2.2 Regression of Lunar nodes

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Abstract

Simple mathematical demonstration solves one of the problems of lunar motion, the regression of lunar nodes, observed more than 2000 years ago. Although their motions draw similar traces (such as retrograde motion in Ptolemy's and Copernicus's models of the universe) we show that celestial bodies do not rotate around their common centre of mass, but, by unified law, one around the other. Due to the rigidity of principle, regardless of the calculation of rounded up values, the range of discrepancy between predicted and observed cycle of regression is in level of magnitude of only 3.4×10^{-5} (due to lunar trajectory perturbation, its perceived values also on a small-scale vary cyclically). Besides the constant π and Terrestrial measure of time, the only variables used in the solution of this dual orbiting system problem are radiuses and surface accelerations of observed bodies.

2.2.1 A brief description of the regression of lunar nodes

The ancient Greeks observed that the positions of ascending and descending nodes at which the Moon passes through the fixed plane of the Earth's orbit around the Sun, the ecliptic, decrease, i.e. orbit the Earth in the opposite direction to the Moon, in such a rate that the cycle of that regression amounts almost exactly 18.6 Earth's years. In other words, if the Moon, during the spring or autumn equinox, when viewed from stationary point on Earth, ascends at a certain position on the east horizon, describes the curve of his path and descends at another particular point on the west horizon, it would take 18.6 years for this trajectory to be repeated. In past centuries, developing lunar theory, many famous mathematicians and astronomers have dealt with described problem (Newton, Clairaut, D'Alembert, Euler, Laplace, Damoiseau, Plana, Poisson, Hansen, De Pontécoulant, J. Herschel, Airy, Delaunay, G.W. Hill, E.W. Brown) indicating its inherent difficulty and the theoretical and practical importance.

2.2.2 Points of acceleration equilibrium

It is valid that the body at a distance n from the celestial body centre at which it falls, every second accelerates at rate of corresponding acceleration a_n. Neglecting the resistance of the media through which it moves, it also applies that after n^2/r^2 seconds its velocity equals to the amount of acceleration measured on the surface of observed celestial body, i.e. distance r from its centre (where the concept of the centre, i.e. the point where $n = 0$, equivalents to an unreachable idea of zero). From this simple principle we derive the universal equality for acceleration a_n at any distance n from the centre of the observed body (2.2.2.1);

$$a_n = \frac{ar^2}{n^2} \qquad 2.2.2.1$$

Presented equality is independent of the direction in relation to the radius, in other words, applies to cases where n is less, equal or greater than r ($n \leq r \leq n$), from which it follows that gravity towards celestial body centre tends to infinity.

From the specified relation (2.2.2.1) follow the equations for orbital and escape velocities v_{no} and v_{nesc} at any orbital distance n from the centre of the celestial body (2.2.2.2, 3);

$$v_{n_o} = \sqrt{na_n} \qquad 2.2.2.2$$

$$v_{n_{esc}} = \sqrt{2na_n} \qquad 2.2.2.3$$

which is equal to (2.2.2.4, 5), wherein the relation (2.2.2.4) is equivalent to excerpt from the formula for the period of the pendulum of maximal ideal trajectory $2l\pi$, where in place l, hence the length of the pendulum, we write n. In other words, the mean speed of an imaginary ideal pendulum which length equals to the radius of observed body at maximal ideal trajectory, equals to the orbital velocity of that body.

