The fine structure constant from the separation distance of two 3-branes in the covering space of a toroidal orbifold

B F Riley[1]

4-branes that wrap the long and short fundamental 1-cycles of a rectangular toroidal orbifold $T^2/(Z_2)^3$, which has Planck-scale compactification radii, intersect after 43 and 109 wrappings, respectively, in a 3-brane upon which the scale factor, equal to a power of the inverse of compactification length, has precisely the same value when measured parallel to either of the fundamental 1-cycles. The scale, 5.12 MeV, of the 3-brane is reduced from Planck scale by a factor 137^{-10}. The distance of the 5.12 MeV intersection from the Planck brane in the orbifold covering space is precisely equal to the length of 40 cycles of the orbifold diagonal, and measures 137 Planck units. A geometric 'Sequence of Scales' extends from 5.12 MeV to both larger and smaller scales, and is associated with the intersections of two 4-branes, with wrapping numbers (1, 2) and (1, -2). All fields may be confined to 3-branes. Weak, strong and electromagnetic 3-branes are identified; all three are associated with ~integer wrappings of the orbifold diagonal. The electromagnetic 3-brane is located adjacent to the 5.12 MeV 3-brane, at a distance of 137.0359 Planck units from the Planck brane in the orbifold covering space. The scale on the electromagnetic 3-brane, 4.927 MeV, is related to the scale on the weak 3-brane, 91.1876 GeV, by a factor 137^{-2}.

1 Introduction

In the Randall and Sundrum RS1 brane world model [1], we live on a 3-brane (the weak brane) at a fixed point of the S^1/Z_2 orbifold; the Planck brane is located at the other fixed point. The natural scale on our brane is suppressed from Planck scale by a warp factor e^{-ky}. In the RS2 model [2], we live on the Planck brane at $y = 0$ in an infinite extra dimension. Motivated by the RS models and multiple-brane extensions thereof [3,4,5], evidence was sought for a geometric sequence of mass levels, such as might derive from the locations of 3-branes at lattice points in a warped extra dimension. A geometric sequence of particle mass levels was identified [6], followed by a Sequence of Scales upon coincidental mass/energy levels within two geometric sequences [7].

In this letter, various scales of physics will be shown to lie upon or adjacent to coincident mass/energy levels within two geometric sequences, of common ratio π^{-1} and $(\pi/2)^{-1}$, that descend from Planck scale. The coincident levels form the Sequence of Scales that ascends and descends from the pre-eminently precise coincidence of levels at a scale of 5.12 MeV. The Sequence of Scales is identified with the scales on 3-branes that arise at the intersections of 4-branes that wrap a rectangular toroidal orbifold $T^2/(Z_2)^3 = S^1/Z_2 \times S^1/(Z_2 \times Z_2')$. The compactification radii are of Planck scale. The 5.12 MeV scale is that of a 3-brane on the fixed point at ($y = 0$, $z = 0$). It was noted in an earlier letter [8] that the distance of the 5.12 MeV intersection from the Planck brane in the orbifold covering space is precisely equal to the length of 40 cycles of the orbifold diagonal. Here, that length is shown to be $\sim\alpha^{-1}$, in Planck units. Conjecturing that all fields are confined to 3-branes, we propose that a 3-brane lying adjacent to the fixed point at ($y = 0$, $z = 0$) is the home of the electromagnetic field. The distance of the 'electromagnetic brane' from the Planck brane in the covering space of the orbifold is α^{-1}. The strong, electromagnetic and weak scales are all shown to be associated with ~integer wrappings of the orbifold diagonal.

[1] bernardriley@hotmail.co.uk

2 The Sequence of Scales

The scales of physics lie upon coincident mass/energy levels within two geometric sequences that descend from Planck scale, 1.220932×10^{19} GeV [9]. Level 43 in Sequence 1, of common ratio π^{-1}, coincides with Level 109 in Sequence 2, of common ratio $(\pi/2)^{-1}$, at 5.12 MeV. The coincidence is of pre-eminent precision: $\pi^{42.9992} = (\pi/2)^{109}$. Because of the approximate equality $\pi^{15} \approx (\pi/2)^{38}$, mass/energy level near-coincidences also occur, for example between Level 28 in Sequence 1 and Level 71 in Sequence 2. Near-coincidences also occur between levels, ½-levels and ¼-levels; for example, Level 35.5 in Sequence 1 coincides with Level 90 in Sequence 2. The Sequence of Scales, of common ratio $\pi^{15} \approx (\pi/2)^{38}$, results; its sublevels are the near-coincident levels and sublevels of Sequence 1 and Sequence 2. The Sequence of Scales extends from 5.12 MeV to both higher and lower scales as shown in Figure 1.

