**A Note on Relativity**

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A note in favor of the correctness of the relativity, special and general.

*Key words*: relativity (special and general).

The special relativity gives us the equations: $E = mc^2$, $E_0 = m_0c^2$, $E = E_0 + T$, $m = \gamma m_0$ and $\gamma = (1 - v^2/c^2)^{-1/2}$, where $E$, $E_0$ and $T$ are the total, rest and kinetic energies of the particle, $m$ and $m_0$ its moving and rest masses, $c$ the speed of the light in the vacuum and $v$ the speed of the particle. For $v^2 << c^2$, $E = m_0c^2 + (1/2)m_0v^2$, which is the correct value for the Newton mechanics, where the kinetic energy is $T = (1/2)m_0v^2$. All these equations are obtained without using relativity, but with $\gamma = (1 - v/c)^{-1}$, which is not the correct value [1]. In addition, for two inertial systems: $f'/f = ((1 - v/c)/(1 + v/c))^{1/2}$ (from the relativistic Doppler effect), where $f'$ and $f$ are the frequencies of the light in the moving and rest frames, respectively, $v$ being the moving speed of the primed frame. But also, $E'/E = ((1 - v/c)/(1 + v/c))^{1/2}$ (from the Lorentz transformation for the energy), then $E'/E = f'/f$, $E' = hf'$ and $E = hf$, which is the Planck-Einstein equation, $h$ being the Planck constant [2]. All this is in favor of the correctness of the special relativity.

From the general relativity: $R_{ik} - (1/2)g_{ik}R = (8\pi G/c^4)T_{ik}$, $(i, k = 0, 1, 2, 3)$, where $R_{ik}$ is the Ricci tensor, $g_{ik}$ the metric tensor, $R$ the scalar curvature, $G$ the universal gravitational constant of Newton and $T_{ik}$ the energy-momentum tensor. This equation is like the Poisson equation: $\nabla^2\varphi = 4\pi G\rho$, where $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$, $x$, $y$ and $z$ being the rectangular coordinates, $\varphi$ the gravitational potential and $\rho$ the mass density. In the vacuum: $R_{ik} - (1/2)g_{ik}R = 0$ (Laplace equation: $\nabla^2\varphi = 0$), and $R_i^k - (1/2)\delta_i^kR = 0$, $R_i^i - (1/2)\delta_i^iR = 0$, $R - (1/2)4R = 0$ and $R = 0$ (where: $R_i^i = R$, $\delta_i^k = 1$ if $k = i$ and $\delta_i^k = 0$ if $k \neq i$, and $\delta_i^i = 4$); then, $R_{ik} = 0$. This equation was solved by Schwarzschild yielding a square space-time interval value of: $ds^2 = (1 - r_g/r)c^2dt^2 - r^2(\sin^2\theta d\phi + d\theta^2) - dr^2/(1 - r_g/r)$, where $r_g = 2GM/c^2$ is the gravitational (or Schwarzschild) radius, $M$ being the rest mass of the particle that produces the gravitational field, $t$ the time, and $r$, $\theta$ and $\phi$ the spherical coordinates. Note that $r > r_g$, since $r = r_g$ and $r = 0$ would yield $ds^2 = -\infty$ and $r < r_g$ would produce a change of sign in the time and in the space. Note also that, we may put $r = 2GM/v^2$, $v$ being like a gravitational escape speed ($E = T + V = (1/2)m_0v^2 - GMm_0/r$, $V$ being the potential energy, and from $E = 0$, the escape velocity would be: $v = (2GM/r)^{1/2}$), and as $r_g = 2GM/c^2$, it would correspond to a gravitational escape speed of $c$. As $r > r_g$, $v < c$, and there is not black holes. In addition, substituting these values in the interval, we would have that: $ds^2 = (1 - v^2/c^2)c^2dt^2 - r^2(\sin^2\theta d\phi + d\theta^2) - dr^2/(1 - v^2/c^2)$, and for given values of $\theta$ and $\phi$ ($\theta = constant$, $\phi = constant$, $d\theta = 0$ and $d\phi = 0$), it would be: $ds^2 = (1 - v^2/c^2)c^2dt^2 - dr^2/(1 - v^2/c^2) = c^2dt'^2 - dr'^2 = ds'^2$, which is a generalization of the special relativity for a radial motion in a gravitational field. Note also that for $|\varphi| << c^2$, which implies that $v^2 << c^2$, it is recovered the Newton gravitation formula [3]: $F = -GMm_0/r^2$, where $F$ is the gravitational attraction force between the masses $M$ and $m_0$ separated a distance $r$ (see the appendix). All this is in favor of the correctness of the general relativity.

