Predicting the Proton and Neutron Masses, Based on Baryons which are Yang-Mills Magnetic Monopoles and Koide Mass Triplets

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Abstract:

We show how the Koide relationships and associated triplet mass matrices can be generalized to derive the observed sum of the free proton and neutron rest masses in terms of the up and down current quark masses and the Fermi vev to six parts in 10,000, which sum can then be solved for the separate neutron and proton masses using the neutron–proton mass difference derived by the author in an recent, separate paper. The opposite charges of the up and down quarks are responsible for the appearance of a complex phase exp(iδ) which in turn can be used to adjust these mass relationships to unlimited accuracy. For the moment, phase angle $\delta=1.9932858^0$ *is an empirical parameter, but it does appear to be possibly related to the CP-violating phase of weak interactions for three fermion generations. The Koide generalizations developed here enable these proton and neutron mass relationships to be given a Lagrangian formulation based on proton and neutron field strength tensors that contain constituent quark wavefunctions and masses. In the course of development, we also uncover new Koide relationships for the neutrinos, the up quarks, and the down quarks.*

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1. Introduction

In an earlier paper [1], the author introduced the thesis that baryons are Yang-Mills magnetic monopoles. One of the relationships predicted in this paper, equation [11.22] therein, predicted the electron rest mass as a function of the up and down quark masses, namely:

$$m_e = 3\left(m_d - m_u\right) / \left(2\pi\right)^{\frac{3}{2}}, \tag{1.1}$$

with the factor of $\left(2\pi\right)^{\frac{3}{2}}$ emerging from a three-dimensional Gaussian integration. Based on a "resonant cavity" analysis of the nucleons whereby the energies released or retained during binding are directly dependent upon the masses of the quarks contained within the nucleons, we also predicted the latent, intrinsic binding energies of a proton and neutron in [12.12] and [12.13] of [1], to be given by:

$$B_P = 2m_u + m_d - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right) / \left(2\pi\right)^{\frac{3}{2}} = 7.640679 MeV \tag{1.2}$$

$$B_N = 2m_d + m_u - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right) / \left(2\pi\right)^{\frac{3}{2}} = 9.812358 MeV\ . \tag{1.3}$$

This predicts a latent binding energy of 8.7625185 MeV per nucleon for a nucleus with an equal number of protons and neutrons, which is remarkably close to what is observed for all but the very lightest nuclides, as well as a total latent binding energy of 493.028394 MeV for ^{56}Fe, in contrast to the empirical binding energy of 492.253892 MeV. This is understood to mean that 99.8429093% of the available binding energy in ^{56}Fe is applied to inter-nucleon binding, with the balance of 0.1570907% retained for the intra-nucleon confinement of quarks. It was also noted that this percentage of energy released for inter-nucleon binding is higher in ^{56}Fe than in any other nuclide, which further explains that although the quarks come closer to de-confinement in ^{56}Fe than in any other nuclide (which also explains the "first EMC effect"), they do always remain confined, as emphasized by the decline in this percentage beyond ^{56}Fe.

In a second paper [2], the author showed how the thesis that baryons are Yang-Mills magnetic monopoles together with the foregoing "resonant cavity" analysis can be used to predict the binding energies of the 1s nuclides, namely ^{2}H, ^{3}H, ^{3}He and ^{4}He, to at least parts per hundred thousand and in most cases parts per million, and also to predict the difference between the proton and neutron masses according to:

$$M_N - M_P = m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right) / \left(2\pi\right)^{\frac{3}{2}}. \tag{1.4}$$

This relationship, originally predicted in [6.16] of [2] to about seven parts per ten million in AMU, was later taken in [9.1] of [2] to be an *exact* relationship, and all of the other prior mass relationships which had been developed were then nominally adjusted to implement (1.4) as an exact relationship. The review of the solar fusion cycle in section 8 of [2] served to emphasize how effectively this resonant cavity analysis can be used to accurately predict empirical binding energies, and suggested how applying gamma radiation with the right resonant harmonics to a store of hydrogen may well have a catalyzing effect for nuclear fusion.

At the heart of these numeric calculations were the two outer products [3.9] and [3.10] in [2] for the proton and the neutron, with components given by [3.11] and related relationships developed throughout section 2 of [2]. In particular, the two matrices which stood at the center of these successful binding calculations were the 3x3 Yang-Mills diagonalized matrices *K* of mass dimension ½ with components $\mathrm{diag}\left(K_P\right) = \left(\sqrt{m_d}, \sqrt{m_u}, \sqrt{m_u}\right)$ for the proton and

$\text{diag}(K_N) = \left(\sqrt{m_u}, \sqrt{m_d}, \sqrt{m_d}\right)$ for the neutron, where m_u is the "current" mass of the up quark and m_d is the current mass of the down quark.

What is very intriguing about these *K* matrices (which we designate as such to reference Koide), is that although they originate out of the thesis that baryons are magnetic monopoles, they have a form very similar to matrices which may be used in the so-called Koide mass formula [3] for the charged leptons, namely:

$$R = \frac{\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2}{m_1 + m_2 + m_3} \cong \frac{3}{2}. \tag{1.5}$$

Above, when we take $m_1 = m_e$, $m_2 = m_\mu$ and $m_3 = m_\tau$ to be the charged lepton masses, the ratio $R \cong 3/2$ gives a very precise relationship among these masses. Indeed, if we use the 2012 PDG data $m_e = 0.510998928 \pm 0.000000011 MeV$, $m_\mu = 105.6583715 \pm 0.0000035 MeV$ and $m_\tau = 1776.82 \pm 0.16 MeV$ [4], we find using the mean experimental data that $R = 1.500022828$ which is very close to 3/2. However, when we use the extremes of the experimental data ranges, specifically, the largest possible tau mass and the lowest possible mu mass, we obtain *R*=1.5000024968. Although this is an order of magnitude closer to 3/2 than the ratio obtained from the mean data, is still *outside* of experimental errors. This means that while $R \cong 3/2$ is a very close relationship, even accounting for experimental error, it is still approximate. For this to be *within* experimental errors, it would have to be possible to obtain some $R \le 3/2$ for some combination of masses at the edges of the experimental ranges, and it is not. So in the application of the Koide relationships to various mass triplets, the question becomes, not *whether* a triplet has a ratio exactly equal to 3/2, because no triplet does have this exact relationship, but rather, how close to 3/2 any given ratio is, and more importantly, what the meaning is of this ratio and deviations from this ratio.

The similarities of the matrices developed by the author in [1] and [2] and those developed by Koide in [3] is highlighted if we define a Koide matrix generally as:

$$K_{AB} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix}. \tag{1.6}$$

Then, the two latent binding energy relationships (1.2) and (1.3) may be written:

$$B_P = K_{AB}K_{BA} - \frac{1}{(2\pi)^{\frac{3}{2}}} K_{AA}K_{BB} = Tr(K^2) - \frac{1}{(2\pi)^{\frac{3}{2}}} Tr(K \otimes K) = 2m_u + m_d - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right) / (2\pi)^{\frac{3}{2}}$$

$$= Tr\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} - \frac{1}{(2\pi)^{\frac{3}{2}}} Tr\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}, \tag{1.7}$$

$$B_N = K_{AB}K_{BA} - \frac{1}{(2\pi)^{\frac{3}{2}}} K_{AA}K_{BB} = Tr(K^2) - \frac{1}{(2\pi)^{\frac{3}{2}}} Tr(K \otimes K) = 2m_d + m_u - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right) / (2\pi)^{\frac{3}{2}}$$

$$= Tr\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} - \frac{1}{(2\pi)^{\frac{3}{2}}} Tr\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}, \quad (1.8)$$

where, based on (1.6), in (1.7) we have set $m_1 \equiv m_d$ and $m_2 = m_3 \equiv m_u$, and in (1.8) we have set $m_1 \equiv m_u$ and $m_2 = m_3 \equiv m_d$. These originate in the author's thesis in [1] that baryons are Yang-Mills magnetic monopoles. Above, $\otimes$ designates an *outer* matrix product.

On the other hand, setting $m_1 = m_e$, $m_2 = m_\mu$ and $m_3 = m_\tau$ in (1.6), we may write:

$$Tr(K^2) = K_{AB}K_{BA} = m_1 + m_2 + m_3, \quad (1.9)$$

$$Tr(K \otimes K) = K_{AA}K_{BB} = \left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2. \quad (1.10)$$

Then, using (1.9) and (1.10), Koide relationship (1.5) for charged leptons may be written as:

$$R = \frac{\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2}{m_1 + m_2 + m_3} = \frac{K_{AA}K_{BB}}{K_{AB}K_{BA}} = \frac{Tr(K \otimes K)}{Tr(K^2)} \cong \frac{3}{2}. \quad (1.11)$$

Clearly then, the Koide matrices (1.6) provide a general form for organizing the study of both binding energy and fermion mass relationships which lead to very accurate empirical results. It thus becomes desirable to understand the physical origin of these matrices and tie them to a Lagrangian formulation so that they are no longer just intriguing curiosities that yield tantalizingly-accurate empirical results. And, it is desirable to see if they can be extended to make additional mass predictions and gain deeper understanding of the particle mass spectrum.