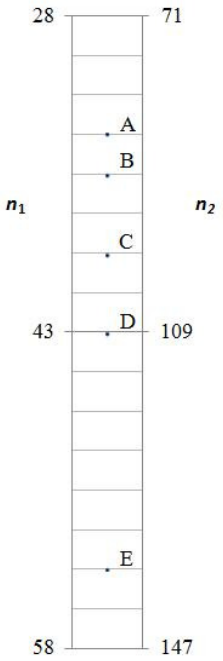

Figure 1 **The Sequence of Scales:** n_1 and n_2 are the level numbers in Sequence 1 and Sequence 2. The sublevels shown lie within a sequence of common ratio $\pi^{-15/8} \approx (\pi/2)^{-38/8}$.

- A Higgs field VEV, 246 GeV
- B b-t scale, equal to the geometric mean of the bottom and top quark masses (PDG central values, 2012), 26.9 GeV
- C s-c scale, equal to the geometric mean of the strange and charm quark masses (PDG central values, 2012), 348 MeV
- D 5.12 MeV
- E Rydberg constant (energy equivalent), 13.6 eV

Each scale is related to Planck scale by a factor $\pi^{-n_1} \approx (\pi/2)^{-n_2}$. The Sequence of Scales includes, upon or adjacent to its levels, the Rydberg constant, energy equivalent (13.6 eV), the Higgs field VEV (246 GeV) and, adjacent to a ½-level of value 2.2×10^{16} GeV, the MSSM GUT scale (2×10^{16} GeV). Also included in the Sequence of Scales are the strange-charm and bottom-top quark doublet mass scales; these scales are each defined as the geometric mean of the masses of the member quarks, using Particle Date Group evaluations [10]; doublets consistently take up symmetrical arrangements about mass levels and sublevels in Sequence 1 and Sequence 2 [6].

3 The 5.12 MeV brane

The values of the coincident levels of Sequence 1 and Sequence 2 are equal to the scales on 3-branes that arise at the intersections of 4-branes that wrap 1-cycles of the toroidal orbifold $T^2/(Z_2)^3 = S^1/Z_2 \times S^1/(Z_2 \times Z_2')$, which is obtained by T^2 moduloing the equivalent classes: $y \sim -y$, $z \sim -z$, $z' \sim -z'$, where

$z' = z - \pi/2$ [11]. The compactification radii are of Planck scale. In Planck units, used throughout, the fixed points of the $T^2/(Z_2)^3$ orbifold are at ($y = 0$, $z = 0$), ($y = 0$, $z = \pi/2$), ($y = \pi$, $z = 0$) and ($y = \pi$, $z = \pi/2$). The extra space is rectangular and will be confirmed as such.

The compactification length scales of the orbifold are π and $\pi/2$; the compactification mass scales are π^{-1} and $(\pi/2)^{-1}$. Scale factors π^{-1} and $(\pi/2)^{-1}$ are associated with single wrappings of the long and short sides of the orbifold, respectively. The scale factors π^{-1} and $(\pi/2)^{-1}$ may be combined to find the diagonal scale factor f associated with a single wrapping of the orbifold diagonal. In a rectangular space, $f = exp\left[-([ln(\pi)]^2 + [ln(\pi/2)]^2)^{1/2}\right] = 3.423223^{-1}$.

4-branes that wrap 1-cycles of the orbifold intersect in 3-branes that can support physics as long as the scale factors π^{-n_1} and $(\pi/2)^{-n_2}$ are equal. Here, n_1 and n_2 refer to the numbers of times the long and short sides of the orbifold are wrapped, up to the point of the intersection. Two 4-branes may intersect many times in the covering space of the orbifold but a physical 3-brane, characterised by a single scale factor, may arise upon an intersection only if the scale factors measured parallel to the fundamental 1-cycles are equal.

4-branes that wrap the long and short fundamental 1-cycles of the $T^2/(Z_2)^3$ orbifold 43 times and 109 times, respectively, intersect in the '5.12 MeV brane' upon the fixed point at ($y = 0$, $z = 0$). The scale on the 5.12 MeV brane is reduced from Planck scale by a factor $\pi^{-43} \approx (\pi/2)^{-109}$. Equivalently, since $\pi^{-43} = f^{40.00007}$, the diagonal of the orbifold is wrapped 40 times; the integer number of wrappings confirms that the orbifold is rectangular.

