Colored and electrically charged gauge bosons and their related quarks

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We propose a model of baryon and lepton number conserving interactions in which the two states of a quark, a colored and electrically charged state and a colorless and electrically neutral state, can transform into each other through the emission or absorption of a colored and electrically charged gauge boson. A novel feature of the model is that the colorless and electrically neutral quarks carry away the missing energy in decay processes as do neutrinos.

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I. INTRODUCTION

The known gauge bosons can be classified according to their color charge and electric charge into four types:

1. Colorless and electrically neutral: photon and Z^0.
2. Colorless and electrically charged: $W^{\pm}$.
3. Colored and electrically neutral: gluons.
4. Colored and electrically charged: X and Y bosons.

All the known gauge bosons of the first, second and third types mediate the baryon and lepton number conserving interactions, whereas the X and Y bosons of the fourth type, i.e., the colored and electrically charged gauge bosons, mediate the baryon and lepton number violating interactions [1, 2].

The question arises as to whether there exist the colored and electrically charged gauge bosons which mediate the baryon and lepton number conserving interactions like the known gauge bosons of the first, second and third types. In this paper, the possibility is explored of existence of such colored and electrically charged gauge bosons.

In section II, we consider the various cases of transitions of a quark from one state to another with the emission or absorption of a colored and electrically charged gauge boson. For the description of the transitions with which we are to deal, we postulate the existence of the colorless and electrically neutral quarks. In section III, we discuss the properties of the colorless and electrically neutral quarks. In section IV, we construct a model of gauge invariant Lagrangian which involves the newly introduced quark and gauge boson fields. In section V, we show the application of the model to the problem of the measured $K^+ \to \pi^+ +$ 'missing energy' branching ratio.

We shall hereafter denote by $p_c^{c'}$ the state of a particle p, of which the color charge and electric charge are c and $c'e$ respectively, and denote by C_0 colorless. For any color c, the relation $c+\bar{c} = C_0$, $\bar{c}$ being the anti-c, holds .

II. QUARKS AND GAUGE BOSONS

Let us consider a transition of a quark from a colored and electrically charged state $q_{c_1}^{e_1}$ to another state $q_{c_2}^{e_2}$ with the emission of a colored and electrically charged gauge boson $b_{c_b}^{e_b}$, $q_{c_1}^{e_1} \to q_{c_2}^{e_2} + b_{c_b}^{e_b}$, and its reverse transition with the absorption of the gauge boson, $q_{c_2}^{e_2} + b_{c_b}^{e_b} \to q_{c_1}^{e_1}$:

$$q_{c_1}^{e_1} \rightleftarrows q_{c_2}^{e_2} + b_{c_b}^{e_b}, \tag{1}$$

where

$$c_1 \neq C_0, \quad e_1 \neq 0, \quad (c_1 = r, g, b, \quad e_1 = -\frac{1}{3}, \ +\frac{2}{3}), \tag{2}$$

$$c_b \neq C_0, \quad e_b \neq 0. \tag{3}$$

It can easily be shown that in the interactions of the quark currents $q^{e_1}_{c_1} \rightleftarrows q^{e_2}_{c_2}$ mediated by the colored and electrically charged gauge boson $b^{e_b}_{c_b}$, the baryon and lepton numbers are strictly conserved.

We shall now determine the color and electric charges of the gauge boson $b^{e_b}_{c_b}$, and those of the quark $q^{e_2}_{c_2}$. The transitions (1) must satisfy the law of conservation of color and electric charges:

$$c_1 = c_2 + c_b, \quad e_1 = e_2 + e_b, \tag{4}$$

which gives

$$c_b = c_1 + \bar{c_2}, \quad e_b = e_1 - e_2. \tag{5}$$

Substituting for c_b and e_b in (1) their values, we have

$$q^{e_1}_{c_1} \rightleftarrows q^{e_2}_{c_2} + b^{e_1 - e_2}_{c_1 + \bar{c_2}}. \tag{6}$$

Since we are considering the case $c_b \neq C_0$, $e_b \neq 0$, (i.e., $c_1 \neq c_2$, $e_1 \neq e_2$), and $c_1 \neq C_0$, $e_1 \neq 0$, we may consider four cases of equality and inequality between the c_2 and C_0, and e_2 and 0:

Case I. When $c_1 \neq c_2 = C_0$ and $e_1 \neq e_2 = 0$,

$$q^{e_1}_{c_1} \rightleftarrows q^{0}_{C_0} + b^{e_1}_{c_1}. \tag{7}$$

Case II. When $c_1 \neq c_2 = C_0$ and $e_1 \neq e_2 \neq 0$,

$$q^{e_1}_{c_1} \rightleftarrows q^{e_2}_{C_0} + b^{e_1 - e_2}_{c_1}. \tag{8}$$

Case III. When $c_1 \neq c_2 \neq C_0$ and $e_1 \neq e_2 = 0$,

$$q^{e_1}_{c_1} \rightleftarrows q^{0}_{c_2} + b^{e_1}_{c_1 + \bar{c_2}}. \tag{9}$$

Case IV. When $c_1 \neq c_2 \neq C_0$ and $e_1 \neq e_2 \neq 0$,

$$q^{e_1}_{c_1} \rightleftarrows q^{e_2}_{c_2} + b^{e_1 - e_2}_{c_1 + \bar{c_2}}. \tag{10}$$

Each case will require considerable discussion and lead to many theories.

In this paper, we shall restrict ourselves to the first case, and construct a model of interactions based on the first case: The transition of a quark from a colored and electrically charged state

$q_{c_1}^{e_1}$ to a colorless and electrically neutral state $q_{C_0}^{0}$ with the emission of a colored and electrically charged gauge boson $b_{c_1}^{e_1}$, and its reverse transition with the absorption of the gauge boson:

$$q_{c_1}^{e_1} \rightleftarrows q_{C_0}^{0} + b_{c_1}^{e_1}, \tag{11}$$

which also mean that the colored and electrically charged quark $q_{c_1}^{e_1}$ and the colorless and electrically neutral quark $q_{C_0}^{0}$ transform into each other through the emission or absorption of the colored and electrically charged gauge boson $b_{c_1}^{e_1}$. Of course, in our case (11), the existence is postulated of colorless and electrically neutral quarks. The extension of the transitions (11) to the cases of involving anti-particles may be made as follows:

$$q_{c_1}^{\bar{e}_1} \rightleftarrows q_{C_0}^{\bar{0}} + b_{c_1}^{\bar{e}_1}, \quad q_{c_1}^{e_1} + b_{c_1}^{\bar{e}_1} \rightleftarrows q_{C_0}^{0}, \quad b_{c_1}^{e_1} \rightleftarrows q_{c_1}^{e_1} + q_{C_0}^{\bar{0}}, \quad q_{c_1}^{\bar{e}_1} + b_{c_1}^{e_1} \rightleftarrows q_{C_0}^{\bar{0}}, \tag{12}$$

etc., where $\bar{A}$ denotes the anti-particle of A.

It should be noted that the colored and electrically charged gauge boson $b_{c_1}^{e_1}$ in (11) has the same color and electric charges as the colored and electrically charged quark $q_{c_1}^{e_1}$, i.e., $b_{c_b}^{e_b} = b_{c_1}^{e_1}$, it thus has the color charges r, g, b and the electric charges $-\frac{1}{3}e$, $+\frac{2}{3}e$.

The transitions (11) can be rewritten in the form

$$q_i^{Q/e} \rightleftarrows \kappa_q + W_i^{Q/e}, \tag{13}$$

where we have put $\kappa_q = q_{C_0}^{0}$, $W = b$, $i = c_1$, $Q/e = e_1$. The (13) takes the form when $q = u$, c or t, $(Q/e = +\frac{2}{3})$,

$$q_i^{+\frac{2}{3}} \rightleftarrows \kappa_q + W_i^{+\frac{2}{3}}, \tag{14}$$

and when $q = d$, s or b, $(Q/e = -\frac{1}{3})$,

$$q_i^{-\frac{1}{3}} \rightleftarrows \kappa_q + W_i^{-\frac{1}{3}}. \tag{15}$$

III. COLORLESS AND ELECTRICALLY NEUTRAL QUARKS

We may see from (14) and (15) that there can be six colorless and electrically neutral quarks, which we shall call *cen-quarks*,

$$\kappa_q \ (q = u, c, t, d, s, b): \ \kappa_u, \ \kappa_c, \ \kappa_t, \ \kappa_d, \ \kappa_s, \ \kappa_b, \tag{16}$$

and six pairs of (q, κ_q): (u, κ_u), (c, κ_c), (t, κ_t), (d, κ_d), (s, κ_s), (b, κ_b).