Because the binding energy formulation in (1.7) and (1.8) has its roots in the thesis that baryons are Yang-Mills magnetic monopoles and specifically emerges from the calculation of energies via $E = -\iiint \mathfrak{L} d^3x$, see [11.7] of [1] et. seq., the author's previous findings will provide us with the means to anchor the Koide relationships in a Lagrangian formulation. And, because Koide provides a generalization of the mass matrices derived by the author, these matrices will provide us with the means to derive additional mass relationships as well.

Most importantly, in this paper, we shall combine the author's previous work in [1] and [2] as well as [5], using the generalization provided by Koide triplet mass matrices of the form (1.6), to deduce the observed rest masses 938.272046 MeV and 939.565379 MeV of the free proton and free neutron, as a function of the up and down quark masses and the Fermi vev. The next two sections will lay the foundation for doing this, and the mass derivation will then commence in section 4.

2. Statistical Reformulation of the Koide Mass Relationship

Let us begin by couching the Koide mass relationship (1.5) for the charged leptons in statistical terms, using $m_1 = m_e$, $m_2 = m_\mu$ and $m_3 = m_\tau$ in (1.6). First, using (1.9), we write the average of the masses in a Koide mass triplet m_1, m_2, m_3, i.e., the "average of the squares" of the matrix elements in (1.6), as:

$$\left\langle K^2 \right\rangle = Tr(K^2)/3 = K_{AB}K_{BA}/3 = \left(m_1 + m_2 + m_3\right)/3 = \left\langle m_i \right\rangle. \quad (2.1)$$

Next, via (1.10), we write the “square of the average” of these matrix elements as:

$$\langle K\rangle^2 = \frac{Tr(K\otimes K)}{9} = \frac{K_{AA}K_{BB}}{9} = \left(\frac{\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}}{3}\right)^2 = \frac{\left(\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}\right)^2}{9}. \tag{2.2}$$

So, combining (2.1) and (2.2) in the form of (1.5) allows us to write:

$$3\frac{\langle K\rangle^2}{\langle K^2\rangle} = \frac{Tr(K\otimes K)}{Tr(K^2)} = \frac{K_{AA}K_{BB}}{K_{AB}K_{BA}} = \frac{\left(\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}\right)^2}{m_1+m_2+m_3} = R \cong \frac{3}{2}. \tag{2.3}$$

This allows us to extract the relationship:

$$\langle K\rangle^2 = \frac{R}{3}\langle K^2\rangle \cong \frac{1}{2}\langle K^2\rangle, \tag{2.4}$$

which naturally absorbs the 3 from the factor of 3/2.

Now, we simply use (2.4) to form the statistical variance $\sigma(K)$ in the usual way, as:

$$\sigma(K) = \langle K^2\rangle - \langle K\rangle^2 = \left(1-\frac{R}{3}\right)\langle K^2\rangle = \left(\frac{3}{R}-1\right)\langle K\rangle^2 = \left(\frac{3}{R}-1\right)\langle m_i\rangle \cong \frac{1}{2}\langle K^2\rangle = \langle K\rangle^2 = \langle m_i\rangle. \tag{2.5}$$

The key relationship here, using the first and last terms, is:

$$\sigma(K) \cong \langle m_i\rangle. \tag{2.6}$$

So the average charged lepton mass $\langle m_i\rangle$ is approximately (and very closely) equal to the statistical variance $\sigma(K)$ of Koide matrix (1.6) for the charged leptons. This is a much simpler and more transparent way to express the Koide mass relationship (1.5), and it completely absorbs the factor of 3/2.

Of course, as noted after (1.5), this is a very close, but still approximate relationship. The exact relationship, also extracted from (2.5), and using $R = 1.500022828$ based on the mean experimental data, is:

$$\sigma(K) = \left(\frac{3}{R}-1\right)\langle m_i\rangle = 0.999969563\langle m_i\rangle \equiv C\langle m_i\rangle, \tag{2.7}$$

where we have defined the statistical coefficient *C* and the inverted relationship for *R* as:

$$C \equiv \frac{3}{R}-1; \quad R \equiv \frac{3}{1+C}. \tag{2.8}$$

Thus, we rewrite the basic Koide relationship (1.5) more generally as:

$$\frac{\left(\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}\right)^2}{m_1+m_2+m_3} = \frac{3}{1+C} = R. \tag{2.9}$$

In the circumstance where the statistical coefficient *C*=1, i.e., where the average mass is exactly equal to the statistical variance, we have $R = 3/2$. So the variance of the square roots of the three charged lepton masses is just a tiny touch less ($\times 0.999969563$) than the average of the three masses themselves. But the factor of 3/2, which is somewhat mysterious in (1.5), is now more readily understood when we realize that it corresponds with *C*=1 in (2.7).

This means that the Koide relationship for *any* given triplet of numbers with mass dimension ½, may be most transparently characterized by the coefficient *C*. Thus, using (2.7), the coefficient for the charged lepton triplet is (we also include *R* for comparison):

$$C(e\mu\tau) = 0.999969563 \cong 1; \quad R(e\mu\tau) = 1.500022828 \cong 3/2. \tag{2.10}$$

So what about some other Koide triplets? For the neutrinos, PDG in [6] provides upper limits on the neutrino masses whereby $m_{\nu_e} < 2eV$, $m_{\nu_\mu} < 0.19MeV$ and $m_{\nu_\tau} < 18.2MeV$. If we use these mass limits in a Koide triplet, we find that *R*=1.202960231, but the significance of this is more easily seen by using (2.8) to calculate:

$$C\left(\nu_e \nu_\mu \nu_\tau\right) = 1.49384803 \cong 3/2; \quad R\left(\nu_e \nu_\mu \nu_\tau\right) = 1.202960231 \cong 6/5. \tag{2.11}$$

So the variance in the square roots of the neutrino mass limits is very close to being 50% larger than the average of these mass limits, i.e., $\sigma\left(K_\nu\right) \cong (3/2)\left\langle m_\nu \right\rangle$. This in an interesting "coefficient migration" as between the charged and uncharged leptons, wherein for the charged leptons masses $R \cong 3/2$ to parts per 100,000, while for the neutrino lepton upper mass limits, $C \cong 3/2$ within about 0.4%. As we shall see, it is the start of a new Koide pattern.

Turning to quark masses, we use $m_u = 2.223792405MeV$ and $m_d = 4.906470335MeV$ developed in [9.3] and [9.4] of [2] via 1 u=931.494 061(21) MeV/c^2, as well as $m_c = 1.275 \pm 0.025GeV$, $m_s = 95 \pm 5MeV$, $m_t = 173.5 \pm .6 \pm .8GeV$ and $m_b = 4.18 \pm 0.03GeV$ from PDG's [7]. For Koide triplets of a single flavor type, we can calculate that:

$$C\left(uct\right) = 1.54688 \cong 3/2; \quad R\left(uct\right) = 1.177913486 \cong 6/5. \tag{2.12}$$

$$C\left(dsb\right) = 1.18741 \cong 6/5; \quad R\left(dsb\right) = 1.371483911 \cong 15/11. \tag{2.13}$$

So we now see a distinctive pattern among (2.10) through (2.13). For the charged leptons in (2.10) which are the lower members of a weak isospin doublet, $R\left(e\mu\tau\right) \cong 3/2$. For the neutrinos which are the upper member of this doublet, $C\left(\nu_e \nu_\mu \nu_\tau\right) \cong 3/2$, which migrates the $3/2$ from the *R* to the *C* coefficient. Then, for the up quarks, $C\left(uct\right) \cong 3/2$, which is same as the *C* for the neutrinos, and both quarks and neutrinos are the upper members of the isospin doublets. But it is the $R\left(uct\right) \cong 6/5$ coefficient for the up quarks, that migrates to $C\left(dsb\right) \cong 6/5$ for down quarks. So the migration is $R\left(e\mu\tau\right) \cong 3/2 \rightarrow C\left(\nu_e \nu_\mu \nu_\tau\right) \cong 3/2$ for leptons, $C\left(\nu_e \nu_\mu \nu_\tau\right) \cong 3/2 \rightarrow C\left(uct\right) \cong 3/2$ providing a "bridge" from "up" leptons to "up" quarks, and then $R\left(uct\right) \cong 6/5 \rightarrow C\left(dsb\right) \cong 6/5$ migrating the up to the down quarks. The net upshot of this coefficient migration is that we now have Koide-style close relations for all four sets of fermions (and anti-fermions) of like electric charge *Q*, namely:

$$R(Q=0) = \frac{\left(\sqrt{m_{\nu(e)}} + \sqrt{m_{\nu(\mu)}} + \sqrt{m_{\nu(\tau)}}\right)^2}{m_{\nu(e)} + m_{\nu(\mu)} + m_{\nu(\tau)}} \cong \frac{6}{5}. \tag{2.14}$$

$$R(Q=\pm 1) = \frac{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2}{m_e + m_\mu + m_\tau} \cong \frac{3}{2}. \tag{2.15}$$

$$R(Q=\pm \frac{2}{3}) = \frac{\left(\sqrt{m_u} + \sqrt{m_c} + \sqrt{m_t}\right)^2}{m_u + m_c + m_t} \cong \frac{6}{5}. \tag{2.16}$$

$$R(Q=\pm \frac{1}{3}) = \frac{\left(\sqrt{m_d} + \sqrt{m_s} + \sqrt{m_b}\right)^2}{m_d + m_s + m_b} \cong \frac{15}{11}. \tag{2.17}$$

Each of these relationships takes 12 (apparently) independent fermion masses and reduces by 1, their mutual independence. So with (2.14) through (12.17), to first approximation, we have now eight, rather than 12 independent fermion masses.

