}=\boldsymbol{\gamma}\frac{\partial f}{\partial w} \tag{2.5.6}$$

B)

$$\frac{\partial f}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f=c\frac{\partial f}{\partial r} \tag{2.5.7}$$

$$\frac{\partial}{\partial t}\left(f\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\left(f\frac{\boldsymbol{\upsilon}}{c}\right)\right)\boldsymbol{\upsilon}=\boldsymbol{\upsilon}\frac{\partial f}{\partial r} \tag{2.5.8}$$

$$\frac{\partial}{\partial t}\left(f\boldsymbol{\beta}\right)+\left(grad\left(f\boldsymbol{\beta}\right)\right)\boldsymbol{\upsilon}=\boldsymbol{\beta}\frac{\partial f}{\partial r} \tag{2.5.9}$$

$$\frac{\partial}{\partial t}\left(f\boldsymbol{\gamma}\right)+\left(grad\left(f\boldsymbol{\gamma}\right)\right)\boldsymbol{\upsilon}=\boldsymbol{\gamma}\frac{\partial f}{\partial r} \tag{2.5.10}$$

We prove equations (2.5.3), (2.5.4) and (2.5.7). The rest of the equations of the fundamental mathematical theorem are proven similarly. For the proof of equation (2.5.3) we have

$$\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\nabla f=\frac{\partial f}{\partial w}\frac{\partial w}{\partial t}+\frac{\partial f}{\partial \delta}\frac{\partial \delta}{\partial t}+\frac{\partial f}{\partial \omega}\frac{\partial \omega}{\partial t}+\frac{\partial f}{\partial r}\frac{\partial r}{\partial t}$$

$$+\boldsymbol{u}\cdot\left(\frac{\partial f}{\partial w}\nabla w+\frac{\partial f}{\partial \delta}\nabla \delta+\frac{\partial f}{\partial \omega}\nabla \omega+\frac{\partial f}{\partial r}\nabla r\right)$$

$$=\frac{\partial f}{\partial w}\left(\frac{\partial w}{\partial t}+\boldsymbol{u}\nabla w\right)+\frac{\partial f}{\partial \delta}\left(\frac{\partial \delta}{\partial t}+\boldsymbol{u}\cdot\nabla \delta\right)$$

$$+\frac{\partial f}{\partial \omega}\left(\frac{\partial \omega}{\partial t}+\boldsymbol{u}\cdot\nabla \omega\right)+\frac{\partial f}{\partial r}\left(\frac{\partial r}{\partial t}+\boldsymbol{u}\cdot\nabla r\right)$$

and taking into account equations (2.5.1) we obtain $\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\nabla f=\frac{\partial f}{\partial w}$, which is equation (2.5.3).

In order to prove equation (2.5.4) we use the identity

$$grad\left(f\boldsymbol{\alpha}\right)=\nabla f\otimes\boldsymbol{\alpha}+fgrad\boldsymbol{\alpha} \tag{2.5.11}$$

which holds for every vector $\boldsymbol{\alpha}$ and scalar function f. We can now prove equation (2.5.4) as:

$$\frac{\partial}{\partial t}\left(f\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\left(f\frac{\boldsymbol{\upsilon}}{c}\right)\right)\boldsymbol{u}=$$

$$\frac{\partial f}{\partial t}\frac{\boldsymbol{\upsilon}}{c}+f\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)+\left(fgrad\frac{\boldsymbol{\upsilon}}{c}+\nabla f\otimes\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{u}$$

Using identity (2.3.13) $\left(\boldsymbol{\alpha}\otimes\boldsymbol{b}\right)\boldsymbol{c}=\left(\boldsymbol{\alpha}\cdot\boldsymbol{c}\right)\boldsymbol{b}$ we obtain

$$\frac{\partial f}{\partial t}\frac{\boldsymbol{\upsilon}}{c}+f\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)+\left(fgrad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{u}+\left(\boldsymbol{u}\cdot\nabla f\right)\frac{\boldsymbol{\upsilon}}{c}=$$
$$\left(\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\nabla f\right)\frac{\boldsymbol{\upsilon}}{c}+f\left(\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{u}\right)=$$
$$\frac{\partial f}{\partial w}\frac{\boldsymbol{\upsilon}}{c}$$
since $\frac{\partial f}{\partial t}+\boldsymbol{u}\cdot\nabla f=\frac{\partial f}{\partial w}$, according to equation (2.5.3) and furthermore
$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{u}=$$
$$\frac{\partial\delta}{\partial t}\boldsymbol{\beta}+\sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma}+\left(\nabla\delta\otimes\boldsymbol{\beta}+\sin\delta\nabla\omega\otimes\boldsymbol{\gamma}\right)\boldsymbol{u}$$
according to equations (2.3.6)(b), (d). Hence we obtain
$$\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)+\left(grad\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{u}=$$
$$\frac{\partial\delta}{\partial t}\boldsymbol{\beta}+\sin\delta\frac{\partial\omega}{\partial t}\boldsymbol{\gamma}+\left(\boldsymbol{u}\cdot\nabla\delta\right)\boldsymbol{\beta}+\sin\delta\left(\boldsymbol{u}\cdot\nabla\omega\right)\boldsymbol{\gamma}=$$
$$\left(\frac{\partial\delta}{\partial t}+\boldsymbol{u}\cdot\nabla\delta\right)\boldsymbol{\beta}+\sin\delta\left(\frac{\partial\omega}{\partial t}+\boldsymbol{u}\cdot\nabla\omega\right)\boldsymbol{\gamma}=0$$
according to equations (2.5.1)(b), (c).
The proof of equation (2.5.7) goes as follows:
$$\frac{\partial f}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f=\frac{\partial f}{\partial w}\frac{\partial w}{\partial t}+\frac{\partial f}{\partial\delta}\frac{\partial\delta}{\partial t}+\frac{\partial f}{\partial\omega}\frac{\partial\omega}{\partial t}+\frac{\partial f}{\partial r}\frac{\partial r}{\partial t}$$
$$+\boldsymbol{\upsilon}\cdot\left(\frac{\partial f}{\partial w}\nabla w+\frac{\partial f}{\partial\delta}\nabla\delta+\frac{\partial f}{\partial\omega}\nabla\omega+\frac{\partial f}{\partial r}\nabla r\right)$$
$$=\frac{\partial f}{\partial w}\left(\frac{\partial w}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla w\right)+\frac{\partial f}{\partial\delta}\left(\frac{\partial\delta}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\delta\right)$$
$$+\frac{\partial f}{\partial\omega}\left(\frac{\partial\omega}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\omega\right)+\frac{\partial f}{\partial r}\left(\frac{\partial r}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla r\right)$$
Taking into consideration equations (2.5.2) we get $\frac{\partial f}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla f=c\frac{\partial f}{\partial r}$, which is equation (2.5.7).
An immediate consequence of the fundamental theorem is the following lemma:
For every vector function $\boldsymbol{F}=\boldsymbol{F}\left(w,\delta,\omega,r\right)$ the following relations hold:
$$\frac{\partial\boldsymbol{F}}{\partial t}+\left(grad\boldsymbol{F}\right)\cdot\boldsymbol{u}=\frac{\partial\boldsymbol{F}}{\partial w}\tag{2.5.12}$$
$$\frac{\partial\boldsymbol{F}}{\partial t}+\left(grad\boldsymbol{F}\right)\boldsymbol{\upsilon}=c\frac{\partial\boldsymbol{F}}{\partial r}\tag{2.5.13}$$
The proof is done by writing the vector function $\boldsymbol{F}$ in the form
$$\boldsymbol{F}=F_1\left(w,\delta,\omega,r\right)\frac{\boldsymbol{\upsilon}}{c}+F_2\left(w,\delta,\omega,r\right)\boldsymbol{\beta}+F_3\left(w,\delta,\omega,r\right)\boldsymbol{\gamma}$$
and applying the theorem.

The fundamental mathematical theorem determines the variation of any scalar, vectorial and tensorial physical quantity, both as defined on the material particle, as well as on the surrounding spacetime. Of special interest are the applications of this theorem for the variations of the rest mass, the electric charge, the energy, the linear momentum, the angular momentum, and any other conserved physical quantity, for the system "material particle-generalized photon". The fundamental theorem allows us to correlate the variations that take place on the material particle with the corresponding variations that take place in the surrounding spacetime.

2.6 The properties of the vector basis $\{\frac{\mathbf{\upsilon}}{c},\mathbf{\beta},\mathbf{\gamma}\}$

The properties of the right-handed orthonormal vector basis $\{\frac{\mathbf{\upsilon}}{c},\mathbf{\beta},\mathbf{\gamma}\}$ are given by equations (2.3.6), (2.3.7) and (2.3.8). In these equations we already know their second parts from the study conducted in the preceding paragraphs. Thus, we can express them in a simpler form.

The first of equations (2.3.6), (2.3.7) and (2.3.8) can be written as:

$$\nabla\cdot\left(\frac{\mathbf{\upsilon}}{c}\right)=\frac{2}{r} \tag{2.6.1}$$

$$\nabla\cdot\mathbf{\beta}=-\frac{\mathbf{u}\cdot\mathbf{\beta}}{cr\left(1-\frac{\mathbf{\upsilon u}}{c^2}\right)}+\frac{\cos\delta}{r\sin\delta} \tag{2.6.2}$$

$$\nabla\cdot\mathbf{\gamma}=-\frac{\mathbf{u\gamma}}{cr\left(1-\frac{\mathbf{\upsilon u}}{c^2}\right)} \tag{2.6.3}$$

Equation (2.6.1) results directly from equation (2.2.21). But we can also prove it in a different way, starting from the first of equations (2.3.6)

$$\nabla\cdot\left(\frac{\mathbf{\upsilon}}{c}\right)=\mathbf{\beta}\cdot\nabla\delta+\sin\delta\mathbf{\gamma}\cdot\nabla\omega$$

With the help of equations (2.3.19) and (2.3.20) we obtain

$$\nabla\cdot\left(\frac{\mathbf{\upsilon}}{c}\right)=\frac{1}{r}+\frac{1}{r}=\frac{2}{r}$$

taking into account that the set of the vectors $\{\frac{\mathbf{\upsilon}}{c},\mathbf{\beta},\mathbf{\gamma}\}$ form a right-handed, orthonormal vector basis.

From the first of equations (2.3.7) we obtain

$$\nabla\cdot\mathbf{\beta}=-\frac{\mathbf{\upsilon}}{c}\nabla\delta+\cos\delta\mathbf{\gamma}\cdot\nabla\omega$$

Through equations (2.3.19) and (2.3.20) we get

$$\nabla\cdot\mathbf{\beta}=-\frac{\mathbf{u\beta}}{cr\left(1-\frac{\mathbf{\upsilon u}}{c^2}\right)}+\frac{\cos\delta}{r\sin\delta}$$

From the first of equations (2.3.8) we have that

$$\nabla\cdot\mathbf{\gamma}=-\sin\delta\frac{\mathbf{\upsilon}}{c}\nabla\omega-\cos\delta\mathbf{\beta}\cdot\nabla\omega$$

Using equation (2.3.20) we see that

$$\nabla\cdot\boldsymbol{\gamma}=-\frac{\mathbf{u}\cdot\boldsymbol{\gamma}}{cr\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)}$$

Accordingly we can write in a simpler form the rest of the equations (2.3.6), (2.3.7) and (2.3.8), whenever it is demanded by the mathematical calculations performed.

2.7 List of auxiliary equations

We prove the following auxiliary equations:

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{\partial t}=\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}+\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})^2-c^2u^2}{c^3r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{2.7.1}$$

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u})=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon}+\frac{c}{r}\boldsymbol{u}+\frac{u^2-(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c} \tag{2.7.2}$$

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{\partial t}=\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{b}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}+\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})-c^2(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{2.7.3}$$

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})=-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{b}}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon}+\frac{c}{r}\boldsymbol{\alpha}+\frac{\boldsymbol{u}\cdot\boldsymbol{\alpha}-\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{cr\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon} \tag{2.7.4}$$

where $\boldsymbol{\alpha}=\boldsymbol{\alpha}(w)=\frac{d\boldsymbol{u}(w)}{dw}$ and $\boldsymbol{b}=\boldsymbol{b}(w)=\frac{d\boldsymbol{\alpha}(w)}{dw}$ and $u^2=\|\boldsymbol{u}\|^2$.

Indeed, it holds that

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{\partial t}=\boldsymbol{u}\frac{\partial\boldsymbol{\upsilon}}{\partial t}+\boldsymbol{\upsilon}\frac{\partial\boldsymbol{u}}{\partial t}$$

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{\partial t}=\boldsymbol{u}\frac{\partial\boldsymbol{\upsilon}}{\partial t}+\boldsymbol{\upsilon}\frac{\partial\boldsymbol{u}}{\partial w}\cdot\frac{\partial w}{\partial t}$$

Through equations (2.2.24) and (2.2.11) we obtain

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{\partial t}=\boldsymbol{u}\frac{\partial\boldsymbol{\upsilon}}{\partial t}+\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}$$

With the help of equation (2.2.16) we get

$$\frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{\partial t}=\frac{c}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})^2}{c^2}-u^2\right]+\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}$$

and performing the necessary algebraic transformations we obtain equation (2.7.1).
In order to prove equation (2.7.2) we start from the identity

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u})=\left(grad^T\boldsymbol{\upsilon}\right)\boldsymbol{u}+\left(grad^T\boldsymbol{u}\right)\boldsymbol{\upsilon}$$

where $grad^T\boldsymbol{\upsilon}$ and $grad^T\boldsymbol{u}$ are the transpose matrices of $grad\boldsymbol{\upsilon}$ and $grad\boldsymbol{u}$.
From equations (2.2.19) and (2.2.27) we obtain

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u})=\left[\frac{c}{r}I+\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}\otimes(\boldsymbol{u}-\boldsymbol{\upsilon})\right]^T\boldsymbol{u}-\frac{1}{c\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(\frac{\boldsymbol{\upsilon}}{c}\otimes\boldsymbol{\alpha}\right)^T\boldsymbol{\upsilon}$$

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u})=\left[\frac{c}{r}I+\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}(\boldsymbol{u}-\boldsymbol{\upsilon})\otimes\frac{\boldsymbol{\upsilon}}{c}\right]^T\boldsymbol{u}-\frac{1}{c\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(\boldsymbol{\alpha}\otimes\frac{\boldsymbol{\upsilon}}{c}\right)\boldsymbol{\upsilon}$$

Using identity (2.3.13) we get

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u})=\frac{c}{r}\boldsymbol{u}+\frac{\boldsymbol{u}\cdot(\boldsymbol{u}-\boldsymbol{\upsilon})}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\cdot\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}}{c\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}$$

which is equation (2.7.2). We can similarly prove equations (2.7.3) and (2.7.4). In order to prove the last equation we use equation (2.2.31), in exactly the same manner we used equation (2.2.27). In the same way, we can prove corresponding equations for all of the inner products such as $\boldsymbol{\upsilon}\cdot\boldsymbol{b}$, $\boldsymbol{u}\cdot\boldsymbol{\alpha}$ etc., that appear in the equations of the theory of selfvariations.

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CHAPTER 3

The study of the selfvariations for a material point particle moving with constant speed

3.1. Introduction

In this chapter we present the study of the selfvariations for a material point particle moving with constant speed. This study was regarded as necessary for two reasons. The first is that constant-speed motion is the simplest possible and, therefore, we are studying the consequences of the selfvariations in their simplest version. The second reason is that arbitrary motion can be considered as a multitude of successive constant-speed motions.

By studying the constant-speed motion of a material particle we can derive the Lorentz-Einstein transformations for the physical quantities w,δ,ω,r that appear in the equations of the theory of selfvariations. Of special interest is the transformation of the volume of the generalized photon, which differs from the volume transformation of material particles as we know it within the framework of Special Relativity. After having studied both the arbitrary motion, as well as the constant-speed motion of the material particle, we have the knowledge necessary for advancing our study in the forthcoming chapters.

3.2 The case of a material point particle moving with constant speed

We consider a material point particle with rest mass m_0 and electric charge q, which moves with velocity $\boldsymbol{u}=\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$ in the inertial frame of reference $S\left(0,x,y,z,t\right)$, as depicted in figure 3.2.1

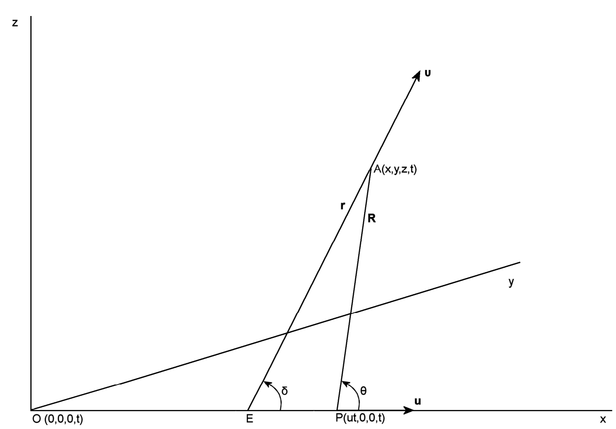

Figure 3.2.1 Material point particle moving with constant speed along the x axis of the inertial reference frame $S\left(0,x,y,z,t\right)$. As the material particle moves from point E to point P, during the time interval $\Delta t=\frac{r}{c}$, a generalized photon moves from point E to point A.

At moment t when the material particle is at point $P(ut,0,0,t)$, the rest mass m_0 and the electric charge q of the material particle act at point $A(x,y,z,t)$ through the generalized photon that was emitted from point E and arrived at point A moving with velocity c. Therefore, the coordinates of point E are

$$E\left(ut-\frac{u}{c}r,0,0,t-\frac{r}{c}\right) \tag{3.2.1}$$

where $r=\|\boldsymbol{r}\|=\|\overline{EA}\|$. Due to the selfvariations, the rest mass m_0 and the electric charge q of the material particle act at point $A(x,y,z,t)$ with the value they had at time

$$w=t-\frac{r}{c} \tag{3.2.2}$$

at point $E\left(ut-\frac{u}{c}r,0,0,t-\frac{r}{c}\right)$, and not with the value they have at point $P(ut,0,0,t)$ at time t . For the vector $\boldsymbol{r}$ we have

$$\boldsymbol{r}=\overrightarrow{EA}=\begin{bmatrix} x-ut+\frac{u}{c}r \\ y \\ z \end{bmatrix} \tag{3.2.3}$$

The magnitude of $\|\boldsymbol{r}\|=r$ can be derived from equations (3.2.3) as

$$\|\boldsymbol{r}\|=r=\gamma^2\frac{u}{c}(x-ut)+\gamma\sqrt{\gamma^2(x-ut)^2+y^2+z^2} \tag{3.2.4}$$

where $\gamma=\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

Combining equations (3.2.3) and (3.2.4) we obtain

$$\boldsymbol{r}=\begin{bmatrix} \gamma^2(x-ut)+\frac{u}{c}\gamma\sqrt{\gamma^2(x-ut)^2+y^2+z^2} \\ y \\ z \end{bmatrix} \tag{3.2.5}$$

The velocity $\boldsymbol{\upsilon}$ of the selfvariations has magnitude $\|\boldsymbol{\upsilon}\|=c$, and is parallel to the vector $\boldsymbol{r}$, thus we have

$$\boldsymbol{\upsilon}=\frac{c}{r}\boldsymbol{r}=\frac{c}{r}\begin{bmatrix} \gamma^2(x-ut)+\frac{u}{c}\gamma\sqrt{\gamma^2(x-ut)^2+y^2+z^2} \\ y \\ z \end{bmatrix} \tag{3.2.6}$$

The position vector $\boldsymbol{R}$ of point $A(x,y,z,t)$ with respect to point $P(ut,0,0,t)$, where the material particle is located, is

$$\boldsymbol{R}=\overrightarrow{PA}=\begin{bmatrix} x-ut \\ y \\ z \end{bmatrix} \quad (3.2.7)$$

From equation (3.2.7) we obtain

$$\|\boldsymbol{R}\|=R=\sqrt{(x-ut)^2+y^2+z^2} \quad (3.2.8)$$

From figure 3.2.1 we see that

$$\boldsymbol{r}=\overrightarrow{EA}+\boldsymbol{R}$$

$$\boldsymbol{r}=\frac{r}{c}\boldsymbol{u}+\boldsymbol{R}$$

Finally, we obtain

$$\boldsymbol{\upsilon}=\boldsymbol{u}+\frac{c}{r}\boldsymbol{R} \quad (3.2.9)$$

$$\boldsymbol{R}=r\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right) \quad (3.2.10)$$

Combining equations (3.2.1) and (3.2.2) we have for the coordinates of point E

$$E(uw,0,0,w) \quad (3.2.11)$$

The relations between the scalar, vectorial and tensorial quantities of this paragraph can be derived by the corresponding relations proven in the second chapter, considering that the acceleration of the material body vanishes, that is $\boldsymbol{\alpha}=\boldsymbol{\alpha}(w)=0$, and that the velocity of the material particle is $\boldsymbol{u}=\boldsymbol{u}(w)=\begin{bmatrix} u(w) \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$.

3.3 The case of a material point particle at rest

We consider an inertial reference frame $S'(0',x',y',z',t')$ moving with velocity $\boldsymbol{u}=\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$ with respect to the inertial reference frame $S(0,x,y,z,t)$ of the previous paragraph. We also suppose that for $t=t'=0$ the origins of the axes of coordinates 0 και $0'$ of these two frames coincide. In the way we have chosen these two inertial frames, the material particle is at rest in frame S' or, equivalently, frame S' accompanies the material particle during its motion. Figure 3.3.1 is the one corresponding to figure 3.2.1 for reference frame S'.

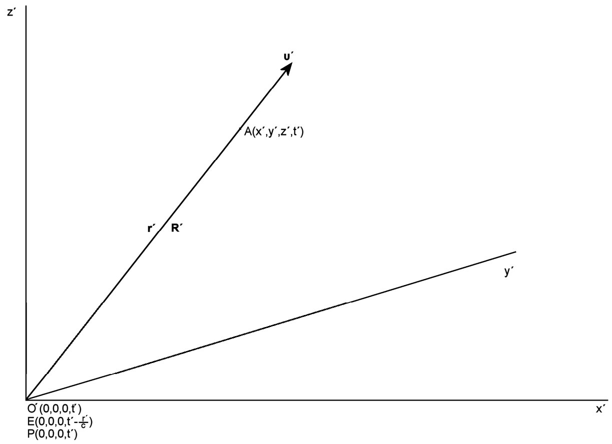

Figure 3.3.1 A material point particle remains at rest at the origin $O'(0,0,0,0,t')$ of the inertial reference frame $S(0',x',y',z',t')$. A generalized photon moves from point $E\left(0,0,0,0,t'-\frac{r'}{c}\right)$ and arrives at point $A(x',y',z',t')$, during the time interval $\Delta t' = \frac{r'}{c}$.

At moment t', when the material particle is located at point $P(0,0,0,t')$, the mass m_o and the electric charge q of the material particle act at point $A(x',y',z',t')$ through the generalized photon that was emitted from point $E\left(0,0,0,t'-\frac{r'}{c}\right)$ and arrived at point $A(x',y',z',t')$ moving with velocity c. Therefore, the coordinates of point E are

$$E\left(0,0,0,t'-\frac{r'}{c}\right) \tag{3.3.1}$$

where $r' = \|\boldsymbol{r}'\| = \|\overline{EA}\|$. Due to the selfvariations, the rest mass m_o and the electric charge q of the material particle act at point $A(x',y',z',t')$ with the value they had at time

$$w' = t' - \frac{r'}{c} \tag{3.3.2}$$

and not with the value they have at $P(0,0,0,t')$.
For the vector $\boldsymbol{r}'$ it holds that

$$\boldsymbol{r}' = \overrightarrow{EA} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{3.3.3}$$

while its magnitude $\|\boldsymbol{r}'\| = r'$ is given by (3.3.4)

$$\|\boldsymbol{r}'\| = r' = \sqrt{x'^2 + y'^2 + z'^2} \tag{3.3.4}$$

The velocity of the selfvariations $\boldsymbol{\upsilon}'$ has magnitude $\|\boldsymbol{\upsilon}'\| = c$, and is parallel to the vector $\boldsymbol{r}'$, therefore it is

$$\boldsymbol{\upsilon}' = \frac{c}{r'}\boldsymbol{r}' = \frac{c}{r'}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{3.3.5}$$

The position vector $\boldsymbol{R}'$ of point $A(x', y', z', t')$ with respect to $P(0,0,0,t')$, where the material particle is located, is given by

$$\boldsymbol{R}' = \overrightarrow{PA} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \boldsymbol{r}' \tag{3.3.6}$$

From equation (3.3.6) we get

$$\|\boldsymbol{R}'\| = R' = \|\boldsymbol{r}'\| = r' = \sqrt{x'^2 + y'^2 + z'^2} \tag{3.3.7}$$

Combining equations (3.3.1) and (3.3.2) we obtain for the coordinates of point E

$$E(0,0,0,w') \tag{3.3.8}$$

The relations between the scalar, vectorial and tensorial quantities of this paragraph can be derived from the corresponding relations we proved in the second chapter, considering that the acceleration and the velocity of the material particle vanish, that is $\boldsymbol{\alpha} = \boldsymbol{\alpha}[w] = \boldsymbol{0}$ και $\boldsymbol{u} = \boldsymbol{u}[w] = \boldsymbol{0}$.

3.4 Lorentz-Einstein transformations of the quantities w, δ, ω, r

In this paragraph we shall study the way in which the fundamental physical quantities appearing in the equations of the theory of selfvariations transform under the action of the Lorentz-Einstein transformations.

In the way we have chosen the inertial reference frames S and S', the transformations of the coordinates in the four-dimensional spacetime are given by the set of equations

$$\begin{aligned} x &= \gamma(x' + ut') & x' &= \gamma(x - ut) \\ y &= y' & y' &= y \\ z &= z' & z' &= z \\ t &= \gamma\left(t' + \frac{u}{c^2}x'\right) & t' &= \gamma\left(t - \frac{u}{c^2}x\right) \end{aligned} \tag{3.4.1}$$

where $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{u^2}{c^2}}}$.

The coordinates of point E are given by relation (3.2.11), and are $E(uw,0,0,w)$ for inertial frame S, and by relation (3.3.8), and are $E(0,0,0,w')$ for inertial frame S'. Applying transformations (3.4.1) we obtain

$$w = \gamma w' \tag{3.4.2}$$

Indeed, based on the fourth equation of the first column of transformations (3.4.1) for the coordinates of point E, we get

$$w = \gamma\left(w' + u \cdot 0\right)$$
$$w = \gamma w'$$
We now consider the trigonometric form of the velocity $\boldsymbol{\upsilon}$, as defined in paragraph 2.2 of the second chapter. From equations (2.3.2) we get for reference frames S and S' respectively

$$\cos\delta = \frac{\upsilon_x}{c}$$
$$\sin\delta\cos\omega = \frac{\upsilon_y}{c} \qquad (3.4.2)$$
$$\sin\delta\sin\omega = \frac{\upsilon_z}{c}$$

$$\cos\delta' = \frac{\upsilon'_x}{c}$$
$$\sin\delta'\cos\omega' = \frac{\upsilon'_y}{c} \qquad (3.4.3)$$
$$\sin\delta'\sin\omega' = \frac{\upsilon'_z}{c}$$

From the Lorentz-Einstein transformations for the velocity we have

$$\upsilon_x = \frac{\upsilon'_x + u}{1 + \frac{u\upsilon'_x}{c^2}} \qquad \upsilon'_x = \frac{\upsilon_x - u}{1 - \frac{u\upsilon_x}{c^2}}$$
$$\upsilon_y = \frac{\upsilon'_y}{\gamma\left(1 + \frac{u\upsilon'_x}{c^2}\right)} \qquad \upsilon'_y = \frac{\upsilon_y}{\gamma\left(1 - \frac{u\upsilon_x}{c^2}\right)} \qquad (3.4.4)$$
$$\upsilon_z = \frac{\upsilon'_z}{\gamma\left(1 + \frac{u\upsilon'_x}{c^2}\right)} \qquad \upsilon'_z = \frac{\upsilon_z}{\gamma\left(1 - \frac{u\upsilon'_x}{c^2}\right)}$$

From transformation (3.4.4) and from equations (3.4.2) and (3.4.3) the following transformations are derived for the functions $\delta = \delta\left(x, y, z, t\right)$ και $\omega = \omega\left(x, y, z, t\right)$:

$$\cos\delta' = \frac{\cos\delta - \frac{u}{c}}{1 - \frac{u}{c}\cos\delta} \qquad \cos\delta = \frac{\cos\delta' + \frac{u}{c}}{1 + \frac{u}{c}\cos\delta'}$$
$$\sin\delta' = \frac{\sin\delta}{\gamma\left(1 - \frac{u}{c}\cos\delta\right)} \qquad \sin\delta = \frac{\sin\delta'}{\gamma\left(1 + \frac{u}{c}\cos\delta'\right)} \qquad (3.4.5)$$
$$\omega' = \omega \qquad \omega = \omega'$$

We shall prove the first equation. The rest are proven similarly.
From the first equation of the second column of transformations (3.4.4) we obtain

$$\upsilon'_x = \frac{\upsilon_x - u}{1 - \frac{u\upsilon_x}{c^2}}$$

$$\frac{\upsilon'_x}{c} = \frac{\frac{\upsilon_x}{c} - \frac{u}{c}}{1 - \frac{u}{c}\frac{\upsilon_x}{c}}$$

Through equations (3.4.3) and (3.4.2) we get

$$\cos\delta' = \frac{\cos\delta - \frac{u}{c}}{1 - \frac{u}{c}\cos\delta}$$

From equation (3.3.7) and transformations (3.4.1) we see that

$$r' = \sqrt{\gamma^2 \left(x - ut\right)^2 + y^2 + z^2} \tag{3.4.6}$$

Combining equations (3.2.4) and (3.4.6) we get

$$r = \gamma^2 \frac{u}{c}\left(x - ut\right) + \gamma r'$$

and since

$$\gamma\left(x - ut\right) = x'$$

from transformations (3.4.1) we obtain

$$r = \gamma \frac{u}{c} x' + \gamma r' \tag{3.4.7}$$

From equation (3.3.5) we see that

$$\upsilon'_x = \frac{c}{r'} x'$$

$$x' = r' \frac{\upsilon'_x}{c}$$

Substituting into equation (3.4.7) we get

$$r = \gamma \frac{u\upsilon'_x}{c^2} r' + \gamma r'$$

$$r = \gamma r' \left(1 + \frac{u\upsilon'_x}{c^2}\right)$$

From equation (3.4.3) we obtain

$$r = \gamma r' \left(1 + \frac{u}{c}\cos\delta'\right)$$

and with the help of transformations (3.4.5) we get

$$r = \gamma r' \left(1 + \frac{u}{c} \frac{\cos\delta - \frac{u}{c}}{1 - \frac{u}{c}\cos\delta} \right)$$

$$r = \gamma r' \frac{1 - \frac{u^2}{c^2}}{1 - \frac{u}{c}\cos\delta}$$

$$r = \frac{r'}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)}$$

$$r' = \gamma r \left(1 - \frac{u}{c}\cos\delta \right) = \gamma r \left(1 - \frac{\boldsymbol{\upsilon} \cdot \boldsymbol{u}}{c^2} \right) \qquad (3.4.8)$$

From transformations (3.4.5) we obtain

$$\sin\delta' = \frac{\sin\delta}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)}$$

$$\cos\delta' \frac{d\delta'}{d\delta} = \frac{\cos\delta \left(1 - \frac{u}{c}\cos\delta \right) - \sin\delta \frac{u}{c}\sin\delta}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)^2}$$

$$\cos\delta' \frac{d\delta'}{d\delta} = \frac{\cos\delta - \frac{u}{c}}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)^2}$$

$$\frac{\cos\delta - \frac{u}{c}}{1 - \frac{u}{c}\cos\delta} \frac{d\delta'}{d\delta} = \frac{\cos\delta - \frac{u}{c}}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)^2}$$

$$\frac{d\delta'}{d\delta} = \frac{1}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)}$$

$$d\delta' = \frac{1}{\gamma \left(1 - \frac{u}{c}\cos\delta \right)} d\delta \qquad (3.4.9)$$

Repeating the same procedure we also arrive at relation

$$\frac{\partial}{\partial\delta'} = \gamma \left(1 - \frac{u}{c}\cos\delta \right) \frac{\partial}{\partial\delta} \qquad (3.4.10)$$

among the operators $\frac{\partial}{\partial\delta'}$ and $\frac{\partial}{\partial\delta}$.