The scale factor on the 5.12 MeV brane is $f^{40} = \alpha^{10.004}$, where α is the fine structure constant, equal to 1/137.035999074(44) [9]. The distance of the 5.12 MeV brane from the Planck brane, measured in the covering space of the orbifold, is $40f^{-1} = 136.929$.

4 Mass/energy sublevels and the locations of physics

Particles lie upon coincident mass/energy levels and sublevels (½-levels, ¼-levels, ⅛-levels, etc)[2] within Sequences 1 and 2; to explain this finding, we have conjectured that particles live on 3-branes at the intersections of 4-branes that wrap 1-cycles of the $T^2/(Z_2)^3$ orbifold [7].

Particles that lie on ½-levels live on 3-branes at the intersection, for example, of 4-branes a and b with wrapping numbers (n_a, m_a) = (0, 1) and (n_b, m_b) = (2, 1) or of 4-branes c and d with wrapping numbers (n_c, m_c) = (1, 1) and (n_d, m_d) = (1, -1). Particles that lie on ¼-levels live on 3-branes at the intersection, for example, of 4-branes g and h with wrapping numbers (n_g, m_g) = (1, 0) and (n_h, m_h) = (1, 4) or 4-branes i and j with wrapping numbers (n_i, m_i) = (1, -1) and (n_j, m_j) = (1, 3). All 1-cycles are wrapped many times and many 4-brane intersections occur in the covering space of the orbifold but physical 3-branes will only occur upon those intersections on which the scale factors measured parallel to the fundamental 1-cycles are equal. In many instances, a particle mass or other physical scale is the scale on a 3-brane at the intersection between a low wrapping number 4-brane and a high wrapping number 4-brane, adjacent to the intersection of two low wrapping number 4-branes. In general, a ground state particle is found to lie upon a 'low order' sublevel, e.g. a ¼-level, in one of the sequences; in the other sequence the particle lies adjacent to a low order sublevel but actually upon a 'higher order' sublevel, e.g. a 1/32-level [8]. The intersections of 4-branes of small wrapping number appear to be favourable locations for physics, which takes place as close to the intersections as is allowed.

The MSSM GUT scale, Rydberg constant (energy equivalent) and the strange-charm and bottom-top quark doublet scales are all associated with the orthogonal intersections of two 4-branes, with wrapping numbers (1, 2) and (1, -2).

Since $f^{14} \approx \pi^{15} \approx (\pi/2)^{38}$, particles that occupy, or lie equally close to, coincident levels and sublevels within Sequence 1 and Sequence 2 will occupy the levels and sublevels of a 'diagonal sequence' of

[2] ⅓-levels may also be present [6].

common ratio *f*. The Z-boson, electron and lightest hadrons are shown in Figure 2 to lie upon the levels and sublevels of the diagonal sequence. Many other particles occupy sublevels within the diagonal sequence, which is incorporated into the Sequence of Scales.

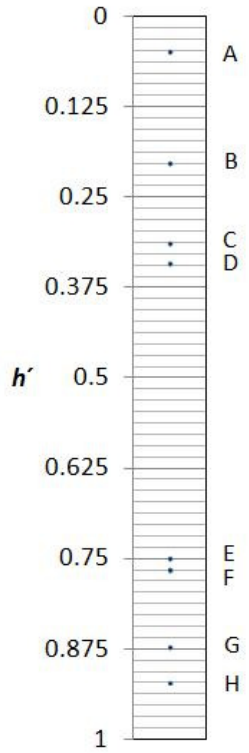

Figure 2 **Sublevel occupation within the diagonal sequence of common ratio *f*:** h' is the fractional part of the exponent h in the diagonal scale factor f^h; h equals the equivalent number of orbifold diagonal wrappings, up to the location of the particle 3-brane. A particle occupying a sublevel of low order within the diagonal sequence will lie close to coincident sublevels of low order within Sequence 1 and Sequence 2.