For some other commonly-studied Koide triplets we have:

$$C(uds) = 0.69290 \cong 1/\sqrt{2}; \quad R(uds) = 1.772105341 \cong 3\sqrt{2}/\left(1+\sqrt{2}\right). \tag{2.18}$$

$$C(ctb) = 1.00939 \cong 1; \quad R(ctb) = 1.492994103 \cong 3/2. \tag{2.19}$$

$$C(usc) = 0.86795; \quad R(usc) = 1.606042302. \tag{2.20}$$

$$C(csb) = 1.02783 \cong 1; \quad R(csb) = 1.479416975 \cong 3/2 \quad (\text{with } -\sqrt{m_s}). \tag{2.21}$$

$$C(dcs) = 0.81520; \quad R(dcs) = 1.652718083. \tag{2.22}$$

We note that the relationship (2.14) for $C(uds) \cong 1/\sqrt{2}$ is accurate to *within experimental errors.* Specifically, given the empirical $m_s = 95 \pm 5 MeV$, (2.14) can be made into an *exact* relationship to ten digits (the accuracy of the up and down masses derived in [2]) if we set $m_s = 98.95303495 MeV$. Of course, even the relationship for the charged leptons is a close but not exact relationship, see the discussion following (1.5), so we ought not expect (2.14) to be exactly $C(uds) = 1/\sqrt{2}$. But, similarly to (1.5), see also (2.10), it may well make sense to regard this as a relationship accurate to the first three or four decimal places, which would improve our knowledge of the strange quark mass by four of five orders of magnitude.

But this main point of the foregoing is not about the specific Koide relationships (though (2.14) through (2.17) are important steps forward in their own right), but about how the ratio parameter *R* which for the charged lepton triplet is $R \cong 3/2$, can be reformulated for *any fermion triplet* into the coefficient *C* in the statistical variance relationship $\sigma(K) = C\langle m_i \rangle$, which, for the charged leptons, is $C \cong 1$. And, as we see in (2.14) through (2.17), this can lead to additional relationships and, indeed, a cascading migration of coefficients.

Turning back to the proton and neutron triplets $\text{diag}(K_P) = \left(\sqrt{m_d}, \sqrt{m_u}, \sqrt{m_u}\right)$ and $\text{diag}(K_N) = \left(\sqrt{m_u}, \sqrt{m_d}, \sqrt{m_d}\right)$ which were so central to obtaining accurate binding energy predictions in [1] and [2], we find using the mass values $m_u = 2.223792405 MeV$ and $m_d = 4.906470335 MeV$ obtained in [2] that:

$$C(p = duu) = 0.0387876019; \quad R(p = duu) = 2.8879821000. \tag{2.19}$$

$$C(n = udd) = 0.0298844997; \quad R(n = udd) = 2.9129480061. \tag{2.20}$$

For these triplets which all have a *small* variance in comparison to the earlier triplets which cross generations, the Koide ratio $R \cong 3$. In the circumstance where the variance is *exactly* zero because all three quarks have the same mass, for example, for the triplets $\Delta^{++} = uuu$ and $\Delta^{-} = ddd$, using the Koide mass relationship for parameterization, we have $C = 0$; $R = 3$.

3. Lagrangian / Energy Reformulation of the Koide Mass Relationship

The appearance of Koide triplets originating from the thesis that Baryons are Yang-Mills magnetic monopoles can be seen, for example, by considering equation [11.2] of [1] for the field strength tensor of a Yang-Mills magnetic monopole containing a triplet of colored quarks in the zero-perturbation limit, reproduced below:

$$\mathrm{Tr}F^{\mu\nu} = -i\left(\frac{\overline{\psi}_R\left[\gamma^\mu {}_\vee \gamma^\nu\right]\psi_R}{\text{"}\not{p}_R - m_R\text{"}} + \frac{\overline{\psi}_G\left[\gamma^\mu {}_\vee \gamma^\nu\right]\psi_G}{\text{"}\not{p}_G - m_G\text{"}} + \frac{\overline{\psi}_B\left[\gamma^\mu {}_\vee \gamma^\nu\right]\psi_B}{\text{"}\not{p}_B - m_B\text{"}} \right). \tag{3.1}$$

If we generalize this to any three fermion wavefunctions ψ_1, ψ_2, ψ_3 such that (3.1) represents the specific case $\psi_1 = \psi_R$, $\psi_2 = \psi_G$ and $\psi_3 = \psi_B$, and, as we did prior to [11.19] of [1], if we consider the circumstance in which the interactions shown in Figure 1 at the start of section 3 in [1] occur essentially at a point, then $\left[\gamma^\mu {}_\vee \gamma^\nu\right] \to \left[\gamma^\mu, \gamma^\nu\right]$ approaches an ordinary commutator, each of the $\not{p} \to 0$, and the "quoted" denominator becomes an ordinary denominator, see [3.9] through [3.12] of [1] for further background. So also setting $m_1 = m_R$, $m_2 = m_G$ and $m_3 = m_B$, (3.1) generalizes for a point interaction to a Koide-style field strength tensor:

$$\mathrm{Tr}F^{\mu\nu} = -i\left(\frac{\overline{\psi}_1\left[\gamma^\mu, \gamma^\nu\right]\psi_1}{m_1} + \frac{\overline{\psi}_2\left[\gamma^\mu, \gamma^\nu\right]\psi_2}{m_2} + \frac{\overline{\psi}_3\left[\gamma^\mu, \gamma^\nu\right]\psi_3}{m_3} \right). \tag{3.2}$$

Then, we form a pure gauge field Lagrangian $\mathfrak{L}_{\text{gauge}} = -\frac{1}{2}Tr\left(F_{\mu\nu}F^{\mu\nu}\right) = -\frac{1}{2}Tr\left(F \cdot F\right)$ as in [11.7] of [1]. As discussed in section 2 of [2], we consider both inner and outer products over the Yang-Mills indexes of *F*, i.e., we consider both $TrF^2 = Tr\left(F_{AB} \cdot F_{BC}\right) = F_{AB} \cdot F_{BA}$ and $Tr\left(F \otimes F\right) = Tr\left(F_{AB} \cdot F_{CD}\right) = F_{AA} \cdot F_{BB}$. Note carefully the different index structures in $F_{AB} \cdot F_{BA}$ versus $F_{AA} \cdot F_{BB}$, and also contrast this to (1.7) and (1.10) in this paper, which is where we are headed at the moment. We then use this Lagrangian to calculate energies according to [11.7] of [1], see also [1.1] of [2], which is reproduced below:

$$E = -\iiint \mathfrak{L}_{\text{gauge}} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{\mu\nu}F^{\mu\nu} d^3x \, . \tag{3.3}$$

In the case where $\psi_1 = \psi_d$, $\psi_2 = \psi_3 = \psi_u$ so that $F^{\mu\nu} = F^{\mu\nu}{}_P$ represents the proton, then depending on whether we contact indexes using $F_{AB} \cdot F_{BA}$ or $F_{AA} \cdot F_{BB}$, we obtain the inner and outer products [2.8] and [2.6] of [2], respectively. When $\psi_1 = \psi_u$, $\psi_2 = \psi_3 = \psi_d$ so $F^{\mu\nu} = F^{\mu\nu}{}_N$ represents the neutron, we obtain the inner and outer products [2.9] and [2.7] of [2], respectively. Using (1.6), the Koide-type generalization of the outer products [2.6] and [2.7] of [2] is:

$$E_\otimes = -\iiint \mathfrak{L}_\otimes d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{\mu\nu} \otimes F^{\mu\nu} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{AB} \cdot F_{CD} d^3x = \tfrac{1}{2}\iiint F_{AA} \cdot F_{BB} d^3x = \tfrac{1}{(2\pi)^{\frac{3}{2}}} K_{AA}K_{BB}$$

$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\left[\begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \right] = \frac{1}{(2\pi)^{\frac{3}{2}}}\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2 \quad ,(3.4)$$

while the Koide generalization of the inner products [2.8] and [2.9] of [2] is:

$$E=-\iiint \mathfrak{L} d^3x=\tfrac{1}{2}\mathrm{Tr}\iiint F_{\mu\nu}F^{\mu\nu}d^3x=\tfrac{1}{2}\mathrm{Tr}\iiint F_{AB}\cdot F_{BD}d^3x=\tfrac{1}{2}\iiint F_{AB}\cdot F_{BA}d^3x=\tfrac{1}{(2\pi)^{\frac{3}{2}}}K_{AB}K_{BA}$$

$$=\frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\left[\begin{pmatrix}\sqrt{m_1} & 0 & 0\\ 0 & \sqrt{m_2} & 0\\ 0 & 0 & \sqrt{m_3}\end{pmatrix}\begin{pmatrix}\sqrt{m_1} & 0 & 0\\ 0 & \sqrt{m_2} & 0\\ 0 & 0 & \sqrt{m_3}\end{pmatrix}\right]=\frac{1}{(2\pi)^{\frac{3}{2}}}(m_1+m_2+m_3) \quad ,(3.5)$$