Cen-quarks are colorless and electrically neutral quarks, whereas neutrinos are colorless and electrically neutral leptons. Since cen-quarks have neither color charge nor electric charge, they participate neither in strong interactions nor in electromagnetic interactions. This means that cen-quarks can carry away the 'missing energy' as do neutrinos.

Let us consider two kinds of transitions

$$q_i^{Q/e} \rightleftarrows \kappa_q + W_i^{Q/e}, \tag{17}$$

$$l^- \rightleftarrows \nu_l + W^-, \tag{18}$$

where $Q/e = +\frac{2}{3}$ $(q = u, c, t)$, $Q/e = -\frac{1}{3}$ $(q = d, s, b)$, $l = e, \mu, \tau$. We may see that the transitions (17) take the same form as the transitions (18): A colored and electrically charged quark loses its color and electric charges completely through the emission of a colored and electrically charged gauge boson, just as an electrically charged lepton loses its electric charge completely through the emission of an electrically charged gauge boson. This suggests that the description of the transitions of a quark (17) can be made in the same way as the transitions of a lepton (18), that is, in the form of electroweak theory of leptons.

The cen-quarks must have spin $\frac{1}{2}$ and baryon number $\frac{1}{3}$. To describe within the framework of $SU(2) \times U(1)$ model, the left-handed cen-quarks have isospin $\frac{1}{2}$: Each left-handed colored and electrically charged quark and its left-handed cen-quark have the same magnitude of isospin charge but opposite in sign. Thus from the isospin charge T_3

$$T_3|q, L\rangle = \begin{cases} +\frac{1}{2}|q, L\rangle & \text{for} \quad q = u, c, t, \\ -\frac{1}{2}|q, L\rangle & \text{for} \quad q = d, s, b, \end{cases} \tag{19}$$

and $\frac{Q}{e} = T_3 + \frac{1}{2}Y$, the isospin charge T_3 and hypercharge Y of each left-handed cen-quark are

$$T_3|\kappa_q, L\rangle = \begin{cases} -\frac{1}{2}|\kappa_q, L\rangle & \text{for} \quad q = u, c, t, \\ +\frac{1}{2}|\kappa_q, L\rangle & \text{for} \quad q = d, s, b, \end{cases} \tag{20}$$

and

$$Y|\kappa_q, L\rangle = \begin{cases} +1|\kappa_q, L\rangle & \text{for} \quad q = u, c, t, \\ -1|\kappa_q, L\rangle & \text{for} \quad q = d, s, b. \end{cases} \tag{21}$$

Accordingly, quarks can be classified into four types and three generations:

$$\begin{array}{lcccc} u - \text{type} & : & u, & c, & t \\ \kappa_d - \text{type} & : & \kappa_d, & \kappa_s, & \kappa_b \\ \kappa_u - \text{type} & : & \kappa_u, & \kappa_c, & \kappa_t \\ d - \text{type} & : & d, & s, & b \end{array} \tag{22}$$

and

$$G_{\rm I} = \begin{pmatrix} u \\ \kappa_d \\ \kappa_u \\ d \end{pmatrix}, G_{\rm II} = \begin{pmatrix} c \\ \kappa_s \\ \kappa_c \\ s \end{pmatrix}, G_{\rm III} = \begin{pmatrix} t \\ \kappa_b \\ \kappa_t \\ b \end{pmatrix}. \tag{23}$$

We may see that cen-quarks resemble neutrinos in many respects. We utilize the resemblance between them by determining the masses of cen-quarks from the mass conditions of neutrinos: We assume that the mass of each cen-quark is either zero or very small in comparison to the mass of the corresponding colored and electrically charged quark, i.e., $m_{\kappa_q} = 0$ or $m_{\kappa_q} \ll m_q$.

IV. MODEL

Let us consider ten column vectors

$$\Psi^L_{1i} = \begin{pmatrix} u^L_i \\ c^L_i \\ t^L_i \\ d^L_i \\ s^L_i \\ b^L_i \end{pmatrix}, \Psi^L_{2i} = \begin{pmatrix} u^L_i \\ c^L_i \\ t^L_i \\ \kappa^L_u \\ \kappa^L_c \\ \kappa^L_t \end{pmatrix}, \Psi^L_{3i} = \begin{pmatrix} \kappa^L_d \\ \kappa^L_s \\ \kappa^L_b \\ d^L_i \\ s^L_i \\ b^L_i \end{pmatrix}, \Psi^L_4 = \begin{pmatrix} \kappa^L_d \\ \kappa^L_s \\ \kappa^L_b \\ \kappa^L_u \\ \kappa^L_c \\ \kappa^L_t \end{pmatrix}, \tag{24}$$

where $i = r, g, b$, $q^L_i = P_L q_i$, $\kappa^L_q = P_L \kappa_q$, $(q = u, c, t, d, s, b)$, $P_L = \frac{1-\gamma^5}{2}$.