Appendix

Newton's gravitational attraction force from Einstein's general relativity:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik} \, , \; (i, k = 0,1,2,3)$$

$$R_i^k - \frac{1}{2} \delta_i^k R = \frac{8\pi G}{c^4} T_i^k$$

$$R_i^i - \frac{1}{2} \delta_i^i R = \frac{8\pi G}{c^4} T_i^i \, , \; \left(R_i^i = R, \delta_i^i = 4, T_i^i = T\right)$$

$$R = -\frac{8\pi G}{c^4} T$$

$$R_{ik} = \frac{8\pi G}{c^4} \left( T_{ik} - \frac{1}{2} g_{ik} T \right)$$

$$|\varphi| << c^2 \, , \; v^2 << c^2$$

$$L = -Mc^2 + \frac{1}{2} M v^2 - M\varphi$$

$L$ being the Lagrangian.

$$L = -Mc \left( c - \frac{v^2}{2c} + \frac{\varphi}{c} \right)$$

$$S = \int L dt = -Mc \int \left( c - \frac{v^2}{2c} + \frac{\varphi}{c} \right) dt$$

$S$ being the action.

$$S = -Mc \int ds$$

$$ds = \left( c - \frac{v^2}{2c} + \frac{\varphi}{c} \right) dt$$

$$ds = c \left( 1 - \frac{v^2}{2c^2} + \frac{\varphi}{c^2} \right) dt$$

$$ds^2 = c^2 \left( 1 - \frac{v^2}{2c^2} + \frac{\varphi}{c^2} \right)^2 dt^2$$

$$\xi << 1 \, , \; (1 \pm \xi)^n \approx 1 \pm n\xi$$

$$\left(1-\frac{v^2}{2c^2}+\frac{\varphi}{c^2}\right)^2=(1+\xi)^2\approx 1+2\xi=1+2\left(-\frac{v^2}{2c^2}+\frac{\varphi}{c^2}\right)=1-\frac{v^2}{c^2}+\frac{2\varphi}{c^2}=1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}$$

$$c^2\left(1-\frac{v^2}{2c^2}+\frac{\varphi}{c^2}\right)^2=c^2\left(1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)=c^2\left(1+\frac{2\varphi}{c^2}\right)-v^2$$

$$vdt=dr$$

$$c^2\left(1-\frac{v^2}{2c^2}+\frac{\varphi}{c^2}\right)^2dt^2=c^2\left(1+\frac{2\varphi}{c^2}\right)dt^2-v^2dt^2=c^2\left(1+\frac{2\varphi}{c^2}\right)dt^2-dr^2$$

$$ds^2=c^2\left(1+\frac{2\varphi}{c^2}\right)dt^2-dr^2$$

$$ds^2=g_{00}c^2dt^2-dr^2$$

$$g_{00}=1+\frac{2\varphi}{c^2}$$

$$g_{\alpha\alpha}=-1\,,\ g_{\alpha\beta}=0\ (\alpha\neq\beta),\ g_{0\alpha}=g_{\alpha 0}=0\,,\ (\alpha,\beta=1,2,3)$$

$$T^{ik}=\rho c\frac{dx^i}{ds}\frac{dx^k}{dt}=\rho cu^iu^k\frac{ds}{dt}\,,\ u^i=\frac{dx^i}{ds}\,,\ c=\frac{dx^0}{dt}\,,\ v^\alpha=\frac{dx^\alpha}{dt}\,,\ \gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2},$$