This means that is now becomes possible to express the Koide relationship (2.9) entirely in terms of energies *E* derived from the general integral (3.3) of a Lagrangian density $\mathfrak{L}=-\tfrac{1}{2}Tr(F\cdot F)$ over d^3x. Specifically, combining (2.9) with (3.4) and (3.5) allows us to write:

$$\frac{E_\otimes}{E}=\frac{\iiint \mathfrak{L}_\otimes d^3x}{\iiint \mathfrak{L} d^3x}=\frac{\mathrm{Tr}\iiint F_{\mu\nu}\otimes F^{\mu\nu}d^3x}{\mathrm{Tr}\iiint F_{\mu\nu}F^{\mu\nu}d^3x}=\frac{\iiint F_{AA}\cdot F_{BB}d^3x}{\iiint F_{AB}\cdot F_{BA}d^3x}=\frac{K_{AA}K_{BB}}{K_{AB}\cdot K_{BA}}$$
$$=\frac{\left(\sqrt{m_1}+\sqrt{m_2}+\sqrt{m_3}\right)^2}{m_1+m_2+m_3}=\frac{3}{1+C}=R \quad . \quad (3.6)$$

This expresses the Koide mass relationship in multiple forms, in terms of the energy integral of a Lagrangian density of the general form $\mathfrak{L}=-\tfrac{1}{2}Tr(F\cdot F)$, with the field strength given by (3.2). This means that for *any* Koide triplet of given empirical *R*, there is an energy E_R which vanishes under the condition:

$$E_R=\iiint(\mathfrak{L}_\otimes-R\mathfrak{L})d^3x=\mathrm{Tr}\iiint\left(F\otimes F-RF^2\right)d^3x=0 . \quad (3.7)$$

This is the Lagrangian / energy formulation of the Koide relationship (2.9), and although different in appearance, it is entirely equivalent. So, for example, using the symbol $\therefore$ as in figure 1 and Table 3 of [5] to represent the three generations of the fermions for any given charge, the four Koide relationships (2.14) through (2.17) for the "pole" (low-probe energy) masses, may be written as:

$$E_{\nu\therefore}=\iiint(\mathfrak{L}_\otimes-\tfrac{6}{5}\mathfrak{L})d^3x=\mathrm{Tr}\iiint\left(F\otimes F-\tfrac{6}{5}F^2\right)d^3x\cong 0 . \quad (3.8)$$

$$E_{e\therefore}=\iiint(\mathfrak{L}_\otimes-\tfrac{3}{2}\mathfrak{L})d^3x=\mathrm{Tr}\iiint\left(F\otimes F-\tfrac{3}{2}F^2\right)d^3x\cong 0 . \quad (3.9)$$

$$E_{u\therefore}=\iiint(\mathfrak{L}_\otimes-\tfrac{6}{5}\mathfrak{L})d^3x=\mathrm{Tr}\iiint\left(F\otimes F-\tfrac{6}{5}F^2\right)d^3x\cong 0 . \quad (3.10)$$

$$E_{d\therefore}=\iiint(\mathfrak{L}_\otimes-\tfrac{15}{11}\mathfrak{L})d^3x=\mathrm{Tr}\iiint\left(F\otimes F-\tfrac{15}{11}F^2\right)d^3x\cong 0 . \quad (3.11)$$

Whether these become *exactly* equal to zero for masses at high-probe energies, and whether there is an underlying action principle involved here, are questions beyond the scope of this paper which are worth consideration.

What ties all of this together, is that we *model* the radial behavior of each fermion using the Gaussian *ansatz* introduced in [9.9] of [1] which is reproduced below with an added label $i=1,2,3$ for each of the fermions and masses in (3.2):

$$\psi_i(r)=u_i(p)\left(\pi\lambda_i^2\right)^{-\frac{3}{4}}\exp\left(-\frac{1}{2}\frac{(r-r_{0i})^2}{\lambdabar_i^{\,2}}\right), \quad (3.12)$$

and that we also relate the reduced Compton wavelength $\lambdabar_i$ to mass m_i via the DeBroglie relation $\lambdabar_i=\hbar/m_ic$, see [1] following [11.18]. This is what makes it possible to precisely calculate the energy in integrals of the form (3.3), specifically making use of the basic Gaussian mathematical relation [9.11] of [1]:

$$\iiint \frac{1}{\pi^{\frac{3}{2}} \lambda\!\!\!^{-3}} \exp\left(-\frac{(r-r_0)^2}{\lambda\!\!\!^{-2}} \right) d^3x = 1, \tag{3.13}$$

and variants thereof. It is (3.12) and (3.13) and $\lambda\!\!\!^{-}_i = 1/m_i$ (in $\hbar = c = 1$ units) which tie everything together when (3.2) is employed in (3.3) through (3.7). And this is what leads to the accurate mass relationship (1.1) and binding energy predictions (1.2) and (1.3), as well as the binding energy predictions for ^{2}H, ^{3}H, ^{3}He and ^{4}He and the proton–neutron mass difference (1.4) developed in [2]. And the final piece which also ties this together, is the empirical normalization for fermion wavefunctions developed in [11.30] of [1], namely:

$$N^4 = \frac{1}{n_f} \frac{(E+m)^2}{(2m)^2} = \frac{1}{24} \frac{(E+m)^2}{(2m)^2}, \tag{3.14}$$

where $n_f = 24$ is the total number of fermions including three colors for each quark.

Now, it is important to emphasize that the Gaussian *ansatz* (3.12) is not a *theory*, but rather, it is a *modeling hypothesis* that allows us to perform the necessary integrations and calculate energies that turn out to correlate very well with empirical data. That is, in explicitly in [1] and implicitly in [2], we *hypothesized* that the fermion wavefunctions can be modeled as Gaussians with specific Compton wavelengths $\lambda\!\!\!^{-}_i = 1/m_i$ defined to match the *undressed, current quark masses*, we performed the integrations in (3.3), and we found that the energies predicted matched empirical binding data to – in most cases – parts per million. This, in turn, tells us that *for the purpose of predicting binding energies*, it is possible to model the *current* quarks as Gaussians (which means they act as free fermions), with masses and wavelengths based on their undressed, current masses, and to thereby obtain empirically-validated results. But, as also discussed at the end of section 11 in [1], this use of undressed mass does *not* apply when it comes predicting the short range of the nuclear interaction which we showed at the end of section 10 in [1] is indeed short range with a standard deviation of $\sigma = \frac{1}{\sqrt{2}} \lambda\!\!\!^{-}$. For, if we use the undressed fermion masses that work so well for binding energies, we find $\lambda\!\!\!^{-}_u \sim 85.65F$ and $\lambda\!\!\!^{-}_d \sim 41.04F$, and the predicted short range is still not short enough. If, however, we turn to the *constituent* quark masses which, at the end of section 11, for estimation, we took to be 939 MeV/3=313 MeV, then we have $\lambda\!\!\!^{-} \sim .63F$ and $\sigma = \frac{1}{\sqrt{2}} \lambda\!\!\!^{-} \sim .45F$, which tells us that the nuclear interaction virtually ceases to be effective at about $4\sigma \approx 3\lambda\!\!\!^{-} \sim 2F$. This is exactly what *is* observed.

In both cases – for nuclear binding energies and for the nuclear interaction short range – we found that the Gaussian *ansatz* (3.12) does yield empirically-accurate results. But for binding energies, it was the undressed, current quark masses which gave us the right results, while for nuclear short range, it was the fully dressed, “constituent” quarks masses that were needed to obtain the correct result. Because we shall momentarily embark on a prediction of the fully dressed rest masses 938.272046 MeV and 939.565379 MeV of the free proton and free neutron, what we learn from this is that while we might also be able to approach these masses using the Gaussian *ansatz* for fermion wavefunctions, we will, however, need to be judicious in the fermions we choose and in the *masses* that we assign to the fermions. That is, the focus of our deliberations will be, not *whether* we can use the Gaussian *ansatz*, but on *how to select the fermions and masses that we <u>do</u> use with the Gaussian ansatz*.

Now, based on all of the foregoing development, let us see how to predict the proton and neutron masses.

4. Predicting the Proton Plus Neutron Mass Sum to within about 6 Parts in 10,000

Because we can connect any Koide matrix products to a Lagrangian via (3.4) and (3.5), let us work directly with the Koide matrix (1.6) to determine how to assign the masses m_1, m_2, m_3 so as to predict the proton and neutron masses. Then, at the end, we can backtrack using the development in section 3 to connect these masses to their associated Lagrangian. In other words, we will first fit the empirical mass data, then we will backtrack to the underlying Lagrangian.