From equation (3.2.10) we get

$$R = r\sqrt{1+\frac{u^2}{c^2}-2\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}$$

$$R = r\sqrt{1+\frac{u^2}{c^2}-2\frac{u}{c}\cos\delta} \tag{3.4.11}$$

From equation (3.4.11) we are able, whenever it is necessary, to derive the Lorentz-Einstein transformation of the quantity R through the use of transformations (3.4.5) and (3.4.8).

We consider now the angle θ between the vectors $\boldsymbol{R}$ and $\boldsymbol{u}$, as depicted in figure 3.2.1. From the law of sines for the triangle EAP we have that

$$\frac{\sin\vartheta}{r} = \frac{\sin\delta}{R}$$

$$\sin\vartheta = \frac{r}{R}\sin\delta$$

Using equation (3.4.11) we obtain

$$\sin\vartheta = \frac{\sin\delta}{\sqrt{1++\frac{u^2}{c^2}-2\frac{u}{c}\cos\delta}} \tag{3.4.12}$$

From the familiar identity $\sin^2\vartheta+\cos^2\vartheta=1$ we have that

$$\cos\vartheta = \frac{\cos\delta-\frac{u}{c}}{\sqrt{1++\frac{u^2}{c^2}-2\frac{u}{c}\cos\delta}} \tag{3.4.13}$$

From transformations (3.4.5) we can, after applying equations (3.4.12) and (3.4.13), derive the Lorentz-Einstein transformations for the quantities $\sin\vartheta$ and $\cos\vartheta$. Furthermore, in the inertial reference frame S' it is $\theta'=\delta'$, as can be seen from figure 3.3.1.

3.5 The Lorentz-Einstein transformation of the volume of the generalized photon

The generalized photon moves with velocity $\boldsymbol{\upsilon}$ of magnitude $\|\boldsymbol{\upsilon}\|=c$ in any inertial reference frame. This has as a consequence that the following transformation does not hold:

$$dV' = \gamma dV$$

This transformation holds for the volume dV of a material particle that is at rest in the inertial reference frame S'. We shall prove that the volume of the generalized photon transforms according to relation

$$dV' = \frac{dV}{\gamma\left(1-\frac{u}{c}\cos\delta\right)} = \frac{dV}{\gamma\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{3.5.1}$$

for our chosen inertial reference frames S and S'.

In the region of point $A(x',y',z',t')$ of figure 3.3.1 we consider the elementary area

$$dA' = r'^2\sin\delta' d\delta' d\omega'$$

of a sphere with center O' and radius r'. Furthermore, we consider a point A_1 close to point A on line OA, as depicted in figure 3.5.1

Figure 3.5.1

The elementary volume of the generalized photon in the inertial reference frame S' is

$$dV' = dA' \left\| \overrightarrow{AA_1} \right| = r'^2 \sin\delta' d\delta' d\omega' \left\| \overrightarrow{AA_1} \right| \quad (3.5.2)$$

assuming that $A_1 \to A$.

In figure (3.5.2) we present the volume dV occupied by the generalized photon in the inertial frame of reference S.

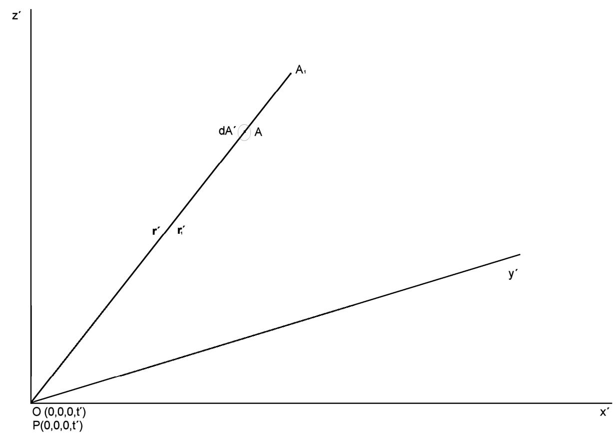

Figure 3.5.1 The infinitesimal volume of the generalized photon in the vicinity of point A of the inertial reference frame $S\left(0', x', y', z', t'\right)$. The material point particle is at position $P\left(0,0,0,t'\right)$. The infinitesimal surface of area dA' is vertical to the vectors $r' = \overrightarrow{PA}$ and $r_1' = \overrightarrow{PA_1}$. The points P, A and A_1 are collinear.

The elementary area dA in S is

$dA = r^2 \sin\delta d\delta d\omega$

while the elementary volume dV is

$$dV = dA \left\| \overrightarrow{HA_1} \right| = r^2 \sin\delta d\delta d\omega \left\| \overrightarrow{HA_1} \right\| \quad (3.5.3)$$

since $A_1 \to A$.

From the Lorentz-Einstein transformations it directly follows that points P, A, A_1, which are collinear in reference frame S' are also collinear in reference frame S. The conclusions of paragraph 2.4 about the representation of the trajectory of the material particle in the surrounding spacetime, also lead to figure 3.5.2. Here, the trajectory of the material particle is on the x axis. We now use the following notation, as depicted in figure 3.5.2

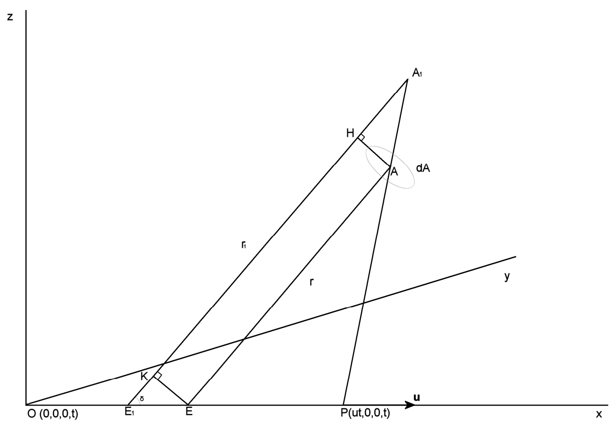

Figure 3.5.2 Figure 3.5.1 as modulated in the inertial reference frame $S(0,x,y,z,t)$, in which the material particle moves with constant speed. The points P, A and A_1 remain collinear, as results from the Lorentz-Einstein transformations.

$$r = \left\|\overrightarrow{EA}\right\| \tag{3.5.4}$$

$$r_1 = \left\|\overrightarrow{E_1A_1}\right\| \tag{3.5.5}$$

according to the notation we have established. Similarly, in figure 3.5.1 we use the notation

$$r' = \left\|\overrightarrow{O'A}\right\| \tag{3.5.6}$$

$$r_1' = \left\|\overrightarrow{O'A_1}\right\| \tag{3.5.7}$$

From figure 3.5.2 we have that

$$\left\|\overrightarrow{E_1A_1}\right\| = \left\|\overrightarrow{E_1K}\right\| + \left\|\overrightarrow{KH}\right\| + \left\|\overrightarrow{HA_1}\right\|$$

and with equations (3.5.4) and (3.5.5) we get

$$r_1 = \left\|\overrightarrow{E_1K}\right\| + r\left\|\overrightarrow{HA_1}\right\|$$

$$\left\|\overrightarrow{HA_1}\right\| = r_1 - r - \left\|\overrightarrow{E_1K}\right\| \tag{3.5.8}$$

From the triangle E_1KE of figure 3.5.2 we see that

$$\cos\delta = \frac{\left\|\overrightarrow{E_1K}\right\|}{\left\|\overrightarrow{E_1E}\right\|}$$

$$\left\|\overrightarrow{E_1K}\right\| = \left\|\overrightarrow{E_1E}\right\|\cos\delta \tag{3.5.9}$$

Similarly, we have that

$$\left\|\overrightarrow{E_1E}\right\| = u\frac{r_1 - r}{c} = udw \tag{3.5.10}$$

since in the time interval $\Delta t = \frac{r_1 - r}{c}dw$ the point particle moved from point E_1 to point E. Combining equations (3.5.9) and (3.5.10) we obtain

$$\left\|\overrightarrow{E_1K}\right\| = udw\cos\delta \tag{3.5.11}$$

Combining equations (3.5.8) and (3.5.11) we also get

$$\left\|\overrightarrow{HA_1}\right\| = cdw\left(1-\frac{u}{c}\cos\delta\right) \tag{3.5.12}$$

since $r_1 - r = cdw$.

Combining equations (3.5.3) and (3.5.12) we get

$$dV = r^2 \sin\delta d\delta d\omega cdw\left(1-\frac{u}{c}\cos\delta\right) \tag{3.5.13}$$

From figure 3.5.1 we have that

$$\left\|AA_1\right\| = \left\|\overrightarrow{O'A_1}\right\| - \left|\overrightarrow{O'A}\right|$$

and with equations (3.5.6) and (3.5.7) we get

$$\left\|\overrightarrow{AA_1}\right\| = r_1' - r' = cdw' \tag{3.5.14}$$

Combining equations (3.5.3) and (3.5.14) we also get

$$dV' = r'^2 \sin\delta' d\delta' d\omega' cdw' \tag{3.5.15}$$

Combining equations (3.5.13) and (3.5.15) we get

$$\frac{dV'}{dV} = \frac{r'^2 \sin\delta' d\delta' d\omega' cdw'}{r^2 \sin\delta d\delta d\omega cdw\left(1-\frac{u}{c}\cos\delta\right)}$$

and with transformations (3.4.8), (3.4.5), (3.4.9) and (3.4.2) we get

$$\frac{dV'}{dV} = \gamma^2\left(1-\frac{u}{c}\cos\delta\right)^2 \frac{1}{\gamma^2\left(1-\frac{u}{c}\cos\delta\right)^2}\frac{1}{\gamma}\frac{1}{1-\frac{u}{c}\cos\delta}$$

$$\frac{dV'}{dV} = \frac{1}{\gamma\left(1-\frac{u}{c}\cos\delta\right)}$$

$$dV' = \frac{dV}{\gamma\left(1-\frac{u}{c}\cos\delta\right)} \tag{3.5.16}$$

This is equation (3.5.1). Given that $\boldsymbol{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$ we arrive at relation

$$\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2} = \frac{u}{c}\frac{\upsilon_x}{c} = \frac{u}{c}\cos\delta \tag{3.5.17}$$

since, according to equation (3.4.2), $\cos\delta = \frac{\upsilon_x}{c}$.

Combining equations (3.5.16) and (3.5.17) we have

$$dV' = \frac{dV}{\gamma\left(1-\frac{u}{c}\cos\delta\right)} = \frac{dV}{\gamma\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}$$

This is the final form of equation (3.5.1).

In the form

$$dV' = \frac{dV}{\gamma\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{3.5.18}$$

transformation (3.5.1) also holds in the case of a material particle in arbitrary motion. In figure 2.4.1 the length of the three-dimensional arc EE_i equals $\left\|\overrightarrow{EE_i}\right\|$ at first approximation, that is, for an infinitesimal displacement of the material particle from point E to point E_i. Thus, we have exactly the situation we describe in figure 3.5.2. On the other hand, for a finite, but not infinitesimal, displacement $\overrightarrow{EE_i}$ of the material particle, the curvature $k_p(w)$ and the torsion $\tau_p(w)$ of curve C_p of figure 2.4.1 enter the transformation of the volume.

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CHAPTER 4

The study of selfvariations at macroscopic scales

4.1 Introduction

In the present chapter we study the consequences of the selfvariations at macroscopic scales. The main conclusion we derive is the existence of energy, momentum, electric charge and electric current in the surrounding spacetime of the material particle as a direct consequence of the selfvariations. We calculate the density of energy, momentum, electric charge and electric current in the surrounding spacetime of an arbitrarily moving material point particle.

We present the four-dimensional electromagnetic potential which is compatible with the selfvariations. An important element that emerges is the splitting of the electromagnetic potential into two individual potentials, where the first one gives the electromagnetic field that accompanies the material particle in its motion, while the second one gives the electromagnetic radiation.

We prove that the selfvariations are compatible with the principles of conservation of electric charge, energy, and momentum. This is accomplished through either direct calculation, based on the continuity equation, and also through the energy-momentum tensor of the generalized photon. These different approaches help the reader comprehend the physical reality that prevails in the surrounding spacetime of material particles.

In the preceding chapters we studied the generalized photon as a geometric object. In this chapter we shall see for the first time that the generalized photon is a carrier of energy, momentum, and electric charge. The density of electric charge and electric current in the surrounding spacetime of the material particle is correlated with the electromagnetic field that accompanies the material particle in its motion. The electromagnetic radiation does not contribute to the density of electric charge and electric current.

We calculate the energy-momentum tensor for the electromagnetic field and for the generalized photon. The energy-momentum tensor describes the energy content of spacetime, but only in macroscopic scales. In microscopic scales, the energy-momentum tensor, as defined by the theory of Special Relativity, cannot describe the energy content of spacetime.

4.2 The density of electric charge and electric current in the surrounding spacetime of an electrically charged point particle

In figure 3.2.1 the electric charge q acts at point $A(x,y,z,t)$ with the value it had at point E. Thus, we have $q=q(w)$. Hence, it follows that

$$\frac{\partial q}{\partial t}=\frac{\partial q}{\partial w}\frac{\partial w}{\partial t}$$

$$\nabla q=\frac{\partial q}{\partial w}\nabla w$$

and with equations (2.2.11) and (2.2.12) we have that

$$\frac{\partial q}{\partial t}=\frac{\partial q}{\partial w}\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}} \tag{4.2.1}$$

$$\nabla q = -\frac{\partial q}{c\partial w}\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c} \tag{4.2.2}$$

According to Special Relativity and the symbols we use in figure 3.2.1, the intensity $\boldsymbol{\varepsilon}$ of the electric field at point A is

$$\boldsymbol{\varepsilon} = \frac{\gamma q}{4\pi\varepsilon_0 r'^3}\boldsymbol{R} \tag{4.2.3}$$

where $\boldsymbol{R}$ is given by equation (3.2.7), r' by equation (3.3.7), and $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$. From Gauss's law we obtain for the electric charge density ρ at point A:

$$\rho = \varepsilon_0 \nabla\cdot\boldsymbol{\varepsilon}$$

$$\rho = \varepsilon_0 \nabla\cdot\left(\frac{\gamma}{4\pi\varepsilon_0 r'^3}\boldsymbol{R}\right)$$

$$\rho = \frac{q\gamma}{4\pi}\nabla\cdot\left(\frac{\boldsymbol{R}}{r'^3}\right) + \frac{\gamma}{4\pi\varepsilon_0 R'^3}\boldsymbol{R}\cdot\nabla q \tag{4.2.4}$$

We can easily prove that

$$\nabla\cdot\left(\frac{\boldsymbol{R}}{r'^3}\right) = 0 \tag{4.2.5}$$

We can avoid the calculation, if we take into account that, ignoring the selfvariations, for constant electric charge q, classical Electromagnetism predicts that $\rho = 0$ at point A. This is equivalent with equation (4.2.5).

Combining equations (4.2.4) and (4.2.5) we get

$$\rho = \frac{\gamma}{4\pi r'^3}\boldsymbol{R}\cdot\nabla q$$

Using equation (4.2.2) we get

$$\rho = -\frac{\partial q}{c\partial w}\frac{\gamma}{4\pi r'^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}\cdot\boldsymbol{R}$$

After applying equation (3.2.10) we have that

$$\rho = -\frac{\partial q}{c\partial w}\frac{\gamma r}{4\pi r'^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\frac{\boldsymbol{\upsilon}}{c}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)$$

$$\rho = -\frac{\partial q}{c\partial w}\frac{\gamma r}{4\pi r'^3}\frac{1}{\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)$$

$$\rho = -\frac{\partial q}{c\partial w}\frac{\gamma r}{4\pi r'^3}$$

Using transformation (3.4.8) we get

$$\rho = -\frac{\partial q}{c\partial w}\frac{1}{4\pi\gamma^2 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3} \tag{4.2.6}$$

We can derive the same equation in a different way. We will develop the second method in the next paragraph for the calculation of the density of energy D due to the selfvariations of the rest mass of the material particle, where we will not be able to use Gauss's law. The reader can easily apply the method of the next paragraph to the electric charge, and still come up with equation (4.2.6).
The generalized photon moves with velocity $\boldsymbol{\upsilon}$, therefore the current density $\boldsymbol{j}$ is given by equation

$$\boldsymbol{j} = \rho\boldsymbol{\upsilon} \tag{4.2.7}$$

where the charge density ρ is given by equation (4.2.6). Equation (4.2.7) can also be easily inferred from Ampere's law

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \frac{\partial \boldsymbol{\varepsilon}}{c^2 \partial t} \tag{4.2.8}$$

The intensity of the magnetic field $\boldsymbol{B}$ at point A of figure 3.2.1 is given initially by the Biot-Savart law:

$$\boldsymbol{B} = \frac{\boldsymbol{u}}{c^2} \times \boldsymbol{\varepsilon} \tag{4.2.9}$$

Combining equations (4.2.3) and (3.2.10) we get

$$\boldsymbol{\varepsilon} = \frac{\gamma q}{4\pi\varepsilon_0 r'^3} r \left(\frac{\boldsymbol{\upsilon}}{c} - \frac{u}{c} \right)$$

and from equation (3.4.8) we have

$$\boldsymbol{\varepsilon} = \frac{q}{4\pi\varepsilon_0 \gamma^2 r^2 \left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3} \left(\frac{\boldsymbol{\upsilon}}{c} - \frac{u}{c} \right) \tag{4.2.10}$$

From equation (4.2.10) we get

$$\frac{\boldsymbol{u}}{c^2} \times \boldsymbol{\varepsilon} = \frac{\boldsymbol{\upsilon}}{c} \times \boldsymbol{\varepsilon}$$

and from equation (4.2.9) we get

$$\boldsymbol{B} = \frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon} \tag{4.2.11}$$

In equation (4.2.11) the velocity $\boldsymbol{\upsilon}$ of the generalized photon refers to point A of figure 3.2.1. This has as a consequence that all physical quantities $\boldsymbol{B}$, $\boldsymbol{\upsilon}$, $\boldsymbol{\varepsilon}$ appearing in equation (4.2.11) refer to the same point in spacetime. On the contrary, in equation (4.2.9) the velocity $\boldsymbol{u}$ of the material particle does not refer to point A, where the electromagnetic field is manifested. Equation (4.2.11) also holds for the case where the material particle is in arbitrary motion, as we shall see in a later paragraph.

4.3 The density of energy and momentum in the surrounding spacetime of a material point particle.

In the case of the rest mass we cannot apply Gauss's law in order to calculate the energy density D in the surrounding spacetime of the material particle. Because of this we will develop a completely different proving procedure. We initially calculate the energy density D' in the inertial reference frame S' in which the material particle is at rest. At point A of figure 3.3.1 the energy density D' due to the selfvariations is

$$D' = c^2 \frac{m_0\left(t' - \frac{r'}{c}\right) - m_0\left(t' - \frac{r' + dr'}{c}\right)}{4\pi r'^2 dr'} \tag{4.3.1}$$

From equation (3.3.2) and for a specific time t' we have that

$$dw' = -\frac{dr'}{c}$$

and equation (4.3.1) becomes

$$D' = c^2 \frac{\frac{dm_0}{dr'}}{4\pi r'^2} = -c\frac{\frac{dm_0}{dw'}}{4\pi r'^2} \tag{4.3.2}$$

We now consider the Lorentz-Einstein transformations for the energy E and the momentum $\boldsymbol{P}$ of the generalized photon:

$$\begin{aligned} &E = \gamma\left(E' + uP'_x\right) && E' = \gamma\left(E - uP_x\right) \\ &P_x = \gamma\left(P'_x + \frac{u}{c^2}E'\right) && P'_x = \gamma\left(P_x - \frac{u}{c^2}E\right) \\ &P_y = P'_y && P'_y = P_y \\ &P_z = P'_z && P'_z = P_z \end{aligned} \tag{4.3.3}$$

Defining as dV the infinitesimal volume occupied by the generalized photon at point A of figure 3.2.1 we have

$$D = \frac{dE}{dV}$$

Applying the transformations (4.3.3) and (3.5.18) we get

$$D = \frac{\gamma\left(dE' + udP'_x\right)}{\gamma\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)dV'}$$

$$D = \frac{dE' + u\frac{\upsilon'_x}{c^2}dE'}{\left(1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)dV'}$$

$$D = \frac{1 + \frac{u\upsilon'_x}{c^2}}{1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{dE'}{dV'}$$

$$D = \frac{1 + \frac{u\upsilon'_x}{c^2}}{1 - \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}D' \tag{4.3.4}$$

From transformations (3.4.4) for the velocity we get

$$1 + \frac{u\upsilon'_x}{c^2} = 1 + \frac{u}{c^2}\frac{\upsilon_x - u}{1 - \frac{u\upsilon_x}{c^2}} =$$

$$\frac{1 - \frac{u^2}{c^2}}{1 - \frac{u\upsilon_x}{c^2}} = \frac{1}{\gamma^2\left(1 - \frac{u\upsilon_x}{c^2}\right)}$$

and since $\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2} = \frac{u}{c}\cos\delta$, we get

$$1+\frac{u\upsilon_x'}{c^2}=\frac{1}{\gamma^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)} \tag{4.3.6}$$

Combining equations (4.3.5) and (4.3.6) we have

$$D=\frac{1}{\gamma^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}D'$$

and with (4.3.2) we get

$$D=-c\frac{1}{\gamma^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\frac{\frac{dm_0}{dw'}}{4\pi r'^2}$$

Applying transformations (3.4.2) and (3.4.8) we obtain

$$D=-c\frac{\partial m_0}{\partial w}\frac{1}{4\pi\gamma^3 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^4} \tag{4.3.7}$$

The generalized photon moves with velocity $\boldsymbol{\upsilon}$, so we have

$$\boldsymbol{J}=D\frac{\boldsymbol{\upsilon}}{c^2} \tag{4.3.8}$$

for the momentum density $\boldsymbol{J}$ at point A of figure 3.2.1.

Factor $\frac{\partial m_0}{\partial w}$, which appears in the equations of this paragraph, corresponds to factor $\frac{\partial q}{\partial w}$ in the equations of the previous paragraph. In figure 3.2.1, the rest mass m_0 of the point particle acts on point $A(x,y,x,t)$ with the value it had at point E, namely $m_0=m_0(w)$. Therefore, we have

$$\frac{\partial m_0}{\partial t}=\frac{\partial m_0}{\partial w}\frac{\partial w}{\partial t}$$

$$\nabla m_0=\frac{\partial m_0}{\partial w}\nabla w$$

and with equations (2.2.11) and (2.2.12), we get

$$\frac{\partial m_0}{\partial t}=\frac{\partial m_0}{\partial w}\frac{1}{1-\frac{\mathbf{\upsilon u}}{c^2}}$$

$$\nabla m_0=-\frac{\partial m_0}{c\partial w}\frac{1}{1-\frac{\mathbf{\upsilon u}}{c^2}}\frac{\mathbf{\upsilon}}{c} \tag{4.3.9}$$

These equations are analogous to equations (4.2.1) and (4.2.2) for the electric charge.

4.4 The selfvariations are in accordance with the principle of conservation of the electric charge

In figure 3.2.1 and for the time interval from $w = t - \frac{r}{c}$ to t, the generalized photons emitted by the material particle are contained within a sphere with centre E and radius r. In order for the conservation of the electric charge to hold, we have to prove the validity of equation:

$$q\left(t - \frac{r}{c}\right) = q(t) + \int_V \rho dV = q(t) + q_i \tag{4.4.1}$$

where V is the volume of the sphere with centre E and radius r, and

$$q_i = \int_V \rho dV \tag{4.4.2}$$

is the electric charge, due to the selfvariations, contained within the sphere. From equation (3.5.13), we get for the infinitesimal volume dV

$$\begin{gathered} dV = b^2\left(1 - \frac{u}{c}\cos\delta\right)\sin\delta d\delta d\omega c dW \\ 0 \le \delta \le \pi \\ 0 \le \omega < 2\pi \\ 0 \le b \le r \\ t - \frac{r}{c} \le w \le t \end{gathered} \tag{4.4.3}$$

Combining equations (4.2.6) and (3.4.10) we get

$$\rho = -\frac{\partial q}{c\partial w}\frac{1}{4\pi\gamma^2 r^2\left(1 - \frac{u}{c}\cos\delta\right)^3} \tag{4.4.4}$$

Combining equations (4.4.2) and (4.4.4) we also get

$$q_i = \int_V \rho dV$$

$$q_i = -\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial q}{c\partial w}\frac{1}{4\pi\gamma^2 b^2\left(1 - \frac{u}{c}\cos\delta\right)^3} b^2\left(1 - \frac{u}{c}\cos\delta\right)\sin\delta d\delta d\omega c dw$$

$$q_i = -\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial q}{\partial w}\frac{\sin\delta}{4\pi\gamma^2\left(1 - \frac{u}{c}\cos\delta\right)^2} d\delta d\omega c dw$$

$$q_i = -\frac{1}{2\gamma^2}\int_0^{\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial q}{\partial w}\frac{\sin\delta}{\left(1 - \frac{u}{c}\cos\delta\right)^2} d\delta dw \tag{4.4.5}$$

We now denote

$$\lambda = 1 - \frac{u}{c}\cos\delta \tag{4.4.6}$$

Thus, we have

$$\frac{c}{u}d\lambda = \sin\delta d\delta \tag{4.4.7}$$

$$1 - \frac{u}{c} \le \lambda \le 1 + \frac{u}{c} \tag{4.4.8}$$

So we have

$$\int_0^{\pi} \frac{\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^2} d\delta = \int_{1-\frac{u}{c}}^{1+\frac{u}{c}} \frac{\frac{c}{u} d\lambda}{\lambda^2} = -\frac{c}{u}\left[\frac{1}{\lambda}\right]_{1-\frac{u}{c}}^{1+\frac{u}{c}} =$$

$$-\frac{c}{u}\left(\frac{1}{1+\frac{u}{c}} - \frac{1}{1-\frac{u}{c}}\right) = -\frac{c}{u}\frac{-2\frac{u}{c}}{1-\frac{u^2}{c^2}} = \frac{2}{1-\frac{u^2}{c^2}} = 2\gamma^2$$

and equation (4.4.5) becomes

$$q_i = -\int_{t-\frac{r}{c}}^{t} \frac{\partial q}{\partial w} dw$$

$$q_i = -\left[q(w)\right]_{t-\frac{r}{c}}^{t}$$

$$q_i = q\left(t-\frac{r}{c}\right) - q(t)$$

$$q(t) + q_i = q\left(t-\frac{r}{c}\right)$$

which is equation (4.4.1).

We can also prove the conservation of the electric charge through the equation of continuity

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = 0 \tag{4.4.9}$$

Indeed, taking into account equation (4.2.7) we have

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{\upsilon})$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial\rho}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla\rho + \rho\nabla\cdot\boldsymbol{\upsilon}$$

and with equation (2.2.21) we get

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial\rho}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla\rho + \frac{2c}{r}\rho$$

Applying equation (2.5.7) of the fundamental mathematical theorem, for $f = \rho$, we get

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = c\frac{\partial\rho}{\partial r} + \frac{2c}{r}\rho \tag{4.4.10}$$

From equation (4.4.4) we have

$$\frac{\partial\rho}{\partial r} = -\frac{2\rho}{r} \tag{4.4.11}$$

Combining equations (4.4.10) and (4.4.11) we finally get

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = 0$$

4.5 The selfvariations are in accordance with the conservation principles of energy and momentum

In figure 3.2.1, for the time interval from $w=t-\frac{r}{c}$ to t, the generalized photons emitted by the material particle due to the selfvariation of the rest mass are contained within the sphere with centre E and radius r. In order for the conservation of energy to hold, it is enough to prove the validity of the following equation:

$$c^2\gamma m_0\left(t-\frac{r}{c}\right)=c^2\gamma m_0\left(t\right)+\int_V DdV=c^2m_0\left(t\right)+E_i \tag{4.5.1}$$

where V is the volume of the sphere with centre E and radius r, and

$$E_i=\int_V DdV \tag{4.5.2}$$

is the energy due to the selfvariation of the rest mass, which is contained within the sphere. Combining equations (4.3.7) and (3.4.10) we get

$$D=-c\frac{\partial m_0}{\partial w}\frac{1}{4\pi\gamma^3r^2\left(1-\frac{u}{c}\cos\delta\right)^4} \tag{4.5.3}$$

Combining equations (4.5.2) and (4.5.3), and following the notation of equation (4.4.3), we get

$$E_i=-c\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t}\frac{\partial m_0}{\partial w}\frac{1}{4\pi\gamma^3b^2\left(1-\frac{u}{c}\cos\delta\right)^4}b^2\left(1-\frac{u}{c}\cos\delta\right)\sin\delta d\delta d\omega cdw$$

$$E_i=-\frac{c^2}{4\pi\gamma^3}\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t}\frac{\partial m_0}{\partial w}\frac{\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3}d\delta d\omega dw$$

$$E_i=-\frac{c^2}{2\gamma^3}\int_0^{\pi}\int_{t-\frac{r}{c}}^{t}\frac{\partial m_0}{\partial w}\frac{\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3}d\delta dw \tag{4.5.4}$$

Using the notation of equations (4.4.6), (4.4.7), and (4.4.8) we have

$$\int_0^{\pi}\frac{\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3}d\delta=\int_{1-\frac{u}{c}}^{1+\frac{u}{c}}\frac{u}{\lambda^3}d\lambda=$$

$$-\frac{c}{2u}\left[\frac{1}{\lambda^2}\right]_{1-\frac{u}{c}}^{1+\frac{u}{c}}=-\frac{c}{2u}\left(\frac{1}{\left(1+\frac{u}{c}\right)^2}-\frac{1}{\left(1-\frac{u}{c}\right)^2}\right)=$$

$$-\frac{c}{2u}\frac{-4\frac{u}{c}}{\left(1-\frac{u^2}{c^2}\right)^2}=\frac{2}{\left(1-\frac{u^2}{c^2}\right)^2}=2\gamma^4$$

Now (4.5.4) becomes

$$E_i = -c^2\gamma \int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w} dw$$

$$E_i = -c^2\gamma \left[m_0 \right]_{t-\frac{r}{c}}^{t}$$

$$E_i = -c^2\gamma m_0(t) + c^2\gamma m_0\left(t - \frac{r}{c}\right)$$

$$c^2\gamma m_0\left(t - \frac{r}{c}\right) = c^2\gamma m_0(t) + E_i$$

which is equation (4.5.1).