By introducing the 6×6 isospin matrices $T_{j\alpha}$ $(j = 1, 2, 3,\ \alpha = 1, 2, 3, 4)$,

$$T_{1\alpha} = \frac{1}{2}\begin{pmatrix} 0 & U_\alpha \\ U^\dagger_\alpha & 0 \end{pmatrix}, T_{2\alpha} = \frac{1}{2}\begin{pmatrix} 0 & -iU_\alpha \\ iU^\dagger_\alpha & 0 \end{pmatrix}, T_{3\alpha} = \frac{1}{2}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{25}$$

the I being the 3×3 unit matrix, the U_α's 3×3 unitary matrices, which satisfy the commutation relations

$$[T_{i\alpha}, T_{j\alpha}] = i\epsilon_{ijk} T_{k\alpha},\ (\alpha : \text{unsummed}), \tag{26}$$

and the 6×6 diagonal hypercharge matrices Y_α

$$Y_1 = \begin{pmatrix} \frac{1}{3}I & 0 \\ 0 & \frac{1}{3}I \end{pmatrix}, Y_2 = \begin{pmatrix} \frac{1}{3}I & 0 \\ 0 & +I \end{pmatrix},$$
$$Y_3 = \begin{pmatrix} -I & 0 \\ 0 & \frac{1}{3}I \end{pmatrix}, Y_4 = \begin{pmatrix} -I & 0 \\ 0 & +I \end{pmatrix}, \tag{27}$$

we may construct the Lagrangian, which is invariant under $T_{j\alpha}$ and Y_α gauge transformations, being of the form

$$\mathscr{L} = \sum_{j=1}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu D^\mu \Psi^L_{ji} + i\bar{\Psi}^L_4\gamma_\mu D^\mu \Psi^L_4 + \sum_{q=u,c,t,d,s,b}\sum_{i=r,g,b} i\bar{q}^R_i\gamma_\mu D^\mu q^R_i + \sum_{q=u,c,t,d,s,b} i\bar{\kappa}^R_q\gamma_\mu D^\mu \kappa^R_q + \mathscr{L}', \tag{28}$$

where $q^R_i = P_R q_i$, $\kappa^R_q = P_R\kappa_q$, $(q = u, c, t, d, s, b)$, $P_R = \frac{1+\gamma^5}{2}$, the $\mathscr{L}'$ involves the terms of free gauge fields and Higgs fields, and the covariant derivatives are defined as

$$D^\mu\Psi^L_{1i} = (\partial^\mu + \sum_{j=1}^{3} igT_{j1}W^\mu_{j1} + ig'\frac{1}{2}Y_1B^\mu)\Psi^L_{1i}, \tag{29}$$

$$D^\mu\Psi^L_{2i} = (\partial^\mu + \sum_{j=1}^{2} igT_{j2}W^\mu_{j2i} + igT_{32}W^\mu_{32} + ig'\frac{1}{2}Y_2B^\mu)\Psi^L_{2i}, \tag{30}$$

$$D^\mu\Psi^L_{3i} = (\partial^\mu + \sum_{j=1}^{2} igT_{j3}W^\mu_{j3i} + igT_{33}W^\mu_{33} + ig'\frac{1}{2}Y_3B^\mu)\Psi^L_{3i}, \tag{31}$$

$$D^\mu\Psi^L_{4} = (\partial^\mu + \sum_{j=1}^{3} igT_{j4}W^\mu_{j4} + ig'\frac{1}{2}Y_4B^\mu)\Psi^L_{4}, \tag{32}$$

$$D^\mu q^R_i = (\partial^\mu + ig'\frac{2}{3}B^\mu)q^R_i, \quad (q = u, c, t), \tag{33}$$

$$D^\mu q^R_i = (\partial^\mu - ig'\frac{1}{3}B^\mu)q^R_i, \quad (q = d, s, b), \tag{34}$$