$$v_{n_o} = r\sqrt{\frac{a}{n}} \qquad 2.2.2.4$$

$$v_{n_{esc}} = r\sqrt{2\frac{a}{n}} \qquad 2.2.2.5$$

Including the light speed c in place of escape velocity v_{nesc} in equality above (2.2.2.5), the value of radius n equals to the Schwarzshild's radius r_s derived from Newton's equation for escape velocity in which the escape velocity is equalized to the speed of light. Demonstrated equivalence (2.2.2.5, 6) indicates the universal validity of the above equalities (2.2.2.1 - 5);

$$n = \frac{2ar^2}{c^2} = \frac{2GM}{c^2} = r_s \qquad 2.2.2.6$$

From the displayed equivalence follows that the GM product, the so-called standard gravitational parameter μ (which is Newton's arbitrary construct of third Kepler's law i.e. $GM = 4\pi^2k$, where $k = r^3/p^2$, where r is any orbital radius of the observed body, and p its corresponding orbital period), equal to the product of surface acceleration a and its corresponding radius r squared (2.2.2.7);

$$\mu = ar^2 \qquad 2.2.2.7$$

At the Earth - Moon line, we calculate common points in which the accelerations of both celestial bodies are equal. According to equality (2.2.2.1) the condition (2.2.2.8) must be satisfied;

$$\frac{a_E r_E^2}{n_E^2} = \frac{a_M r_M^2}{n_M^2} \qquad 2.2.2.8$$

where a_E, r_E, a_M and r_M are the surface accelerations and radiuses of the Earth and the Moon while the values n_E and n_M are the requested distances from their centres. As there are two such points at described line, the values n_E are expressed by relations (2.2.2.9, 10);

$$n_E = r_o - n_M \qquad 2.2.2.9$$

$$n_E = r_o + n_M \qquad 2.2.2.10$$

where the first relation calculate internal and the second external point on the line that connects their centres, and whose distance is the radius r_o of their common observed orbit. Depending on this, the equality (2.2.2.8) is written (2.2.2.11, 12);

$$\frac{a_E r_E^2}{(r_o - n_M)^2} = \frac{a_M r_M^2}{n_M^2} \qquad 2.2.2.11$$

$$\frac{a_E r_E^2}{(r_o + n_M)^2} = \frac{a_M r_M^2}{n_M^2} \qquad 2.2.2.12$$

which equals to (2.2.2.13, 14);

$$\frac{r_E\sqrt{a_E}}{r_o - n_M} = \frac{r_M\sqrt{a_M}}{n_M} \qquad 2.2.2.13$$

$$\frac{r_E\sqrt{a_E}}{r_o + n_M} = \frac{r_M\sqrt{a_M}}{n_M} \qquad 2.2.2.14$$

Therefore, to calculate the distance of required points in relation to the Earth, we derive equalities (2.2.2.15, 16);

$$n_{Ei} = r_o \frac{r_E\sqrt{a_E}}{r_E\sqrt{a_E} + r_M\sqrt{a_M}} \qquad 2.2.2.15$$

$$n_{Eo} = r_o \frac{r_E\sqrt{a_E}}{r_E\sqrt{a_E} - r_M\sqrt{a_M}} \qquad 2.2.2.16$$

and for their distances with respect to the Moon (2.2.2.17, 18);

$$n_{Mi} = r_o \frac{r_M\sqrt{a_M}}{r_E\sqrt{a_E} + r_M\sqrt{a_M}} \qquad 2.2.2.17$$

$$n_{Mo} = r_o \frac{r_M\sqrt{a_M}}{r_E\sqrt{a_E} - r_M\sqrt{a_M}} \qquad 2.2.2.18$$

wherein the n_{Ei}, n_{Eo}, n_{Mi} and n_{Mo} are distances of inner and outer points of the observed orbit and where the sum of their internal values equals to distance of observed bodies centres, i.e. their common orbit r_o (2.2.2.19).

$$r_o = n_{Ei} + n_{Mi} \qquad 2.2.2.19$$

As the roots of accelerations are inversely proportional to their orbital radii, for calculating the value of acceleration a_n of the described points, the condition (2.2.2.20, 21) must be met;

$$\frac{n_E}{r_E} = \frac{\sqrt{a_E}}{\sqrt{a_{En}}} \qquad 2.2.2.20$$

$$\frac{n_M}{r_M} = \frac{\sqrt{a_M}}{\sqrt{a_{Mn}}} \qquad 2.2.2.21$$

where it is worth (2.2.2.22);