The conservation of energy can also be proven using the continuity equation

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = 0 \tag{4.5.5}$$

Indeed, if we take into account equation (4.3.8) we obtain

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial D}{c^2\partial t} + \nabla\cdot\left(D\frac{\boldsymbol{\upsilon}}{c^2}\right)$$

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial D}{c^2\partial t} + \frac{\boldsymbol{\upsilon}}{c^2}\nabla D + \frac{D}{c^2}\nabla\cdot\boldsymbol{\upsilon}$$

and with equation (2.2.21) we have

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = \frac{\partial D}{c^2\partial t} + \frac{\boldsymbol{\upsilon}}{c^2}\nabla D + \frac{D}{c^2}\nabla\frac{2c}{r}$$

Using equation (2.5.7) of the fundamental mathematical theorem for $f = D$, we get

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = c\frac{\partial D}{c^2\partial r} + \frac{D}{c^2}\frac{2c}{r} \tag{4.5.6}$$

From equation (4.5.3) we have

$$\frac{\partial D}{\partial r} = -\frac{2D}{r} \tag{4.5.7}$$

Combining equations (4.5.6) and (4.5.7) we get

$$\frac{\partial D}{c^2\partial t} + \nabla\cdot\boldsymbol{j} = 0$$

In order to prove the conservation of momentum, it suffices to prove the corresponding of equation (4.5.1), that is, it is enough to prove equation

$$\gamma m_0\left(t - \frac{r}{c}\right)\boldsymbol{u} = \gamma m_0(t)\boldsymbol{u} + \int_V \boldsymbol{J}dV = \gamma m_0(t)\boldsymbol{u} + \boldsymbol{P}_i \tag{4.5.8}$$

where

$$\boldsymbol{P}_i = \int_V \boldsymbol{J}dV \tag{4.5.9}$$

is the momentum due to the selfvariation of the rest mass, contained within the sphere of centre E and radius r. Combining equations (4.5.9) and (4.3.8) we obtain

$$\boldsymbol{P}_i = \int_V D\frac{\boldsymbol{\upsilon}}{c^2}dV \tag{4.5.10}$$

We first work on the x-axis:

$$P_{ix} = \int_V D\frac{\upsilon_x}{c^2}dV$$

Using equation (3.4.2) we get

$$P_{ix} = \int_V D\frac{\cos\delta}{c}dV$$

and with equations (4.5.3) and (4.4.3) we get

$$P_{ix} = -\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w}\frac{\cos\delta}{4\pi\gamma^3 b^2\left(1-\frac{u}{c}\cos\delta\right)^4} b^2\left(1-\frac{u}{c}\cos\delta\right)\sin\delta d\delta d\omega c dw$$

$$P_{ix} = -\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w}\frac{\cos\delta\sin\delta}{4\pi\gamma^3 b^2\left(1-\frac{u}{c}\cos\delta\right)^3} d\delta d\omega c dw$$

$$P_{ix} = -\frac{c}{4\pi\gamma^3}\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w}\frac{\cos\delta\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3} d\delta d\omega dw$$

$$P_{ix} = -\frac{c}{2\gamma^3}\int_0^{\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w}\frac{\cos\delta\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3} d\delta dw \qquad (4.5.11)$$

Using the notation appearing in equations (4.4.6), (4.4.7), and (4.4.8) we have

$$\int_0^{\pi}\frac{\cos\delta\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3}d\delta=\int_{1-\frac{u}{c}}^{1+\frac{u}{c}}\frac{c^2}{u^2}\frac{1-\lambda}{\lambda^3}d\lambda=$$

$$\frac{c^2}{u^2}\int_{1-\frac{u}{c}}^{1+\frac{u}{c}}\left(\frac{1}{\lambda^3}-\frac{1}{\lambda^2}\right)d\lambda=\frac{c^2}{u^2}\left(-\frac{1}{2}\left[\frac{1}{\lambda^2}\right]_{1-\frac{u}{c}}^{1+\frac{u}{c}}+\left[\frac{1}{\lambda}\right]_{1-\frac{u}{c}}^{1+\frac{u}{c}}\right)=$$

$$\frac{c^2}{u^2}\left(-\frac{1}{2}\frac{\left(1+\frac{u}{c}\right)^2-\left(1-\frac{u}{c}\right)^2}{\left(1-\frac{u^2}{c^2}\right)^2}+\frac{-2\frac{u}{c}}{1-\frac{u^2}{c^2}}\right)=$$

$$\frac{c^2}{u^2}\left(-\frac{1}{2}\frac{-4\frac{u}{c}}{\left(1-\frac{u^2}{c^2}\right)^2}-\frac{2\frac{u}{c}}{1-\frac{u^2}{c^2}}\right)=$$

$$\frac{2c}{u}\left(\frac{1}{\left(1-\frac{u^2}{c^2}\right)^2}-\frac{1}{1-\frac{u^2}{c^2}}\right)=$$

$$\frac{\frac{2c}{u}}{\left(1-\frac{u^2}{c^2}\right)^2}\left(1-1+\frac{u^2}{c^2}\right)=\frac{\frac{2u}{c}}{\left(1-\frac{u^2}{c^2}\right)^2}2\gamma^4\frac{u}{c}$$

and equation (4.5.1) becomes

$$P_{ix}=-u\gamma\int_{t-\frac{r}{c}}^{t}\frac{\partial m_0}{\partial w}dw=-u\gamma\left[m_0\right]_{t-\frac{r}{c}}^{t}$$

$$P_{ix}=u\gamma m_0\left(t-\frac{r}{c}\right)-u\gamma m_0\left(t\right) \qquad (4.5.12)$$

Similarly for the y -axis we get

$$P_{iy}=-\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t}\frac{\partial m_0}{\partial w}\frac{\sin\delta}{\left(1-\frac{u}{c}\cos\delta\right)^3}\upsilon_y d\delta d\omega dw$$

and with equation (3.4.2)

$$\frac{\upsilon_y}{c}=\sin\delta\cos\omega$$

we get

$$P_{iy} = -\int_0^{\pi}\int_0^{2\pi}\int_{t-\frac{r}{c}}^{t} \frac{\partial m_0}{\partial w} \frac{c\sin^2\delta\cos\omega}{\left(1-\frac{u}{c}\cos\delta\right)^3} d\delta d\omega dw \tag{4.5.13}$$

The presence of factor $\cos\omega$ causes integral (4.5.13) to vanish, and we have

$$P_{iy} = 0 \tag{4.5.14}$$

We can similarly prove that

$$P_{iz} = 0 \tag{4.5.15}$$

Given that

$$\boldsymbol{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$$

equations (4.5.12), (4.5.14) and (4.5.15) can be written as

$$\boldsymbol{P}_i = \boldsymbol{u}\gamma m_0\left(t-\frac{r}{c}\right) - \boldsymbol{u}\gamma m_0(t)$$

which is equation (4.5.8).
From equation (4.5.1) we get

$$E_i = \int_V D dV = c^2\gamma\left(m_0\left(t-\frac{r}{c}\right) - m_0(t)\right) \tag{4.5.16}$$

From equation (4.5.8) we also have

$$\boldsymbol{P}_i = \int_V \boldsymbol{J} dV = \boldsymbol{u}\gamma\left(m_0\left(t-\frac{r}{c}\right) - m_0(t)\right) \tag{4.5.17}$$

Combining equations (4.5.16) and (4.5.17) we get

$$\boldsymbol{P}_i = E_i \frac{\boldsymbol{u}}{c^2} \tag{4.5.18}$$

and

$$\int_V \boldsymbol{J} dV = \frac{\boldsymbol{u}}{c^2}\int_V D dV \tag{4.5.19}$$

Equations (4.5.18) and (4.5.19) hold for every volume V, i.e. for every radius r of the sphere with centre E and radius r of figure 3.2.1. Therefore, they also hold for $r = 0$, that is, on the material particle at time w. Hence, the total energy E_s and the total momentum $\boldsymbol{P}_s$ emitted by the material particle at time w in all directions, are connected through the relation

$$\boldsymbol{P}_s = E_s \frac{\boldsymbol{u}}{c^2} \tag{4.5.20}$$

where $\boldsymbol{u} = \boldsymbol{u}(w)$. This equation has fundamental consequences for the material particle, and we shall encounter them as our study continues.

4.6 The electromagnetic field in the macrocosm. The electromagnetic potential of the selfvariations

Using the symbols at point $A(x, y, z, t)$ of figure 2.2.1, the scalar potential V and the vector potential $\boldsymbol{A}$ of the selfvariations are given by the following equations:

$$V=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}+\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2} \tag{4.6.1}$$

$$\boldsymbol{A}=V\frac{\boldsymbol{\upsilon}}{c^2} \tag{4.6.2}$$

The intensity $\boldsymbol{\varepsilon}$ of the electric field, and the intensity $\boldsymbol{B}$ of the magnetic field arising from these two potentials, are given by

$$\boldsymbol{\varepsilon}=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)+\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)-\boldsymbol{\alpha}\right] \tag{4.6.3}$$

$$\boldsymbol{B}=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}+\frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\alpha}\right] \tag{4.6.4}$$

where $\boldsymbol{u}=\boldsymbol{u}(w)$ is the velocity, and $\boldsymbol{\alpha}=\boldsymbol{\alpha}(w)$ is the acceleration of the material particle. Furthermore, the density of electric charge at point A is

$$\rho=-\frac{\partial q}{\partial w}\frac{1}{4\pi\gamma^2 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3} \tag{4.6.5}$$

exactly as given by equation (4.2.6).

In equations (4.6.3) and (4.6.4) we recognize the electromagnetic field as we know it experimentally, but also as predicted by the Lienard-Wiechert potentials. However, the electromagnetic potentials of the selfvariations have a fundamental characteristic that is not shared by the Lienard-Wiechert potentials. Namely, they split into two individual couples of potentials

$$V_u=\frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2} \tag{4.6.6}$$

$$\boldsymbol{A}_u=V_u\frac{\boldsymbol{\upsilon}}{c^2}$$

and

$$V_\alpha=\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2} \tag{4.6.7}$$

$$\boldsymbol{A}_\alpha=V_\alpha\frac{\boldsymbol{\upsilon}}{c^2}$$

The (4.6.6) potentials express the electromagnetic field

$$\boldsymbol{\varepsilon}_u = \frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)$$

$$\boldsymbol{B}_u = \frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c} \tag{4.6.8}$$

that accompanies the material particle in its motion. The (4.6.7) potentials express the electromagnetic radiation

$$\boldsymbol{\varepsilon}_\alpha = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)-\boldsymbol{\alpha}\right]$$

$$\boldsymbol{B}_\alpha = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\alpha}\right] \tag{4.6.9}$$

The (4.6.7) potential of the electromagnetic radiation does not depend on the distance r, while it vanishes for $\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}=0$. Furthermore, for each couple of the electromagnetic field we can easily prove that equation (4.2.11) holds

$$\boldsymbol{B}_u = \frac{\boldsymbol{\upsilon}}{c^2}\times\boldsymbol{\varepsilon}_u \tag{4.6.10}$$

$$\boldsymbol{B}_\alpha = \frac{\boldsymbol{\upsilon}}{c^2}\times\boldsymbol{\varepsilon}_\alpha \tag{4.6.11}$$

We remind the reader that the electromagnetic field can be calculated from the electromagnetic potentials via equations

$$\boldsymbol{\varepsilon} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t} \tag{4.6.12}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A} \tag{4.6.13}$$

where $\nabla V = \begin{bmatrix}\frac{\partial V}{\partial x}\\ \frac{\partial V}{\partial y}\\ \frac{\partial V}{\partial z}\end{bmatrix}$, and $\nabla\times\boldsymbol{A} = curl\boldsymbol{A}$.

We shall now prove equation

$$\boldsymbol{\varepsilon}_\alpha = -\nabla V_\alpha - \frac{\partial \boldsymbol{A}_\alpha}{\partial t} \tag{4.6.14}$$

and the general equations (4.6.3) and (4.6.4) can be proven similarly. We shall make use of the equations of paragraph 2.7. From the (4.6.7) potentials we obtain

$$\boldsymbol{\varepsilon}_\alpha = -\left[\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\right] - \frac{\partial}{\partial t}\left[\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^5\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\boldsymbol{\upsilon}\right]$$

$$\boldsymbol{\varepsilon}_\alpha = -\nabla\left[\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\right] - \boldsymbol{\upsilon}\frac{\partial}{\partial t}\left[\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^5\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\right] - \frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^5\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\frac{\partial\boldsymbol{\upsilon}}{\partial t}$$

$$\begin{aligned}\boldsymbol{\varepsilon}_\alpha = &-\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left(\nabla q + \frac{\partial q}{c^2\partial t}\boldsymbol{\upsilon}\right) - \frac{q}{4\pi\varepsilon_0\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}) + \frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{c^2\partial t}\boldsymbol{\upsilon}\right] \\ &-\frac{2q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left[\nabla\left(\frac{\boldsymbol{\upsilon}\boldsymbol{u}}{c^2}\right) + \frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{c^4\partial t}\boldsymbol{\upsilon}\right] - \frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^5\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\frac{\partial\boldsymbol{\upsilon}}{\partial t}\end{aligned} \tag{4.6.15}$$

Combining equations (4.2.1) and (4.2.2) we get

$$\nabla q + \frac{\partial q}{c^2\partial t}\boldsymbol{\upsilon} = 0 \tag{4.6.16}$$

Combining equations (2.7.3) and (2.7.4) we get

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha}) + \frac{\partial(\boldsymbol{\upsilon}\boldsymbol{\alpha})}{c^2\partial t} = \frac{c}{r}\boldsymbol{\alpha} - \frac{(\boldsymbol{\upsilon}\boldsymbol{\alpha})}{cr}\boldsymbol{\upsilon} \tag{4.6.17}$$

Combining equations (2.7.1) and (2.7.4) we get

$$\nabla(\boldsymbol{\upsilon}\cdot\boldsymbol{u}) + \frac{\partial(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{c^2\partial t}\boldsymbol{\upsilon} = -\frac{c}{r}\left(\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})}{c^2}\boldsymbol{\upsilon} - \boldsymbol{u}\right) \tag{4.6.18}$$

We substitute equations (4.6.16), (4.6.17) and (4.6.18) into equation (4.6.15) and we obtain

$$\boldsymbol{\varepsilon}_\alpha = -\frac{q}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{c}{r}\boldsymbol{\alpha}-\frac{(\boldsymbol{\upsilon\alpha})}{cr}\boldsymbol{\upsilon}\right]+\frac{2q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^5\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\frac{c}{r}\left[\frac{(\boldsymbol{\upsilon u})}{c^2}\boldsymbol{\upsilon}-\boldsymbol{u}\right]$$

$$-\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^4\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left[\frac{(\boldsymbol{\upsilon u})}{c^2}\boldsymbol{\upsilon}-\boldsymbol{u}\right]$$

$$\boldsymbol{\varepsilon}_\alpha = -\frac{q}{4\pi\varepsilon_0 c^3\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{c}{r}\boldsymbol{\alpha}-\frac{(\boldsymbol{\upsilon\alpha})}{cr}\boldsymbol{\upsilon}\right]+\frac{q(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{4\pi\varepsilon_0 c^4 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left[\frac{(\boldsymbol{\upsilon u})}{c^2}\boldsymbol{\upsilon}-\boldsymbol{u}\right]$$

$$\boldsymbol{\varepsilon}_\alpha = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[-\boldsymbol{\alpha}+\frac{(\boldsymbol{\upsilon\alpha})}{c^2}\boldsymbol{\upsilon}+\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{u})(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{c^4\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{\upsilon}-\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\boldsymbol{u}\right]$$

$$\boldsymbol{\varepsilon}_\alpha = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[-\boldsymbol{\alpha}+\frac{(\boldsymbol{\upsilon\alpha})}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left(\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)\boldsymbol{\upsilon}+\frac{(\boldsymbol{\upsilon u})}{c^2}\boldsymbol{\upsilon}-\boldsymbol{u}\right)\right]$$

$$\boldsymbol{\varepsilon}_\alpha = \frac{q}{4\pi\varepsilon_0 c^2 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[-\boldsymbol{\alpha}+\frac{(\boldsymbol{\upsilon\alpha})}{c^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}(\boldsymbol{\upsilon}-\boldsymbol{u})\right]$$

which is equation (4.6.9) for the electric field $\boldsymbol{\varepsilon}_\alpha$.

In order to prove equations (4.6.8) we also need equations

$$\nabla\left(u^2\right)+\frac{\partial\left(u^2\right)}{c^2\partial t}\boldsymbol{\upsilon}=0 \tag{4.6.19}$$

$$\nabla r+\frac{\partial r}{c^2\partial t}\boldsymbol{\upsilon}=\frac{\boldsymbol{\upsilon}}{c} \tag{4.6.20}$$

We can prove equation (4.6.19) as follows

$$\nabla\left(u^2\right)+\frac{\partial\left(u^2\right)}{c^2\partial t}\boldsymbol{\upsilon}=\frac{\partial u^2}{\partial w}\nabla w+\frac{\partial u^2}{c^2\partial w}\frac{\partial w}{\partial t}\boldsymbol{\upsilon}=\frac{\partial u^2}{\partial w}\left(\nabla w+\frac{\partial w}{c^2\partial t}\cdot\boldsymbol{\upsilon}\right)=0$$

This results immediately from the combination of equations (2.2.11) and (2.2.12). Equation (4.6.20) results from the combination of equations (2.2.9) and (2.2.14).

In order to prove equation (4.6.5), we denote

$$\boldsymbol{f}=\frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)-\boldsymbol{\alpha}\right] \tag{4.6.21}$$

and

$$\boldsymbol{g} = \frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)+\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\alpha}}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{\boldsymbol{u}}{c}\times\frac{\boldsymbol{\upsilon}}{c}\right)-\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\alpha}\right] \quad (4.6.22)$$

Using the notation of equations (4.6.21) and (4.6.22), and from equations (4.6.8) and (4.6.9) we obtain

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_u + \boldsymbol{\varepsilon}_\alpha = q\boldsymbol{f} \quad (4.6.23)$$

$$\boldsymbol{B} = \boldsymbol{B}_U + \boldsymbol{B}_\alpha = q\boldsymbol{g} \quad (4.6.24)$$

From Gauss's law we have

$$\rho = \varepsilon_0 \nabla\cdot\boldsymbol{\varepsilon}$$

and using equation (4.6.23) we have

$$\rho = \varepsilon_0 \nabla\cdot(q\boldsymbol{f})$$

$$\rho = \varepsilon_0 q\nabla\cdot\boldsymbol{f} + \varepsilon_0 \boldsymbol{f}\cdot\nabla q \quad (4.6.25)$$

From classical electromagnetism we know that

$$\nabla\cdot\boldsymbol{f} = \boldsymbol{0}$$

Hence, equation (4.6.25) becomes

$$\rho = \varepsilon_0 \boldsymbol{f}\cdot\nabla q$$

Using equation (4.6.16) we obtain

$$\rho = -\varepsilon_0 \frac{\partial q}{c^2\partial t}\boldsymbol{\upsilon}\cdot\boldsymbol{f} \quad (4.6.26)$$

From equation (4.6.21) we see that

$$\boldsymbol{\upsilon}\cdot\boldsymbol{f} = \frac{1-\frac{u^2}{c^2}}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}\left(\frac{c^2}{c}-\frac{\boldsymbol{\upsilon u}}{c}\right)+\frac{1}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\left[\frac{\left(\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{u}\right)}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(\frac{c^2}{c}-\frac{\boldsymbol{\upsilon u}}{c}\right)-\boldsymbol{\upsilon\alpha}\right]$$

$$\boldsymbol{\upsilon}\cdot\boldsymbol{f} = \frac{c\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}+\frac{1}{r\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)}\left[\frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)-\boldsymbol{\upsilon\alpha}\right]$$

$$\boldsymbol{\upsilon}\cdot\boldsymbol{f} = \frac{c\left(1-\frac{u^2}{c^2}\right)}{r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}+0 \quad (4.6.27)$$

Combining equations (4.6.26) and (4.6.27) we get

$$\rho = -\varepsilon_0 \frac{1-\frac{u^2}{c^2}}{4\pi\varepsilon_0\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^2}\frac{\partial q}{\partial t}$$

and with equation (4.2.1) we finally obtain

$$\rho = -\frac{\partial q}{\partial w}\frac{1-\dfrac{u^2}{c^2}}{4\pi r^2\left(1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}$$

which is equation (4.6.5), since

$$\frac{1}{\gamma^2} = 1-\frac{u^2}{c^2}.$$

Similarly we can prove equation

$$\nabla\cdot\boldsymbol{B} = 0 \tag{4.6.28}$$

From equation (4.2.24) we have that

$$\nabla\cdot\boldsymbol{B} = \nabla\cdot\left(q\boldsymbol{g}\right)$$

$$\nabla\cdot\boldsymbol{B} = q\nabla\cdot\boldsymbol{g} + \boldsymbol{g}\cdot\nabla q \tag{4.6.29}$$

From classical electromagnetism we know that

$$\nabla\cdot\boldsymbol{g} = 0$$

Thus, equation (4.6.29) becomes

$$\nabla\cdot\boldsymbol{B} = \boldsymbol{g}\cdot\nabla q$$

and with equation (4.6.16) we obtain

$$\nabla\cdot\boldsymbol{B} = -\frac{\partial q}{c^2\partial t}\boldsymbol{\upsilon}\cdot\boldsymbol{g} \tag{4.6.30}$$

From equation (4.6.22) it immediately can be seen that

$$\boldsymbol{\upsilon}\cdot\boldsymbol{g} = 0$$

and from equation (4.6.30) we also obtain

$$\nabla\cdot\boldsymbol{B} = 0$$

Combining equations (4.6.30) and (4.6.24) we have that

$$\nabla\cdot\boldsymbol{B} = -\frac{\partial q}{c^2 q\partial t}\boldsymbol{\upsilon}\cdot\boldsymbol{B} \tag{4.6.31}$$

From equation (4.6.31) it follows that

$$\nabla\cdot\boldsymbol{B} = 0$$

if and only if

$$\boldsymbol{\upsilon}\cdot\boldsymbol{B} = 0$$

From equations (4.6.10) and (4.6.11) we get

$$\boldsymbol{B} = \frac{\boldsymbol{\upsilon}}{c^2}\times\boldsymbol{\varepsilon} \tag{4.6.32}$$

Therefore, it holds that

$$\boldsymbol{\upsilon}\cdot\boldsymbol{B} = 0$$

or equivalently

$$\nabla\cdot\boldsymbol{B} = 0$$

Combining equations (4.6.26) and (4.6.23) we get

$$\rho = -\varepsilon_0\frac{\partial q}{c^2 q\partial t}\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} \tag{4.6.33}$$

From equation (4.6.33) it follows that

$$\rho = 0$$

if and only if

$$\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = 0$$

From equation (4.6.9) for the electric field $\boldsymbol{\varepsilon}_\alpha$, we can immediately deduce that

$$\boldsymbol{\upsilon} \cdot \boldsymbol{\varepsilon}_\alpha = 0 \qquad (4.6.34)$$

Therefore, the electromagnetic radiation does not contribute to the charge density ρ. On the contrary, for the electric field $\boldsymbol{\varepsilon}_u$ that accompanies the material particle, it holds that

$$\boldsymbol{\upsilon} \cdot \boldsymbol{\varepsilon}_u \neq 0$$

as follows from equation (4.6.8).
From equation (4.6.32) we obtain

$$\boldsymbol{B} = \frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon}$$

$$\boldsymbol{B}^2 = \left(\frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon} \right)^2$$

$$\boldsymbol{B}^2 = \left(\frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon} \right) \cdot \left(\frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon} \right)$$

After performing the necessary calculations we finally get

$$\boldsymbol{\varepsilon}^2 = c^2 \boldsymbol{B}^2 + \left(\frac{\boldsymbol{\upsilon} \cdot \boldsymbol{\varepsilon}}{c} \right)^2 \qquad (4.6.35)$$

We end this paragraph with an interesting observation. Comparing equations (4.6.8) for the electric field $\boldsymbol{\varepsilon}_u$ with equation (2.4.11), we conclude that the vectors $\boldsymbol{t}$ and $\boldsymbol{\varepsilon}_u$ are parallel. Then, the "trajectory representation theorem" informs us that the direction of the electric field $\boldsymbol{\varepsilon}_u$ represents the tangential vector $\boldsymbol{t}_p$ of the trajectory C_p of the material particle.

4.7 The energy-momentum tensor of the electromagnetic field at macroscopic scales

The equations of this paragraph as well as of the remaining paragraphs of this chapter, could be stated differently, so that they also hold for non-inertial reference frames. However, such a formulation does not serve the purposes of the present edition. Therefore, we will formulate the equations for an inertial reference frame, while simultaneously suggesting the way in which the same equations can also be formulated for a non-inertial reference frame.
From the axiomatic foundation of the theory of selfvariations, as stated in paragraph 2.2, we have that

$$dS^2 = 0$$

or, equivalently,

$$g_{ik} dx^i dx^k = 0 \qquad {}_{i,k=0,1,2,3} \qquad (4.7.1)$$

where

$$\left(x^0, x^1, x^2, x^3 \right) = \left(ct, x, y, z \right) \qquad (4.7.2)$$

and g_{ik} are the components of the metric tensor. In equation (4.7.1) we use the Einstein summation convention for the indices i and k.
We denote

$$\upsilon^i = \frac{dx^i}{dt} \qquad {}_{i=0,1,2,3} \qquad (4.7.3)$$

that is,

$$\left(\upsilon^0,\upsilon^1,\upsilon^2,\upsilon^3\right)=\left(c,\upsilon_x,\upsilon_y,\upsilon_z\right) \tag{4.7.4}$$

From equation (4.7.1) we obtain

$$g_{ik}=\frac{dx^1}{dt}\frac{dx^k}{dt}=0$$

and with equation (4.7.3) we get

$$g_{ik}\upsilon^i\upsilon^k=0 \qquad {\scriptstyle i,k=0,1,2,3} \tag{4.7.5}$$

Using this notation, all the equations we will formulate also hold for non-inertial reference frames if we replace differentiation with respect to x^k with covariant differentiation with respect to x^k, $\quad k=0,1,2,3$.

We now denote the four-vector of velocity as

$$\boldsymbol{\upsilon}=\begin{bmatrix}\upsilon^0\\ \upsilon^1\\ \upsilon^2\\ \upsilon^3\end{bmatrix}=\begin{bmatrix}c\\ \upsilon_x\\ \upsilon_y\\ \upsilon_z\end{bmatrix} \tag{4.7.6}$$

and the four-vector of current density as

$$\boldsymbol{j}=\begin{bmatrix}j^0\\ j^1\\ j^2\\ j^3\end{bmatrix}=\begin{bmatrix}\rho\upsilon^0\\ \rho\upsilon^1\\ \rho\upsilon^2\\ \rho\upsilon^3\end{bmatrix}=\begin{bmatrix}\rho c\\ \rho\upsilon_x\\ \rho\upsilon_y\\ \rho\upsilon_z\end{bmatrix} \tag{4.7.7}$$

as results from equations (4.2.6) and (4.2.7). Also, according to equations (4.6.1) and (4.6.2), the four-vector of the electromagnetic potential is

$$\boldsymbol{A}=\begin{bmatrix}A^0\\ A^1\\ A^2\\ A^3\end{bmatrix}=\begin{bmatrix}\frac{V}{c}\upsilon^0\\ \frac{V}{c}\upsilon^1\\ \frac{V}{c}\upsilon^2\\ \frac{V}{c}\upsilon^3\end{bmatrix}=\begin{bmatrix}V\\ \frac{V}{c}\upsilon_x\\ \frac{V}{c}\upsilon_y\\ \frac{V}{c}\upsilon_z\end{bmatrix} \tag{4.7.8}$$

Subsequently we will symbolize the differentiation with respect to $\dfrac{\partial}{\partial x^k}$ with $(,k)$, $\quad k=0,1,2,3$.

We now consider the tensor of the electromagnetic field

$$T^{\mu\nu}=\frac{I}{4\pi}\left(F^{\mu\alpha}F_\alpha^\nu-\frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{4.7.9}$$

where $g^{\mu\nu}$ is the inverse of the matrix $g_{\mu\nu}$, $g_{\mu k}g^{k\nu}=\delta_{\mu\nu}$

$$\delta_{\mu\nu}=\begin{cases}1 & for \quad \mu=\nu\\ \\ 0 & for \quad \mu\neq\nu\end{cases} \tag{4.7.10}$$

and $F^{\mu\nu}$ is the Maxwell stress tensor

$$F^{\mu\nu}=A^\nu_{,\mu}-A^\mu_{,\nu} \tag{4.7.11}$$

Using this notation and taking into account that in the surrounding spacetime of the material particle there is an electric current $\boldsymbol{j}$, as given by equation (4.7.7), the energy-momentum tensor of the electromagnetic field is given by the tensor

$$\Phi^{\mu\nu} = T^{\mu\nu} - j^{\mu}A^{\nu} \tag{4.7.12}$$

We now write the tensor $T^{\mu\nu}$ in the form

$$T^{ij} = \begin{bmatrix} w & cS_x & cS_y & cS_z \\ cS_x & & & \\ cS_y & & \sigma_{\alpha\beta} & \\ cS_z & & & \end{bmatrix} \tag{4.7.13}$$

$$\boldsymbol{S} = \varepsilon_0 \boldsymbol{\varepsilon} \times \boldsymbol{B} \tag{4.7.14}$$

where $\boldsymbol{S}$ is the Poynting vector, and $\boldsymbol{\varepsilon}$ and $\boldsymbol{B}$ are the intensities of the electric and magnetic field, respectively. Taking into account equations (4.6.10) and (4.6.11), as summarized in equation

$$\boldsymbol{B} = \frac{\boldsymbol{\upsilon}}{c^2} \times \boldsymbol{\varepsilon} \tag{4.7.15}$$

equation (4.7.14) becomes

$$\boldsymbol{S} = \varepsilon_0 \left(\frac{\boldsymbol{\varepsilon}^2}{c^2} \right) \boldsymbol{\upsilon} - \varepsilon_0 \left(\frac{\boldsymbol{\upsilon} \cdot \boldsymbol{\varepsilon}}{c^2} \right) \boldsymbol{\varepsilon} \tag{4.7.16}$$

The Maxwell stress tensor $\sigma_{\alpha\beta}$ is given by relation

$$\sigma_{\alpha\beta} = \varepsilon_0 \left(-\varepsilon_\alpha \varepsilon_\beta - c^2 B_\alpha B_\beta + W \delta_{\alpha\beta} \right) \tag{4.7.17}$$

where $\delta_{\alpha\beta}$ is given by relation (4.7.10), and

$$W = \frac{1}{2} \varepsilon_0 \left(\boldsymbol{\varepsilon}^2 + c^2 \boldsymbol{B}^2 \right) \tag{4.7.18}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

Combining equations (4.7.12) and (4.7.13), we arrive at the energy-momentum tensor

$$\Phi^{ij} = \begin{bmatrix} w & cS_x & cS_y & cS_z \\ cS_x & \sigma_{11} & \sigma_{12} & \sigma_{13} \\ cS_y & \sigma_{21} & \sigma_{22} & \sigma_{23} \\ cS_z & \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} - \frac{\rho V}{c^2} \begin{bmatrix} c^2 & c\upsilon_x & c\upsilon_y & c\upsilon_z \\ \upsilon_x c & \upsilon_x^2 & \upsilon_x \upsilon_y & \upsilon_x \upsilon_z \\ \upsilon_y c & \upsilon_y \upsilon_x & \upsilon_y^2 & \upsilon_y \upsilon_z \\ \upsilon_z c & \upsilon_z \upsilon_x & \upsilon_z \upsilon_y & \upsilon_z^2 \end{bmatrix} \tag{4.7.19}$$

We shall now prove that the scalar potential, as given by equation (4.6.1), satisfies the relation

$$\frac{\partial V}{\partial t} + \boldsymbol{\upsilon} \cdot \nabla V = -\boldsymbol{\upsilon} \cdot \boldsymbol{\varepsilon} \tag{4.7.20}$$

From equation (4.6.12) we have that

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = -\boldsymbol{\upsilon}\left[-\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}\right]$$

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = \boldsymbol{\upsilon}\left(\nabla V + \frac{\partial \boldsymbol{A}}{\partial t}\right)$$

Using equation (4.6.2) we have

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = \boldsymbol{\upsilon}\left(\nabla V + \frac{\partial V}{c\partial t}\frac{\boldsymbol{\upsilon}}{c} + \frac{V}{c}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)\right)$$

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = \boldsymbol{\upsilon}\cdot\nabla V + \frac{\partial V}{\partial t} + \frac{V}{c}\boldsymbol{\upsilon}\cdot\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}}{c}\right)$$

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = \boldsymbol{\upsilon}\cdot\nabla V + \frac{\partial V}{\partial t} + \frac{V}{2c}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{\upsilon}^2}{c}\right)$$

$$-\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon} = \boldsymbol{\upsilon}\cdot\nabla V + \frac{\partial V}{\partial t}$$

since $\boldsymbol{\upsilon}^2 = c^2$.