Each of the proton and neutron contains three quarks. The sum of the quark masses is $2m_u + m_d = 9.35405514MeV$ for the proton and $2m_d + m_u = 12.0367331MeV$ for the neutron. For a *free* proton and neutron, none of their rest mass is released as binding energy, and so these quark mass sums are included in $M_P = 938.272046MeV$ and $M_N = 939.565379MeV$ respectively, where we use an uppercase *M* to denote these fully-dressed, observed masses. As demonstrated in sections 11 and 12 of [1] and throughout [2], these rest masses are reduced when the proton and neutron fuse with other nucleons. But for *free* protons and neutrons, the entire rest mass is retained and all of the latent binding energy is used to confine quarks. Using $m_u = 2.223792405MeV$ and $m_d = 4.906470335MeV$ from [9.3] and [9.4] of [2] as earlier introduced after (2.11), this means that the "mass coverings" *m* (using a lowercase *m*) of the proton and neutron, may be calculated to be:

$$m_P \equiv M_P - 2m_u - m_d = 928.9179915MeV\,, \tag{4.1}$$

$$m_N \equiv M_N - 2m_u - m_d = 927.5286457MeV\,. \tag{4.2}$$

That is, these *m* represent the observed, fully-dressed proton and neutron masses *M*, less the sum $K_{AB}K_{BA} = m_1 + m_2 + m_3$ of the current quark masses, with $m_1 \equiv m_d$, $m_2 = m_3 \equiv m_u$ for the proton, and $m_1 \equiv m_u$, $m_2 = m_3 \equiv m_d$ for the neutron, see (1.9). One may think of this as the weight of the rather heavy "clothing" over the bare quarks. The *sum* of these two mass covers is:

$$m_N + m_P = M_N + M_P - 3m_u - 3m_d = 1856.446637MeV\,. \tag{4.3}$$

At the end of section 9 of [2], after deriving the neutron–proton mass difference (1.4), we noted that the individual masses for the proton and neutron could now be obtained by deriving some independent expression related to the *sum* of their masses, and then solving these two simultaneous equations – sum equation and difference equation – for the two target masses – proton and neutron. We shall do exactly that here. In particular, it will be our goal to derive the *sum* $M_N + M_P$ of these two masses, and then use (1.4) as a simultaneous equation to obtain each separate mass. The benefit of this approach using a sum, referring to the so-called mass "toolbox" in [3.11] of [2] and also the discussion of the alpha nuclide following [4.4] of [2], is that in selecting mass terms to consider, we can eliminate any candidates that are not absolutely symmetric under $p \leftrightarrow n$ and $u \leftrightarrow d$ interchange, because the sum $M_N + M_P$ contains three up quarks and three down quarks, as well as one proton and one neutron. The empirical target, therefore, is $M_N + M_P = 1877.837425MeV$, or, alternatively, $m_P + m_N = 1856.446637MeV$ from (4.3). This is what we seek to predict.

Now let us return to the "clues" we laid out in [3.6] through [3.8] of [5]. Let us start in the simplest way possible by focusing our consideration on [3.8] of [5], reproduced below, but multiplied by a factor of 2 and separated into $\sqrt[4]{vm_u}$ and $\sqrt[4]{vm_d}$ in the second term:

$$2\sqrt{v\cdot\sqrt{m_u m_d}} = 2\sqrt[4]{vm_u}\sqrt[4]{vm_d} = 1803.670518\text{ MeV}\,. \tag{4.4}$$

Here, v_F=246.219651 GeV is the Fermi vev. Because, this is about 3% smaller than $m_P + m_N$ in (4.3) and it is closer to $m_P + m_N$ than either [3.6] or [3.7] of [5], and it is symmetric under $u \leftrightarrow d$ interchange, we shall see if (4.4) can be used, by itself, to provide the foundation for reaching the target. As we shall, see, with $m_P + m_N$ in (4.3) as the target, it can be so used!

Now, in (3.11) of [2], we developed a "toolkit" of masses which we used for calculating several binding and fusion release energies with very close precision. We shall wish to add to this toolkit here, and in particular, will wish to refine our use of the Fermi vev v_F=246.219651 GeV beyond what is shown in (4.4). Specifically, as noted after [3.8] of [5], we need to put (4.4) "and like expressions into the right context and obtain the right coefficients. And where do such coefficients come from? The generators of a GUT!" Now, we need to use the GUT we developed in (4.4) to obtain the needed coefficients needed to bring (4.4) closer to the target mass of $1856.446637 MeV$ in (4.3). Because the vev that seems bring us into the correct "ballpark" is the Fermi vev, we focus on the electroweak symmetry breaking which occurs at the Fermi vev, and which, in [8.2] of [5], is specified by:

$$\text{diag}\left(\Phi_F\right) = \text{diag}\left(T^i\varphi_{iF}\right) \equiv v_F\left(0,\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3},-1,-\tfrac{1}{3},\tfrac{2}{3},\tfrac{2}{3}\right) = v_F\text{diag}Q\,. \tag{4.5}$$

For the proton with a fermion triplet $\left(d,u,u\right)$, the corresponding eigenvalue entries in (4.5) above are $\left(-\tfrac{1}{3}v_F,\tfrac{2}{3}v_F,\tfrac{2}{3}v_F\right)$. For the neutron and its $\left(u,d,d\right)$ triplet, the entries are $\left(\tfrac{2}{3}v_F,-\tfrac{1}{3}v_F,-\tfrac{1}{3}v_F\right)$. We now wish to use these to establish respective Koide triplet matrices for the proton and neutron.

Looking at (4.4) and the vacuum triplets $\left(-\tfrac{1}{3}v_F,\tfrac{2}{3}v_F,\tfrac{2}{3}v_F\right)$ and $\left(\tfrac{2}{3}v_F,-\tfrac{1}{3}v_F,-\tfrac{1}{3}v_F\right)$, we see that to obtain the proper mass dimension of the terms with $\sqrt[4]{vm_u}$ and $\sqrt[4]{vm_d}$ and use these as Koide triplets, we will need to take the fourth roots of these triplets. Let us do exactly that, and pair these triplets with the mass triplets $\left(m_d,m_u,m_u\right)$ and $\left(m_u,m_d,m_d\right)$, for which we also take the fourth root. Thus, we define two Koide triplets, one for the proton and one for the neutron:

$$K_{AB}(P) \equiv \begin{pmatrix} \sqrt[4]{-\tfrac{1}{3}vm_d} & 0 & 0 \\ 0 & \sqrt[4]{\tfrac{2}{3}vm_u} & 0 \\ 0 & 0 & \sqrt[4]{\tfrac{2}{3}vm_u} \end{pmatrix} = \begin{pmatrix} i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} & 0 & 0 \\ 0 & \sqrt[4]{\tfrac{2}{3}vm_u} & 0 \\ 0 & 0 & \sqrt[4]{\tfrac{2}{3}vm_u} \end{pmatrix}, \tag{4.6}$$

$$K_{AB}(N) \equiv \begin{pmatrix} \sqrt[4]{\tfrac{2}{3}vm_u} & 0 & 0 \\ 0 & \sqrt[4]{-\tfrac{1}{3}vm_d} & 0 \\ 0 & 0 & \sqrt[4]{-\tfrac{1}{3}vm_d} \end{pmatrix} = \begin{pmatrix} \sqrt[4]{\tfrac{2}{3}vm_u} & 0 & 0 \\ 0 & i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} & 0 \\ 0 & 0 & i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} \end{pmatrix}. \tag{4.7}$$

We see that because of the negative charge of the down quark, each of these triplets contains $i^{.5} = \tfrac{1}{\sqrt{2}}\left(1+i\right)$ which is a complex number. In recent years, consideration has been given to having *negative* square root terms in Koide mass relations, see for example (2.16) in which one uses $-\sqrt{m_s}$ to derive a close relation for the (csb) triplet. The above, (4.6) and (4.7) take this a step further, because they raise the specter of triplets with *complex* square roots! In the next section we shall explore the implication of these complex components, which arise from the opposite charges of the up and down quarks. But for the moment, let us ignore $i^{.5}$ in the above so we can look at magnitudes only, and let us form the Koide matrix product with $i^{.5}$ excised:

$$K_{AB}(P)K_{BA}(N) = Tr\left[\begin{pmatrix} \sqrt[4]{\frac{1}{3}vm_d} & 0 & 0 \\ 0 & \sqrt[4]{\frac{2}{3}vm_u} & 0 \\ 0 & 0 & \sqrt[4]{\frac{2}{3}vm_u} \end{pmatrix}\begin{pmatrix} \sqrt[4]{\frac{2}{3}vm_u} & 0 & 0 \\ 0 & \sqrt[4]{\frac{1}{3}vm_d} & 0 \\ 0 & 0 & \sqrt[4]{\frac{1}{3}vm_d} \end{pmatrix}\right]. \tag{4.8}$$

$$= 3\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} = \mathbf{1857.570635\ MeV}$$

Comparing to (4.3) which tells us that $m_P + m_N = \mathbf{1856.446637\ MeV}$ we see that we have hit the target to within about 0.06%! That is:

$$\frac{K_{AB}(P)K_{BA}(N)}{\left(m_N + m_P\right)_{\text{Observed}}} = \frac{\mathbf{1857.570635\ MeV}}{\mathbf{1856.446637\ MeV}} = \mathbf{1.000605457}\,! \tag{4.9}$$

This is extremely close, and in particular, we now see that to within about 6 parts in 10,000, the sum of the proton and neutron masses may be expressed completely as a function of the up and down quark masses and the Fermi vev! So if we use this close relationship to hypothesize that a meaningful relationship is given by $\left(m_N + m_P\right)_{\text{Predicted}} \cong K_{AB}(P)K_{BA}(N)$, then using the above with (4.3), we now see that to within about 0.06%:

$$M_N + M_P = m_N + m_P + 3m_u + 3m_d \cong 3\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} + 3m_u + 3m_d\,. \tag{4.10}$$

This expression is symmetric under under $p \leftrightarrow n$ and $u \leftrightarrow d$ interchange, and as a sum of the proton and a neutron mass, it is formed by taking the *inner* product $K_{AB}(P)K_{BA}(N)$ between a Koide proton matrix $\text{diag}(K_P) = \left(\sqrt[4]{\frac{1}{3}vm_d}, \sqrt[4]{\frac{2}{3}vm_u}, \sqrt[4]{\frac{2}{3}vm_u}\right)$ which employs electric charge and mass magnitudes for one down and two up quarks, and $\text{diag}(K_N) = \left(\sqrt[4]{\frac{2}{3}vm_u}, \sqrt[4]{\frac{1}{3}vm_d}, \sqrt[4]{\frac{1}{3}vm_d}\right)$ which is a Koide neutron matrix employing electric charge and mass magnitudes for one up and two down quarks.