$$a_{En} = a_{Mn} = a_n \qquad 2.2.2.22$$

As according to the equality (2.2.2.19), the sum of distances from Earth n_{Ei} and Moon n_{Mi} equals to the radius of their mutual orbit r_o, it applies (2.2.2.23);

$$r_o = \frac{r_E\sqrt{a_E} + r_M\sqrt{a_M}}{\sqrt{a_n}} \qquad 2.2.2.23$$

Therefore, the acceleration values a_n are expressed by relations (2.2.2.24, 25);

$$a_{ni} = \left(\frac{r_E\sqrt{a_E} + r_M\sqrt{a_M}}{r_o}\right)^2 \qquad 2.2.2.24$$

$$a_{no} = \left(\frac{r_E\sqrt{a_E} - r_M\sqrt{a_M}}{r_o}\right)^2 \qquad 2.2.2.25$$

where a_{ni} and a_{no} are required acceleration values of the internal and external point on a line of their common orbit r_o.

Described points are specific positions of the space-time equilibrium. The body left at the point n_i will not fall either on Earth or the Moon. Relations between points n_{Ei} and n_{Mi} and n_{Eo} and n_{Mo} are equal to relation of Moon's orbital velocity v_{oME} in Earth's and Earth's orbital velocity v_{oEM} in Moon's orbit (2.2.2.26) where their velocities are calculated by the relation (2.2.2.4);

$$\frac{n_{Ei}}{n_{Mi}} = \frac{n_{Eo}}{n_{Mo}} = \frac{v_{oME}}{v_{oEM}} \qquad 2.2.2.26$$

Also, square roots of these relations correspond to the difference of their orbital velocities at measured positions while the squares of the same relations are equivalent to ratio of distances between the points where orbital velocities of both bodies are equal, and whose positions are anticipated by the same principle described. Therefore, the relations for positions of these points are written (2.2.2.27 - 30);

$$n_{Evoi} = r_o \frac{r_E^2 a_E}{r_E^2 a_E + r_M^2 a_M} \qquad 2.2.2.27$$

$$n_{Evoo} = r_o \frac{r_E^2 a_E}{r_E^2 a_E - r_M^2 a_M} \qquad 2.2.2.28$$

$$n_{Mvoi} = r_o \frac{r_M^2 a_M}{r_E^2 a_E + r_M^2 a_M} \qquad 2.2.2.29$$

$$n_{Mvoo} = r_o \frac{r_M^2 a_M}{r_E^2 a_E - r_M^2 a_M} \qquad 2.2.2.30$$

Where n_{Evoi}, n_{Evoo}, n_{Mvoi} and n_{Mvoo} are the inner and outer points with respect to the Earth and the Moon at which the orbital speed of both bodies are equal and whose inner and outer values, v_{oi} and v_{oo} are expressed by relations (2.2.2.31, 32);

$$v_{oi} = \frac{r_E^2 a_E + r_M^2 a_M}{r_o} \qquad 2.2.2.31$$

$$v_{oo} = \frac{r_E^2 a_E + r_M^2 a_M}{r_o} \qquad 2.2.2.32$$

Listed equalities are universal for all orbiting systems.

2.2.3 Principle of regression of lunar nodes

The ratio of relation (2.2.2.26) for the specific case of the Earth - Moon dual orbiting system is 9.023. In other words, the point of acceleration equilibrium at line Earth - Moon is 9.023 times closer to Moon than to Earth. That relation, according to expressions (2.2.2.15 - 18) is constant and independent of their distance $r_{o.}$ Consequently, to demonstrate regression of lunar nodes, trajectories of both bodies are treated as circles which radiuses equal to semi-major axis of the observed Moon's orbit ellipse.

So as gravity, orbital velocity is universal space-time property of all space-time entities.
Implicitly, being in Moons orbit, the Earth revolves around the Moon by the same universal law (2.2.2.2, 4), by which the Moon revolves around the Earth.