We will now prove the conservation of energy and momentum, as expressed by equation

$$\Phi^{ij}_{,j} = \frac{\partial \Phi^{ij}}{\partial x^j} = 0 \qquad (4.7.21)$$

We begin with an observation which allows us to avoid complex calculations. Equation (4.7.21) holds in classical electromagnetic theory, i.e. if we ignore the consequences of the selfvariations and consider the electric charge q constant, both in the electromagnetic potential, as well as in the intensity of the electromagnetic field. Furthermore, $\rho = 0$ in equation (4.7.19). Therefore, it is enough to prove that in equation (4.7.21) the factors resulting from the selfvariation of the electric charge q, also vanish. Certainly, in equation (4.7.19) it holds that $\rho \neq 0$, where the charge density ρ is given by equation (4.6.5).

The energy density W of the electromagnetic field as given by equation (4.7.18), as well as the Poynting vector S, given by equation (4.7.16), are proportional to q^2. Therefore, in our calculations we will have to take into consideration the rate of change of the factor q^2. From equations (4.2.1) and (4.2.2) we have

$$\frac{\partial q^2}{\partial t} = 2q\frac{\partial q}{\partial t} = \frac{2q}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{\partial w}$$

$$\nabla q^2 = 2q\nabla q = -\frac{2q}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{\partial w}\frac{\boldsymbol{\upsilon}}{c^2}$$

Thus, we arrive at equations

$$\frac{\partial q^2}{\partial t} = \frac{2}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{q\partial w}q^2 = -2\lambda q^2$$

$$\frac{\partial q^2}{\partial x} = 2\lambda\frac{\upsilon_x}{c^2}q^2$$

$$\frac{\partial q^2}{\partial y} = 2\lambda\frac{\upsilon_y}{c^2}q^2 \qquad (4.7.22)$$

$$\frac{\partial q^2}{\partial z} = 2\lambda\frac{\upsilon_z}{c^2}q^2$$

$$\lambda = -\frac{1}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{q\partial w}$$

From equation (4.7.20), and for $i=0$, we have that

$$\frac{\partial\Phi^{0j}}{\partial x^j} = \frac{\partial\Phi^{00}}{\partial x^0}+\frac{\partial\Phi^{01}}{\partial x^1}+\frac{\partial\Phi^{02}}{\partial x^2}+\frac{\partial\Phi^{03}}{\partial x^3} =$$

$$\frac{\partial}{c\partial t}(w)\frac{\partial}{\partial x}(cS_x)+\frac{\partial}{\partial y}(cS_y)+\frac{\partial}{\partial z}(cS_z)$$

$$-\frac{1}{c^2}\left[\frac{\partial}{c\partial t}(\rho Vc^2)+\frac{\partial}{\partial x}(\rho Vc\upsilon_x)+\frac{\partial}{\partial y}(\rho Vc\upsilon_y)+\frac{\partial}{\partial z}(\rho Vc\upsilon_z)\right]$$

and using relations (4.7.22), which we apply on the quantities W, S_x, S_y, S_z, which are proportional to q^2, we get

$$\frac{\partial\Phi^{0j}}{\partial x^j} = -2\lambda W + 2\lambda\upsilon_x S_x + 2\lambda\upsilon_y S_y + 2\lambda\upsilon_z S_z$$

$$-\frac{V}{c}\left[\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho\upsilon_x)+\frac{\partial}{\partial y}(\rho\upsilon_y)+\frac{\partial}{\partial z}(\rho\upsilon_z)\right]$$

$$-\frac{\rho}{c}\left(\frac{\partial V}{\partial t}+\upsilon_x\frac{\partial V}{\partial x}+\upsilon_y\frac{\partial V}{\partial y}+\upsilon_z\frac{\partial V}{\partial z}\right)$$

$$\frac{\partial\Phi^{0j}}{\partial x^j} = 2\lambda\left(-W+\upsilon_x S_x+\upsilon_y S_y+\upsilon_z S_z\right)$$

$$-\frac{V}{c}\left[\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{\upsilon})\right]-\frac{\rho}{c}\left(\frac{\partial V}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla V\right)$$

and from the equation of continuity, as well as equation (4.7.19), we get

$$\frac{\partial\Phi^{0j}}{\partial x^j} = 2\lambda\left(-W+\upsilon_x S_x+\upsilon_y S_y+\upsilon_z S_z\right)+\frac{\rho}{c}(\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon})$$

and with equations (4.7.16) and (4.7.17) we get

$$\frac{\partial \Phi^{0j}}{\partial x^j}=2\lambda\varepsilon_0\left[-\frac{1}{2}\boldsymbol{\varepsilon}^2-\frac{1}{2}\boldsymbol{B}^2+\frac{\upsilon_x^2}{c^2}\boldsymbol{\varepsilon}^2-\upsilon_x\varepsilon_x\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c^2}\right)+\frac{\upsilon_y^2}{c^2}\boldsymbol{\varepsilon}^2\right.$$

$$\left.-\upsilon_y\varepsilon_y\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c^2}\right)+\frac{\upsilon_z^2}{c^2}\boldsymbol{\varepsilon}^2-\upsilon_z\varepsilon_z\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c^2}\right)\right]+\rho\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)$$

$$\frac{\partial \Phi^{0j}}{\partial x^j}=-2\lambda\varepsilon_0\left[-\frac{1}{2}\boldsymbol{\varepsilon}^2-\frac{1}{2}c^2\boldsymbol{B}^2+\frac{\upsilon_x^2+\upsilon_y^2+\upsilon_z^2}{c^2}\boldsymbol{\varepsilon}^2-(\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon})(\upsilon_x\varepsilon_x+\upsilon_y\varepsilon_y+\upsilon_z\varepsilon_z)\right]$$

$$+\rho\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)$$

and since it is $\upsilon_x^2+\upsilon_y^2+\upsilon_z^2=c^2$ and also $\upsilon_x\varepsilon_x+\upsilon_y\varepsilon_y+\upsilon_z\varepsilon_z=\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}$, we see that

$$\frac{\partial \Phi^{0j}}{\partial x^j}=-2\lambda\varepsilon_0\left[-\frac{1}{2}\boldsymbol{\varepsilon}^2-\frac{1}{2}c^2\boldsymbol{B}^2-\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)^2\right]+\rho\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)$$

From equation (4.6.35) we obtain

$$\frac{\partial \Phi^{0j}}{\partial x^j}=-2\lambda\varepsilon_0\frac{1}{2}\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)^2+\rho\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)$$

$$\frac{\partial \Phi^{0j}}{\partial x^j}=\left(\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right)\left[\rho-\lambda\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}\right]=0$$

since

$$\rho-\lambda\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}=0 \qquad (4.7.23)$$

Indeed, substituting the factor λ from equations (4.2.22), we have

$$\lambda\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}=-\frac{1}{1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{q\partial w}\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}$$

$$\lambda\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}=-\frac{1}{1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{q\partial w}\varepsilon_0\frac{\boldsymbol{\upsilon}}{c}(\boldsymbol{\varepsilon}_u+\boldsymbol{\varepsilon}_\alpha)$$

From equation (4.6.34) we have

$$\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}_\alpha=0$$

Hence, we see that

$$\lambda\varepsilon_0\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c}=-\frac{\varepsilon_0}{1-\dfrac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}}\frac{\partial q}{q\partial w}\frac{\boldsymbol{\upsilon}}{c}\boldsymbol{\varepsilon}_u$$

and with equation (4.6.8) for the electric field $\boldsymbol{\varepsilon}_u$ we get

$$\lambda\varepsilon_0 \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c} = -\frac{\varepsilon_0}{1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}} \frac{\partial q}{q\partial w} \frac{q\left(1-\frac{u^2}{c^2}\right)}{4\pi\varepsilon_0 r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3} \frac{\boldsymbol{\upsilon}}{c}\left(\frac{\boldsymbol{\upsilon}}{c}-\frac{\boldsymbol{u}}{c}\right)$$

$$\lambda\varepsilon_0 \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c} = -\frac{\partial q}{\partial w} \frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^4}\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c}\right)$$

$$\lambda\varepsilon_0 \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c} = -\frac{\partial q}{\partial w} \frac{1-\frac{u^2}{c^2}}{4\pi r^2\left(1-\frac{\boldsymbol{\upsilon}\cdot\boldsymbol{u}}{c^2}\right)^3}$$

Applying equation (4.6.5) we get

$$\lambda\varepsilon_0 \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c} = \rho$$

$$\rho - \lambda\varepsilon_0 \frac{\boldsymbol{\upsilon}\cdot\boldsymbol{\varepsilon}}{c} = 0$$

The validity of equation (4.7.21) for $i = 1,2,3$ is proven similarly.

In paragraph 4.5 we proved that the selfvariations are in agreement with the conservation of energy and momentum. The proof was done in two different ways: by direct calculation, and by applying the continuity equation. While it is of interest that the two different proofs, both lead to the conclusion that the selfvariations are compatible with the conservation principles of Physics, the calculation for the energy-momentum tensor was done for a completely different, and very substantial, reason. At macrocosmic scales, that is at large distances from the material particle, where equations (4.2.1) and (4.2.2) hold, the energy-momentum tensor Φ^{ij}, as given by equation (4.7.19), indeed contains all the information about the energy content of spacetime. At microcosmic scales the equations of the theory of selfvariations highlight additional parameters about the energy content of spacetime. These parameters bring the quantum phenomena to the forefront.

4.8 The energy-momentum tensor of the generalized photon at macrocosmic scales

In this paragraph we shall study the energy-momentum tensor for the generalized photon that balances the selfvariation of the rest mass of the material particle. Using our notation the energy-momentum tensor is given by

$$\Phi^{ij} = \frac{D}{c^2}\begin{bmatrix} c^2 & c\upsilon_x & c\upsilon_y & c\upsilon_z \\ \upsilon_x c & \upsilon_x^2 & \upsilon_x\upsilon_y & \upsilon_x\upsilon_z \\ \upsilon_y c & \upsilon_y\upsilon_x & \upsilon_y^2 & \upsilon_y\upsilon_z \\ \upsilon_z c & \upsilon_z\upsilon_x & \upsilon_z\upsilon_y & \upsilon_z^2 \end{bmatrix} \tag{4.8.1}$$

with the energy density D given by equation (4.3.7).

We shall prove the conservation of energy and momentum as given by equation

$$\Phi^{ij}_{,j} = \frac{\partial\Phi^{ij}}{\partial x^j} = 0 \tag{4.8.2}$$

For $i=0$ we have

$$\frac{\partial\Phi^{0j}}{\partial x^j}=\frac{\partial\Phi^{00}}{\partial x^0}+\frac{\partial\Phi^{01}}{\partial x^1}+\frac{\partial\Phi^{02}}{\partial x^2}+\frac{\partial\Phi^{03}}{\partial x^3}$$

$$\frac{\partial\Phi^{0j}}{\partial x^j}=\frac{\partial}{c\partial t}\left(\frac{Dc^2}{c^2}\right)+\frac{\partial}{\partial x}\left(\frac{D}{c^2}c\upsilon_x\right)+\frac{\partial}{\partial y}\left(\frac{D}{c^2}c\upsilon_y\right)+\frac{\partial}{\partial z}\left(\frac{D}{c^2}c\upsilon_z\right)$$

$$\frac{\partial\Phi^{0j}}{\partial x^j}=\frac{1}{c}\left[\frac{\partial D}{\partial t}+\nabla\cdot\left(D\boldsymbol{\upsilon}\right)\right]$$

and with equation (4.3.8) we get

$$\frac{\partial\Phi^{0j}}{\partial x^j}=0$$

For $i=1$ we have

$$\frac{\partial\Phi^{ij}}{\partial x^j}=\frac{\partial\Phi^{10}}{\partial x^0}+\frac{\partial\Phi^{11}}{\partial x^1}+\frac{\partial\Phi^{12}}{\partial x^2}+\frac{\partial\Phi^{13}}{\partial x^3}$$

$$\frac{\partial\Phi^{ij}}{\partial x^j}=\frac{1}{c^2}\left[\frac{\partial}{\partial t}\left(D\upsilon_x\right)+\frac{\partial}{\partial x}\left(D\upsilon_x\upsilon_x\right)+\frac{\partial}{\partial y}\left(D\upsilon_x\upsilon_y\right)+\frac{\partial}{\partial z}\left(D\upsilon_x\upsilon_z\right)\right]$$

$$\frac{\partial\Phi^{ij}}{\partial x^j}=\frac{1}{c^2}\left[\upsilon_x\left(\frac{\partial D}{\partial t}+\nabla\cdot\left(D\boldsymbol{\upsilon}\right)\right)+D\left(\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x\right)\right]$$

and with equation (4.3.8) we get

$$\frac{\partial\Phi^{ij}}{\partial x^j}=\upsilon_x\left(\frac{\partial D}{\partial t}+\nabla\cdot\boldsymbol{j}\right)+\frac{1}{c^2}D\left(\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x\right)$$

and with (4.5.5) we arrive at

$$\frac{\partial\Phi^{ij}}{\partial x^j}=\frac{D}{c^2}\left(\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x\right)\tag{4.8.3}$$

From equation (2.3.1) we have

$$\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x=\frac{\partial}{\partial t}\left(c\cos\delta\right)+\boldsymbol{\upsilon}\cdot\nabla\left(c\cos\delta\right)$$

$$\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x=-c\sin\delta\left(\frac{\partial\delta}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\delta\right)$$

and with equation (2.5.2)(b) we get

$$\frac{\partial\upsilon_x}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla\upsilon_x=0\tag{4.8.4}$$

Combining equations (4.8.3) and (4.8.4), we see that

$$\frac{\partial\Phi^{ij}}{\partial x^j}=0$$

We can similarly prove the validity of equation (4.8.1) for $i=2,3$.

By comparing the results of the last two paragraphs we find substantial differences between the generalized photon that counterbalances the selfvariation of the electric charge and the generalized photon that counterbalances the selfvariation of the rest mass of the material particle. Within the energy-momentum tensor of the first, there appears the electromagnetic field, as expressed by the first matrix of the second part of equation (4.7.19). On the contrary, in the expression of the energy-momentum tensor of equation (4.8.1), no corresponding matrix appears. Therefore, the generalized photon counterbalancing the rest mass does not correspond to a kind of

field with the structure and content of the electromagnetic field. Furthermore, by comparing the second matrix of equation (4.7.19) with the matrix of equation (4.8.1), we observe that in place of the potential V in the first, the factor c^2 appears in the second. These observations hold even if we formulate the equations for a non-inertial reference frame (we have already suggested a way for formulating the equations in non-inertial reference frames). By careful observation of the equations appearing in paragraphs 4.2, 4.3 and 4.4, we realize that the difference in the "behavior" of the couples $(\rho, \boldsymbol{j})$ and $(D, \boldsymbol{J})$ is the result of the different way the electric charge and the energy transform according to Lorentz-Einstein. It is exactly this difference that is captured on tensors (4.7.19) and (4.8.1). The generalized photon gives us the exact mechanism of transport of energy and momentum from one material particle to the other. At the same time, it highlights the similarities and differences between the electromagnetic and the gravitational interaction.
We could call the generalized photon that counterbalances the selfvariation of the rest mass by a different name. In any case it is obvious when we refer to the electric charge and when we refer to the rest mass. We shall, therefore, keep the name "generalized photon" for both cases.
The observation we made at the end of the previous paragraph regarding the tensor given by equation (4.7.19), also holds for tensor (4.8.1). It is valid at macrocosmic scales. At microcosmic scales, further parameters emerge from the theory of selfvariations, which cannot be given by the energy-momentum tensor.

4.9 The internality of the universe to the measurement procedure

The selfvariations hypothesis brings to the foreground the "internality of the Universe to the measurement procedure". Usually, in order to measure a physical quantity, we define as unit an arbitrary quantity with which we compare other physical quantities of the same kind. If the defined unit of measurement depends on the rest mass or the electric charge, then it is itself subject to the selfvariations. This fact must be taken into account every time we perform a measurement.
The photon does not have rest mass or electric charge and is, therefore, not affected by the selfvariations. The evidence we have suggests that the selfvariations take place at extremely slow rates. Therefore, the first consequence of the selfvariations we expect to observe is the following: photons with great lifetimes will be measured to have less energy than expected.
The extremely slow rate of evolution of the selfvariations, combined with the "internality of the Universe to the measurement procedure", do not allow their immediate observation in the laboratory. In the laboratory we only observe the consequences of the selfvariations. These consequences are the potential fields and the quantum phenomena.

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CHAPTER 5

The quantitative determination of the selfvariations

5.1 Introduction

In the present chapter we develop the main axis of the structure of the theory of selfvariations. We determine quantitatively the rate of evolution of the selfvariations, and formulate the *law of selfvariations*.

The law of selfvariations dominates from the microcosmic scales up to the observations we conduct billions of light years away. It reveals the causes of quantum phenomena, while it contains as physical information the totality of the cosmological observational data. At the same time, it sets the path for understanding the interactions between material particles.

The equations resulting from the law of selfvariations are of fundamental nature for the science of Physics and the related Physical Sciences. They contain a large amount of physical information, which permits the full understanding of physical reality.

5.2 The law of selfvariations

The conclusions derived in the previous chapters refer to the surrounding spacetime of the material particle. These conclusions are grounded on the second proposition-axiom of the theory of selfvariations, which states that

$$dS^2 = 0 \qquad (5.2.1)$$

This proposition is equivalent to the relation $\|\mathbf{\upsilon}\| = c$ which holds in every inertial system of reference.

In figure 2.2.1 the rest mass m_0 and the electric charge q of the material particle act at point $A(x, y, z, t)$ with the value they acquired at the moment $w = t - \frac{r}{c}$. Thus, we have that $m_0 = m_0(w)$ and $q = q(w)$. For the relevant calculations and proofs we have taken into consideration the axioms of the theory of selfvariations, but we have not yet defined the rate of evolution of their manifestation. In order to study the consequences of the selfvariations we have to determine quantitatively the first proposition-axiom of the theory of selfvariations.

Equation (5.2.1), combined with the first proposition-axiom of the selfvariations, leads directly to the concept of the *"generalized photon"*. The material particle emits generalized photons, and each generalized photon carries energy E and momentum $\mathbf{P}$, in order to counterbalance the change in energy and momentum that results from the selfvariations of the rest mass of the material particle. If the material particle also carries electric charge, then the generalized photon carries electric charge as well, in order to counterbalance the variation of the electric charge of the material particle due to the selfvariations.

The rate of evolution of the selfvariations is determined axiomatically with the help of the total energy E_s and the total momentum $\mathbf{P}_s$, which is emitted simultaneously and in all directions by the material particle, according to the following proposition-axiom:

«The rest mass m_0 and the electric charge q of every material particle vary according to the action of the operators

$$\frac{\partial}{\partial t} \rightarrow \frac{i}{\hbar} E_s \qquad \nabla \rightarrow \frac{i}{\hbar} \mathbf{P}_s \qquad (5.2.2)$$

where E_s and $\mathbf{P}_s$ denote the total energy and total momentum of the generalized photons emitted simultaneously by the material particle in all directions, and $\hbar = \frac{h}{2\pi}$, where h is Planck's constant » .

Stated in the form of equations, relations (5.2.2) can be written as

$$\frac{\partial m_0}{\partial t} = -\frac{i}{\hbar} E_s m_0 \qquad \nabla m_0 = \frac{i}{\hbar} \mathbf{P}_s m_0 \qquad (5.2.3)$$

and

$$\frac{\partial q}{\partial t} = -\frac{i}{\hbar} E_s q \qquad \nabla q = \frac{i}{\hbar} \mathbf{P}_s q \qquad (5.2.4)$$

In equations (5.2.3) and (5.2.4) we use the same symbol for the energy E_s and the momentum $\mathbf{P}_s$. But these are not the same physical quantities. In equations (5.2.3) the energy E_s and the momentum $\mathbf{P}_s$ counterbalance the consequences of the selfvariations of the rest mass. In equations (5.2.4) they counterbalance the consequences of the selfvariations of the electric charge. Later, we shall modify equation (5.2.4) in order to make this difference transparent.

The emission of generalized photons by the material particle comes about, initially, as a consequence of the principles of conservation of energy, momentum and electric charge. The operators given in relations (5.2.2) determine the relation between the material particle and the generalized photons, independently from the principles of conservation. Equations (5.2.3) and (5.2.4) express in a quantitative manner the *law of selfvariations.*

According to the law of selfvariations the rest mass m_0 and the electric charge q are functions of time t, as well as of the position of the material particle

$$m_0 = m_0(X_p, Y_p, Z_p, t) \qquad q = q(X_p, Y_p, Z_p, t) \qquad (5.2.5)$$

The dependence of the rest mass and the electric charge, not only on time, but also on the spatial position, is to be expected. Even if in some inertial frame of reference they only depend on time, in another inertial frame of reference they will also depend on the position, according to the Lorentz-Einstein transformations.

From equation (4.5.20), and for $\mathbf{u}=0$, we take that $\mathbf{P}_s=\mathbf{0}$, so that the second equation of the couple of equations (5.2.3) gives $\nabla m_0=\mathbf{0}$, whereas the first equation can be written as

$$\frac{dm_0}{dt}=-\frac{i}{\hbar}E_0 m_0$$

$$\dot{m}=-\frac{i}{\hbar}E_0 m_0$$

$$E_0=i\hbar\frac{\dot{m}_0}{m_0} \qquad (5.2.6)$$

Here, we denote the differentiation with respect to time by $(\bullet)$, and we set $E_s=E_0$ (the necessity of denoting $E_s=E_0$ will become apparent later on).

Furthermore, from the principle of conservation of energy at the instant of emission of the generalized photons, we obtain that

$$(m_0c^2+E_0)^{\bullet}=0 \qquad (5.2.7)$$

Combining equations (5.2.6) and (5.2.7) we arrive at equation

$$\left(m_0c^2+i\hbar\frac{\dot{m}_0}{m_0}\right)^{\bullet}=0 \qquad (5.2.8)$$

Equation (5.2.8) both contains as physical information, and justifies, the whole corpus of the current cosmological observational data, as described in chapter 7.

5.3 The "percentage function" Φ

The law of selfvariations expresses the total interaction of the generalized photons, which are emitted simultaneously by the material particle, with its rest mass and electric charge. However, in a particular direction $\frac{\boldsymbol{\upsilon}}{c}$, the material particle emits generalized photons of energy E and momentum $\mathbf{P}$. Therefore, we have to derive quantitatively the partial contribution of a single generalized photon of energy E and momentum $\mathbf{P}$ to the law of selfvariations.

We have to answer the following question:

"Which mathematical equation correlates the energy E and the momentum $\mathbf{P}$ of a single generalized photon emitted towards a particular direction $\frac{\boldsymbol{\upsilon}}{c}$, to the selfvariations of the rest mass m_0 and the electric charge q of the material particle?"

Thus, we are seeking the form of equations (5.2.3) and (5.2.4) that correspond to a single generalized photon.

Based on the law of selfvariations, the answer to this physical problem can only be given by the following statement:

"The partial contribution of a single generalized photon to the selfvariations of the rest mass m_0 and the electric charge q of the material particle is given by any mathematical expression which agrees with the operators defined in equations (5.2.2). If we sum the contributions of the single generalized photons towards all directions, during their simultaneous emission by the material particle, we have to end up with the equations given in (5.2.3) and (5.2.4)".

Considering this physical problem from its mathematical aspect, we can choose arbitrarily any mathematical expression giving the partial contribution of a single generalized photon according to the law of selfvariations, which satisfies the operators (5.2.2). Then, we can compare the results obtained by our particular choice with physical reality. On the other hand, we can choose the mathematical expression taking into account some specific *physical* criteria beforehand.

A fundamental case for the partial contribution of a generalized photon according to the law of selfvariations arises from the following observation: A single generalized photon counterbalances only a percentage of the total energy, momentum and electric charge that result from the selfvariations. Therefore, we must examine whether the contribution of a single generalized photon to the law of selfvariations is correlated with a *percentage* Φ of the rest mass m_0 and electric charge q. In this case, the partial contribution to the law of selfvariations for a single generalized photon of energy E and momentum $\mathbf{P}$ will be given by the set of equations

$$\begin{aligned} \frac{\partial(\Phi m_0)}{\partial t} &= -\frac{i}{\hbar} E m_0 \\ \nabla(\Phi m_0) &= \frac{i}{\hbar} \mathbf{P} m_0 \end{aligned} \qquad (5.3.1)$$

$$\begin{aligned} \frac{\partial(\Phi q)}{\partial t} &= -\frac{i}{\hbar} E q \\ \nabla(\Phi q) &= \frac{i}{\hbar} \mathbf{P} q \end{aligned} \qquad (5.3.2)$$

Summing in all directions of emission of generalized photons in the first equation of the set of equations (5.3.1), we obtain relations

$$\sum \frac{\partial(\Phi m_0)}{\partial t} = -\frac{i}{\hbar} \sum E m_0$$

$$\frac{\partial}{\partial t}\left(\sum \Phi m_0\right) = -\frac{i}{\hbar} m_0 \sum E$$

$$\frac{\partial}{\partial t}\left(m_0 \sum \Phi\right) = -\frac{i}{\hbar} m_0 \sum E$$

Since it holds that $\sum E = E_s$ and the total percentage of the contributions is 1, that is $\sum \Phi = 1$, we get

$$\frac{\partial m_0}{\partial t} = -\frac{i}{\hbar} m_0 E_s$$

This is the first equation of the set of equations (5.2.3).

Also, from the second equation of the set of equations (5.3.1) we obtain relations

$$\sum \nabla(\Phi m_0) = \frac{i}{\hbar} \sum \mathbf{P} m_0$$

$$\nabla\left(\sum (\Phi m_0)\right) = \frac{i}{\hbar} m_0 \sum \mathbf{P}$$

$$\nabla\left(m_0 \sum \Phi\right) = \frac{i}{\hbar} m_0 \sum \mathbf{P}$$

Since $\sum \Phi = 1$ and $\sum \mathbf{P} = \mathbf{P}_s$, we see that

$$\nabla m_0 = \frac{i}{\hbar} m_0 \mathbf{P}_s$$

This is the second of the equations given in (5.2.3).

We can perform the same procedure for equations (5.3.2) as well. Therefore, a single generalized photon can contribute to the selfvariation with a percentage Φ of the rest mass or the electric charge, and then this contribution is expressed by equations (5.3.1) and (5.3.2).

From equations (5.3.1) we obtain

$$\Phi \frac{\partial m_0}{\partial t} + m_0 \frac{\partial \Phi}{\partial t} = -\frac{i}{\hbar} E m_0$$

$$\Phi \nabla m_0 + m_0 \nabla \Phi = \frac{i}{\hbar} \mathbf{P} m_0$$

From equations (4.3.9) we also obtain

$$\Phi \frac{1}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{\partial w} + m_0 \frac{\partial \Phi}{\partial t} = -\frac{i}{\hbar} E m_0$$

$$-\Phi \frac{1}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{c \partial w} \frac{\boldsymbol{\upsilon}}{c} + m_0 \nabla \Phi = \frac{i}{\hbar} \mathbf{P} m_0$$

Eliminating from the equations the quantity m_0, we obtain

$$\frac{1}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + \frac{\partial \Phi}{\partial t} = -\frac{i}{\hbar} E$$

$$-\frac{1}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} + \nabla \Phi = \frac{i}{\hbar} \mathbf{P}$$

Finally, we arrive at the set of equations

$$E = \Phi \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + i\hbar \frac{\partial \Phi}{\partial t}$$

$$\mathbf{P} = \Phi \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \nabla \Phi \qquad (5.3.3)$$

The function Φ can be any mathematical function, defined on the material particle and obeying relation

$$\sum \Phi = 1 \qquad (5.3.4)$$

However, it has to be considered a function depending on the direction in space, since this is implied by the summation given in equation (5.3.4).

According to the operators defined in (5.2.2), the continuous evolution of the selfvariations with the passage of time is assured by the condition

$$E_s \neq 0 \qquad (5.3.5)$$

This condition is a straightforward consequence of the first proposition-axiom of the theory of selfvariations.

We are seeking now to derive the relation between the total momentum $\mathbf{P}_s$ and the total energy E_s. According to equation (4.5.20) this relation can be written as

$$\mathbf{P}_s = E_s \frac{\mathbf{u}}{c^2} \qquad (5.3.6)$$

Here, $\mathbf{u}$ denotes the velocity of the material particle at the moment of the emission of the generalized photons.