A	Z^0	**E**	η'
B	η	**F**	p, n
C	$\pi^{\pm}$	**G**	e
D	π^0	**H**	ρ

The evidence suggests that all fields except gravity are confined to 3-branes. As the scale factor ($\sim\alpha^{10}$) and distance of the 5.12 MeV brane from the Planck brane in the orbifold covering space ($\sim\alpha^{-1}$) are closely related to the value of the fine structure constant, the 5.12 MeV brane might be thought to be the domain of the electromagnetic field.[3] However, one should expect that nature avoids the curvature singularities upon the fixed points of the orbifold [13], and consequently it may be upon a 3-brane adjacent to the 5.12 MeV brane in the covering space of the orbifold that the electromagnetic field is confined. That 3-brane would be expected to occupy coincident mass levels within Sequence 1 and Sequence 2, and therefore occupy a sublevel within the diagonal sequence.

Like the scale, ~5 MeV, on the conjectured 'electromagnetic brane', the QCD scale, ~200 MeV, and the weak scale, m_Z = 91.1876 GeV, are both associated with ~integer wrappings of the orbifold diagonal up to the location of the 'force brane'. The scale factors on the 'strong brane' and 'weak brane' are $\sim f^{37.02}$ and $f^{32.046}$, respectively. The weak brane occupies a 1/64-level in the diagonal

[3] In another scenario [12], the gauge field on a set of wrapped D4-branes has a gauge coupling with fine structure constant inversely proportional to the length of the wrapped cycle.

sequence, as shown in Figure 2, while the electromagnetic brane may occupy a 1/32-level in the same sequence. An electromagnetic brane upon which the scale factor is $f^{40.03125}$ would be located at a distance of $40.03125f^{-1}$ = 137.035904 Planck units from the Planck brane. The corresponding electromagnetic scale is 4.927 MeV. Neither the weak scale nor the electromagnetic scale, nor perhaps any other scale, lies exactly upon a prominent mass/energy sublevel in the diagonal sequence, such as a 1/32-level or a 1/64-level, so the fine structure constant α may be exactly equal to the inverse length of wrapped diagonal up to the location of the electromagnetic brane. The scale of a 3-brane located at a distance of exactly α^{-1} Planck units from the Planck brane is also 4.927 MeV, which is related to the scale on the weak 3-brane, 91.1876 GeV, by a factor $\alpha^{1.997}$.

The exponent h of the scale factor f^h is of slightly greater than integer value for the purported electromagnetic and weak branes. The fractional parts, h', of the exponents are shown for the electromagnetic and weak branes in Figure 3. If the QCD scale is ≤205.4 MeV, the strong brane will lie close to the Planck brane in the h' ladder. A QCD scale of 201.5 MeV would signify that the strong brane lies on the vacant level between the Planck brane and the electromagnetic brane.

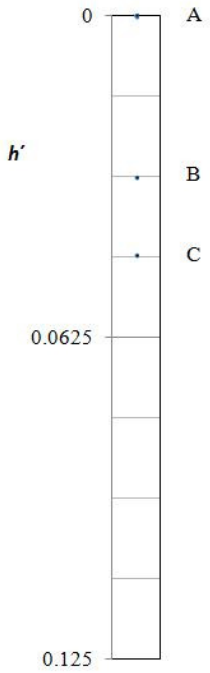

Figure 3 **The 'force branes':** h' is the fractional part of the number of orbifold diagonal wrappings, up to the location of the force brane. The vacant site between A and B would be filled by a physical 3-brane of scale 201.5 MeV, which could be the 'strong brane'.

A Planck brane
B 'Electromagnetic brane'
C 'Weak brane'

In Figure 4, the force branes, the Rydberg constant (energy equivalent), the $W^{\pm}$ boson, the charged leptons and various ground state hadrons, including light and charmed pseudoscalar mesons, light and charmed $J^P = ½^+$ baryons, and $q\bar{q}$ states, are shown to lie adjacent to the 1/64-levels within Sequences 1 and 2, and in some instances clearly upon coincident sublevels. The electron, for example, lies upon coincident 1/64-levels, while the proton lies at the coincidence of a 1/64-level with a 1/32-level.