According to mentioned equivalence (2.2.2.26), their orbital velocities travelled on the same scopes differ by the same described ratio (9.023). Therefore, the Earth's period around Moon, is 9.023 times longer than the Moon's around the Earth.

The described model can be presented by two spheres in whose centres are Earth and Moon, and whose radii differ for the said ratio. These spheres are touching at the inner acceleration equilibrium point (2.2.2.15, 17) and according to aforementioned periods are rolling around each other (Figure 2.2.3.a).

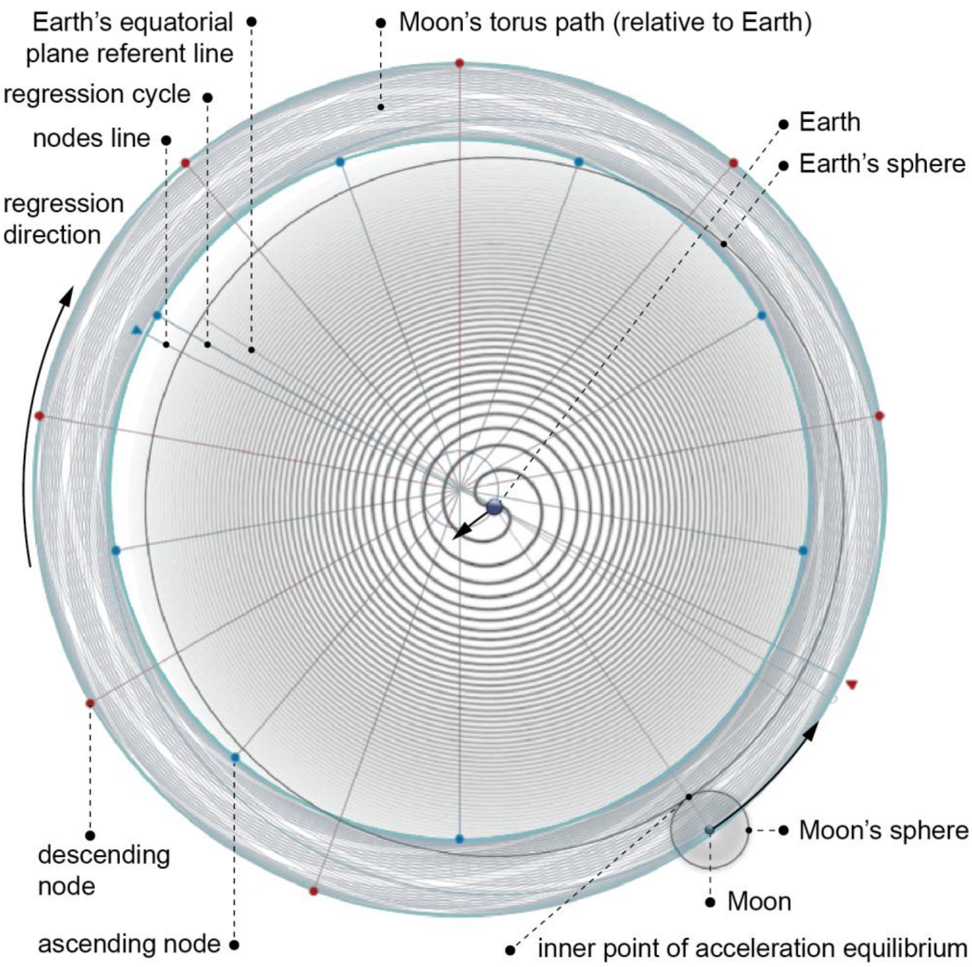

Figure 2.2.3.a *Geometrical presentation of regression of lunar nodes principle. Moon is converging to the next descending node of described cycle. Animated model is shown in movie clip regression of lunar nodes.*

Accordingly, as the lunar period at circle of semi-major axis radius lasts 27.44 Earth days, which is 2.846 hours longer than sidereal (27.322 days), in the described model, the Earth's 9.023 times longer period around the Moon amounts 247.588 days.
Time position after which the cycle of their rotation ratio will be repeated is their first common multiple (27.322 x 247.588) and amounts 6793.851 days, which is almost exactly 18.6 years. The obtained amount entirely matches the observed regression of lunar nodes cycle.