This relation has to be reconsidered for the following reason: During the proof of this relation in paragraph 4.3 of chapter 4, we have taken into consideration equation (4.3.8), that is equation

$$\mathbf{J} = D\frac{\boldsymbol{\upsilon}}{c^2}$$

This equation presupposes the validity of the condition

$$\mathbf{P} = E\frac{\boldsymbol{\upsilon}}{c^2} \qquad (5.3.7)$$

for every single generalized photon emitted towards any direction defined by $\frac{\boldsymbol{\upsilon}}{c}$, as depicted in figure 3.2.1. However, equations (5.3.3) reveal a more complex, and certainly different relation, between the momentum $\mathbf{P}$ and the energy E of a single generalized photon. Therefore, we have to reconsider the validity of equation (5.3.6), since we cannot base its proof on equation (5.3.7). As we shall see immediately, equation (5.3.6) is of general validity, and is compatible with the set of equations (5.3.3).

We consider a material point particle at rest, as depicted in figure 3.3.1. In order for this particle to remain at rest, the total momentum emitted simultaneously and towards all directions has to vanish, that is

$$\mathbf{P}_s^{'} = \mathbf{0} \quad (5.3.8)$$

If the case were different, the material particle would undergo an arbitrary motion, as a consequence of the principle of conservation of momentum. From equation (5.3.8), and from the set of transformations (4.3.3) for the total energy E_s and the total momentum $\mathbf{P}_s$, we arrive at equation (5.3.6). Thus, we have

$$E_s = \gamma\left(\mathrm{E}_s^{'} + uP_{sx}^{'}\right)$$

$$P_{sx} = \gamma\left(P_{sx}^{'} + \frac{u}{c^2}\mathrm{E}_s^{'}\right)$$

$$P_{sy} = P_{sy}^{'}$$

$$P_{sz} = P_{sz}^{'}$$

Since, according to equation (5.3.8) it holds that $\left(P_{sx}^{'}, P_{sy}^{'}, P_{sz}^{'}\right) = (0,0,0)$, we obtain the following relations

$E_s = \gamma E_s^{'}$

$P_{sx} = \gamma E_s^{'} \frac{u}{c^2}$

$P_{sy} = 0$

$P_{sz} = 0$

We also have that $\mathbf{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$, thus we obtain

$E_s = \gamma E_s^{'}$

$\mathbf{P}_s = \gamma E_s^{'} \frac{\mathbf{u}}{c^2}$

Finally, we have

$E_s = \gamma E_s^{'}$

$\mathbf{P}_s = E_s \frac{\mathbf{u}}{c^2}$

This is equation (5.3.6). Furthermore, we also obtain equation

$$E_s = \gamma E_s^{'} = \gamma E_0 = \frac{E_0}{\sqrt{1 - \frac{u^2}{c^2}}} \qquad (5.3.9)$$

Here, we denote

$$E_s^{'} = E_0 \qquad (5.3.10)$$

A material particle at rest can emit generalized photons of different energies for different directions. If the generalized photons emitted in opposite directions have opposite momenta, the material particle will remain at rest. But the momentum of a generalized photon can also be balanced by two other generalized photons emitted towards appropriate directions and with appropriate energies. In reality, there is an infinite number of combinations of emmision of generalized photons, with infinite combinations of energies and directions of emission. In each of these cases where equation (5.3.8) holds, the particle remains at rest. The case of emission of identical generalized photons in all directions by a material particle at rest is only one among the infinite number of cases satisfying equation (5.3.8).

Therefore, by rotating the unit vector $\frac{\mathbf{\upsilon}^{'}}{c}$ around the point particle at rest, as depicted in figure 3.3.1, we expect a change in the energy of the generalized photons. Exactly this is shown by equations (5.3.3), while at the same time they highlight the factors defining the energy and momentum of each single generalized photon.

5.4 The accompanying particle

In the previous paragraph we proved equations (5.3.6) and (5.3.9):

$$\mathbf{P}_s = E_s \frac{\mathbf{u}}{c^2}$$
$$E_s = \frac{E_0}{\sqrt{1-\frac{u^2}{c^2}}} \qquad (5.4.1)$$

Equations (5.4.1) show that the total energy and momentum emitted simultaneously and in all directions by the material particle behaves as a particle moving with velocity $\mathbf{u}$, and accompanying the material particle. There is a definite correspondence between equations (5.4.1) and equations

$$\mathbf{P} = m\mathbf{u}$$
$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$$

which give the momentum $\mathbf{P}$ and the mass m of the material particle.

According to equations (5.4.1), the accompanying particle has rest energy E_0. This is the rest energy $E_s^{'}$ from equation (5.3.10). Therefore, the accompanying particle has a rest mass given by $\frac{E_0}{c^2}$.

According to the first proposition-axiom of the theory of selfvariations, the rest mass $\frac{E_0}{c^2}$ of the accompanying particle changes with the passage of time. Hence, we seek the counterparts of equations (5.2.3), which define the rate of change of the rest mass $\frac{E_0}{c^2}$, or equivalently the rest energy E_0. As such, we obtain the corresponding form of equations (5.2.3)

$$\frac{\partial E_0}{\partial t} = \frac{i}{\hbar} mc^2 E_0 = \frac{i}{\hbar}\gamma m_0 c^2 E_0 = \frac{i}{\hbar}\frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}} E_0$$
$$\nabla E_0 = -\frac{i}{\hbar} m\mathbf{u} E_0 = -\frac{i}{\hbar}\gamma m_0 \mathbf{u} E_0 = -\frac{i}{\hbar}\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\mathbf{u} E_0 \qquad (5.4.2)$$

Equations (5.2.3) describe the effect of the generalized photons on the rest mass of the material particle. In nature, though, effects are always mutual. Hence, just as the generalized photons affect the material particle, the material particle in turn affects the generalized photons, and these mutual interactions must occur in the framework of the same physical law. Therefore, from the outset the issue arises of the existence of a rest mass concealed within the operators (5.2.2), and of a corresponding equation symmetrical to (5.2.3). The quest for the partial contribution of a single generalized photon to the law of selfvariations revealed the existence of the rest mass $\frac{E_0}{c^2}$ and equations (5.4.2). The existence of the rest mass $\frac{E_0}{c^2}$ is predicted by the initial equations we formulated for the macrocosmic scales, through equation (4.5.20).

A large part of the predictions of the theory of selfvariations can be made without the aid of equations (5.4.2). For example, the justification of the observational cosmological data can be obtained from (5.2.8), which is proven independently without resorting to equations (5.4.2). The same holds for equations (5.3.3). However, the accompanying particle is a direct consequence of the selfvariations. Indeed, if we combine the second of equations (5.4.1) with relation (5.3.5) we see immediately that

$$E_0 \neq 0 \qquad (5.4.3)$$

The rest mass $\frac{E_0}{c^2}$ of the accompanying particle cannot vanish. Therefore, in order to study the consequences of the selfvariations in their totality, we have to take into account the existence and the properties of the accompanying particle. In nature there is always the system "material particle-accompanying particle".

Let M_0 be the rest mass of the system "material particle-accompanying particle", given by

$$M_0 = m_0 + \frac{E_0}{c^2} \qquad (5.4.4)$$

We have that

$$\frac{\partial M_0}{\partial t} = \frac{\partial}{\partial t}\left(m_0 + \frac{E_0}{c^2}\right)$$

$$\frac{\partial M_0}{\partial t} = \frac{\partial m_0}{\partial t} + \frac{\partial E_0}{c^2 \partial t}$$

Using the first equations of the sets of equations given in (5.2.3) and (5.2.4), we obtain relation

$$\frac{\partial M_0}{\partial t} = -\frac{i}{\hbar} E_s m_0 + \frac{i}{\hbar} \gamma m_0 E_0$$

And using equation (5.3.9) we get

$$\frac{\partial M_0}{\partial t} = -\frac{i}{\hbar} \gamma E_0 m_0 + \frac{i}{\hbar} \gamma m_0 E_0$$

$$\frac{\partial M_0}{\partial t} = 0 \qquad (5.4.5)$$

Similarly, using the second equations of the sets of equations (5.2.3) and (5.4.2) we have that

$$\nabla M_0 = \mathbf{0} \qquad (5.4.6)$$

From equations (5.4.5) and (5.4.6) we conclude that the rest mass M_0 of the system "material particle-accompanying particle" is a physical quantity not affected by the process of the selfvariations. Therefore, we can use the rest mass M_0 and the rest energy $M_0 c^2$ as a unit of measurement of mass and energy, respectively. By this approach we circumvent the methodological problems stemming from the principle of the *"internality of the universe with respect to the measurement procedure"*, as stated in paragraph 4.9.

The quantitative mathematical description of physical reality depends on our ability to include in our equations the consequences of the internality of the universe to the measurement procedure. In the macrocosmic scales there is a very simple way

to accomplish this, as described in chapter 7. In the microcosmic scale we use as units of measurement of mass and energy the quantities M_0 and M_0c^2, respectively.
We rewrite now equations (5.2.3) in the form

$$\frac{\partial}{\partial t}\left(\frac{m_0}{M_0}\right)=-\frac{i}{\hbar}E_s\left(\frac{m_0}{M_0}\right)$$
$$\nabla\left(\frac{m_0}{M_0}\right)=\frac{i}{\hbar}\mathbf{P}_s\left(\frac{m_0}{M_0}\right) \qquad (5.4.7)$$

These equations have the exact same physical content as equations (5.2.3). They give the rate of change of the rest mass m_0, since the rest mass M_0 is not affected by the selfvariations, according to equations (5.4.5) and (5.4.6). At the same time, these equations highlight the action of the operators (5.2.2) on the complex number $\frac{m_0}{M_0}\in\mathbb{C}$, since the complex unit i appears within the expressions of the operators. The same procedure can be repeated for the case of equations (5.4.2) as well, by introducing the number $\frac{E_0}{M_0c^2}\in\mathbb{C}$, and for the whole list of equations we have stated.

The accompanying particle has rest mass of magnitude $\frac{E_0}{c^2}$, which comes from the sum of the contributions of the generalized photons emitted simultaneously by the material particle. This is the physical content of equations (5.4.1). Therefore, the mechanism through which the selfvariations occur plays a fundamental role for the determination of the physical properties of the accompanying particle, and eventually for the physical properties of the actual system "material particle-accompanying particle".

5.5 The symmetrical law for the electric charge

From the study already conducted in paragraph 4.2 it follows that the generalized photons counterbalancing the selfvariation of the electric charge q carry electric charge. Therefore, the physical object resulting from their aggregation carries electric charge q_i.
The law of the selfvariations for the electric charge q is given by equations (5.2.4)

$$\frac{\partial q}{\partial t}=-\frac{i}{\hbar}E_s q$$
$$\nabla q=\frac{i}{\hbar}\mathbf{P}_s q \qquad (5.5.1)$$

In these equations we denote with E_s and $\mathbf{P}_s$ the total energy and momentum emitted by the material particle simultaneously in all directions, and which counterbalances the variations in energy resulting from the selfvariation of the electric charge. Although we have kept the same notation, these quantites are not the same as the ones appearing in equations (5.2.3).

In order to repeat the study conducted for the rest mass for the case of the electric charge, we have to define the equations symmetrical to (5.2.4). That is, we have to formulate the counterparts of equations (5.2.4) for the electric charge q_i.

The law of selfvariations for the electric charge (5.5.1) has to be modified so that it will define the interaction of the electric charges q and q_i, exactly as the law stated in equation (5.2.3) determines the interaction of the rest masses m_0 and $\frac{E_0}{c^2}$. Therefore, the second part of equation (5.5.1) has to be expressed such that the electric charge q_i appears. This can only be accomplished by the introduction of an electric potential V through equation

$$E_s = Vq_i \quad (5.5.2)$$

With this notation, and taking into account equation (5.3.6), equations (5.5.1) can be written as

$$\begin{aligned} \frac{\partial q}{\partial t} &= -\frac{i}{\hbar} V q_i q \\ \nabla q &= \frac{i}{\hbar} V q_i \frac{\mathbf{u}}{c^2} q \end{aligned} \quad (5.5.3)$$

Equations (5.5.3) and (5.5.1) have the same physical content, if and only if the electric potential V is independent of the selfvariations.

Starting from equation (5.5.3), we can also deduce all equations inferred in the previous paragraphs for the rest mass, except now for the electric charge. The proof follows similar paths, and we shall note repeat it here in full.

Firstly, it can be deduced that the potential V can be written in the form

$$V = \gamma V_0 = \frac{V_0}{\sqrt{1-\frac{u^2}{c^2}}} \quad (5.5.4)$$

The potential V_0 stays invariant under the action of the Lorentz-Einstein transformations, and is independent of the selfvariations. The corresponding expressions of equations (5.2.6) and (5.2.8) are

$$\begin{aligned} q_i V_0 &= i\hbar \frac{\dot{q}}{q} \\ \left(q + \frac{i\hbar}{V_0} \frac{\dot{q}}{q} \right)^{\cdot} &= 0 \end{aligned} \quad (5.5.5)$$

The corresponding equations to the ones given in (5.3.3) for the generalized photon, can be formulated as

$$\begin{aligned} E &= \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial q}{q\partial w} + i\hbar \frac{\partial \Phi}{\partial t} \\ \mathbf{P} &= \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial q}{qc\partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \nabla \Phi \end{aligned} \quad (5.5.6)$$

The corresponding equations to the equations (5.4.2), that is, the corresponding form of the law expressed in (5.5.3), are

$$\frac{\partial q_i}{\partial t} = \frac{i}{\hbar}\gamma V_0 q q_i$$
$$\nabla q_i = -\frac{i}{\hbar}\gamma V_0 q \frac{\mathbf{u}}{c^2} q_i \qquad (5.5.7)$$

The corresponding relation to relation (5.4.3) is

$$q_i \neq 0 \qquad (5.5.8)$$

The corresponding expression of equation (5.4.4), that is, the electric charge Q of the system "material particle-accompanying particle" is

$$Q = q + q_i \qquad (5.5.9)$$

The corresponding equations to equations (5.4.5) and (5.4.6) take the form

$$\frac{\partial Q}{\partial t} = 0$$
$$\nabla Q = \mathbf{0} \qquad (5.5.10)$$

The electric charge Q is not affected by the selfvariations.

5.6. Fundamental study of the generalized photon

In chapter 4 we studied the consequences of the selfvariations in the surrounding spacetime of the material particle. In that study we considered the validity of equation (4.3.8)

$$\mathbf{J} = D\frac{\boldsymbol{\upsilon}}{c^2}$$

which presupposes the validity of equation

$$\mathbf{P} = E\frac{\boldsymbol{\upsilon}}{c^2} \qquad (5.6.1)$$

for the generalized photon.

We know by now that the energy E and the momentum $\mathbf{P}$ of the generalized photon are not correlated through the simple relation (5.6.1). For the generalized photon that results from the selfvariation of the rest mass, equations (5.3.11) hold

$$E = \Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial m_0}{m_0\partial w} + i\hbar\frac{\partial\Phi}{\partial t}$$
$$\mathbf{P} = \Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial m_0}{m_0 c\partial w}\frac{\boldsymbol{\upsilon}}{c} - i\hbar\nabla\Phi \qquad (5.6.2)$$

For the generalized photon that results from the selfvariation of the electric charge, equations (5.5.6) hold

$$E = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial q}{q \partial w} + i\hbar \frac{\partial \Phi}{\partial t}$$
$$\mathbf{P} = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial q}{q c \partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \nabla \Phi \tag{5.6.3}$$

Equations (5.6.2) and (5.6.3) lead to a completely different relation from (5.6.1), between the energy E and the momentum $\mathbf{P}$ of a generalized photon.

We will study the generalized photon, as given in equations (5.6.2). The study of equations (5.6.3) is exactly the same.

The percentage-function Φ depends on the direction $\frac{\boldsymbol{\upsilon}}{c}$ and can, therefore, be written as $\Phi = \Phi(\delta, \omega)$, and can also depend on the moment, $w = t - \frac{r}{c}$, of emission of the generalized photon, so that

$$\Phi = \Phi(w, \delta, \omega) \tag{5.6.4}$$

From the first of equations (5.6.2) we have

$$E = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + i\hbar \frac{\partial \Phi}{\partial t}$$

and with equation (5.6.4), we get

$$E = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + i\hbar \left(\frac{\partial \Phi}{\partial w} \frac{\partial w}{\partial t} + \frac{\partial \Phi}{\partial \delta} \frac{\partial \delta}{\partial t} + \frac{\partial \Phi}{\partial \omega} \frac{\partial \omega}{\partial t} \right)$$

and with equations (2.2.11), (2.3.9) and (2.3.10) we get

$$E = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} + \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \left(\frac{\partial \Phi}{\partial w} - \frac{\mathbf{u}\boldsymbol{\beta}}{r} \frac{\partial \Phi}{\partial \delta} - \frac{\mathbf{u}\boldsymbol{\gamma}}{r \sin \delta} \frac{\partial \Phi}{\partial \omega} \right) \tag{5.6.5}$$

From the second of equations (5.6.2), we have

$$\mathbf{P} = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \nabla \Phi$$

and with equation (5.6.4) we get

$$\mathbf{P} = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} - i\hbar \left(\frac{\partial \Phi}{\partial w} \nabla w + \frac{\partial \Phi}{\partial \delta} \nabla \delta + \frac{\partial \Phi}{\partial \omega} \nabla \omega \right)$$

and with equations (2.2.12), (2.3.19) and (2.3.20), we get

$$\mathbf{P} = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} + \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial \Phi}{\partial w} \frac{\boldsymbol{\upsilon}}{c^2} - \frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \left(\frac{\mathbf{u}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\beta} \right) - \frac{i\hbar}{r \sin \delta} \frac{\partial \Phi}{\partial \omega} \left(\frac{\mathbf{u}\boldsymbol{\gamma}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\gamma} \right) \tag{5.6.6}$$

We now denote

$$E_i = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w}$$
$$P_i = \Phi \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \frac{\boldsymbol{\upsilon}}{c} \tag{5.6.7}$$

$$E_\Phi = i\hbar \frac{\partial \Phi}{\partial t} = \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial \Phi}{\partial w} - \frac{i\hbar \mathbf{u}\boldsymbol{\beta}}{r\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)} \frac{\partial \Phi}{\partial \delta} - \frac{i\hbar \mathbf{u}\boldsymbol{\gamma}}{r\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)\sin \delta} \frac{\partial \Phi}{\partial \omega}$$
$$\mathbf{P}_\Phi = -i\hbar \nabla \Phi = \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial \Phi}{\partial w} \frac{\boldsymbol{\upsilon}}{c^2} - \frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \left(\frac{\mathbf{u}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\beta} \right) - \frac{i\hbar}{r \sin \delta} \frac{\partial \Phi}{\partial \omega} \left(\frac{\mathbf{u}\boldsymbol{\gamma}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\gamma} \right) \tag{5.6.8}$$

With this notation, equations (5.6.5) and (5.6.6) can be written as

$$E = E_i + E_\Phi$$
$$\mathbf{P} = \mathbf{P}_i + \mathbf{P}_\Phi \tag{5.6.9}$$

Combining equations (5.6.5) and (5.6.6), we obtain relation

$$\mathbf{P} = E \frac{\boldsymbol{\upsilon}}{c^2} - \frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \boldsymbol{\beta} - \frac{i\hbar}{r \sin \delta} \frac{\partial \Phi}{\partial \omega} \boldsymbol{\gamma} \tag{5.6.10}$$

relating the energy E and momentum $\mathbf{P}$ of the generalized photon.

The energy-momentum pair $\left(E_\Phi, \mathbf{P}_\Phi\right)$ can be decomposed into three partial pairs

$$E_w = \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial \Phi}{\partial w}$$
$$\mathbf{P}_w = \frac{i\hbar}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial \Phi}{\partial w} \frac{\boldsymbol{\upsilon}}{c^2} \tag{5.6.11}$$

$$E_\delta = -\frac{i\hbar \mathbf{u}\boldsymbol{\beta}}{r\left(1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)} \frac{\partial \Phi}{\partial \delta}$$
$$\mathbf{P}_\delta = -\frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \left(\frac{\mathbf{u}\boldsymbol{\beta}}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\beta} \right) \tag{5.6.12}$$

$$E_\omega = -\frac{i\hbar \mathbf{u}\boldsymbol{\gamma}}{r\left(1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)\sin\delta} \frac{\partial \Phi}{\partial \omega}$$
$$\mathbf{P}_\omega = -\frac{i\hbar}{r\sin\delta} \frac{\partial \Phi}{\partial \omega} \left(\frac{\mathbf{u}\boldsymbol{\gamma}}{1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\gamma} \right) \tag{5.6.13}$$

$$E_\Phi = E_w + E_\delta + E_\omega$$
$$\mathbf{P}_\Phi = \mathbf{P}_w + \mathbf{P}_\delta + \mathbf{P}_\omega \tag{5.6.14}$$

It is easy to prove that, in the case of constant-speed motion with velocity $\mathbf{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$, each of the energy-momentum pairs $(E_i, \mathbf{P}_i), (E_w, \mathbf{P}_w), (E_\delta, \mathbf{P}_\delta), (E_\omega, \mathbf{P}_\omega)$ transforms autonomously, independently of the rest, according to the Lorentz-Einstein transformations. Furthermore, an invariant amount of energy corresponds to each pair.

We shall calculate the four invariant amounts of energy. In the same way, we can prove the independent Lorentz-Einstein transformations of the four energy-momentum pairs.

From equation (5.6.8) we have

$$E_i^2 - c^2\mathbf{P}_i^2 = E_i^2 - c^2 E_i^2 \left(\frac{\boldsymbol{\upsilon}}{c^2}\right)^2$$

and since $\boldsymbol{\upsilon}^2 = c^2$, we get

$$E_i^2 - c^2\mathbf{P}_i^2 = 0 \tag{5.6.15}$$

From equation (5.6.11) we have

$$E_w^2 - c^2\mathbf{P}_w^2 = E_w^2 - c^2E_w^2\left(\frac{\boldsymbol{\upsilon}}{c^2}\right)^2$$

and from $\boldsymbol{\upsilon}^2 = c^2$, we get

$$E_w^2 - c^2\mathbf{P}_w^2 = 0 \tag{5.6.16}$$

From equation (5.6.12) we have

$$E_\delta^2 - c^2\mathbf{P}_\delta^2 = -\frac{\hbar^2(\mathbf{u\beta})^2}{r^2\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)^2}\left(\frac{\partial\Phi}{\partial\delta}\right)^2 + \frac{c^2\hbar^2}{r^2}\left(\frac{\partial\Phi}{\partial\delta}\right)^2\left[\frac{\mathbf{u\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}+\boldsymbol{\beta}\right]^2$$

and since it is $\mathbf{u}\cdot\boldsymbol{\beta} = 0, \boldsymbol{\upsilon}^2 = c^2, \boldsymbol{\beta}^2 = 1$, we get

$$E_\delta^2 - c^2\mathbf{P}_\delta^2 = -\frac{\hbar^2(\mathbf{u\beta})^2}{r^2\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)^2}\left(\frac{\partial\Phi}{\partial\delta}\right)^2 + \frac{\hbar^2(\mathbf{u\beta})^2}{r^2\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)^2}\left(\frac{\partial\Phi}{\partial\delta}\right)^2 + \left(\frac{c\hbar}{r}\frac{\partial\Phi}{\partial\delta}\right)^2$$

$$E_\delta^2 - c^2\mathbf{P}_\delta^2 = \left(\frac{c\hbar}{r}\frac{\partial\Phi}{\partial\delta}\right)^2 \tag{5.6.17}$$

Similarly, from equations (5.6.13) we get

$$E_\omega^2 - c^2\mathbf{P}_\omega^2 = \left(\frac{c\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega}\right)^2 \tag{5.6.18}$$

From the transformations (3.4.8) we get

$$\frac{c\hbar}{r'}\frac{\partial\Phi}{\partial\delta'} = \frac{c\hbar}{\gamma r\left(1-\frac{u}{c}\cos\delta\right)}\frac{\partial\Phi}{\partial\delta}\gamma\left(1-\frac{u}{c}\cos\delta\right)$$

$$\frac{c\hbar}{r'}\frac{\partial\Phi}{\partial\delta'} = \frac{c\hbar}{\gamma r}\frac{\partial\Phi}{\partial\delta} \tag{5.6.19}$$

Therefore, the second part of equation (5.6.17) remains invariant according to the Lorentz-Einstein transformations.

From transformations (3.4.5) and (3.4.8) we have

$$\frac{c\hbar}{r'\sin\delta'}\frac{\partial\Phi}{\partial\omega'}=\frac{c\hbar}{\gamma r\left(1-\frac{u}{c}\cos\delta\right)\frac{\sin\delta}{\gamma\left(1-\frac{u}{c}\cos\delta\right)}}\frac{\partial\Phi}{\partial\omega}$$

$$\frac{c\hbar}{r'\sin\delta'}\frac{\partial\Phi}{\partial\omega'}=\frac{c\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega} \tag{5.6.20}$$

Therefore, the second part of equation (5.6.18) remains invariant under the Lorentz-Einstein transformations.

From equation (5.6.10) we can calculate the total invariant energy of the generalized photon

$$E^2-c^2\mathbf{P}^2=E^2-c^2\left(E\frac{\boldsymbol{\upsilon}}{c^2}-\frac{i\hbar}{r}\frac{\partial\Phi}{\partial\delta}\boldsymbol{\beta}-\frac{i\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\gamma}\right)^2$$

and taking into consideration that the set of vectors $\frac{\boldsymbol{\upsilon}}{c},\boldsymbol{\beta},\boldsymbol{\gamma}$ constitute an orthonormal basis, we get

$$E^2-c^2\mathbf{P}^2=E^2-E^2+\left(\frac{c\hbar}{r}\frac{\partial\Phi}{\partial\delta}\right)^2+\left(\frac{c\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega}\right)^2$$

$$E^2-c^2\mathbf{P}^2=\left(\frac{c\hbar}{r}\frac{\partial\Phi}{\partial\delta}\right)^2+\left(\frac{c\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega}\right)^2 \tag{5.6.21}$$

According to equations (5.6.19) and (5.6.20), the second part of equation (5.6.21) remains invariant under the Lorentz-Einstein transformations.

We will now prove that:

"In the case of constant-speed motion with velocity $\mathbf{u}=\begin{bmatrix}u\\0\\0\end{bmatrix}$, pairs $\left(E_i,\mathbf{P}_i\right),\left(E_w,\mathbf{P}_w\right)$ correspond to a flow of energy and momentum into the surrounding spacetime. On the contrary, pairs $\left(E_\delta,\mathbf{P}_\delta\right)$ and $\left(E_\omega,\mathbf{P}_\omega\right)$ correspond to a redistribution of energy and momentum in the surrounding spacetime".

From equation (3.2.10) together with the second of equations (5.6.11), we get

$$\mathbf{P}_i \cdot \mathbf{R} = \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 \partial w} \frac{\boldsymbol{\upsilon}}{c^2} r \left(\frac{\boldsymbol{\upsilon}}{c} - \frac{\mathbf{u}}{c} \right)$$

$$\mathbf{P}_i \cdot \mathbf{R} = \frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\partial m_0}{m_0 c \partial w} \left(1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2} \right)$$

$$\mathbf{P}_i \cdot \mathbf{R} = i\hbar r \frac{\partial m_0}{m_0 c \partial w}$$

$$\mathbf{P}_i \cdot \frac{\mathbf{R}}{r} = i\hbar \frac{\partial m_0}{m_0 c \partial w} \tag{5.6.22}$$

Similarly, from equation (3.2.10) together with the second of equations (5.6.11) we get

$$\mathbf{P}_w \cdot \frac{\mathbf{R}}{r} = i\hbar \frac{\partial \Phi}{c \partial w} \tag{5.6.23}$$

We conclude that both the momentum $\mathbf{P}_i$, as well as the momentum $\mathbf{P}_w$, have a component along the direction of vector $\mathbf{R}$, as depicted in Figure 3.2.1.

Combining equation (3.2.10) with the second of equations (5.6.12), we get

$$\mathbf{P}_\delta \cdot \mathbf{R} = -\frac{i\hbar}{r} \frac{\partial \Phi}{\partial \delta} \left(\frac{\mathbf{u}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} + \boldsymbol{\beta} \right) r \left(\frac{\boldsymbol{\upsilon}}{c} - \frac{\mathbf{u}}{c} \right)$$

and since $\boldsymbol{\upsilon}^2 = c^2$ and $\boldsymbol{\upsilon}\boldsymbol{\beta} = 0$, we obtain

$$\mathbf{P}_\delta \cdot \mathbf{R} = -i\hbar \frac{\partial \Phi}{\partial \delta} \left(\frac{\mathbf{u}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \frac{\boldsymbol{\upsilon}}{c^2} \left(\frac{\boldsymbol{\upsilon}}{c} - \frac{\mathbf{u}}{c} \right) - \frac{\mathbf{u}\boldsymbol{\beta}}{c} \right)$$

$$\mathbf{P}_\delta \cdot \mathbf{R} = -i\hbar \frac{\partial \Phi}{\partial \delta} \left(\frac{\frac{\mathbf{u}}{c}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}} \left(1 - \frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2} \right) - \frac{\mathbf{u}\boldsymbol{\beta}}{c} \right)$$

$$\mathbf{P}_\delta \cdot \mathbf{R} = 0 \tag{5.6.24}$$

Similarly, from equation (3.2.10) and the second of equations (5.6.13) we get

$$\mathbf{P}_\varpi \cdot \mathbf{R} = 0 \tag{5.6.25}$$

Both the momentum $\mathbf{P}_\delta$, and the momentum $\mathbf{P}_\omega$, are vertical to the vector $\mathbf{R}$ of Figure 3.2.1.

We will now prove that:

"The generalized photon carries intrinsic angular momentum $\mathbf{S}$, independent of the distance r. The component $\mathbf{S}_u$ of the intrinsic angular momentum $\mathbf{S}$ along the direction of the motion of the material particle does not depend upon the velocity $\mathbf{u}$ of the motion".