Furthermore, if we divide (4.8) by 2, we see that:

$$K_{AB}(P)K_{BA}(N)/2 = \frac{3}{2}\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} = 928.7853174\,MeV\,. \tag{4.11}$$

This actually falls *between* $m_P = 928.9179915\,MeV$ and $m_N = 927.5286457\,MeV$ from (4.1) and (4.2), and so (4.10) clearly appears to be the correct expression for the leading terms in the proton and neutron masses. Based on this close concurrence and "threading the needle" between the proton and neutron masses with (4.11), we now regard (4.10) as a *meaningful* (rather than coincidental) close expression for $M_P + M_N$ to 0.06%, and embark in the next section on the task of overcoming this final 0.06% and developing an *exact* expression for $M_P + M_N$. Then, we shall be able to use that in combination with the $M(n) - M(p)$ difference (1.4) to specify the proton and neutron masses individually, as a function of the up and down quark masses. That, in turn, will enable us to return to relationships such as [7.1], [7.3], [7.5] and [7.6] in [2] which still contain the proton and neutron masses, and express the nuclear weights of these composite ^{2}H, ^{3}H, ^{3}He and ^{4}He nuclides, and others developed in the future, strictly in terms of quark masses.

Now, let us see how to close the remaining 0.06% gap.

5. Exact Characterization of the Proton and Neutron Masses, with a Phase Parameter *δ*

In (4.8) we neglected the factors of $i^{.5} = \frac{1}{\sqrt{2}}(1+i)$ in order to examine the magnitude of the predicted $m_P + m_N$. If we now restore this factor, (4.8) becomes:

$$K_{AB}(P)K_{BA}(N) = Tr\left[\begin{pmatrix} i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} & 0 & 0 \\ 0 & \sqrt[4]{\tfrac{2}{3}vm_u} & 0 \\ 0 & 0 & \sqrt[4]{\tfrac{2}{3}vm_u} \end{pmatrix}\begin{pmatrix} \sqrt[4]{\tfrac{2}{3}vm_u} & 0 & 0 \\ 0 & i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} & 0 \\ 0 & 0 & i^{.5}\sqrt[4]{\tfrac{1}{3}vm_d} \end{pmatrix}\right]. \quad (5.1)$$

$$= 3\frac{1}{\sqrt{2}}(1+i)\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} = \frac{1}{\sqrt{2}}(1+i)\cdot 1857.570635\, MeV$$

Having now found the right magnitude, we could make use of a $\sqrt{2}$ factor and continue to match the empirical data by writing $\sqrt{2}\,\mathrm{Re}\left(K_{AB}(P)K_{BA}(N)\right) \cong m_P + m_N$. But this just sidesteps some questions because a) it introduces a substantial imaginary component to this mass sum and b) it does not help us past the 0.06% difference that still remains. Let us therefore deal with the $i^{.5} = \frac{1}{\sqrt{2}}(1+i)$ factor a little differently.

Instead, let us write this factor $i^{.5} = \frac{1}{\sqrt{2}}(1+i)$ in terms of a phase δ such that:

$$i^{.5} = \tfrac{1}{\sqrt{2}}(1+i) = \exp(i\delta) = \cos\delta + i\sin\delta; \quad \delta = \pi/4. \quad (5.2)$$

So now, we write the proton plus neutron mass sum (5.1) as:

$$m_N + m_P = K_{AB}(P)K_{BA}(N) = 3\exp(i\delta)\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} = \exp(i\delta)\cdot 1857.570635\, MeV, \quad (5.3)$$

with $\delta = \pi/4$. Now, let us rotate the proton and neutron masses to a different phase $\delta \to \delta'$ such that the real part of the rotated mass sum is exactly equal to $m_P + m_N = 1856.446637 MeV$ from (4.3). In other words, we rotate the phase such that:

$$\begin{aligned}(m_N + m_P)' &= K'_{AB}(P)K'_{BA}(N) = 3\exp(i\delta')\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} = \exp(i\delta')\cdot 1857.570635\, MeV \\ &= (\cos\delta' + i\sin\delta')\cdot 1857.570635\, MeV \equiv \mathbf{1856.446637 MeV} + i\sin\delta'\cdot 1857.570635\, MeV\end{aligned} \quad .(5.4)$$

Above, we have highlighted the *empirical* value in bold type. This means that δ' will, for now, need to be an *empirical* phase parameter *defined* such that:

$$\cos\delta' = \frac{1856.446637 MeV}{1857.570635\, MeV} = 0.9993949098. \quad (5.5)$$

It is fortunate, and a further indication that (4.10) is indeed a meaningful and not merely a coincidental relationship, that the predicted $m_N + m_P$ is *larger* than the empirical $m_N + m_P$, because this satisfies the required constraint that $\cos\delta' \le 1$. Had the prediction been *smaller*, then we would have $\cos\delta' > 1$, which is mathematically impossible without using imaginary arguments to convert over to hyperbolic cosines. So from (5.5), we readily deduce:

$$\delta' = 0.0347894\ \text{rad} = 1.9932858^0 . \tag{5.6}$$

Consequently, we have $\sin\delta = 0.034782383$, so that (5.4) becomes:

$$\left(m_N + m_P\right)' = 1856.446637 MeV + i \cdot 64.61073342\ MeV . \tag{5.7}$$

The real part of this expression, by design, now precisely matches the empirical data.

Now let us solve the simultaneous equations (5.4) and (1.4) to obtain the *separate* masses of the proton and neutron. First, as in (4.10), we add the quark masses back in to (5.4), and also we remove the primes from (5.4) and thus establish $\delta = 1.9932858^0$ rather than the original $\delta = \pi / 4$ (see (5.2)) as the unprimed phase angle. Thus, we recast (5.4) as:

$$M_N + M_P = 3\exp(i\delta) \cdot \sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}} + 3m_u + 3m_d . \tag{5.8}$$

Now, we simultaneously solve (5.8) and (1.4) to obtain the separate expressions:

$$M_N = \frac{1}{2}\left(3\exp(i\delta) \cdot \sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}} + 4m_u + 3m_d - \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}} \right). \tag{5.9}$$

$$M_P = \frac{1}{2}\left(3\exp(i\delta) \cdot \sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}} + 2m_u + 3m_d + \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}} \right). \tag{5.10}$$

Now, as a double check, let us solve for the phase using the separate, *observed* masses of the neutron and proton in each of (5.9) and (5.10). Using Γ to represent the magnitude of the imaginary term that arises from the phase, respectively, we first obtain:

$$\begin{aligned} M_N &= 939.5653788 MeV + i\Gamma_N \\ &= \frac{1}{2}\left(3(\cos\delta + i\sin\delta) \cdot \sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}} + 4m_u + 3m_d - \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}} \right). \end{aligned} \tag{5.11}$$

$$\begin{aligned} M_P &= 938.2720466 MeV + i\Gamma_P \\ &= \frac{1}{2}\left(3(\cos\delta + i\sin\delta) \cdot \sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}} + 2m_u + 3m_d + \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}} \right). \end{aligned} \tag{5.12}$$

Then we segregate out the real terms and reorder to write and calculate:

$$\cos\delta = \frac{939.5653788 MeV - \frac{1}{2}\left(4m_u + 3m_d - \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}} \right)}{\frac{3}{2}\sqrt[4]{\frac{2}{9}} \sqrt{v \cdot \sqrt{m_u m_d}}} = 0.9993949098 . \tag{5.13}$$

$$\cos\delta = \frac{938.2720466MeV - \frac{1}{2}\left(2m_u + 3m_d + \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}}\right)}{\frac{3}{2}\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}}} = 0.9993949098. \qquad (5.14)$$

Because (1.4) for the neutron–proton mass difference has been defined as an exact relationship with all other masses adjusted to ensure this, see section 9 of [2], the numerical value of $\cos\delta$ is exactly the same in each of the above, and also matches the magnitude calculated in (5.5). So our check confirms that all of the calculations fit together correctly.