$$D^\mu\kappa^R_q = \partial^\mu\kappa^R_q. \tag{35}$$

The terms

$$\sum_{i=r,g,b} i\bar{\Psi}^L_{1i}\gamma_\mu D^\mu\Psi^L_{1i} + \sum_{q=u,c,t,d,s,b}\sum_{i=r,g,b} i\bar{q}^R_i\gamma_\mu D^\mu q^R_i \tag{36}$$

are well-known. Newly introduced terms are

$$\sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu D^\mu\Psi^L_{ji} + i\bar{\Psi}^L_4\gamma_\mu D^\mu\Psi^L_4 + \sum_{q=u,c,t,d,s,b} i\bar{\kappa}^R_q\gamma_\mu D^\mu\kappa^R_q + \mathscr{L}''. \tag{37}$$

Since the charged currents constructed out of Ψ^L_{1i} do not carry color charges, they are coupled not to colored and electrically charged gauge bosons, but to colorless and electrically charged gauge bosons like $W^\pm$. Whereas, the charged currents constructed out of Ψ^L_{2i} or Ψ^L_{3i} carry color and electric charges, and are coupled to colored and electrically charged gauge bosons.

The first term of (37) can be written in the form

$$\sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu D^\mu\Psi^L_{ji} = \sum_{j=2}^{3}\sum_{i=r,g,b} i\bar{\Psi}^L_{ji}\gamma_\mu \partial^\mu\Psi^L_{ji} + \mathscr{L}_{\mathrm{IC}} + \mathscr{L}_{\mathrm{IN}}, \tag{38}$$

where

$$\mathscr{L}_{\rm IC} = -g\sum_{j=2}^{3}\sum_{k=1}^{2}\sum_{i=r,g,b}\bar{\Psi}^{L}_{ji}\gamma_\mu T_{kj}W^{\mu}_{kji}\Psi^{L}_{ji}, \tag{39}$$

$$\mathscr{L}_{\rm IN} = -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}^{L}_{ji}\gamma_\mu(gT_{3j}W^{\mu}_{3j} + g'\frac{1}{2}Y_jB^\mu)\Psi^{L}_{ji}. \tag{40}$$

The $\mathscr{L}_{\rm IC}$ describes the interactons in which each current involving a cen-quark and a colored and electrically charged quark is coupled to a colored and electrically charged gauge boson:

$$\mathscr{L}_{\rm IC} = -\frac{g}{2\sqrt{2}}\sum_{i=r,g,b}(W^{\mu}_{2i}J_{2i\mu} + W^{\mu\dagger}_{2i}J^{\dagger}_{2i\mu} + W^{\mu\dagger}_{3i}J_{3i\mu} + W^{\mu}_{3i}J^{\dagger}_{3i\mu}), \tag{41}$$

where $W^{\mu}_{2i} = \frac{1}{\sqrt{2}}(W^{\mu}_{12i} - iW^{\mu}_{22i})$, $W^{\mu}_{3i} = \frac{1}{\sqrt{2}}(W^{\mu}_{13i} + iW^{\mu}_{23i})$,

$$J_{ji\mu} = 2\bar{\Psi}^{L}_{ji}\gamma_\mu H_j\Psi^{L}_{ji},\ H_j = \begin{pmatrix} 0 & U_j \\ 0 & 0 \end{pmatrix}. \tag{42}$$

We shall denote the quanta of the field W^{μ}_{2i} and those of the field W^{μ}_{3i} by $W_i^{+\frac{2}{3}}$ (or $W^{+\frac{2}{3}}$) and $W_i^{-\frac{1}{3}}$ (or $W^{-\frac{1}{3}}$) respectively. The $\mathscr{L}_{\rm IC}$ describes the processes such as

$$u_i \rightleftarrows \kappa_u + W_i^{+\frac{2}{3}},\ d_i \rightleftarrows \kappa_d + W_i^{-\frac{1}{3}}, \tag{43}$$

where $i = r, g, b$.