In Figure 2.2.1, the angular momentum $\mathbf{S}$ of the generalized photon with respect to the (constant) point of emission $E\left(x_p\left(w\right), y_p\left(w\right), z_p\left(w\right), w\right)$ is

$$\mathbf{S} = \mathbf{r} \times \mathbf{P}$$

and with equation (2.2.6) written in the form

$$\mathbf{r} = \frac{r}{c}\boldsymbol{\upsilon}$$

we get

$$\mathbf{S} = \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P} = \frac{r}{c}\boldsymbol{\upsilon} \times \left(\mathbf{P}_i + \mathbf{P}_w + \mathbf{P}_\delta + \mathbf{P}_\omega\right) \qquad (5.6.26)$$

Denoting

$$\begin{aligned} \mathbf{S}_i &= \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P}_i \\ \mathbf{S}_w &= \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P}_w \\ \mathbf{S}_\delta &= \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P}_\delta \\ \mathbf{S}_\omega &= \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P}_\omega \end{aligned} \qquad (5.6.27)$$

equation (5.6.26) can be written as

$$\mathbf{S} = \mathbf{S}_i + \mathbf{S}_w + \mathbf{S}_\delta + \mathbf{S}_\omega \qquad (5.6.28)$$

From the first of equations (5.6.27) we have

$$\mathbf{S}_i = \frac{r}{c}\boldsymbol{\upsilon} \times \mathbf{P}_i$$

and with the second of equations (5.6.7) we get

$$\mathbf{S}_i = 0 \qquad (5.6.29)$$

From the second of equations (5.6.27) we have

$$\mathbf{S}_w = \frac{r}{c}\boldsymbol{\upsilon}\times\mathbf{P}_w$$

and with the second of equations (5.6.11) we get

$$\mathbf{S}_w = \mathbf{0} \tag{5.6.30}$$

From the third of equations (5.6.27) we have

$$\mathbf{S}_\delta = \frac{r}{c}\boldsymbol{\upsilon}\times\mathbf{P}_\delta$$

and with the second of equations (5.6.12) we have

$$\mathbf{S}_\delta = -i\hbar\frac{\partial\Phi}{\partial\delta}\frac{\boldsymbol{\upsilon}}{c}\times\left(\frac{\mathbf{u}\boldsymbol{\beta}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}+\boldsymbol{\beta}\right)$$

$$\mathbf{S}_\delta = -i\hbar\frac{\partial\Phi}{\partial\delta}\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\beta}$$

and since it is $\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\beta}=\boldsymbol{\gamma}$, we get

$$\mathbf{S}_\delta = -i\hbar\frac{\partial\Phi}{\partial\delta}\boldsymbol{\gamma} \tag{5.6.31}$$

From the third of equations (6.2.27) we have

$$\mathbf{S}_\omega = \frac{r}{c}\boldsymbol{\upsilon}\times\mathbf{P}_\omega$$

and with the second of equations (5.6.13) we get

$$\mathbf{S}_\omega = -\frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\frac{\boldsymbol{\upsilon}}{c}\times\left(\frac{\mathbf{u}\boldsymbol{\gamma}}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\boldsymbol{\upsilon}}{c^2}+\boldsymbol{\gamma}\right)$$

and since $\frac{\boldsymbol{\upsilon}}{c}\times\boldsymbol{\gamma}=-\boldsymbol{\beta}$, we get

$$\mathbf{S}_\omega = \frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\beta} \tag{5.6.32}$$

Equation (5.6.28) can now be written as

$$\mathbf{S} = \frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\beta} - i\hbar\frac{\partial\Phi}{\partial\delta}\boldsymbol{\gamma} \tag{5.6.33}$$

We now calculate the component $\mathbf{S}_u$ of the angular momentum $\mathbf{S}$ along the direction of motion of the material particle.

For $\mathbf{u} \neq 0$ we have

$$\mathbf{S}_u = \frac{\mathbf{u}}{\|\mathbf{u}\|}\mathbf{S}$$

and with equation (5.6.33) we get

$$\mathbf{S}_u = \frac{\mathbf{u}}{\|\mathbf{u}\|}\left(\frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}\boldsymbol{\beta} - i\hbar\frac{\partial\Phi}{\partial\delta}\boldsymbol{\gamma}\right) \tag{5.6.34}$$

For constant-speed motion with velocity $\mathbf{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$, and taking into consideration equations (2.3.3) and (2.3.4), we obtain from equation (5.6.34)

$$S_u = \frac{i\hbar}{\sin\delta}\frac{\partial\Phi}{\partial\omega}(-\sin\delta)$$

$$S_u = -i\hbar\frac{\partial\Phi}{\partial\omega} \tag{5.6.35}$$

In the case of constant-speed motion with velocity $\mathbf{u} = \begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}$, from the transformations of equations (3.4.5) $\omega' = \omega$, we conclude that the angular momentum S_u does not depend on the inertial reference frame. Furthermore, it does not depend on the angle δ, i.e. the angle formed between the direction of emission $\frac{\boldsymbol{\upsilon}}{c}$ of the generalized photon and the velocity $\mathbf{u}$ of the material particle in Figure 3.2.1.

We will now study the changes in energy and momentum that take place during the motion of the generalized photon with velocity $\boldsymbol{\upsilon}$, after its emission by the material particle.

From the fundamental mathematical theorem, specifically from equation (2.5.7) for $f = E_i, f = E_w, f = E_\delta$ and $f = E_\omega$, we have

$$\frac{\partial E_i}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_i = c \frac{\partial E_i}{\partial r}$$
$$\frac{\partial E_w}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_w = c \frac{\partial E_w}{\partial r}$$
$$\frac{\partial E_\delta}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_\delta = c \frac{\partial E_\delta}{\partial r}$$
$$\frac{\partial E_\omega}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_\omega = c \frac{\partial E_\omega}{\partial r}$$

and with the first of equations (5.6.7), (5.6.11), (5.6.12) and (5.6.13), we get

$$\begin{aligned} &\frac{\partial E_i}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_i = 0 \\ &\frac{\partial E_w}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_w = 0 \\ &\frac{\partial E_\delta}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_\delta = -\frac{c}{r} E_\delta \\ &\frac{\partial E_\omega}{\partial t} + \mathbf{\upsilon} \cdot \nabla E_\omega = -\frac{c}{r} E_\omega \end{aligned} \tag{5.6.36}$$

Similarly, after combining equations (2.5.8), (2.5.9), (2.5.10) with the second parts of equations (5.6.7), (5.6.11), (5.6.12) and (5.6.13), we get

$$\begin{aligned} &\frac{\partial \mathbf{P}_i}{\partial t} + \left(grad\mathbf{P}_i\right)\mathbf{\upsilon} = \mathbf{0} \\ &\frac{\partial \mathbf{P}_w}{\partial t} + \left(grad\mathbf{P}_w\right)\mathbf{\upsilon} = \mathbf{0} \\ &\frac{\partial \mathbf{P}_\delta}{\partial t} + \left(grad\mathbf{P}_\delta\right)\mathbf{\upsilon} = -\frac{c}{r} \mathbf{P}_\delta \\ &\frac{\partial \mathbf{P}_\omega}{\partial t} + \left(grad\mathbf{P}_\omega\right)\mathbf{\upsilon} = -\frac{c}{r} \mathbf{P}_\omega \end{aligned} \tag{5.6.37}$$

From the equations of this paragraph we conclude that there are physical quantities that do not depend on the distance r. Such physical quantities are the energy-momentum pairs $\left(E_i, \mathbf{P}_i\right)$ and $\left(E_w, \mathbf{P}_w\right)$, as well as the angular momenta $\mathbf{S}$ and S_u. These quantities are defined for $r = 0$, that is, on the material particle. On the contrary, the energy-momentum pairs $\left(E_\delta, \mathbf{P}_\delta\right)$ and $\left(E_\omega, \mathbf{P}_\omega\right)$, as well as the rest energies $\frac{c\hbar}{r}\frac{\partial \Phi}{\partial \delta}$ and $\frac{c\hbar}{r\sin\delta}\frac{\partial \Phi}{\partial \omega}$, are defined only in the surrounding spacetime of the material particle, due to the appearance of the factor $\frac{1}{r}$. Furthermore, they vanish for $r \to +\infty$, while they attain large values for small values of r, i.e. close to the material particle.

5.7 The simplest case of a generalized photon

The simplest generalized photon arises in the case where the percentage Φ is constant:

$$\begin{aligned}&\frac{\partial\Phi}{\partial t}=0\\&\nabla\Phi=0\end{aligned}\tag{5.7.1}$$

In this case, equations (5.6.2) and (5.6.3) are rewritten, respectively

$$\begin{aligned}&E=\Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial m_0}{m_0\partial w}\\&\mathbf{P}=\Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial m_0}{m_0 c\partial w}\frac{\boldsymbol{\upsilon}}{c}\end{aligned}\tag{5.7.2}$$

$$\begin{aligned}&E=\Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial q}{q\partial w}\\&\mathbf{P}=\Phi\frac{i\hbar}{1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}}\frac{\partial q}{qc\partial w}\frac{\boldsymbol{\upsilon}}{c}\end{aligned}\tag{5.7.3}$$

From the second of equations (5.7.1) we obtain

$$\nabla\Phi=0$$

and from equation (5.6.4) we get

$$\frac{\partial\Phi}{\partial w}\nabla w+\frac{\partial\Phi}{\partial\delta}\nabla\delta+\frac{\partial\Phi}{\partial\omega}\nabla\omega=0$$

and from the linear independence of the vectors $\nabla w,\nabla\delta,\nabla\omega$ (paragraph 2.5) we get

$$\begin{aligned}&\frac{\partial\Phi}{\partial w}=0\\&\frac{\partial\Phi}{\partial\delta}=0\\&\frac{\partial\Phi}{\partial\omega}=0\end{aligned}\tag{5.7.4}$$

Replacing equations (5.7.4) into the equations of the last paragraph causes the energy-momentum pairs $(E_w,\mathbf{P}_w),(E_\delta,\mathbf{P}_\delta),(E_\omega,\mathbf{P}_\omega)$ to become zero, the angular momentum

$\mathbf{S}$ becomes zero, and so do the rest energies $\frac{c\hbar}{r}\frac{\partial\Phi}{\partial\delta}$ and $\frac{c\hbar}{r\sin\delta}\frac{\partial\Phi}{\partial\omega}$. The energy-momentum pair $(E_i, \mathbf{P}_i)$, as given by equations (5.7.2), does not become zero. Therefore, the generalized photon is defined for $r = 0$, i.e. on the material particle.

We shall now prove that the interaction of the material particle with every generalized photon is instantaneous during the moment w of the emission of the generalized photon. More specifically, we shall prove that the generalized photon keeps its energy E and moment $\mathbf{P}$ constant, after its emission by the material particle.

From equation (2.5.7) of the fundamental mathematical theorem, and for $f = E$, we have

$$\frac{\partial E}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla E = c\frac{\partial E}{\partial r} \qquad (5.7.5)$$

From the first of equations (5.7.2), and since it holds that $m_0 = m_0(w)$, we get

$$\frac{\partial E}{\partial r} = 0 \qquad (5.7.6)$$

and from equation (5.7.5) we see that

$$\frac{\partial E}{\partial t} + \boldsymbol{\upsilon}\cdot\nabla E = 0 \qquad (5.7.7)$$

From equation (5.7.7) we conclude that the energy E of the generalized photon remains constant during its motion with velocity $\boldsymbol{\upsilon}$, after its emission by the material particle.

Combining equations (5.7.2) we obtain relation

$$\mathbf{P} = E\frac{\boldsymbol{\upsilon}}{c^2} \qquad (5.7.8)$$

between the momentum $\mathbf{P}$ and energy E of the generalized photon.

From equation (2.5.8) for $f = \frac{E}{c}$, we obtain

$$\frac{\partial}{\partial t}\left(E\frac{\boldsymbol{\upsilon}}{c^2}\right) + \left(grd\left(E\frac{\boldsymbol{\upsilon}}{c^2}\right)\right)\boldsymbol{\upsilon} = \frac{\boldsymbol{\upsilon}}{c}\frac{\partial E}{\partial r}$$

and with equations (5.7.6) and (5.7.8) we get

$$\frac{\partial \mathbf{P}}{\partial t} + (grad\mathbf{P})\boldsymbol{\upsilon} = \mathbf{0} \qquad (5.7.9)$$

From equation (5.7.9) we conclude that the momentum $\mathbf{P}$ of the generalized photon remains constant during its motion with velocity $\boldsymbol{\upsilon}$, after its emission by the material particle.

According to equations (5.7.7) and (5.7.9), the generalized photon does not exchange energy and momentum with the material particle after its emission. The interaction between the material particle and every generalized photon takes place instantaneously at the moment of emission of the generalized photon. Furthermore, according to equation (5.7.8), there is a continuous flow of generalized photons moving with velocity $\boldsymbol{\upsilon}$, from the material particle into the surrounding spacetime, on the condition, of course, that the percentage Φ remains constant.

We can undertake a similar study for the generalized photon resulting from the selfvariation of the electric charge. It suffices to replace equations (5.7.2) with equations (5.7.3) in the above study.

5.8 The cosmological data "condensed" into a single equation

In the inertial frame of reference S', where the material particle is at rest, the first of equations (6.2.2) can be written as

$$E' = \Phi i\hbar \frac{\partial m_0}{m_0 \partial w'} + i\hbar \frac{\partial \Phi}{\partial t'} \qquad (5.8.1)$$

Summing in all directions of emission of generalized photons, and taking into consideration that $\sum E' = E_0$ and $\sum \Phi = 1$, from equation (5.8.1) we obtain

$$E_0 = i\hbar \frac{\partial m_0}{m_0 \partial w'} \qquad (5.8.2)$$

During the emission of the generalized photons by the material particle it is $r' = 0$, and equation (2.2.3) can be written as $w' = t'$, therefore we get
$\frac{\partial m_0}{\partial w'} = \frac{dm_0}{dw'} = \frac{dm_0}{dt'} = \dot{m}_0$, and equation (5.8.2) can be written as

$$E_0 = i\hbar \frac{\dot{m}_0}{m_0} \qquad (5.8.3)$$

which is equation (5.2.6).

In the inertial reference frame S', where the material particle is at rest, and for $r' = 0$, hence for $w' = t'$, the first of equations (5.4.2) can be written as

$$\dot{E}_0 = \frac{i}{\hbar} m_0 c^2 E_0 \qquad (5.8.4)$$

Eliminating the rest energy E_0, we get

$$\left(i\hbar\frac{\dot{m}_0}{m_0}\right)^{\bullet}=\frac{i}{\hbar}m_0c^2 i\hbar\frac{\dot{m}_0}{m_0}$$

$$\left(i\hbar\frac{\dot{m}_0}{m_0}\right)^{\bullet}=-\dot{m}_0c^2$$

$$\left(i\hbar\frac{\dot{m}_0}{m_0}+m_0c^2\right)^{\bullet}=0 \qquad (5.8.5)$$

which is equation (5.2.8).

In paragraph 5.2 we derived equation (5.2.8) by combining equation (5.8.3) with the principle of conservation of energy. In the derivation we conducted in this paragraph we combined equation (5.8.3) with the symmetric law (5.4.2). Furthermore, from the derivation procedure we have followed, it becomes obvious that the percentage-function Φ does not play any role in equation (5.8.5), i.e. in equation (5.2.8).

If we borrow equation (7.3.15), $E_0 = i\hbar H$, from chapter 7, and combine it with equation (5.8.3), we obtain $\frac{\dot{m}_0}{m_0}=H \sim 2\times10^{-18}s^{-1}$. In the cosmological data we observe the consequences of the real increase of the rest masses of the material particles, which takes place at an extremely slow rate.

In chapter 7 the differential equation (5.8.5) is solved. As we shall see, this equation contains as information the totality of the cosmological data. The cosmological data are "condensed" within a single equation.

5.9 The generalized particle

From the previous study it becomes evident that the selfvariations correlate every material particle with the surrounding spacetime. Fundamental physical characteristics of the material particle, like the rest mass and the electric charge, are correlated with spacetime. Furthermore, each material particle contributes to the energy content of spacetime in a strictly defined manner.

The relation between the material particle and the surrounding spacetime is determined by two fundamental physical objects predicted by the theory of selfvariations: the generalized photon and the accompanying particle. These two physical objects are related to each other since the accompanying particle results from the aggregation of the generalized photons. All the equations we have stated in the preceding paragraphs and preceding chapters, concern the relation of the material particle either with the generalized photon, or with the accompanying particle.

In the surrounding spacetime of the material particle, and for each generalized photon, we know exactly what is expressed by equation (5.2.1), $dS^2=0$: the generalized photon moves with velocity $\mathbf{\upsilon}$ of magnitude $\|\mathbf{\upsilon}\|=c$ in every inertial frame of reference. According to the second statement-axiom we have posed, equation $dS^2 = 0$ also holds for the accompanying particle, which, as an aggregation of generalized photons, is related with the propagation of the selfvariations in the four-dimensional spacetime. The question then arises, as to how equation $dS^2 = 0$ is expressed in the part of spacetime where the generalized photons aggregate.

The accompanying particle has rest energy E_0 and, therefore, rest mass $\frac{E_0}{c^2} \neq 0$. The combination $dS^2 = 0$ and $\frac{E_0}{c^2} \neq 0$ renders the accompanying particle an intermediate state between "matter" and the "photon". It is a completely new physical object predicted by the theory of selfvariations, which introduces us into an unknown territory of physical reality. The first question we have to answer is how do the relations $dS^2 = 0$ and $\frac{E_0}{c^2} \neq 0$ become compatible with each other.

About the intermediate state of matter we can give the following interpretation:

The aggregation of the generalized photons implies the co-incidence of different points ($dS^2 = 0$) in the part of spacetime where the aggregation takes place. This interpretation is in agreement with the strict application of the axioms of the theory of selfvariations.

At this point we are required to make two observations about the relation of the theory of selfvariations with the theory of relativity. These observations have to do with the relation between the energy content and the properties of spacetime.

For the derivation of the Lorentz-Einstein transformations we consider two observers who exchange signals moving with velocity c. If we consider the exchange of signals moving with a different velocity, for example acoustic signals, we end up with different transformations. Judging by the result, both on theoretical, and on experimental grounds, we know that the transformations derived by the first method are correct, whereas the transformations derived by the second method are wrong.

The theory of selfvariations predicts the generalized photon in the surrounding spacetime of the material particles. There is a continuous exchange of generalized photons between the material particles, in other words, a continuous exchange of signals moving with velocity c. The exchange of signals with velocity c is not simply a hypothesis we can make for the derivation of the Lorentz-Einstein transformations, but a continuous physical reality. Therefore, the theory of selfvariations strengthens the theoretical background of the special theory of relativity.

The general theory of relativity correlates the properties of spacetime with its energy content. The theory of selfvariations gives us the detailed contribution of each material particle to the energy content of spacetime. In the part of spacetime where the aggregation of generalized photons takes place, the material particle interacts with the accompanying particle. This interaction concerns a strictly distinct subset of the total energy content of spacetime. While we assume a unified spacetime, whose properties are defined by its total energy content, each particle interacts and is correlated with only a subset of the energy content of spacetime. In reality, every material particle occupies its "own" spacetime. For every material particle the properties of its "own" spacetime are determined by the generalized photons with which it interacts. Therefore, the co-incidence of different points of spacetime concerns the accompanying particle for every material particle, and does not constitute a general property of spacetime.

The law of selfvariations has been stated based on the accompanying particle. Relation (5.2.2), in combination with the symmetric laws (5.4.2) and (5.5.3), expresses the continuous interaction of the rest mass m_0 and the electric charge q of the material particle with the energy E_0 of the accompanying particle. Therefore, we cannot refer just to the material particle, or just to the accompanying particle. What exists in nature is the system of the two particles, which behaves as a "generalized particle" that occupies a part of spacetime.

The co-incidence of different points in the part of spacetime occupied by the generalized particle alters the trajectories and velocities of the generalized photons compared to the strictly defined trajectories and velocities we studied in the preceding chapters. In the case of co-incidence of all points belonging to this part of spacetime, the concepts of trajectory and velocity of the generalized photons loose their meaning. The trajectory and velocity of the material particle will suffer the same consequences, if the material particle belongs to the part of spacetime where the aggregation of the generalized photons takes place.

In Figures 2.2.1 and 3.2.1 imagine that, for the material particle, the points of spacetime within the interior of a sphere of centre E and radius r coincide. The physical object in the interior of the sphere constitutes a generalized particle with a specific rest mass. In every point of the spherical surface, the generalized photon moves with velocity $\mathbf{\upsilon}$ of magnitude $\|\mathbf{\upsilon}\| = c$. None of the axioms of special relativity and of the theory of selfvariations are violated. Furthermore, the co-incidence of different points of spacetime within the interior of the sphere, concerns the material particle, and does not constitute a general property of spacetime.

The investigation of the internal structure and physical properties of the generalized particle is the central issue for the theory of selfvariations. We have to answer specific questions regarding the generalized particle, and develop specific methods for the study of its physical properties.

A fundamental question concerns the distribution of the total rest mass M_0 of the generalized particle, between the material particle (m_0) and the accompanying particle $\left(\frac{E_0}{c^2}\right)$. Of equal importance is the size of the portion of spacetime occupied by the generalized particle.

A basic method for the study of the generalized particle is the elimination of the velocity, which also represents the trajectory, from the equations of the theory of selfvariations. It is not the only method, though. In the following chapter we present the basic study for the generalized particle.

CHAPTER 6

The quantum phenomena as a consequence of the selfvariations

6.1 Introduction

The intermediate state between "matter" and "photon" predicted by the theory of selfvariations, is responsible for the quantum phenomena. The study of the generalized photon leads to the Schrödinger and the Klein-Gordon equations, as well as to the wave equation of Maxwell's theory of electromagnetism.
The elimination of the kinematic characteristics of the material particle from the equations of the selfvariations, emerges as the fundamental method for the study of the generalized particle and, eventually, of quantum phenomena. This is what is actually done by all the theories developed during the last century in order to interpret quantum phenomena.
The basic method for the study of the generalized particle is complemented by the percentage-function Φ. The Φ function has to do with the generalized photon and, by extension, with the generalized particle. Furthermore, it is related with the interactions of the material particles. Function Φ inextricably links the quantum phenomena with the interactions of the material particles. The investigation of its properties furthers the theory of selfvariations beyond the bounds of the present edition.

6.2 The distribution functions of the rest mass

According to equation (5.4.4)

$$M_0 = m_0 + \frac{E_0}{c^2} \tag{6.2.1}$$

the rest mass M_0 of the generalized particle is equal to the sum of the rest masses of the material particle (m_0) and the accompanying particle $\left(\frac{E_0}{c^2}\right)$. One way of studying the inner structure of the generalized particle is to study how the rest mass M_0 is distributed to each of the two particles. Knowing the sum of the rest masses m_0 and $\frac{E_0}{c^2}$, it suffices to calculate one of the "distribution functions", that is, one of the complex numbers $X = \frac{m_0}{M_0}, \Psi = \frac{E_0}{M_0 c^2}, Z = \frac{m_0 c^2}{E_0}$.

But it is

$$X + \Psi = \frac{m_0 c^2}{M_0 c^2} + \frac{E_0}{M_0 c^2} = \frac{m_0 c^2 + E_0}{M_0 c^2}$$

and with equation (6.2.1) we get $X + \Psi = 1$. Therefore, it suffices to study either function Ψ

$$\Psi = \frac{E_0}{M_0 c^2} \tag{6.2.2}$$

or function Z

$$Z = \frac{m_0 c^2}{E_0} \tag{6.2.3}$$

in order to determine the distribution of the rest mass M_0 into m_0 and $\frac{E_0}{c^2}$.

Initially, we will study the effects of the selfvariations on the function Z. From equation (6.2.3) we have

$$\frac{\partial Z}{\partial t}=\frac{1}{E_0}\frac{\partial m_0c^2}{\partial t}-\frac{m_0c^2}{E_0^2}\frac{\partial E_0}{\partial t}$$

and with the firsts of equations (5.2.3) and (5.4.2) we get

$$\frac{\partial Z}{\partial t}=-\frac{1}{E_0}\frac{i}{\hbar}E_s m_0c^2-\frac{m_0c^2}{E_0^2}\frac{i}{\hbar}\gamma m_0c^2E_0$$

and with equation (5.3.9) we get

$$\frac{\partial Z}{\partial t}=-\frac{1}{E_0}\frac{i}{\hbar}\gamma E_0 m_0c^2-\frac{m_0c^2}{E_0^2}\frac{i}{\hbar}\gamma m_0c^2E_0$$

$$\frac{\partial Z}{\partial t}=-\frac{i}{\hbar}\frac{m_0c^2}{E_0}\gamma\left(m_0c^2+E_0\right)$$

and with equation (6.2.1) we get

$$\frac{\partial Z}{\partial t}=-\frac{i}{\hbar}\frac{m_0c^2}{E_0}\gamma M_0c^2$$

and with equation (6.2.3)

$$\frac{\partial Z}{\partial t}=-\frac{i}{\hbar}\gamma M_0c^2Z \qquad (6.2.4)$$

From equation (6.2.3) we obtain

$$\nabla Z=\frac{1}{E_0}\nabla m_0c^2-\frac{m_0c^2}{E_0^2}\nabla E_0$$

and with the second of equations (5.2.3) and also (5.2.4) we get

$$\nabla Z=\frac{1}{E_0}\frac{i}{\hbar}\mathbf{P}_s m_0c^2+\frac{m_0c^2}{E_0^2}\frac{i}{\hbar}\gamma m_0\mathbf{u}E_0$$

and with equation (5.3.6) we have

$$\nabla Z=\frac{1}{E_0}\frac{i}{\hbar}E_s\frac{\mathbf{u}}{c^2}m_0c^2+\frac{m_0c^2}{E_0^2}\frac{i}{\hbar}\gamma m_0\mathbf{u}E_0$$

Using equation (5.3.9) we get

$$\nabla Z=\frac{1}{E_0}\frac{i}{\hbar}\gamma E_0\frac{\mathbf{u}}{c^2}m_0c^2+\frac{m_0c^2}{E_0^2}\frac{i}{\hbar}\gamma m_0\mathbf{u}E_0$$

$$\nabla Z=\frac{i}{\hbar}\frac{m_0c^2}{E_0^2}\gamma\left(\frac{E_0}{c^2}+m_0\right)\mathbf{u}$$

Through equation (6.2.1) we get

$$\nabla Z=\frac{i}{\hbar}\frac{m_0c^2}{E_0^2}\gamma M_0\mathbf{u}$$

and with equation (6.2.3) we get

$$\nabla Z=\frac{i}{\hbar}\gamma M_0\mathbf{u}Z \qquad (6.2.5)$$

The differential equations (6.2.4) and (6.2.5) offer the advantage that the rest mass M_0 that appears on their second part, does not depend on the selfvariations. On the other hand, they also have a disadvantage. We do not know the additional conditions we have to introduce for the rest mass M_0 in order to solve the system of differential equations (6.2.4) and (6.25). These additional conditions are related to a more general investigation of the equations of the theory of selfvariations, which is not included in the present edition.

We shall now study how the selfvariations affect function Ψ. From equation (6.2.2) we have

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\left(\frac{E_0}{M_0 c^2}\right)$$

and with equation (5.4.5) we obtain

$$\frac{\partial \Psi}{\partial t} = \frac{1}{M_0 c^2}\frac{\partial E_0}{\partial t}$$

and with the first of equations (5.4.2) we get

$$\frac{\partial \Psi}{\partial t} = \frac{1}{M_0 c^2}\frac{i}{\hbar}\gamma m_0 c^2 E_0$$

and from equation (6.2.2) we get

$$\frac{\partial \Psi}{\partial t} = \frac{i}{\hbar}\gamma m_0 c^2 \Psi \qquad (6.2.6)$$

From equation (6.2.2) we have

$$\nabla \Psi = \nabla\left(\frac{E_0}{M_0 c^2}\right)$$

and with equation (5.4.6) we obtain

$$\nabla \Psi = \frac{1}{M_0 c^2}\nabla\left(E_0\right)$$

and using the second of equations (5.4.2) we get

$$\nabla \Psi = \frac{1}{M_0 c^2}\left(-\frac{i}{\hbar}\gamma m_0 \mathbf{u} E_0\right)$$

and with equation (6.2.2) we arrive at

$$\nabla \Psi = -\frac{i}{\hbar}\gamma m_0 \mathbf{u} \Psi \qquad (6.2.7)$$

The differential equations (6.2.6) and (6.2.7) have the advantage that the rest mass m_0 of the material particle appears in their second part. This fact allows us to introduce additional conditions in order to solve the system of differential equations (6.2.6) and (6.2.7). We present this study in the following two paragraphs.

The distribution functions determine the distribution of the rest mass of the generalized particle between the material particle and the accompanying particle. For every point $A(x, y, z, t)$ in the part of spacetime where the generalized particle can reside, these distribution functions acquire specific values. These values, in turn, define the values of the rest masses m_0 and $\frac{E_0}{c^2}$.

The behavior of the generalized particle can be influenced by any cause that interacts with the generalized particle in the part of spacetime it occupies. An external cause can redistribute the rest mass of the generalized particle, directing it either to the material particle, or to the accompanying particle. In the first case, the generalized particle will behave as a material particle with a well-defined trajectory, energy, etc.

In the second case, the generalized particle will spread out in spacetime, while the consequences resulting from the aggregation of the generalized photons will be strengthened and intensified. We observe such a case in the double-slit experiment for the electron and for material particles in general (we assume that the reader is familiar with the double-slit experiment).
The study of the distribution functions is a fundamental goal in order to understand the behavior of the generalized particle.

6.3 The Schrödinger equation

From equation (6.2.6) we have

$$\frac{\partial^2\Psi}{\partial t^2}=\frac{i}{\hbar}\gamma m_0c^2\frac{\partial\Psi}{\partial t}+\frac{i}{\hbar}\gamma c^2\Psi\frac{\partial m_0}{\partial t}$$

and with equation (6.2.6) and the first of equations (5.2.3), we get

$$\frac{\partial^2\Psi}{\partial t^2}=-\frac{\gamma^2m_0^2c^4}{\hbar^2}\Psi+\frac{i}{\hbar}\gamma c^2\Psi\left(-\frac{i}{\hbar}E_s m_0\right)$$

and with equation (5.3.9) we get

$$\frac{\partial^2\Psi}{\partial t^2}=-\frac{\gamma^2m_0^2c^4}{\hbar^2}\Psi+\frac{i}{\hbar}\gamma c^2\Psi\left(-\frac{i}{\hbar}\gamma E_0 m_0\right)$$

$$\frac{\partial^2\Psi}{\partial t^2}=-\frac{\gamma^2m_0^2c^4}{\hbar^2}\Psi+\frac{\gamma^2m_0c^2E_0}{\hbar^2}\Psi$$

$$\frac{\partial^2\Psi}{\partial t^2}=-\frac{\gamma^2m_0c^4}{\hbar^2}\left(m_0-\frac{E_0}{c^2}\right)\Psi \qquad (6.3.1)$$

From equation (6.2.7) we have

$$\nabla^2\Psi=-\frac{i}{\hbar}\gamma m_0\mathbf{u}\nabla\Psi-\frac{i}{\hbar}\gamma\Psi\mathbf{u}\nabla m_0$$

and with equation (6.2.7) together with the second of equations (5.2.3), we get

$$\nabla^2\Psi=-\frac{\gamma^2m_0^2u^2}{\hbar^2}\Psi-\frac{i}{\hbar}\gamma\Psi\mathbf{u}\left(\frac{i}{\hbar}E_s\frac{\mathbf{u}}{c^2}m_0\right)$$

and with equation (5.3.9) we get

$$\nabla^2\Psi=-\frac{\gamma^2m_0^2u^2}{\hbar^2}\Psi-\frac{i}{\hbar}\gamma\Psi\mathbf{u}\left(\frac{i}{\hbar}\gamma E_0\frac{\mathbf{u}}{c^2}m_0\right)$$

$$\nabla^2\Psi=-\frac{\gamma^2m_0^2u^2}{\hbar^2}\Psi+\frac{\gamma^2m_0E_0u^2}{c^2\hbar^2}\Psi$$

$$\nabla^2\Psi=-\frac{\gamma^2m_0u^2}{\hbar^2}\left(m_0-\frac{E_0}{c^2}\right)\Psi \qquad (6.3.2)$$

We now consider the case where the rest mass M_0 is mainly distributed to the material particle. This happens when $\left\|\frac{E_0}{m_0c^2}\right\|<<1$ or when $E_0\to 0$. Under these conditions equation (6.3.2) can be written as

$$\nabla^2\Psi=-\frac{\gamma^2m_0^2u^2}{\hbar^2}\Psi \qquad (6.3.3)$$

We will now eliminate the velocity u from equation (6.3.3), within the framework of the analysis we performed in paragraph 5.9 for the generalized particle. For small velocities u, it is $\gamma \sim 1$, and equation (6.3.3) can be written as

$$\nabla^2\Psi = -\frac{m_0^2 u^2}{\hbar^2}\Psi \tag{6.3.4}$$

Furthermore, denoting by ε the constant sum of the kinetic energy $\frac{1}{2}m_0u^2$ and the potential energy $U = U(x, y, z)$ of the material particle, we have

$$\frac{1}{2}m_0u^2 + U = \varepsilon$$

$$u^2 = \frac{2(\varepsilon - U)}{m_0}$$

Replacing factor u^2 into equation (6.3.4) we obtain

$$\nabla^2\Psi = -\frac{2m_0(\varepsilon - U)}{\hbar^2}\Psi \tag{6.3.5}$$

which is the time-independent Schrödinger wave-function.