Now, using $\sin\delta = 0.034782383$, see just after (5.6), the magnitudes Γ of the imaginary portion of (5.11) and (5.12) may be calculated via:

$$\Gamma_N = \frac{1}{2}\left(3\sin\delta\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} + 4m_u + 3m_d - \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}}\right) = 43.6474269MeV, \quad (5.15)$$

$$\Gamma_P = \frac{1}{2}\left(3\sin\delta\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} + 2m_u + 3m_d + \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}}\right) = 42.3540947MeV. \quad (5.16)$$

Note that the difference $\Gamma_N - \Gamma_P = 1.2933322MeV$ is also the same as the neutron minus proton mass difference $M_N - M_P$. Finally, using $\delta = 0.0347894$ rad from (5.6), our theoretical expressions (5.11) and (5.12) for the proton and neutron masses become:

$$\begin{aligned} M_N &= \frac{1}{2}\left(3\exp(i\cdot 0.0347894)\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} + 4m_u + 3m_d - \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}}\right), \qquad (5.17)\\ &= (939.5653788 + i43.6474269)MeV \end{aligned}$$

$$\begin{aligned} M_P &= \frac{1}{2}\left(3\exp(i\cdot 0.0347894)\cdot\sqrt[4]{\frac{2}{9}}\sqrt{v\cdot\sqrt{m_u m_d}} + 2m_u + 3m_d + \frac{3m_d + 2\sqrt{m_\mu m_d} - 3m_u}{(2\pi)^{\frac{3}{2}}}\right), \qquad (5.18)\\ &= (938.2720466 + i42.3540947)MeV \end{aligned}$$

with $\delta = 0.0347894 \text{ rad} = \pi/90.3031564$ being an *empirical* parameter, and with the real part of the masses matching precisely what is observed. Now let us discuss some of the implications of these results.

6. Vacuum-Amplified (Constituent) Quark Masses, Meaning of the Phase Parameter *δ*, and the Lagrangian Formulation of the Proton Plus Neutron Mass Sum

The expressions (5.17), (5.18) for the neutron and proton masses are exact expressions, but in order to close the 0.06% gap between theory and experiment that emerged at the end of section 4, we were required to utilize a phase angle with an *empirically, not theoretically-obtained* value of $\cos\delta = 0.9993949098$, which, because $1/\cos\delta = 1.000605457$, essentially represents and closes this 0.06% gap. So while we are able to *predict* the proton plus neutron mass sum to 6 parts in 10,000, we needed an empirical parameter δ to represent and close the balance of this gap. This also has the consequence of introducing an imaginary component for each of the proton and neutron masses. Thus, we should discuss this phase angle, and try to gain some sense as to how this phase might itself be understood from a theoretical viewpoint.

Theoretically, of course, this phase angle is not pasted on in any way, but naturally results from the fact that the down quark has a negative electric charge (or more accurately, a charge that is oppositely-signed from that of the up quark), and so is a consequence of Koide matrices (4.6) and (4.7) with complex components generalized via $i^{.5} = \frac{1}{\sqrt{2}}(1+i) \rightarrow \exp(i\delta)$ which are associated with the down quark mass. So the *existence* of this phase angle has a fully theoretical basis, but its actual value $\cos\delta = 0.9993949098$ is what is empirical and so requires close consideration. Especially, the question arises, is this phase angle a *new* parameter, or is related in some way to a phase parameter that is *already known to exist* elsewhere in elementary particle physics? The obvious candidate for an *already existing phase* is the phase angle that is responsible for CP violation, and which arises in the Cabibbo-type mixing of quarks and leptons for three generations. Might there be some definitive relationship between the phase uncovered here, and this phase that arises from generational mixing, so that they are really one and the same in different guises? Certainly, economy would suggest that this question be explored.

While it is beyond the scope of this paper to fully explore this question, let us at least explore its plausibility. We start with the Koide matrices (4.6) and (4.7) from whence this phase originated, use the replacement $i^{.5} = \frac{1}{\sqrt{2}}(1+i) \rightarrow \exp(i\delta) = \sqrt[4]{\exp(4i\delta)}$, and connect this, in turn, to the generalized Koide matrix (1.6) to write:

$$K_{AB}(P) = \begin{pmatrix} \sqrt[4]{\frac{1}{3}\exp(4i\delta)vm_d} & 0 & 0 \\ 0 & \sqrt[4]{\frac{2}{3}vm_u} & 0 \\ 0 & 0 & \sqrt[4]{\frac{2}{3}vm_u} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix}. \tag{6.1}$$

$$K_{AB}(N) = \begin{pmatrix} \sqrt[4]{\frac{2}{3}vm_u} & 0 & 0 \\ 0 & \sqrt[4]{\frac{1}{3}\exp(4i\delta)vm_d} & 0 \\ 0 & 0 & \sqrt[4]{\frac{1}{3}\exp(4i\delta)vm_d} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix}. \tag{6.2}$$

Let us then develop a correspondence between these, and the generalized Koide matrix (1.6) as applied in (1.7) and (1.8) to obtain nuclear binding energies, namely:

$$K_{AB}(P) = K_{AB} = \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \tag{6.3}$$

$$K_{AB}(N) = K_{AB} = \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \tag{6.4}$$

Comparing (6.1) with (6.3) we see the correspondence ($\Leftrightarrow$):

$$m_1 = \sqrt{\tfrac{1}{3}\exp(4i\delta)vm_d} \Leftrightarrow m_d$$
$$m_2 = m_3 = \sqrt{\tfrac{2}{3}vm_u} \Leftrightarrow m_u \quad . \tag{6.5}$$

Similarly, comparing (6.2) with (6.4):

$$m_1 = \sqrt{\tfrac{2}{3}vm_u} \Leftrightarrow m_u$$
$$m_2 = m_3 = \sqrt{\tfrac{1}{3}\exp(4i\delta)vm_d} \Leftrightarrow m_d \quad . \tag{6.6}$$

This leads to several points. First, let us *define* what we shall refer to as the "vacuum-amplified" masses M_u and M_d for the up and down quarks according to:

$$M_u \equiv \sqrt{\tfrac{2}{3}vm_u} = 604.1751345MeV \Leftrightarrow m_u \, . \tag{6.7}$$

$$M_d \equiv \sqrt{\tfrac{1}{3}vm_d} = 634.5784463MeV \Leftrightarrow m_d \, . \tag{6.8}$$

These should be compared to [3.6] and [3.7] of [5], which in the above have now acquired the desired coefficients based on the magnitude of their electric charges. With this, we start to use the other two "clues" that we left in [3.6] to [3.8] of [5]. Now we make use of (6.7) and (6.8) to rewrite (6.1) and (6.2) as:

$$K_{AB}(P) = \begin{pmatrix} \exp i\delta\sqrt{M_d} & 0 & 0 \\ 0 & \sqrt{M_u} & 0 \\ 0 & 0 & \sqrt{M_u} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix}, \tag{6.9}$$

$$K_{AB}(N) = \begin{pmatrix} \sqrt{M_u} & 0 & 0 \\ 0 & \exp(i\delta)\sqrt{M_d} & 0 \\ 0 & 0 & \exp(i\delta)\sqrt{M_d} \end{pmatrix} \equiv \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix}. \tag{6.10}$$

So we see that indeed, the Koide matrices now have *complex* components, associated in particular with the vacuum-amplified down quark masses. This takes yet another step in the development of the Koide relationships, by introducing complex mass square roots.

Second, the correspondences in (6.7) and (6.8) may now be written as:

$$\exp(2i\delta)\sqrt{\tfrac{1}{3}vm_d} = \exp(2i\delta)M_d \Leftrightarrow m_d$$
$$\sqrt{\tfrac{2}{3}vm_u} = M_u \Leftrightarrow m_u \quad . \tag{6.11}$$

So the vacuum-amplified down quark mass with the correspondence $\exp(2i\delta)M_d \Leftrightarrow m_d$ now carries a phase. But in the Cabibbo mixing scheme for quarks with three colors $C = R, G, B$, the down quarks mix according to [7.14] of [5], namely:

$$d'_{\mathbb{C}} = \begin{pmatrix} d'_C \\ s'_C \\ b'_C \end{pmatrix} \equiv \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & -c_1s_2c_3 - c_2s_3e^{i\delta} & -c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix}_q \begin{pmatrix} d_C \\ s_C \\ b_C \end{pmatrix} = U_q d_{Ci} \, . \tag{6.12}$$

This is of course one of several mixing parameterizations and is well-known. But in [5], the author derived the very *existence* of three generations and their mixing on a strictly theoretical foundation based on the GUT symmetry breaking of an SU(8) octuplet of fermions which removes two degrees of freedom which are then used for the horizontal freedom of three generations. While this particular parameterization (6.12) places all of the phase into the strange and bottom quarks s'_C and b'_C, this is simply one representation, and it also uses the convention in which all of the mixing occurs for the lower, versus the upper, members of the weak isospin doublets (u,d). Additionally, as to the up and down quarks, what matters is not that the down quark has a negative charge and the up quark a positive charge, but the fact that these have *opposite* charges. That is what injects the complex phase into (6.9) and (6.10). The sign is a matter of convention, and were we to reverse the sign convention, the vacuum-amplified up quark masses would instead be the ones with an associated complex phase.