The $\mathscr{L}_{\rm IN}$ describes the interactions in which neutral currents of quarks are coupled to colorless and electrically neutral gauge bosons:

$$\mathscr{L}_{\rm IN} = -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}^{L}_{ji}\gamma_\mu(gT_3W^{\mu}_{3j} + g'\frac{1}{2}Y_jB^\mu)\Psi^{L}_{ji}, \tag{44}$$

where $T_3 \equiv T_{3j}$. By introducing Hermitian fields Z^{μ}_{j} and A^μ

$$W^{\mu}_{3j} = \cos\theta_j Z^{\mu}_{j} + \sin\theta_j A^\mu, \tag{45}$$

$$B^\mu = -\sin\theta_j Z^{\mu}_{j} + \cos\theta_j A^\mu, \tag{46}$$

substituting for W^{μ}_{3j} and B^μ in (44) their values, we obtain

$$\begin{aligned}\mathscr{L}_{\rm IN} &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}^{L}_{ji}\gamma_\mu[gT_3(\cos\theta_j Z^{\mu}_{j} + \sin\theta_j A^\mu) \\ &\quad + g'\frac{1}{2}Y_j(-\sin\theta_j Z^{\mu}_{j} + \cos\theta_j A^\mu)]\Psi^{L}_{ji} \\ &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}^{L}_{ji}\gamma_\mu[(g\cos\theta_j T_3 - g'\sin\theta_j\frac{1}{2}Y_j)Z^{\mu}_{j} \\ &\quad + (g\sin\theta_j T_3 + g'\cos\theta_j\frac{1}{2}Y_j)A^\mu]\Psi^{L}_{ji}.\end{aligned} \tag{47}$$

Since $\frac{Q_j}{e} = T_3 + \frac{1}{2}Y_j$ and $g\sin\theta_j T_3 + g'\cos\theta_j \frac{1}{2}Y_j = Q_j$, they agree if we take $g\sin\theta_j = g'\cos\theta_j = e$. Thus it becomes

$$\begin{aligned}\mathscr{L}_{\mathrm{I_N}} &= -\sum_{j=2}^{3}\sum_{i=r,g,b}\bar{\Psi}^L_{ji}\gamma_\mu[\frac{g}{\cos\theta_j}(T_3 - \sin^2\theta_j\frac{Q_j}{e})Z^\mu_j + Q_jA^\mu]\Psi^L_{ji} \\ &= -\sum_{j=2}^{3}[\frac{g}{\cos\theta_j}(J^{(T_3)}_{\mu j} - \sin^2\theta_j\frac{J^{(Q_j)}_{\mu j}}{e})Z^\mu_j + J^{(Q_j)}_{\mu j}A^\mu],\end{aligned} \tag{48}$$

where $J^{(T_3)}_{\mu j} = \sum_{i=r,g,b}\bar{\Psi}^L_{ji}\gamma_\mu T_3\Psi^L_{ji}$ and $J^{(Q_j)}_{\mu j} = \sum_{i=r,g,b}\bar{\Psi}^L_{ji}\gamma_\mu Q_j\Psi^L_{ji}$.

From (41) and (48), we have

$$\begin{aligned}\mathscr{L}_{\mathrm{I_C}} + \mathscr{L}_{\mathrm{I_N}} = &-\frac{g}{2\sqrt{2}}\sum_{i=r,g,b}(W^\mu_{2i}J_{2i\mu} + W^{\mu\dagger}_{2i}J^\dagger_{2i\mu} + W^{\mu\dagger}_{3i}J_{3i\mu} + W^\mu_{3i}J^\dagger_{3i\mu}) \\ &-\sum_{j=2}^{3}[\frac{g}{\cos\theta_j}(J^{(T_3)}_{\mu j} - \sin^2\theta_j\frac{J^{(Q_j)}_{\mu j}}{e})Z^\mu_j + J^{(Q_j)}_{\mu j}A^\mu].\end{aligned} \tag{49}$$

V. APPLICATION

It should be noted that cen-quarks can be produced in non-leptonic decays, and they can carry away 'missing energy' as do neutrinos. Thus we must take into account the processes involving the cen-quarks in non-leptonic weak interactions where missing energies occur.

For example, for the description of the rare kaon decay $K^+ \to \pi^+ +$ 'missing energy', we must take into account the following quark level processes:

$$\text{(i) } \bar{s}_i \to \bar{d}_i\nu_l\bar{\nu}_l \ (l = e, \mu, \tau), \tag{50}$$

$$\text{(ii) } \bar{s}_i \to \bar{d}_i\kappa'_q\bar{\kappa}'_q \ (q = u, c, t, d, s, b), \tag{51}$$

$$\text{(iii)} \bar{s}_i \to \bar{d}_i\kappa'_d\bar{\kappa}'_s, \tag{52}$$

where $i = r, g, b$, and κ'_d and κ'_s are mixed states of κ_d, κ_s and κ_b. We may see that the process (iii) is similar to the muon decay $\mu^- \to \nu_\mu e^-\bar{\nu}_e$ in many respects.