Thus, the cycle of regression of lunar nodes P_{Ln} equals to the product of Moon's rotation period around the Earth p_M and the Earth's around the Moon p_E (2.2.3.1);

$$P_{Ln} = p_M p_E \qquad 2.2.3.1$$

where periods p_M and p_E, are calculated as the relation of their trajectory and speed, i.e. the scope o_{ro} of the orbital radius r_o by their correlated orbital velocities v_{oME} and v_{oEM} (2.2.3.2);

$$P_{Ln} = \frac{o_{ro}^2}{v_{oME}\, v_{oEM}} \qquad 2.2.3.2$$

which is according to universal equations for the circumference of circle and orbital velocities (2.2.2.4) equal to (2.2.3.3);

$$P_{Ln} = \frac{4 r_o^2 \pi^2}{r_E r_M \frac{\sqrt{a_E a_M}}{r_o}} \qquad 2.2.3.3$$

As the first common multiplier we look for is for periods expressed in days, and previous equalities are expressed in seconds, the mutual factor of periods p_M and p_E in equalities (2.2.3.2, 3) is the number of seconds in Earth's day d_{sE}.
Therefore, the equality (2.2.3.3) is divided by square of mentioned amount d_{sE} so we get the formula for calculating the cycle of regression of lunar nodes P_{Ln} expressed in Earth's days (2.2.3.4);

$$P_{Ln} = \frac{4 r_o^3 \pi^2}{r_E r_M \sqrt{a_E a_M}\, d_{sE}^2} \qquad 2.2.3.4$$

where r_o is the radius of common orbit of Earth and Moon, r_E, r_M, a_E and a_M are their radii and surface accelerations, and d_{sE} is the period of Earth's day expressed in seconds.
In presented equation evident is the aforementioned Kepler's ratio ($4\pi^2 r^3/p^2$) misinterpreted by Newton's arbitrary *GM* invention, according to which formulation $r_E r_M \sqrt{a_E a_M}$ can be written as geometric mean of standard gravitational parameter μ for Earth and Moon ($\sqrt{\mu_E\, \mu_M}$ or $G\sqrt{M_E M_M}$) where variables *M* are therefore incorrectly calculated Newton's masses. As mentioned before the nature of *GM* construction describes the relation (2.2.2.7) which is constant for all orbital radiuses r_o and their corresponding accelerations a_o of observed body.

Prediction of cycle of lunar nodes regression and relations of the results obtained with those observed, are shown in Table 2.2.3.e;

Regression of Lunar nodes	data
Earth's Siderial year (days)	365,256363004000000
day (seconds)	86.400,000000000000000
Earth - Moon semi mayor axis (m)	384.399.000,000000000000000
Earth's equatorial radius (m)	6.378.100,000000000000000
Earth's surface acceleration	9,806700000000000
Moon's equatorial radius (m)	1.738.140,000000000000000
Moon's surface acceleration	1,622000000000000
	predicted
regression cycle (days)	6.793,851521784520000
regression cycle (years)	18,600227702837100
	observed
regression cycle (days)	6.793,622249329200000
regression cycle (years)	18,599600000000000
level of tolerance	1,000033748190130

Table 2.2.3.a *Predicted cycle of regression of lunar nodes and its perceived value*

2.2.4 Conclusion

By use of elementary mathematical logic, without calculations of "universal constants of nature" demonstrated is the nature of regression of lunar nodes. Precision and rigidity of the predicting method described, indicates the universal nature of orbiting systems where the orbiting bodies are rotating around each other, which is the correction of paradigm in which their rotation takes place around their common centre of mass.
Also demonstrated, universal equalities for acceleration and orbital velocities (2.2.2.1 - 5) change the perspective of understanding the structure of celestial entities, and the nature of gravity, mass and speed of light.