From the initial conditions, $\left\|\frac{E_0}{m_0c^2}\right\| << 1$ or $E_0 \to 0$, we set, and from equation (6.2.1) we obtain $m_0 \to M_0$, therefore equation (6.3.5) can be written in the form

$$\nabla^2\Psi = -\frac{2M_0(\varepsilon - U)}{\hbar^2}\Psi \tag{6.3.6}$$

From the derivation process we have followed it becomes obvious that the Schrödinger equation only approximately describes the internal structure of the generalized particle.

6.4 The Klein-Gordon equation

The way in which we chose to eliminate the velocity from equation (6.3.3) had as a consequence the appearance of the potential energy U in Schrödinger's equation (6.3.5). We will now eliminate the velocity u from function Ψ in a different manner. Combining equations (6.3.1) and (6.3.2), we obtain

$$\frac{\partial^2\Psi}{\partial t^2} - c^2\nabla^2\Psi = -\frac{\gamma^2 m_0 c^4}{\hbar^2}\left(m_0 - \frac{E_0}{c^2}\right)\Psi + \frac{\gamma^2 m_0 c^2 u^2}{\hbar^2}\left(m_0 - \frac{E_0}{c^2}\right)\Psi$$

$$\frac{\partial^2\Psi}{\partial t^2} - c^2\nabla^2\Psi = -\frac{\gamma^2 m_0 c^4}{\hbar^2}\left(1 - \frac{u^2}{c^2}\right)\left(m_0 - \frac{E_0}{c^2}\right)\Psi$$

and since $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$, we get

$$\frac{\partial^2\Psi}{\partial t^2} - c^2\nabla^2\Psi = -\frac{m_0 c^4}{\hbar^2}\left(m_0 - \frac{E_0}{c^2}\right)\Psi$$

$$\frac{\partial^2\Psi}{\partial t^2} - c^2\nabla^2\Psi + \frac{m_0 c^4}{\hbar^2}\left(m_0 - \frac{E_0}{c^2}\right)\Psi = 0 \tag{6.4.1}$$

In the case where $\left\| \frac{E_0}{m_0 c^2} \right\| << 1$ or $E_0 \to 0$, equation (6.4.1) can be written as

$$\frac{\partial^2 \Psi}{\partial t^2} - c^2 \nabla^2 \Psi + \frac{m_0 c^4}{\hbar^2} \Psi = 0 \qquad (6.4.2)$$

which is the Klein-Gordon equation. With the conditions we posed, it follows that $m_0 \to M_0$ in equation (6.4.2).

Of particular interest is the case $m_0 = 0$, where from equation (6.4.1) we obtain

$$\frac{\partial^2 \Psi}{\partial t^2} - c^2 \nabla^2 \Psi = 0$$

$$\nabla^2 \Psi - \frac{\partial^2 \Psi}{c^2 \partial t^2} = 0 \qquad (6.4.3)$$

From equation (6.2.1) for $m_0 = 0$ we get $E_0 = M_0 c^2$. Therefore, all of the rest energy of the generalized particle has shifted to the accompanying particle. Furthermore, we get $\|\Psi\| = \left\| \frac{E_0}{M_0 c^2} \right\| = 1$. In every case we solve the differential equation (6.4.3), we should modify the final solution such that the wave-like behavior of a scalar quantity Ψ appears, for which we demand that $\|\Psi\| = 1$.

6.5 The central role of the percentage function Φ in the internal structure and the physical properties of the generalized particle.

According to equations (5.6.5) και (5.6.6) the energy E and the momentum $\mathbf{P}$ of a single generalized photon depends on the percentage function Φ. Furthermore, according to equation (5.6.33), the intrinsic angular momentum $\mathbf{S}$ of a single generalized particle depends exclusively on the percentage function Φ. The generalized particle emerges in the part of spacetime where the aggregation of the generalized photons takes place. Therefore, the percentage function Φ plays a fundamental role, both for the internal structure, as well as for the physical properties of the generalized particle.

Function Φ allows the comprehension of the extent of the portion of spacetime occupied by the generalized particle. In paragraph 5.6 we determined the physical quantities that can only be defined in the surrounding spacetime of the material particle. These physical quantities are inversely proportional to the distance r. Therefore, the space occupied by the generalized photon can extend to infinity, with the consequences, of course, predicted by the corresponding equations for its energy, momentum, and angular momentum. Since each generalized photon can extend to infinity, the same also holds for the part of space where the aggregation of the generalized photons takes place. Therefore, the generalized particle can extend to infinity.

In the case of the simplest generalized photon, as we studied it in paragraph 5.7, there results an instantaneous interaction of the material particle with the accompanying particle. This interaction takes place at the instant of emission of the generalized photon, exactly at the point where the material point particle resides. Therefore, in this case the generalized particle is a point particle.

In conclusion, we can say that the generalized particle can extend from a point of spacetime up to an infinite distance from the material particle. Furthermore, in each

case, the extent of the part of spacetime in which the generalized particle extends, is determined by the percentage function Φ.
For the derivation of the Schrödinger and the Klein-Gordon equations, we based our investigation on equation (6.2.1), $M_0 = m_0 + \frac{E_0}{c^2}$. A fundamental piece of information, related with the function Φ, is missing from this equation. The generalized photon carries rest energy, according to equations (5.6.17) and (5.6.18), which depends on the function Φ and the distance r. In other words, right from the start, the generalized photon, and therefore the generalized particle, are correlated with a rest energy in the surrounding spacetime of the material particle. The rest mass corresponding to this rest energy does not appear in equation (6.2.1). For the same reason, the angular momentum does not appear in the Schrödinger and the Klein-Gordon equations, since the internal angular momentum of the generalized photon depends exclusively on function Φ, according to equation (5.6.33).
Function Φ expresses the potential of a material particle to emit generalized photons of different energies for different directions. Theoretically, we cannot predict exactly how function Φ depends on the internal structure of the material particle. Quite likely we can do this by performing some measurements. But we can predict theoretically an important factor on which function Φ depends, that results from the continuous exchange of generalized photons between material particles. This exchange of generalized photons is equivalent to a variation of function Φ. According to equations (5.6.5), (5.6.6) and (5.6.33), the energy, momentum and intrinsic angular momentum of the generalized photon are exactly correlated with the variation of function Φ. We, therefore, come to the conclusion that the quantum phenomena are interrelated with the interactions of the material particles, the connecting link being function Φ. Function Φ is related with the interactions between material particles, but also with the energy of the generalized photons and, by extension, with the generalized particle.
In paragraph 5.9 we referred to the fundamental method for studying the generalized particle. We analyzed the reasons for which we have to expunge the velocity from the equations of the theory of selfvariations in order to study the internal structure and the physical properties of the generalized particle. Of equal importance is the inclusion of function Φ in the study of the generalized particle.
Observing the Schrödinger operators, as used in quantum mechanics, we realize that the first consequence of their use is the elimination of the kinematic characteristics of the material particle from the resulting differential equations. Function Φ does not appear in the final equations, since it does not exist as a concept within the physical theories of the last century. It is represented, though, by the physical quantities related with the interactions in which the material particle participates, by the potential energy or the generalized momentum of the material particle. Analogous is the procedure followed by Dirac for the derivation of his eponymous equation.
One of the questions about the generalized particle, to which we deliberately did not refer in paragraph 5.9, is the probability of finding the material particle at a specific moment, in a specific position in the part of spacetime occupied by the generalized particle. There are many physical quantities related with the Schrödinger operators. Judging by the success of quantum mechanics, one way to study the generalized particle is through statistical interpretation. We must not forget, though, that a single cause suffices in order to shift the rest energy of the generalized particle, either towards the material particle, or towards the accompanying particle. One and only

cause is sufficient for the corpuscular or wave-like behavior of the generalized particle to emerge.
By investigating the properties of function Φ or by making concrete hypotheses regarding function Φ, we can extend our study of quantum phenomena and the interactions of particles. On the contrary, in paragraph 5.8 we showed that equation (5.8.5) does not depend on function Φ. This allows us to solve it and investigate it completely. We present that study in the next chapter.

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CHAPTER 7

The cosmological data as a consequence of the selfvariations

7.1 Introduction

The origin of matter is already recorded in the cosmological observational data. We just lacked a fundamental piece of information in order to decode it: the law of selfvariations.

The redshift of distant astronomical objects, the cosmic microwave background radiation and the information obtained by the analysis of this radiation, the increased luminosity distances of supernovae, the large-scale, as well as small-scale, structure of matter in the universe, the large-scale isotropy and flatness of the universe, the slight variation of the fine structure constant, and the arrow of time, all share the law of selfvariations as a common cause.

The law of selfvariations contains as information the entire corpus of the cosmological observational data, as we observe and record them since the time of Hubble. Behind the barrage of interventions made in order to bring the Standard Cosmological Model in agreement with the cosmological observational data, lies our ignorance about the fundamental law of selfvariations. The physical theories of the past century do not possess the necessary completeness in order to explain the cosmological observational data.

The improved scientific observation instruments we possess record persistently, and with ever increasing detail, the consequences of the law of selfvariations.

7.2 The fundamental equations

The cosmological data concern the observation of the Universe at long distances, that is, in the past. At a distant astronomical object, located at a distance r from Earth, the rest mass $m_0(r)$ of a material particle in the past is smaller, compared to the laboratory rest mass m_0 of the same material particle we measure "now" on Earth. The electric charge $q(r)$ also differs from the laboratory value q of the electric charge as measured "now" on Earth. We calculate the quantity $m_0(r)$ as a function of m_0, and $q(r)$ as a function of q. In this manner, we incorporate into our equations the consequences resulting from the internality of the Universe to the process of measurement.

In the following, and using the known physical laws, we determine the consequences of the selfvariations for distant astronomical objects. Furthermore, we can determine the consequences of the selfvariations in the electromagnetic spectra of the astronomical objects we receive "now" on Earth. We shall prove that equation (5.2.8)

$$\left(m_0c^2 + i\hbar\frac{\dot{m}_0}{m_0}\right)^{\bullet} = 0 \qquad (7.2.1)$$

which holds for every material particle contains as information the entirety of the cosmological data.

We will solve equation (7.2.1) for a material particle in the case of a flat and static universe. This equation contains as information the redshift of distant astronomical objects. Furthermore, it predicts that the gravitational interaction cannot play the role attributed to it by the Standard Cosmological Model. It informs us that the gravitational interaction cannot lead the Universe either to collapse or to expansion.

Consequently, there is no point of solving equation (7.2.1) within an expanding Universe.
Equation (7.2.1) contains as information the fact that the total energy of the Universe is zero. Therefore, after solving the equation, it can be verified a posteriori that the Universe is flat.
From equation (7.2.1) we have that

$$\left(m_0 c^2 + i\hbar \frac{\dot{m}_0}{m_0}\right)^{\bullet} = 0$$

$$\left(\frac{i}{\hbar} m_0 c^2 - \frac{\dot{m}_0}{m_0}\right)^{\bullet} = 0$$

$$\frac{i}{\hbar} m_0 c^2 - \frac{\dot{m}_0}{m_0} = k \qquad (7.2.2)$$

Here, k is the constant of integration. From equation (7.2.2) we see that

$$m_0 = -\frac{ik\hbar}{c^2} \frac{1}{1 - e^{kt+\mu}} \qquad (7.2.3)$$

Here, μ is the constant of integration.
Let us suppose that we observe "now" on Earth, the electromagnetic spectrum of an astronomical object located at a distance r away from Earth. The emission of the electromagnetic spectrum from the astronomical object took place before a time interval $\Delta t = \frac{r}{c}$. According to equation (7.2.3) the rest mass $m_0(r)$ of the material particle at the moment of the emission of the corresponding electromagnetic spectrum was

$$m_0 = -\frac{ik\hbar}{c^2} \frac{1}{1 - e^{k\left(t - \frac{r}{c}\right)+\mu}} \qquad (7.2.4)$$

Combining equations (7.2.3) and (7.2.4) we have that

$$m_0(r) = m_0 \frac{1 - e^{kt+\mu}}{1 - e^{k\left(t - \frac{r}{c}\right)+\mu}}$$

Setting

$$A = e^{kt+\mu} \qquad (7.2.5)$$

we obtain

$$m_0(r) = m_0 \frac{1 - A}{1 - Ae^{-\frac{kr}{c}}} \qquad (7.2.6)$$

Equation (7.2.6) expresses the rest mass $m_0(r)$ of the material particle in the distant astronomical object and before a time interval $\Delta t = \frac{r}{c}$, compared with the laboratory value of the rest mass m_0 of the same material particle. In this way we include in the equations we state the consequences of the internality of the Universe with respect to the measurement process, as set forth in paragraph 4.9.
If we remove the imaginary unit i from equation (7.2.1), or replace it by any arbitrary constant $b \neq 0$, we will again end up with equations (7.2.5) and (7.2.6). The problems caused by the internality of the Universe with respect to the measurement

procedure can only be evaded through equation (7.2.6). Only after comparing the rest masses $m_0(r)$ and m_0 can we measure the consequences of the selfvariations.
From equation (7.2.5) we obtain for the parameter A

$$\frac{dA}{dt}=\dot{A}=kA \qquad (7.2.7)$$

From equation (7.2.3) we also obtain

$$\dot{m}_0=m_0\frac{ke^{kt+\mu}}{1-e^{kt+\mu}}$$

Through equation (7.2.5) we see that

$$\dot{m}_0=m_0\frac{kA}{1-A} \qquad (7.2.8)$$

Combining equations (5.2.6) and (7.2.8) we obtain

$$E_0=i\hbar\frac{kA}{1-A} \qquad (7.2.9)$$

In the case of the electric charge the corresponding equation to equation (7.2.1) is the second of equations (5.5.5)

$$\left(q+\frac{i\hbar}{V_0}\frac{\dot{q}}{q}\right)^{\cdot}=0 \qquad (7.2.10)$$

This gives us the corresponding solution

$$q(r)=q\frac{1-B}{1-Be^{-\frac{k_1r}{c}}} \qquad (7.2.11)$$

$$B=e^{k_1t+\mu_1} \qquad (7.2.12)$$

$$\frac{dB}{dt}=\dot{B}=k_1B \qquad (7.2.13)$$

Here, k_1 and μ_1 are the constants of integration.
The corresponding equation to equation (7.2.8) is equation

$$\dot{q}=q\frac{k_1B}{1-B} \qquad (7.2.14)$$

Combining the first of equations (5.5.5) with equation (7.2.14) we obtain

$$q_iV_0=i\hbar\frac{k_1B}{1-B} \qquad (7.2.15)$$

This equation is the corresponding equation to equation (7.2.9).
If we remove from equation (7.2.10) the imaginary unit i, or if we replace it by any arbitrary constant $b\neq 0$, we will still arrive at equations (7.2.11) and (7.2.12). The problems caused by the internality of the Universe with respect to the measurement procedure can only be evaded through equation (7.2.11). We can only measure the consequences of the selfvariations by comparing the electric charges $q(r)$ and q.

7.3 The redshift of the far distant astronomical objects

The wavelength λ of the linear spectrum of an atom is inversely proportional to the factor m_0q^4, where m_0 is the rest mass and q the electric charge of the electron. We denote by λ the wavelength of the linear spectrum we observe "now" on Earth, and which originates from the atoms of an astronomical object located at distance r. With λ_0 we denote the wavelength of the same kind of atom as measured in the laboratory "now" on Earth.

We have that

$$\frac{\lambda}{\lambda_0}=\frac{m_0 q^4}{m_0(r)q^4\left(r\right)}$$

Using equations (7.2.6) and (7.2.11) we obtain

$$\frac{\lambda}{\lambda_0}=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}\left(\frac{1-Be^{-\frac{k_1 r}{c}}}{1-B}\right)^4 \tag{7.3.1}$$

For the redshift z of the astronomical object we obtain

$$z=\frac{\lambda-\lambda_0}{\lambda_0}$$

$$z=\frac{\lambda}{\lambda_0}-1$$

Using equation (7.3.1) we see that

$$z=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}\left(\frac{1-Be^{-\frac{k_1 r}{c}}}{1-B}\right)^4-1 \tag{7.3.2}$$

This equation constitutes the full mathematical expression for the redshift z of the linear spectrum of distant astronomical objects.

We shall now perform an approximation. From the cosmological data we know that the fine structure constant

$$\alpha=\frac{e^2}{4\pi\varepsilon_0 c\hbar}$$

remains constant for observations we make at very large distances from Earth. Therefore, the value of the electric charge $q\left(r\right)$ differs minimally from the laboratory value q in the region of the Universe we observe. Therefore, we can write equation (7.3.2) in a simpler form, that is

$$z=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}-1 \tag{7.3.3}$$

Here, we used the approximation $q\left(r\right)=q$.

Equation (7.3.3) holds for the regions of the Universe that can be surveyed by the scientific observation instruments we currently have at our disposal. We shall return to the issue of the fine structure constant in another paragraph.

From equation (7.2.5) we see that

$$A>0 \tag{7.3.4}$$

According to equation (7.3.3), and since the value of the redshift z increases with the distance r , it holds that

$$k>0 \tag{7.3.5}$$

From equation (7.3.3), and for $r\to+\infty$, we obtain

$$z_{\max}=\frac{1}{1-A}-1$$

$$z_{\max}=\frac{A}{1-A} \tag{7.3.6}$$

We have that $z_{\max}>0, A>0$, as given in relation (7.3.4), thus we get

$1 - A > 0$

$A < 1$ (7.3.7)

Now, it holds that

$z < z_{max}$

Using equation (7.3.6) we obtain

$$z < \frac{A}{1-A}$$

Due to relation (7.3.7) we obtain

$z(1-A) < A$

$z - zA < A$

$z < (1+z)A$

$$\frac{z}{1+z} < A$$

Through relation (7.3.7) we finally arrive at

$$\frac{z}{1+z} < A < 1 \qquad (7.3.8)$$

This inequality holds for every redshift z, and allows us to estimate the range of values the parameter A acquires.

From equation (7.3.3), and for $r = 0$, we obtain $z = 0$, thus

$$z' = \frac{dz}{dr} = \frac{1 - Ae^{-\frac{kr}{c}}}{1-A} \frac{kAe^{-\frac{kr}{c}}}{c(1-A)}$$

For $r = 0$ we get

$$z'(0) = \left.\frac{dz}{dr}\right]_{r=0} = \frac{kA}{c(1-A)}$$

We expand equation (7.3.3) giving $z = z(r)$ into a Taylor series, and only to first order terms

$z(r) = z(0) + z'(0)$

$$z(r) = 0 + \frac{kA}{c(1-A)} r$$

$$cz = \frac{kA}{1-A} r$$

Comparing with Hubble's law $cz = Hr$, we obtain

$$\frac{kA}{1-A} = H \qquad (7.3.9)$$

where H is the Hubble parameter.

From equation (7.3.9) we obtain $k = H\frac{1-A}{A}$. The range of values of parameter A, as determined from inequality (7.3.8), allows us to estimate the extremely small value of the constant k. Now, according to equation (7.2.7), the parameter A increases at an extremely slow rate, and remains practically constant in the measurements we conduct.

For the energy E, which results during nuclear fission, nuclear fusion, and more generally, every case where the conversion of rest mass into energy takes place, we obtain

$$\frac{E(r)}{E}=\frac{m_0(r)c^2}{m_0c^2}$$

Using equation (7.2.6) we see that

$$\frac{E(r)}{E}=\frac{1-A}{1-Ae^{-\frac{kr}{c}}}$$

$$E(r)=E\frac{1-A}{1-Ae^{-\frac{kr}{c}}} \tag{7.3.10}$$

For the photons which result from the conversion of mass into energy we have

$$\frac{\lambda_\gamma}{\lambda_{0\gamma}}=\frac{\frac{ch}{E(r)}}{\frac{ch}{E}}=\frac{E}{E(r)}$$

Using equation (7.3.10) we obtain

$$\frac{\lambda_\gamma}{\lambda_{0\gamma}}=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}$$

$$\frac{\lambda_\gamma-\lambda_{0\gamma}}{\lambda_{0\gamma}}=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}-1$$

$$z_\gamma=\frac{1-Ae^{-\frac{kr}{c}}}{1-A}-1 \tag{7.3.11}$$

Equations (7.3.11) and (7.3.3) are identical. However, beyond the limits reached by our current observations, the redshift z of the linear spectrum is given in general by equation (7.3.2).

From equation (7.3.3) we obtain

$$1+z=\frac{1-Ae^{-\frac{kr}{c}}}{1-A} \tag{7.3.12}$$

Combining equations (7.2.6) and (7.3.12) we have that

$$m_0(z)=\frac{m_0}{1+z} \tag{7.3.13}$$

Combining equations (7.3.10) and (7.3.12) we see that

$$E(z)=\frac{E}{1+z} \tag{7.3.14}$$

Combining equations (7.2.9) and (7.3.9) we obtain

$$E_0=i\hbar H \tag{7.3.15}$$

for the laboratory value of the energy E_0.

7.4 The graphs of the functions $r=r(z)$ and $R=R(z)$

From equation (7.3.3) we have that

$$z = \frac{1 - Ae^{-\frac{kr}{c}}}{1 - A} - 1$$

$$z = \frac{A}{1 - A} e^{-\frac{kr}{c}}$$

Solving for r we obtain

$$r = \frac{c}{k} \ln\left(\frac{A}{A - z(1 - A)} \right) \qquad (7.4.1)$$

This equation gives the distance r of the astronomical object as a function of the redshift z.

From equation (7.3.9) we obtain $k = H\frac{1 - A}{A}$, and after replacing the constant k into equation (7.4.1), we get

$$r = \frac{c}{H} \frac{A}{1 - A} \ln\left(\frac{A}{A - z(1 - A)} \right) \qquad (7.4.2)$$

This equation is more convenient than equation (7.4.1), since we know the value of the Hubble parameter H, as well as the range of values of the parameter A from inequality (7.3.8), that is

$$\frac{z}{1 + z} < A < 1$$

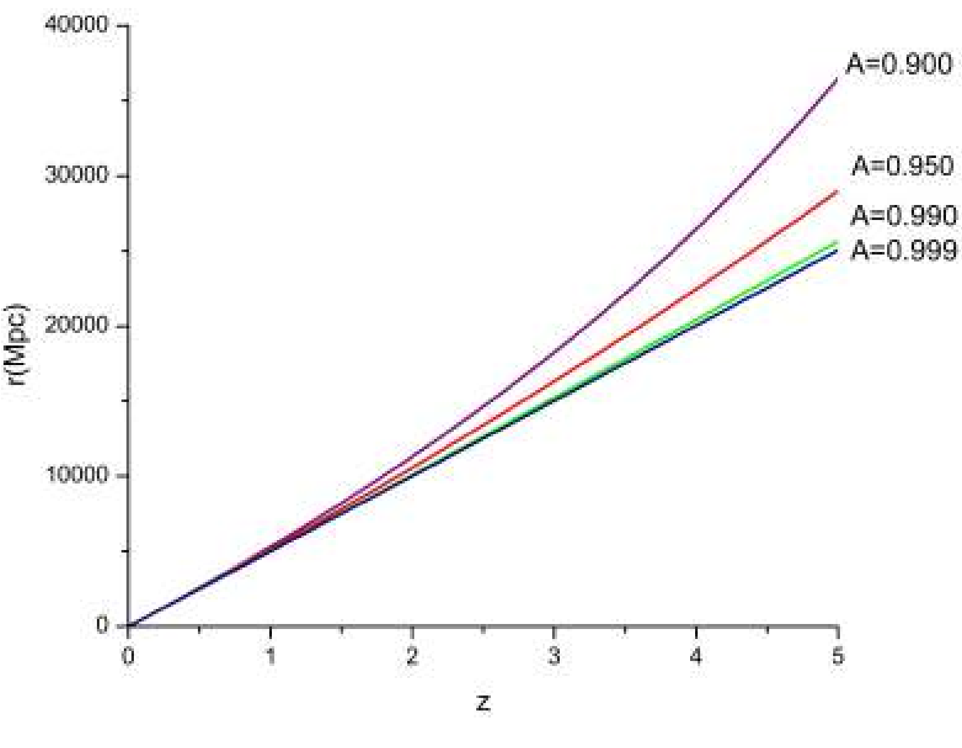

Diagram 7.4.1 The graph of distance r of a distant astronomical object as a function of the redshift z. As we increase the value of the parameter A from 0.900 to 0.999,

the curve $r = r(z)$ approaches a straight line. The graph has been made with $H = 60\frac{km}{sMpc}$ as the value of Hubble's constant.

In Figure 7.4.1 we present the graph of the curve $r = r(z)$ for $H = 60\frac{km}{sMpc}$, and for the values of $A = 0.900, A = 0.950, A = 0.990, A = 0.999$ up to $z = 5$. We observe that as the value of the parameter A increases, the curve tends to be a straight line.

We shall now prove that for $A \to 1^-$ the equivalent equations (7.3.3) and (7.4.2) tend to Hubble's law

$$cz = Hr \qquad (7.4.3)$$

From equation (7.3.9) we have $k = \frac{1-A}{A}H$, and after substituting into equation (7.3.3), we obtain

$$z = \frac{1 - Ae^{-\frac{1-A}{A}\frac{H}{c}r}}{1-A} - 1$$

We denote by $x = \frac{1-A}{A}$, therefore $x \to 0^+$ for $A \to 1^-$, and $A = \frac{1}{x+1}$, so we have

$$z = \frac{1 - \frac{1}{x+1}e^{-x\frac{Hr}{c}}}{1 - \frac{1}{x+1}} - 1 = \frac{x + 1 - e^{-x\frac{Hr}{c}}}{x} - 1$$

$$\lim_{A \to 1^-}(z) = \lim_{x \to 0^+}(z) = \lim_{x \to 0^+}\left(\frac{x + 1 - e^{-x\frac{Hr}{c}}}{x} - 1\right) \overset{\frac{0}{0}}{=} \lim_{x \to 0^+}\left(1 + \frac{Hr}{c}e^{-x\frac{Hr}{c}} - 1\right) = \frac{Hr}{c}$$

Equation (7.4.2) gives the distance r of the astronomical object, when we know the value of its redshift z. On the other hand, if we measure the distance based on the luminosity of the astronomical object, we shall always find it to be larger than the one given by equation (7.4.2). The reason is simple: The energy feeding the radiation of the astronomical objects originates from nuclear fusion, and more generally, from the conversion of rest mass into energy. According to equation (7.3.10), this energy $E(r)$ is less than the expected energy E. Therefore, the luminosity of the astronomical object is itself lower than the standard luminosity we use.

The luminosity distance R of an astronomical object is defined by equation

$$J = \frac{1}{4\pi R^2}\frac{dE}{dt} \qquad (7.4.4)$$

In this equation, J denotes the power per unit surface we receive from the astronomical object, whereas the power $\frac{dE}{dt}$ refers to the "standard candle" we are using.

If the real distance of the astronomical object is r, then we obtain for the power per unit surface J

$$J = \frac{1}{4\pi r^2}\frac{dE(r)}{dt} \tag{7.4.5}$$

From equations (7.4.4) and (7.4.5) we get

$$\frac{1}{R^2}\frac{dE}{dt} = \frac{1}{r^2}\frac{dE(r)}{dt}$$

Using equation (7.3.14) we have that

$$\frac{1}{R^2}\frac{dE}{dt} = \frac{1}{r^2}\frac{1}{1+z}\frac{dE}{dt}$$

$$R^2 = r^2(1+z)$$

$$R = r\sqrt{1+z} \tag{7.4.6}$$

Combining equations (7.4.6) and (7.4.2) we see that

$$R = \frac{c}{H}\frac{A}{1-A}\sqrt{1+z}\ln\left(\frac{A}{A-z(1-A)}\right) \tag{7.4.7}$$

The measurements conducted for the determination of Hubble's constant H, have not taken into account the consequences of equation (7.4.6). Even for the case of small values of the redshift z it holds that $R > r$. The measurement of Hubble's parameter H with the use of the luminosity distances of astronomical objects is correct only for very small values of z, where it holds that $R \sim r$. Such measurements result in a value of $H = 60\frac{km}{sMpc}$. Measurements performed have included astronomical objects with large values of the redshift z, thus increasing the value of the parameter H to values ranging between 72 and $74\frac{km}{sMpc}$.

Today, we perform measurements with high accuracy. Taking into consideration the consequences of equation (7.4.6) we expect the parameter H to be measured close to $60\frac{km}{sMpc}$, independently of the redshift z of the astronomical object. We, of course, refer to measurements of the parameter H, on the basis of the luminosity distances of astronomical objects.

Equally well to equation (7.4.7) we can also use the equation which results after combining equations (7.4.6) and (7.4.3), that is

$$R = \frac{c}{H}z\sqrt{1+z} \tag{7.4.8}$$

For $H = 60\frac{km}{sMpc}$ and $c = 3\times10^5\frac{km}{s}$ this can be written as

$$R = 5000z\sqrt{1+z} \tag{7.4.9}$$

The luminosity distance R is given in Mpc. In the graph 7.4.2 we present the graph of the function $R = R(z)$, as given in equation (7.4.9) and up to values of the redshift $z = 1.5$.

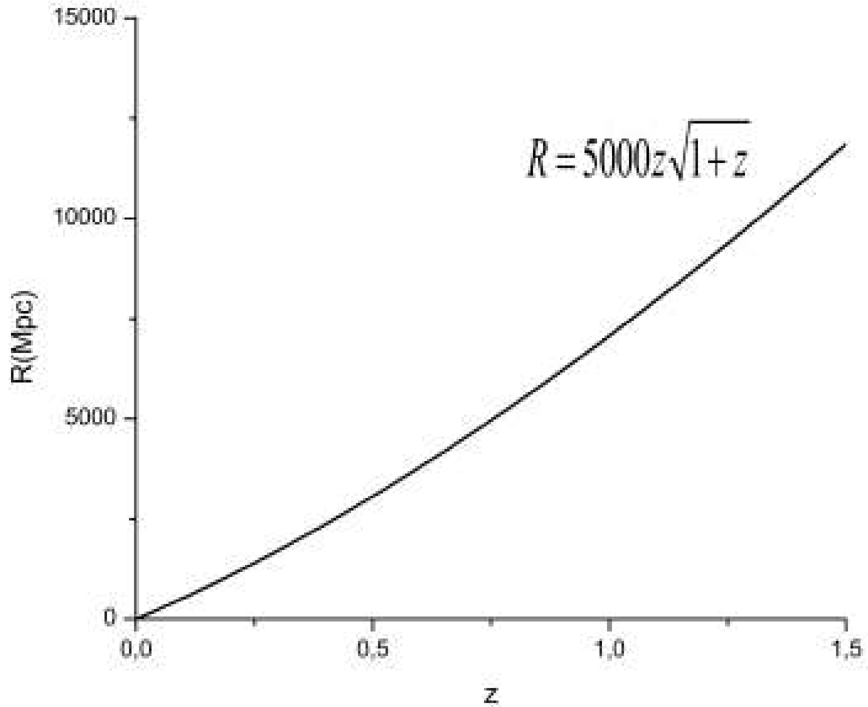

Diagram 7.4.2 The graph of the luminosity distance R of astronomical objects as a function of the redshift z. The measurement of the luminosity distances of type Ia supernova confirms the theoretical prediction of the law of selfvariations.