The interactions of the well-known processes (i) are mediated by the gauge bosons $W^\pm$ and Z^0. Whereas the interactions of (ii) and (iii) are mediated by the gauge bosons $W^{-\frac{1}{3}}$, $\bar{W}^{-\frac{1}{3}}$ and Z^0_3. We may infer from extremely short range of the interactions of (ii) and (iii) that the mass of the gauge bosons $W^{-\frac{1}{3}}$, $\bar{W}^{-\frac{1}{3}}$ and Z^0_3 must be very massive.

The decay rate of $K^+ \to \pi^+ +$ 'missing energy', i.e., $\Gamma(K^+ \to \pi^+ + \text{Nothing})$, can be written from (50), (51) and (52) as

$$\Gamma(K^+ \to \pi^+ + \text{Nothing}) = \Gamma_{\text{(i)}} + \Gamma_{\text{(ii)}} + \Gamma_{\text{(iii)}}, \tag{53}$$

where

$$\Gamma_{(\mathrm{i})} = \Gamma(K^+ \to \pi^+ \nu\bar{\nu})_{\nu=\nu_e,\nu_\mu,\nu_\tau}, \tag{54}$$

$$\begin{aligned}\Gamma_{(\mathrm{ii})} &= \Gamma(K^+ \to \pi^+ \kappa'\bar{\kappa'})_{\kappa'=\kappa'_u,\kappa'_c,\kappa'_t,\kappa'_d,\kappa'_s,\kappa'_b} \\ &= \Gamma(\bar{s}_i \to \bar{d}_i \kappa'\bar{\kappa'})_{\kappa'=\kappa'_u,\kappa'_c,\kappa'_t,\kappa'_d,\kappa'_s,\kappa'_b},\end{aligned} \tag{55}$$

$$\Gamma_{(\mathrm{iii})} = \Gamma(K^+ \to \pi^+ \kappa'_d\bar{\kappa'_s}) = \Gamma(\bar{s}_i \to \bar{d}_i \kappa'_d\bar{\kappa'_s}). \tag{56}$$

The terms among the Lagrangian terms in (49) responsible for the processes (ii) and (iii) are

$$\begin{aligned}\mathscr{L}_{\mathrm{I}_3} = &-\frac{g}{2\sqrt{2}} \sum_{i=r,g,b} (W^{\mu\dagger}_{3i} J_{3i\mu} + W^{\mu}_{3i} J^\dagger_{3i\mu}) \\ &-[\frac{g}{\cos\theta_3}(J^{(T_3)}_{\mu 3} - \sin^2\theta_3 \frac{J^{(Q_3)}_{\mu 3}}{e}) Z^\mu_3 + J^{(Q_3)}_{\mu 3} A^\mu],\end{aligned} \tag{57}$$

where

$$J_{3i\mu} = 2\bar{\Psi}^L_\kappa \gamma_\mu U_3 \Psi^L_{q_i} = 2\bar{\Psi}^{L\prime}_\kappa \gamma_\mu \Psi^L_{q_i}, \tag{58}$$

where $\Psi^{L\prime}_\kappa = U_3^\dagger \Psi^L_\kappa$, and

$$\Psi^L_\kappa = \begin{pmatrix} \kappa^L_d \\ \kappa^L_s \\ \kappa^L_b \end{pmatrix}, \Psi^L_{q_i} = \begin{pmatrix} d^L_i \\ s^L_i \\ b^L_i \end{pmatrix}. \tag{59}$$

In the limit $m_{W^{-\frac{1}{3}}} \to \infty$, the $W^{-\frac{1}{3}}$ propagator reduces to

$$i g^{\mu\nu} m^{-2}_{W^{-\frac{1}{3}}}. \tag{60}$$

Similarly, in the limit $m_{Z^0_3} \to \infty$, the Z^0_3 propagator reduces to

$$i g^{\mu\nu} m^{-2}_{Z^0_3}. \tag{61}$$

In the lowest order, the process (iii) has one internal $W^{-\frac{1}{3}}$ boson line, whereas each of the processes (ii) has two internal $W^{-\frac{1}{3}}$ boson lines for the box diagram, or at least one internal $W^{-\frac{1}{3}}$ boson line plus one internal Z^0_3 boson line for the Z^0_3-penguin diagrams. Thus from $\frac{g^2_W}{m^2_{W^{-\frac{1}{3}}}} \ll 1$ and $\frac{8g^2_W}{\cos^2\theta_3 m^2_{Z^0_3}} \ll 1$, we have

$$\Gamma_{(\mathrm{ii})} \ll \Gamma_{(\mathrm{iii})}, \tag{62}$$

and

$$\Gamma(K^+ \to \pi^+ + \text{Nothing}) \approx \Gamma_{(\mathrm{i})} + \Gamma_{(\mathrm{iii})}. \tag{63}$$