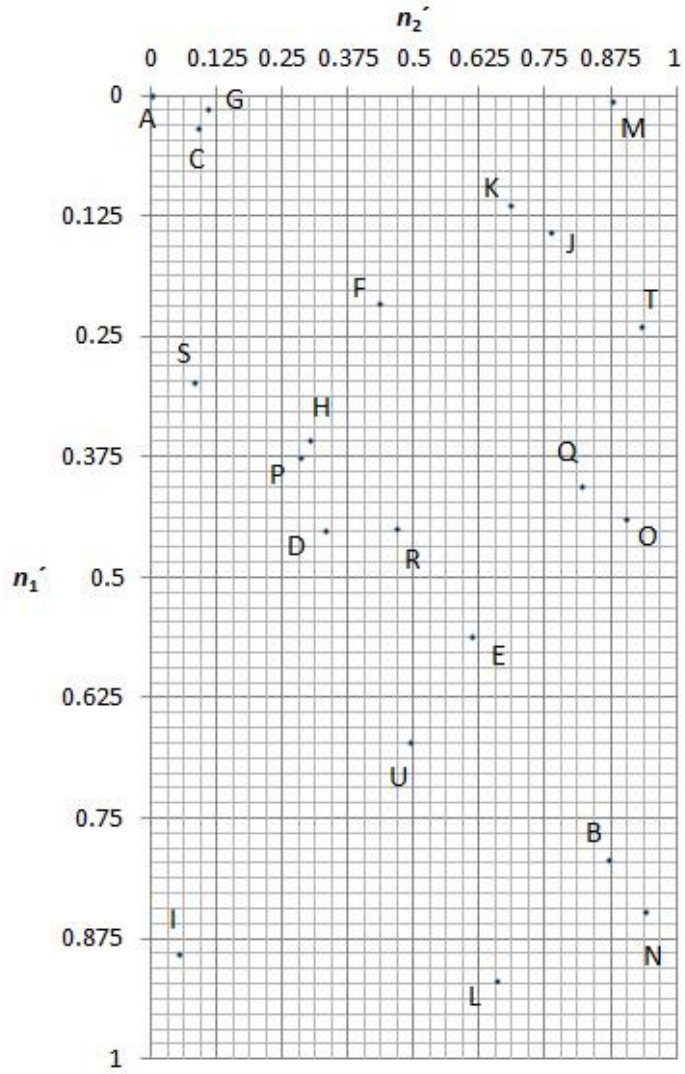

Figure 4 **The locations of force and particle 3-branes on the orbifold:** n_1' and n_2' are the fractional parts of n_1 and n_2, the level numbers in Sequence 1 and Sequence 2, calculated from the scale upon the 3-brane on which the field is confined. The strange and charmed pseudoscalar isospin doublets are each represented in mass by the geometric mean of the neutral and charged meson masses.

A	Planck scale	**H**	μ	**O**	η_c
B	'QCD scale' (201.5 MeV)	**I**	τ	**P**	ϕ
C	'Electromagnetic scale' (4.927 MeV)	**J**	π^0	**Q**	J/ψ
D	Z^0	**K**	$\pi^{\pm}$	**R**	p
E	$W^{\pm}$	**L**	η	**S**	Λ
F	R_∞ (13.6 eV)	**M**	K	**T**	Σ^0
G	e	**N**	D	**U**	Λ_c^+

6 Discussion

The 5.12 MeV mass level coincidence within Sequences 1 and 2 is of unsurpassed precision and is the central term in the Sequence of Scales that extends to both larger and smaller scales. After the Planck scale, it is perhaps the most significant of all scales. In tandem with the Planck scale, the 5.12 MeV scale sets the scale of the physical world. A geometric basis for the value of the gravitational coupling constant suggests itself: $\alpha_G = (M_P/M_{5.12})^2 = f^{80}$ ($\approx \alpha^{20}$). The value of f^{80} is 1.759 x 10^{-43}.

The scales of physics are associated with the intersections of small wrapping number 4-branes. Since viable intersections of such 4-branes are rare, physics is located upon the intersections of higher wrapping number 4-branes, near to the intersections of small wrapping number 4-branes. In the mass sequences, the various scales of physics and the particle masses lie close to mass/energy levels and low order sublevels but upon higher order sublevels. Ground state particles, either as singlets or in doublets, tend to occupy coincident mass sublevels of relatively (to other particles) low order. Other states occupy coincident sublevels of higher order in a framework of levels and sublevels that implies the four-dimensional world is filled with 3-branes.

The location of a 3-brane on the orbifold may be calculated by using the scale factors π^{-n_1} and $(\pi/2)^{-n_2}$. A physical 3-brane appears to take up a location which minimises its distance from preferred locations within Sequence 1 and Sequence 2. The distance of the 3-brane from the Planck brane may be calculated by using the diagonal scale factor f^h.

The value of the fine structure constant may be understood within a six-dimensional model, as presented here, but other aspects of physics may require the introduction of more than two extra dimensions [14].

7 References

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