So, speaking in representation and convention-independent language, each of the observed quark wavefunctions (and each of the lepton wavefunctions given that the neutrino mass is not *exactly* equal to zero) do carry a complex component in their *wavefunction* which emanates from a phase that is indicative of weak CP violation. This phase should then be expected to appear in the *masses* of the fermions as well. And, with that being the case, when using conventions which reflect the phase in the down quarks (and elsewhere), one would expect these phases to make their way through into the vacuum-amplified down quark mass, which appears as $\exp(2i\delta)M_d \Leftrightarrow m_d$ in the Koide matrices. So it is indeed plausible to expect that the phase δ developed here, will come to be understood as bearing a direct and precise relationship to the weak CP-violation phase, and thus is not a *new* parameter, but a *known* parameter stumbled upon via the independent line of inquiry that we have developed here to obtain the proton and neutron masses on a fully theoretical basis. In short, the phase we came upon here, may well be an indication of CP violation arrived at from a completely different direction.

Third, we note via (6.7) that $M_u/2 = 302.0875673 MeV$; $M_d/2 = 317.2892232 MeV$. So if we add these two numbers and multiply by 3, we find $3(M_u + M_d)/2 = 1858.130371 MeV$. This is not far from the empirical $m_N + m_P = 1856.446637 MeV$ of (4.3) or the predicted $m_N + m_P = 1857.570635 MeV$ of (4.8). This suggests that (6.8) may be directly related to the *constituent* masses of the up and down quarks which specify how much of the proton and neutron masses arise from the down quarks and their interactions with the vacuum, versus from the up quarks and their interactions with the vacuum.

Finally, we as noted at the start of section 4, we can connect *any* Koide matrix products to a Lagrangian via (3.4) and (3.5). Now that we have obtained a theoretical expression for the proton and neutron masses up to a phase that looks to be related to the CP-violating phase of weak interactions among three generations, it is time to backtrack using the development in section 3 to connect these masses to their associated Lagrangian expression.

To start, we use (6.9) and (6.10) together with (6.7) and (6.8) and (5.8) to write:

$$\begin{aligned} K_{AB}(P)K_{BA}(N) &= \mathrm{Tr}\begin{pmatrix} \exp i\delta\sqrt{M_d} & 0 & 0 \\ 0 & \sqrt{M_u} & 0 \\ 0 & 0 & \sqrt{M_u} \end{pmatrix}\begin{pmatrix} \sqrt{M_u} & 0 & 0 \\ 0 & \exp(i\delta)\sqrt{M_d} & 0 \\ 0 & 0 & \exp(i\delta)\sqrt{M_d} \end{pmatrix} \\ &= 3\exp i\delta\sqrt{M_u M_d} = 3\exp i\delta\sqrt{\sqrt{\tfrac{2}{3}vm_u}\sqrt{\tfrac{1}{3}vm_d}} = 3\exp i\delta\cdot\sqrt[4]{\tfrac{2}{9}}\sqrt{v\sqrt{m_u m_d}} \qquad (6.13) \\ &= M_N + M_P - 3m_u - 3m_d \end{aligned}$$

Referring to (3.4) and (3.5), this means that we can write the mass sum $m_N + m_P$ as:

$$
\begin{aligned}
(2\pi)^{\frac{3}{2}}\mathcal{E} &= -(2\pi)^{\frac{3}{2}}\iiint \mathfrak{L}d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\,\mathrm{Tr}\iiint \mathfrak{F}_{\mu\nu}(P)\mathfrak{F}^{\mu\nu}(N)d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\,\mathrm{Tr}\iiint \mathfrak{F}_{AB}(P)\cdot\mathfrak{F}_{BD}(N)d^3x \\
&= \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\iiint \mathfrak{F}_{AB}(P)\cdot\mathfrak{F}_{BA}(N)d^3x = K_{AB}(P)K_{BA}(N) \\
&= \mathrm{Tr}\left[\begin{pmatrix} \exp i\delta\sqrt{M_d} & 0 & 0 \\ 0 & \sqrt{M_u} & 0 \\ 0 & 0 & \sqrt{M_u} \end{pmatrix}\begin{pmatrix} \sqrt{M_u} & 0 & 0 \\ 0 & \exp(i\delta)\sqrt{M_d} & 0 \\ 0 & 0 & \exp(i\delta)\sqrt{M_d} \end{pmatrix}\right] \\
&= M_N + M_P - 3m_u - 3m_d = m_N + m_P
\end{aligned} \tag{6.14}
$$

by introducing two new field strength tensors defined in the manner of (3.2), namely:

$$\mathrm{Tr}\mathfrak{F}^{\mu\nu}(P) \equiv -i\left(\frac{\overline{\Psi}_d\left[\gamma^\mu,\gamma^\nu\right]\Psi_d}{M_d} + 2\frac{\overline{\Psi}_u\left[\gamma^\mu,\gamma^\nu\right]\Psi_u}{M_u}\right), \tag{6.15}$$

$$\mathrm{Tr}\mathfrak{F}^{\mu\nu}(N) \equiv -i\left(\frac{\overline{\Psi}_u\left[\gamma^\mu,\gamma^\nu\right]\Psi_u}{M_u} + 2\frac{\overline{\Psi}_d\left[\gamma^\mu,\gamma^\nu\right]\Psi_d}{M_d}\right). \tag{6.16}$$

Above, the "vacuum-amplified" masses M_u and M_d are defined as in (6.7) and (6.8), and Ψ_u and Ψ_d represent wavefunctions for the vacuum-amplified up and down quarks (which as noted just above should bear a relationship to the so-called "constituent quark" wavefunctions). Lastly, we apply the Gaussian *ansatz* (3.12), in the form:

$$\Psi_u(r) = U\left(\pi\lambda\!\!\!^{-}{}_u^{\,2}\right)^{-\frac{3}{4}}\exp\left(-\frac{1}{2}\frac{\left(r-r_{0u}\right)^2}{\lambda\!\!\!^{-}{}_u^{\,2}}\right), \tag{6.17}$$

$$\Psi_d(r) = D\left(\pi\lambda\!\!\!^{-}{}_d^{\,2}\right)^{-\frac{3}{4}}\exp\left(-\frac{1}{2}\frac{\left(r-r_{0d}\right)^2}{\lambda\!\!\!^{-}{}_d^{\,2}}\right), \tag{6.18}$$

and for the reduced Compton wavelengths $\lambda\!\!\!^{-}$, we also make use M_u and M_d defined in (6.7) and (6.8), to specify:

$$\lambda\!\!\!^{-}{}_u = \hbar / M_u c = 1/M_u, \tag{6.19}$$

$$\lambda\!\!\!^{-}{}_d = \hbar / M_d c = 1/M_d. \tag{6.20}$$

So, referring back to the discussion at the end of section 3, as was the case with the short range of the nuclear interaction, we can indeed use the Gaussian *ansatz* to model fermion wavefunctions as Gaussians and obtain the fully-dressed proton and neutron masses. But to do so, we are not using the undressed "current" quarks which yielded binding energies in [1] and [2], but are instead using vacuum-amplified quark wavefunctions and masses and wavelengths associated with the fully-dressed, *constituent* quark masses, which also are responsible for yielding the correct magnitude of the short range of nuclear interactions. So here too, it is not a question of *whether* we can use a Gaussian *ansatz*, but rather, it is a question of *which* wavefunctions with *which* masses and wavelengths we need to use in the Gaussian *ansatz*, in order to obtain a precise concurrence with empirical data. So, insofar as fully covered protons and neutrons are concerned, it looks as if the *constituent* quarks are behaving as free fermions, in contrast to when we derived nuclear binding energies for which the *current* quarks behave as free fermions. This underscores the role of the Gaussian *ansatz* as a modeling tool use to derive effective concurrence with empirical data, rather than as a part of the theory per se. The theory is centered on baryons being Yang-Mills magnetic monopoles, and nucleons which release or retain binding energies based on a their resonant properties which in turn depend upon the current quark content of those nucleons. For calculations which involve the components and emissions of protons and neutrons such as their quarks and their binding energies, the *current* quarks can be modeled as free fermions to obtain empirically-accurate results. For other calculations which involve the bulk behavior of protons and neutrons, accurate results may be obtained by modeling *constituent* quarks as free fermions.

7. Conclusion

We have shown how the Koide relationships and associated triplet mass matrices can be generalized to derive the observed sum of the free proton and neutron rest masses in terms of the up and down current quark masses and the Fermi vev to six parts in 10,000, which *sum* can then be solved for the separate neutron and proton masses using the neutron–proton mass *difference* earlier derived in [2]. The opposite charges of the up and down quarks are responsible for the appearance of a complex phase $\exp(i\delta)$ which in turn can be used to adjust these mass relationships to unlimited accuracy. For the moment, phase angle $\delta=1.9932858^0$ is an empirical parameter, but it does appear to be possibly related to the CP-violating phase of weak interactions for three fermion generations. The Koide generalizations developed here enable these proton and neutron mass relationships to be given a Lagrangian formulation based on proton and neutron field strength tensors that contain constituent quark wavefunctions and masses. In the course of development, we have also uncovered new Koide relationships for the neutrinos, the up quarks, and the down quarks.

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