$$ds=cdt\gamma^{-1}\,,\ T_0^0=\gamma\mu c^2\approx\mu c^2\,,\ T_\alpha^\alpha=\gamma\mu v^\alpha v_\alpha=\gamma\mu v^2\approx\mu v^2$$

$$T=T_i^i=T_0^0+T_\alpha^\alpha=\rho c^2+\rho v^2\approx\rho c^2$$

$$R_i^k=\frac{8\pi G}{c^4}\left(T_i^k-\frac{1}{2}\delta_i^kT\right)$$

$$R_0^0=\frac{8\pi G}{c^4}\left(T_0^0-\frac{1}{2}\delta_0^0T\right)=\frac{8\pi G}{c^4}\left(T_0^0-\frac{1}{2}T\right)=\frac{8\pi G}{c^4}\left(\rho c^2-\frac{1}{2}\rho c^2\right)=\frac{4\pi G\rho}{c^2}$$

$$R_{00}=R_0^0=\frac{4\pi G\rho}{c^2}$$

$$R_{00}=\frac{\partial\Gamma_{00}^\alpha}{\partial x^\alpha}$$

$\Gamma_{00}{}^\alpha$ being a Christoffel symbol.

$$\Gamma_{00}^\alpha\approx-\frac{1}{2}g^{\alpha\beta}\frac{\partial g_{00}}{\partial x^\beta}=-\frac{1}{2}(-1)\frac{\partial g_{00}}{\partial x^\alpha}=\frac{1}{2}\frac{\partial}{\partial x^\alpha}\left(1+\frac{2\varphi}{c^2}\right)$$

$$\Gamma_{00}^\alpha=\frac{1}{c^2}\frac{\partial\varphi}{\partial x^\alpha}$$

$$R_{00}=\frac{1}{c^2}\frac{\partial^2\varphi}{\partial x^{\alpha 2}}$$

$$\frac{\partial^2\varphi}{\partial x^{\alpha 2}}=4\pi G\rho$$

$$\nabla^2\varphi = 4\pi G\rho$$

$$\varphi = -\frac{1}{4\pi}\int\frac{4\pi G\rho dV}{r} = -G\int\frac{\rho dV}{r}$$

$V$ being the volume, since $\nabla^2(1/r) = -4\pi\delta(r)$, where $\delta(r)$ is the Dirac delta function: $\delta(r) = +\infty$ for $r = 0$ and $\delta(r) = 0$ for $r \neq 0$ and $\int\delta(r)dV = 1$; and $\nabla^2\varphi = -G\int\rho\nabla^2(1/r)dV = 4\pi G\int\rho\delta(r)dV = 4\pi G\rho\int\delta(r)dV = 4\pi G\rho$. For a group of $n$ particles

$$\varphi = -G\sum_n\frac{M_n}{r_n}$$

$M_n$ being the masses of the particles and $r_n$ the distances from them to the field points. And for a single particle

$$\varphi = -G\frac{M}{r}$$

$$F = -m_0\frac{\partial\varphi}{\partial r}$$

$$F = -G\frac{Mm_0}{r^2}$$

$F$ being the Newton gravitational attraction force between two particles of masses $M$ and $m_0$ separated a distance $r$.

[1] José Francisco García Juliá, A Note on the Mass-Energy Relation, January 10, 2012. http://vixra.org/pdf/1109.0057v2.pdf

[2] James H. Smith, Introducción a la Relatividad Especial, pp. 142-143, Reverté, Barcelona, 1969. Original edition, Introduction to Special Relativity, Benjamin, New York.

[3] L. D. Landau and E. M. Lifshitz, Teoría Clásica de los Campos, pp. 31, 116, 344, 384, 390, in Spanish, Reverté, Barcelona, 1973. Original edition by Nauka, Moscow, 1967.