Type Ia supernova are cosmological objects for which we can measure their luminosity distance R. Furthermore, this measurement can be conducted at large distances, where the consequences of equation (7.3.14) are measurable.
At the end of the last century this kind of measurements were conducted by independent scientific groups. The graph thet results from these measurements exactly matches graph 7.4.2 which is predicted theoretically by the theory of selfvariations. In order to explain the inconsistency of the Standard Cosmological Model with graph 7.4.2, the existence of dark energy was invented and introduced.

7.5 Gravity cannot play the role attributed to it by the Standard Cosmological Model

From equation (7.3.9) we obtain $k=\frac{1-A}{A}H$, and

$$\frac{k}{c}=\frac{1-A}{A}\frac{H}{c} \tag{7.5.1}$$

For $H=60\frac{km}{sMpc}$, $A=0.999$, $c=3\times10^{5}\frac{km}{s}$ we have that

$$\frac{k}{c}=2\times10^{-7}\frac{1}{Mpc} \tag{7.5.2}$$

We replace this value of $\frac{k}{c}$ into equation (7.2.6) and obtain

$$m_0(r) = m_0 \frac{0.001}{1-0.999e^{-2\times10^{-7}r}} \tag{7.5.3}$$

Here, the distance r is measured in Mpc.

For values of r of the order of magnitude of kpc, equation (7.5.3) gives that $m_0(r) = m_0$. Therefore, the strength of the gravitational interaction is not affected in the scale of galactic distances. For example, for distance $r = 100kpc$, equation (7.5.3) gives $m_0(r) = 0.99999m_0$. Therefore, the selfvariations do not affect the stability of the solar system, galaxies, and galaxy clusters.

On the contrary, for distances of order of magnitude of Mpc, equation (7.5.3) predicts a clearly smaller value of $m_0(r)$, compared to m_0. For example, for $r = 100Mpc$ equation (7.5.3) gives $m_0(r) = 0.98m_0$. The strength of the gravitational interaction exerted on our galaxy by a galaxy from a distance of $100Mpc$ is 98% of the expected. For $r = 2\times10^3 Mpc$ equation (7.5.3) gives $m_0(r) = 0.7145m_0$. The strength of the gravitational interaction exerted by a galaxy, which is located at a distance of $2000Mpc$, on our galaxy is only 71.45% of the expected.

Therefore, we conclude that due to the selfvariations the gravitational interaction is weakened at cosmological distances and cannot play the role attributed to it by the Standard Cosmological Model. The gravitational interaction dominates and rules at a local level, at scales of a few hundreds or thousands of kpc.

We note that if we chose a different value for the parameter A, from the values permitted by inequality (7.3.8), all the arithmetic values appearing in equation (7.5.3) shall be altered. However, the same conclusions will be drawn about the relation between rest masses $m_0(r)$ and m_0.

The rest mass is given as a function of the redshift z from equation (7.3.13)

$$m_0(z) = \frac{m_0}{1+z}$$

For $z = 0.1$ we get $m_0(z) = 0.9091m_0$, for $z = 1$ we have $m_0(z) = 0.5m_0$, and for $z = 9$ we see that $m_0(z) = 0.1m_0$. The strength of the gravitational interaction exerted by an astronomical object with redshift $z = 9$ on our galaxy is only 10% of the expected.

For even greater distances the gravitational interaction practically vanishes.

7.6 The very early Universe

All the equations we have stated in this chapter are compatible with the condition $r \to \infty$. The equations are stated in such a way that the condition $r \to \infty$ offers us information about the very early Universe.

For $r \to \infty$ equation (7.2.6) gives

$$m_0(r \to \infty) \to m_0(1-A) \tag{7.6.1}$$

The inequality (7.3.8)

$$\frac{z}{1+z} < A < 1$$

holds for every value of the redshift z, hence $A \to 1$. Therefore, from relation (7.6.1) we conclude that the initial form of the Universe only slightly differs from the vacuum.
Similarly, from equation (7.2.11) we have that

$$q(r \to \infty) \to q(1-B) \tag{7.6.2}$$

This relation does not lead to the same consequences as relation (7.6.1). We know that the electric charge exists in opposite physical quantities in the Universe. Because of this, the total electric charge of the Universe is zero. Relation (7.6.1) informs as that the energy content of the very early Universe also tends to zero. The very early Universe only slightly differs from the vacuum. It possesses, though, a very important property which determines its evolution. It is temporally variable due to the selfvariations.
The increase of the rest masses and the electric charges destroys the initial homogeneity and state of rest, induces the first minute motions of the particles, and shifts the system to a temperature slightly above $0K$ (temperature reflects the kinetic state of the particles in the system). The evolution of the selfvariations with the passage of time leads the Universe to the form in which we observe it today.
In general, this is the prediction for the begining and evolution of the Universe from the equations we have stated. This prediction is also verified from the calculations presented in the following paragraphs.

7.7 The Universe is flat

From the principle of conservation of energy we conclude that the total energy content of the Universe is constant, and remains the same at every moment. Relation (7.6.1) informs us that the energy content of the very early Universe tends to zero. Therefore, the same holds today as we observe the Universe. Because of this, the Universe is flat.
The difference between the current state of the Universe and its initial state is the following: The rest masses of particles have increased, but this increase is counterbalanced by the generalized photons that flood spacetime, and by the strengthening of all kinds of negative potential energies that result as a consequence.
The observations conducted by the COBE and WMAP satellites confirm that the Universe is flat. Other observational data lead us to the same conclusion.

7.8 The origin of the cosmic microwave background radiation

The laboratory value for the Thomson scattering coefficient is

$$\sigma_\tau = \frac{8\pi}{3}\frac{q^4}{m_0{}^2c^2} \tag{7.8.1}$$

Here, q and m_0 are the electric charge and the rest mass of the electron, respectively.
At a distant astronomical object the Thomson coefficient is

$$\sigma_\tau(r) = \frac{8\pi}{3}\frac{q^4(r)}{m_0{}^2(r)c^2} \tag{7.8.2}$$

Combining these equations we get that

$$\frac{\sigma_\tau(r)}{\sigma_\tau} = \left(\frac{m_0}{m_0(r)}\right)^2\left(\frac{q(r)}{q}\right)^4 \tag{7.8.3}$$

From the observations we have made on the variation of the fine structure constant we know that, for large distances r, it holds that $q(r)=q$. Therefore, at a very good approximation, equation (7.8.3) can be written as

$$\frac{\sigma_\tau(r)}{\sigma_\tau}=\left(\frac{m_0}{m_0(r)}\right)^2$$

Using equation (7.2.6) we obtain that

$$\frac{\sigma_\tau(r)}{\sigma_\tau}=\left(\frac{1-Ae^{-\frac{kr}{c}}}{1-A}\right)^2 \tag{7.8.4}$$

For very large distances $(r\to\infty)$ very close to the initial state of the Universe, and at a temperature of about $0K$, equation (7.8.4) gives

$$\frac{\sigma_\tau(r\to\infty)}{\sigma_\tau}=\left(\frac{1}{1-A}\right)^2 \tag{7.8.5}$$

But according to inequality (7.3.8), $A\to 1$. Therefore, in the very distant past, and for a temperature of the Universe just slightly above $0K$, the Thomson scattering coefficient acquires enormous values, rendering the Universe opaque. The cosmic microwave background radiation we observe today, originates in this phase of the evolution of the Universe. The conditions we described refer to the whole expanse of the Universe. That is why the cosmic microwave background radiation seems to originate "from everywhere".

Equation (7.8.4) gives the value of the scattering coefficient at distant astronomical objects. Combining this equation with equation (7.3.3) gives

$$\frac{\sigma_\tau(z)}{\sigma_\tau}=(1+z)^2$$

$$\sigma_\tau(z)=\sigma_\tau(1+z)^2 \tag{7.8.6}$$

This equation is easier to use, since it expresses the Thomson scattering coefficient as a function of the redshift z of the distant astronomical object. We can also write equation (7.8.6) in the form

$$\sigma_\tau(z)=\frac{8\pi}{3}\frac{e^4}{m_e^{\,2}c^2}(1+z)^2 \tag{7.8.7}$$

where e and m_e denote the electric charge and the mass of the electron, respectively.

The Thomson coefficient concerns the scattering of photons of low energy E. For high energy photons it is replaced by the Klein-Nishina coefficient, given in the laboratory by

$$\sigma=\frac{3}{8}\sigma_\tau\frac{m_0c^2}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right)+\frac{1}{2}\right] \tag{7.8.8}$$

and by relation

$$\sigma(z)=\frac{3}{8}\sigma_\tau(z)\frac{m_0(z)c^2}{E(z)}\left[\ln\left(\frac{2E(z)}{m_0(z)c^2}\right)+\frac{1}{2}\right] \tag{7.8.9}$$

for the distant astronomical object.

From equations (7.3.13) and (7.3.14) we obtain

$$\frac{m_0(z)c^2}{E(z)} = \frac{m_0c^2}{E}$$

Therefore, equation (7.8.9) can be written as

$$\sigma(z) = \frac{3}{8}\sigma_\tau(z)\frac{m_0c^2}{E}\left[\ln\left(\frac{2E}{m_0c^2}\right) + \frac{1}{2}\right]$$

Using equation (7.8.8) we have

$$\frac{\sigma}{\sigma(z)} = \frac{\sigma_\tau}{\sigma_\tau(z)}$$

Using equation (7.8.6) we take that

$$\sigma(z) = \sigma(1+z)^2 \qquad (7.8.10)$$

The two scattering coefficients depend in the same way upon the redshift z, and the distance r.

7.9 The decrease of the atomic ionization energies at distant astronomical objects

The ionization and excitation energy X_n of the atoms is proportional to the factor m_0q^4, where m_0 is the rest mass of the electron and q is its electric charge. Thus, we have

$$\frac{X_n(r)}{X_n} = \frac{m_0(r)}{m_0}\left(\frac{q(r)}{q}\right)^4$$

After applying the familiar approximation $q(r) = q$ we obtain

$$\frac{X_n(r)}{X_n} = \frac{m_0(r)}{m_0}$$

Using equation (7.2.6) we have

$$\frac{X_n(r)}{X_n} = \frac{1-A}{1-Ae^{-\frac{kr}{c}}} \qquad (7.9.1)$$

Through equation (7.3.3) we see that

$$X_n(z) = \frac{X_n}{1+z} \qquad (7.9.2)$$

According to this equation the redshift z affects the rate of ionization of the atoms in distant astronomical objects. Boltzmann's equation

$$\frac{N_n}{N_1} = \frac{g_n}{g_1}e^{-\frac{X_n}{KT}} \qquad (7.9.3)$$

expresses the number of the ionized atoms N_n occupying the energy level n in a stellar surface which is at thermodynamic equilibrium. With X_n we denote the excitation energy from the energy level 1 to the level n, T stands for the temperature of the stellar surface in K, $K = 1.38\times10^{-23}\frac{J}{K}$ is Boltzmann's constant, and g_n is the degree of degeneracy multiplicity of level n, that is, the number of energy levels into which level n splits in the presence of a magnetic field.

Combining equations (7.9.2) and (7.9.3), we obtain for the distant astronomical object relation

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\frac{X_n}{KT(1+z)}} \qquad (7.9.4)$$

In the case of the hydrogen atom, for $n = 2, X_2 = 10.15eV = 16.24 \times 10^{-19} J, g_1 = 2, g_2 = 8$ and for a solar surface temperature $T \sim 6000K$, equation (7.9.3) shows that only one atom out of 10^8 occupies the $n = 2$ state. At the same temperature, equation (7.9.4) gives that for a redshift value of $z = 1$ we have $\frac{N_2}{N_1} = 2.2 \times 10^{-4}$, for $z = 2$ we have $\frac{N_2}{N_1} = 5.8 \times 10^{-3}$, and for $z = 5$ we have $\frac{N_2}{N_1} = 0.15$.

The conclusions drawn from the current and the previous paragraph demand a reexamination of the conclusions we have drawn from the observation of the electromagnetic spectrum of distant astronomical objects.

For very large distances, that is, in the very distant past, equation (7.9.1) gives

$$X_n(r \to \infty) = X_n(1 - A) \qquad (7.9.5)$$

This equation informs us that the very early Universe was ionized at some stage. The ionization energies of the atoms had very small values. We can reach the same conclusion if we substitute into equation (7.9.2) very large values of the variable z, or if in equation (7.9.3) we replace the energy X_n with $X_n(1 - A)$.

7.10 On the fine structure constant

In the preceding chapters we saw that due to the manifestation of the selfvariations, energy, momentum, angular momentum and electric charge flow from the material particles to the surrounding spacetime. The first consequence of the selfvariations is the potential to transfer energy, momentum, angular momentum and electric charge from one material particle to another, i.e. the interaction between the material particles. The gravitational and electromagnetic interactions determine the starting point for the quantitative determination of the selfvariations. Because of this, we supposed that the rest masses and the electric charges, and not any other physical quantity, vary with the passage of time. We offer this remark since, at cosmological scales, equation (7.2.1) justifies all of the cosmological observational data we possess, and it could be supposed that the electric charge remains constant. Such an assumption cannot hold within the framework of the theory of selfvariations, where the selfvariations of the electric charge are responsible for the electromagnetic field.

By analyzing the electromagnetic spectra reaching Earth from distant quasars from distances up to $6 \times 10^9 ly$, the value of the fine structure constant α remains constant. More precisely, there are indications of a very slight variation of the parameter α.

The parameter α depends on the electron charge q, as given in

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 c\hbar} \qquad (7.10.1)$$

Therefore, this parameter is not constant. We have

$$\frac{\alpha(r)}{\alpha} = \left(\frac{q(r)}{q}\right)^2$$

Using equation (7.2.11) we also have

$$\frac{\alpha(r)}{\alpha}=\left(\frac{q(r)}{q}\right)^2=\left(\frac{1-B}{1-Be^{-\frac{k_1 r}{c}}}\right)^2 \qquad (7.10.2)$$

From this equation it can be inferred that the parameter $\alpha(r)$ (essentially the electric charge $q(r)$), remains constant for large distances r when the constant k_1 or the parameter B acquire extremely small values. According to relation (7.6.2) we have that

$$q(r\to\infty)\to q(1-B)$$

This relation can be written as

$$\alpha(r\to\infty)\to\alpha(1-B)^2$$

Therefore, the value of the electric charge and of the parameter α in the very early Universe are only determined by the value of the parameter B. Hence, the parameter B has a very small value, independently of the value of constant k_1.

For very small values of the parameter B we see that

$$q(r\to\infty)\to q(1-B)\to q$$

This prediction does not cause any problems at the initial state of the Universe, since the electric charge exists in couples of opposite physical quantities. Such a relation cannot hold for the case of the rest mass, and indeed we know that

$$\frac{z}{1+z}<A<1$$

$$m_0(r\to\infty)\to m_0(1-A)\to 0$$

From equation (7.2.12) we obtain $B>0$. Thus, we arrive at the conclusion that the parameter B acquires extremely small positive values.

The extremely small value of the parameter B assures the stability of the value of the parameter α for large distances r. Hence, we turn our attention not to the arithmetic value (which is likely to be extremely small, as is the case for the constant $k=\frac{1-A}{A}H$), but to the sign of the constant k_1.

For $k_1>0$ we obtain successively that

$$k_1>0$$

$$-\frac{k_1 r}{c}<0$$

$$e^{-\frac{k_1 r}{c}}<1 \quad ,(B>0)$$

$$Be^{-\frac{k_1 r}{c}}<B$$

$$-Be^{-\frac{k_1 r}{c}}>-B$$

$$1-Be^{-\frac{k_1 r}{c}}>1-B \qquad ,\left(1-Be^{-\frac{k_1 r}{c}}>0\right)$$

$$\frac{1-B}{1-Be^{-\frac{k_1 r}{c}}}<1$$

From equation (7.10.2) we have that

$$\frac{\alpha(r)}{\alpha}=\left(\frac{q(r)}{q}\right)^2<1, k_1>0$$

Therefore, for $k_1>0$ we will measure a slight decrease of the parameter α at large distances. Similarly, it turns out that for $k_1<0$ we will measure a slight increase of the parameter α at large distances.

$$\frac{\alpha(r)}{\alpha}=\left(\frac{q(r)}{q}\right)^2>1,\ k_1<0$$

Based on the observational data we currently have, measurements of the variation of the parameter α have to be conducted for distances greater than $6\times10^9 ly$. The extremely small value of the (positive) parameter B renders these measurements difficult, in both cases.

7.11 The large structures in the Universe

The increase of the rest masses with the passage of time strengthens the gravitational interaction and accumulates matter towards various directions. The consequences of the accumulation of matter depend upon the quantity of the accumulated matter, as well as on the volume it occupies. In all cases, the total initial energy of the accumulated matter is zero, according to relation (7.6.1).

At large scales, at distances of order of magnitude $10^9 ly$, the distribution of matter must have been determined by a large-scale destruction of the absolute homogeneity of the vacuum in the very early Universe. This explains the colossal webs of matter through vast expances of empty space that we observe with the modern observational instruments.

At smaller scales, within the dimensions of a galaxy, the accumulation of matter increases the temperature, as a result of the conversion of the gravitational potential energy into heat. A percentage of the particles of matter accumulates in a first central core of high temperature, while the remaining percentage remains distributed in the surrounding space during the period of accumulation. The slow rate at which the selfvariations occur, strengthens, also at a slow rate, the magnitude of the gravitational interaction, and allows a considerable percentage of the particles to remain in the surrounding space.

A further accumulation of the first core will lead to the formation of a second, more centralized core, until the temperature reaches the point where nuclear fusion starts. The initiation of nuclear fusion prevents the further accumulation of matter.

We separated the process of the accumulation into two phases, and we mentioned two cores for the following reason: The initial percentage of matter which remained outside the initial central core concerns the initial phase of the accumulation and is at a low temperature, slightly above $0K$. However, the percentage of matter which stays outside the second, and real central core, already has a high temperature. If we take into account the very high value of the Reynolds coefficient in this region, turbulent vortices will be generated. Therefore, the formation of stars should occur in this region. In the final central core, the density of matter should be larger than in the rest of the galaxy. Clusters of galaxies are formed through similar processes.

Rough calculations give an equation correlating the mass and the volume of a galaxy. This relation is consistent with the data we possess about galaxies (and galaxy clusters). But in reality, the process of accumulation is not separated into phases, but evolves in a continuous manner, from its beginning up to the formation of a galaxy.

Therefore, we can only reach safe conclusions on the issue through computer simulations.

7.12 The origin of matter and the arrow of time

The equations of the theory of selfvariations predict at the limit, in the very distant past, that the beginning of the Universe was the vacuum. Therefore, we cannot consider a point to be the beginning of the universe, as proposed by the Standard Cosmological Model. All the points within the Universe are equivalent. The Universe originates " from everywhere", exactly as the cosmic microwave background radiation does (paragraph 7.8). Which physical mechanism can lead to such a result?

The theory of selfvariations predicts that the generalized particle can behave in such a way. The correlation of the vacuum with the condition $dS^2 = 0$ leads to such an interpretation, as we analyzed it in paragraph 5.9 and in chapter 6.

What happens at the microcosm is a repetition at a local level, in a region of spacetime, of the condition that dominated throughout the spacetime occupied by the Universe during its emergence from the vacuum. That is how the slight perturbations of enormous spatial dimensions emerged within the initial homogeneity of the vacuum.

These perturbations were recorded on the cosmic microwave background radiation that followed ($2.74K$) and which also originates from the whole Universe, as discussed in paragraph 7.8. Moreover, these perturbations are responsible for the large-scale distribution of matter in the Universe (paragraph 7.11).

The theory of selfvariations solves a fundamental problem of physical reality, which the physical theories of the last century are unable to solve. The equations of the theory of selfvariations include the arrow of time. The Universe originates from the vacuum and evolves towards a particular direction, which is determined by the selfvariations. The selfvariations continuously "distance" the Universe from the state of vacuum, but the Universe remains consistent with its origin:

The origin of matter from the vacuum, combined with the principles of conservation, has as a consequence that the energy content of the Universe is zero.

In the laboratory, the internality of the Universe to the process of measurement apparently "freezes" the time evolution of the selfvariations. On the contrary, the consequences of the selfvariations are directly imprinted on the observations we conduct at large distances. The Universe we observe today, and the complex processes taking place in Nature, are the results of the evolution of the selfvariations with the passage of time.

7.13 The future evolution of the Universe

The range of values parameter A takes is given by inequality (7.3.8)

$$\frac{z}{1+z} < A < 1$$

Furthermore, equation (7.2.7) informs us that the parameter A approaches unity at an exceptionally slow rate, due to the extremely small value of the constant $k = \frac{1-A}{A}H$.

The parameter A appears in all of the equations we have stated. Because of this, the evolution of this parameter through time also determines the future evolution of the Universe, at least in the observations we will conduct in the far future.

From equation (7.3.9) we have that

$$\dot{H} = k\frac{\dot{A}(1-A)+A\dot{A}}{(1-A)^2}$$

Using equation (7.2.7) we obtain

$$\dot{H} = k\frac{kA}{(1-A)^2}$$

$$\dot{H} = \frac{1}{A}\left(\frac{kA}{1-A}\right)^2$$

$$\dot{H} = \frac{1}{A}H^2$$

For $A \sim 1, H = 60\frac{km}{sMpc} = 2\times10^{-18}s^{-1}$

$$\dot{H} = 4\times10^{-36}s^{-2}$$

The Hubble parameter varies at an extremely slow rate.

We shall now see how the redshift z varies with the passage of time. From equation (7.3.3) we get

$$z = \frac{1-Ae^{-\frac{kr}{c}}}{1-A} - 1 \qquad (7.13.1)$$

$$z = \frac{A}{1-A}\left(1-e^{-\frac{kr}{c}}\right)$$

For the same distance r we have that

$$\dot{z} = \left(\frac{A}{1-A}\right)^{\bullet}\left(1-e^{-\frac{kr}{c}}\right)$$

$$\dot{z} = \frac{\dot{A}}{(1-A)^2}\left(1-e^{-\frac{kr}{c}}\right)$$

Using equation (7.2.7) we see that

$$\dot{z} = \frac{kA}{(1-A)^2}\left(1-e^{-\frac{kr}{c}}\right)$$

Considering equation (7.13.1) we obtain

$$\dot{z} = \frac{k}{1-A}z$$

$$\dot{z} = \frac{1}{A}\frac{kA}{1-A}z$$

Through equation (7.3.9) we arrive at

$$\dot{z} = \frac{H}{A}z \qquad (7.13.2)$$

For $H = 2\times10^{-18}s^{-1} = 6.3\times10^{-11}year^{-1}$ and $A \sim 1$ we obtain

$$\dot{z} = z\cdot 6.3\times10^{-11}year^{-1} \qquad (7.13.3)$$

The rate of increase of the redshift z is a measure with which to evaluate the future evolution of the Universe.

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Appendix

The Topographic Theorem

For a material point particle, the velocity $\boldsymbol{\upsilon}$ of the selfvariations is defined by equation (2.2.6)

$$\boldsymbol{\upsilon} = \frac{c}{r}\mathbf{r} \qquad (1)$$

This equation refers solely to the material point particle. On the contrary, equation (2.3.1)

$$\frac{\boldsymbol{\upsilon}}{c} = \begin{bmatrix} \cos\delta \\ \sin\delta\cos\omega \\ \sin\delta\sin\omega \end{bmatrix} \qquad (2)$$

has more general validity. The velocity in equation (2) satisfies the relation $\|\boldsymbol{\upsilon}\| = c$, without necessarily having the form (1). Therefore, we have to study the properties of the velocity $\boldsymbol{\upsilon}$, as they follow from equation (2). The differentiation between the two equations occurs in equations (2.3.11) and (2.3.12).

$$\nabla\delta = \lambda_1 \frac{\boldsymbol{\upsilon}}{c} + K\boldsymbol{\beta} + L\boldsymbol{\gamma}$$

$$\nabla\omega = \lambda_2 \frac{\boldsymbol{\upsilon}}{c} + M\boldsymbol{\beta} + N\boldsymbol{\gamma}$$

which take the form

$$\begin{aligned} \nabla\delta &= -\frac{\partial\delta}{c\partial t}\frac{\boldsymbol{\upsilon}}{c} + K\boldsymbol{\beta} + L\boldsymbol{\gamma} \\ \nabla\omega &= -\frac{\partial\omega}{c\partial t}\frac{\boldsymbol{\upsilon}}{c} + M\boldsymbol{\beta} + N\boldsymbol{\gamma} \end{aligned} \qquad (3)$$

We will mention the general properties of the velocity $\boldsymbol{\upsilon}$, without citing the relevant proofs.

The coefficients $\frac{\partial\delta}{c\partial t}, K, L, \frac{\partial\omega}{c\partial t}, M, N$ are not independent from each other, but are constrained by the following compatibility equations:

$$\frac{\partial\delta}{c\partial t}\left(L-M\sin\delta\right)+\left(KM+LN\right)\cos\delta-\boldsymbol{\gamma}\cdot\nabla K+\boldsymbol{\beta}\cdot\nabla L=0$$
$$\frac{\partial\omega}{c\partial t}\left(L-M\sin\delta\right)+\left(M^2+N^2\right)\cos\delta-\boldsymbol{\gamma}\cdot\nabla M+\boldsymbol{\beta}\cdot\nabla N=0$$
$$\frac{\partial K}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla K=-c\left(K^2+LM\sin\delta\right)$$
$$\frac{\partial L}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla L=-cL\left(K+N\sin\delta\right)$$
$$\frac{\partial M}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla M=-cM\left(K+N\sin\delta\right)$$
$$\frac{\partial N}{\partial t}+\boldsymbol{\upsilon}\cdot\nabla N=-c\left(LM+N^2\sin\delta\right)$$
$$\frac{\partial}{\partial t}\left(\frac{\partial\delta}{c\partial t}\right)+\boldsymbol{\upsilon}\cdot\nabla\left(\frac{\partial\delta}{c\partial t}\right)=-K\frac{\partial\delta}{\partial t}-L\sin\delta\frac{\partial\omega}{\partial t}$$
$$\frac{\partial}{\partial t}\left(\frac{\partial\omega}{c\partial t}\right)+\boldsymbol{\upsilon}\cdot\nabla\left(\frac{\partial\omega}{c\partial t}\right)=-M\frac{\partial\delta}{\partial t}-N\sin\delta\frac{\partial\omega}{\partial t} \qquad (4)$$

These equations are valid in every inertial frame of reference.

For the inertial reference frames S and S', as we defined them in chapter 3, the following Lorentz-Einstein transformations hold

$$\frac{\partial\delta'}{c\partial t'}=\frac{\partial\delta}{c\partial t}+\frac{u}{c}\frac{K\sin\delta}{1-\frac{u}{c}\cos\delta}$$
$$K'=\frac{K}{\gamma\left(1-\frac{u}{c}\cos\delta\right)}$$
$$L'=\frac{L}{\gamma\left(1-\frac{u}{c}\cos\delta\right)}$$
$$\frac{\partial\omega'}{c\partial t'}\sin\delta'=\frac{\partial\omega}{c\partial t}\sin\delta-\frac{u}{c}M\sin\delta\frac{\sin\delta}{1-\frac{u}{c}\cos\delta}$$
$$M'\sin\delta'=\frac{M\sin\delta}{\gamma\left(1-\frac{u}{c}\cos\delta\right)}$$
$$N'\sin\delta'=\frac{N\sin\delta}{\gamma\left(1-\frac{u}{c}\cos\delta\right)} \qquad (5)$$

We define the vector

$$\mathbf{t} = \nabla\delta \times \sin\delta\nabla\omega = t_1\frac{\boldsymbol{\upsilon}}{c} + t_2\boldsymbol{\beta} + t_3\boldsymbol{\gamma} =$$

$$= \left(KN\sin\delta - LM\sin\delta\right)\frac{\boldsymbol{\upsilon}}{c} + \left(\frac{\partial\delta}{c\partial t}N\sin\delta - L\frac{\partial\omega}{c\partial t}\sin\delta\right)\boldsymbol{\beta} \qquad (6)$$

$$+\left(K\frac{\partial\omega}{c\partial t}\sin\delta - \frac{\partial\delta}{c\partial t}M\sin\delta\right)\boldsymbol{\gamma}$$

The topography of the generalized photon is defined by the following theorem:

The Topographic Theorem

«For every inertial frame of reference and for every generalized photon, the following hold:

a. If it is $(t_1, t_2, t_3) \neq (0,0,0)$, then the generalized photon is of one spatial dimension. The material points of the generalized photon are arranged on a curve. At each point of the curve the vector $\mathbf{t}$ is tangent on the curve.
b. The generalized photon can have two spatial dimensions, with its material points arranged on a surface. Then at each point of the surface, the vector $\mathbf{n}$, vertical to the surface, is given by $\mathbf{n} = \frac{\nabla\delta}{\|\nabla\delta\|} = \frac{\nabla\omega}{\|\nabla\omega\|}$.
c. If the material points of the generalized photon are arranged in the three-dimensional space, then it is $K = L = M\sin\delta = N\sin\delta = \frac{\partial\delta}{c\partial t} = \frac{\partial\omega}{c\partial t}\sin\delta = 0$»

For the material point particle and for the velocity vector (1), we obtain from equations (2.3.19) and (2.3.2)

$$\frac{\partial\delta}{c\partial t} = -\frac{\mathbf{u}\cdot\boldsymbol{\beta}}{cr\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)}$$

$$K = \frac{1}{r}$$

$$L = 0$$

$$\sin\delta\frac{\partial\omega}{c\partial t} = -\frac{\mathbf{u}\cdot\boldsymbol{\gamma}}{cr\left(1-\frac{\boldsymbol{\upsilon}\mathbf{u}}{c^2}\right)}$$

$$M\sin\delta = 0$$

$$N\sin\delta = \frac{1}{r}$$

Thus, we get

$$t_1 = KN\sin\delta - LM\sin\delta = \frac{1}{r^2} \neq 0$$

and, therefore, it is $\mathbf{t} \neq (0,0,0)$. Consequently, in the case of equation (1) the generalized photon is of one spatial dimension. Therefore, the trajectory representation theorem emerges, as we saw in paragraph 2.4 of chapter 2.

The topographic theorem permits the study of the selfvariations for non-point material particles.