From (57), we may construct the invariant amplitude of the lowest order for the process (iii) which takes in the form

$$\mathscr{M} \approx -i\frac{4g_W^2}{m^2_{W^{-\frac{1}{3}}}}\bar{s}_i^L\gamma^\mu\kappa_s^{L\prime}\bar{\kappa}_d^{L\prime}\gamma_\mu d_i^L, \tag{64}$$

where $g_W = \frac{g}{2\sqrt{2}}$ and

$$\kappa_s^{L\prime} = \sum_{\kappa=\kappa_d^L,\kappa_s^L,\kappa_b^L} U^\dagger_{3(s^L,\kappa)}\kappa, \quad \kappa_d^{L\prime} = \sum_{\kappa=\kappa_d^L,\kappa_s^L,\kappa_b^L} U^\dagger_{3(d^L,\kappa)}\kappa. \tag{65}$$

Assuming that $U_3 \approx I$, $m_{\kappa_d} \approx 0$, $m_{\kappa_s} \approx 0$, $m_d \ll m_s$, and that the process (iii) is unaffected by strong and electromagnetic interactions except some negligible higher order corrections, we have from (64)

$$\Gamma(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s') \approx \frac{g_W^4 m_s^5}{96\pi^3 m^4_{W^{-\frac{1}{3}}}}, \tag{66}$$

where $g_W^2 \approx \frac{1}{\sqrt{2}}Gm^2_{W^\pm}$ well-known in electro-weak theory.

The (63) suggests that the branching ratio $B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s')$ can be determined by the discrepancy between the measured $K^+ \to \pi^+ +$ 'missing energy' branching ratio and the predicted $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio. Thus if the discrepancy is determined, from

$$B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s') = \tau_{K^+}\Gamma(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s') \approx \frac{\tau_{K^+}G^2 m^4_{W^\pm} m_s^5}{192\pi^3 m^4_{W^{-\frac{1}{3}}}}, \tag{67}$$

we may calculate the mass of $W^{-\frac{1}{3}}$

$$m_{W^{-\frac{1}{3}}} \approx \left(\frac{\tau_{K^+}G^2 m^4_{W^\pm} m_s^5}{192\pi^3 B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s')}\right)^{\frac{1}{4}}, \tag{68}$$

where $G \approx 1.16639 \times 10^{-5}\mathrm{GeV}^{-2}$, $m_{W^\pm} \approx 80.399\mathrm{GeV}$, $m_s \approx 100\mathrm{MeV}$, $\tau_{K^+} \approx 1.238 \times 10^{-8}\mathrm{s}$.

However, at the present stage, if we compare the measured $K^+ \to \pi^+ +$ 'missing energy' branching ratio [3] with the predicted $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio [4],

$$B(K^+ \to \pi^+ + \mathrm{Nothing})_{\mathrm{Exp}} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}, \tag{69}$$

$$B(K^+ \to \pi^+\nu\bar{\nu}) = (0.781^{+0.080}_{-0.071} \pm 0.029) \times 10^{-10}, \tag{70}$$

the range of the experimental uncertainty is so wide that we cannot know the exact value of $B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s')$, i.e., the discrepancy between the (69) and (70).

In this situation, if we postulate that $B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s')$ is approximately the discrepancy between the representative values of (69) and (70), i.e.,

$$B(K^+ \to \pi^+\kappa_d'\bar{\kappa}_s') \approx 0.95 \times 10^{-10}, \tag{71}$$

the value for the $m_{W^{-\frac{1}{3}}}$ becomes from (68)

$$m_{W^{-\frac{1}{3}}} \approx 6.594 \text{ TeV}. \tag{72}$$

To confirm our speculation, we should look for colored and electrically charged bosons with spin-1 consistent with the properties described in this paper.

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