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General Relativity in Microscopic Scale (GRMS)

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ABSTRACT

We use the Modified Reissner-Nordstrom solution to Einstein's field equation for static charged spherical black hole in microscopic scale, by redefine the potential. Therefore, we obtained circular orbits and horizons in atomic domain. Generalization of special relativity in the sense of the model, predicted proton radius, mass defect, bending of light and precession of perihelion. Mass defect used in estimation electron, positron, neutrino and anti-neutrino mass, in addition of proton decaying energy and mass charge equivalent relation.

KEYWORDS: General Relativity ; Microscopic scale; Generalization ;special relativity ; Mass defect; beta decay; radius

1. INTRODUCTION

The universality of the wave nature of any physical system keeps the properties of the system when we measured them. According to a principle of scale relativity, the laws of physics should be applied to systems of references whatever their scales. But when the total mass, m, becomes larger than Planck's mass, its Compton length becomes smaller than Schwarzschild's radius. Thus becoming unmeasureable, since it enters into black hole horizon. On the contrary, for total mass less than Planck' mass, the Schwarzschild's radius becomes smaller than Compton length, where the concept of position loses its physical meaning [1].It appears that the notion of Schwarzschild's radius is also loses its physical meaning.

Nottale suggested the fractal nature of space time [2,3,4], attempted to connect the fractal approach to general relativity by considering a simple model of fractal metric as:

$$ds^2 = \zeta^2(t, \Delta t)\left(1-\frac{\tau}{\Delta t}\right)^2 c^2 dt^2 - dl^2$$

Where, ζ is some finite fractional function with $\zeta = 1$for $\Delta t > \tau$, $\tau = \frac{\hbar}{mc^2}$ is De Broglie time of a system of inertial mass, m, and dl^2 is a fractal spatial element.

On the other hand,various authors [5,6,7] have suggested that the black hole is a normal quantum system with discrete energy levels.

Therefore, it is easy to compare between the Black hole and the nucleus and put forward a model using general relativity metric in fraction space. This could be done by transforming the macroscopic scale constant (universal gravitational constant G) to microscopic scale constant (Planck constant,$\hbar$) through the transformation $\frac{GM}{c^2} \to \frac{\hbar}{mc}$[8]. This demands the conversion of the effectiveness of macroscopic scale with mass M to microscopic scale with mass m.

2. MicroscopicGeneral Relativity Model (MGR)

In this model we applied the scale transformation to Reissner-Nordstrom metric to obtain the microscopic scale metric.

Reissner-Nordstrom metric in spherical coordinate symmetric is given by:

$$d\tau^2 = -\left(1-\frac{2GM}{rc^2}+\frac{GQ^2}{r^2c^4}\right)dt^2 + \frac{1}{c^2}\left(1-\frac{2GM}{rc^2}+\frac{GQ^2}{r^2c^4}\right)^{-1} dr^2 + \frac{1}{c^2}r^2(d\theta^2 + sin^2\theta d\emptyset^2) \qquad (1)$$

By substituting $\frac{GM}{c^2} \to \frac{\hbar}{mc}$ (Compton's length) in (1).then, the metric becomes:

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$$d\tau^2 = -\left(1-\frac{2\hbar}{m_s cr}+\frac{\hbar Q^2}{4\pi\epsilon c^3 m^2 r^2}\right)dt^2+\frac{1}{c^2}\left(1-\frac{2\hbar}{m_s cr}+\frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right)^{-1}dr^2 + \frac{1}{c^2}r^2(d\theta^2+sin^2\theta d\emptyset^2)$$

We restore $\frac{1}{4\pi\epsilon}$ in the equation above to the electric term. For Simplicity we can rewrite the metric equation (1) as follows:

$$d\tau^2 = -\left(1-\frac{r_s}{r}+\frac{r_Q^2}{r^2}\right)dt^2+\frac{1}{c^2}\left(1-\frac{r_s}{r}+\frac{r_Q^2}{r^2}\right)^{-1}dr^2+\frac{1}{c^2}r^2(d\theta^2+sin^2\theta d\emptyset^2) \quad (2)$$

Where: $r_s = \frac{2\hbar}{m_s c}$, $r_Q^2 = \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2}$,m_s is the total mass of the black hole and m_Q is the electro-mass. The scale transformation can be obtained as:

$$GM \to \frac{\hbar c}{m} \quad (3)$$

There is true singularity in (2)at $m = 0$, mass singularity, which leads to redefining the microscopic scale as the scale of mass in the range , $0 < m_0 < m_{pl}$, where m_0 is the rest mass of the microscopic Particle and m_{pl} is Planck mass. In addition, there are two singularities (horizons) at:

$$r_\pm = \frac{\hbar}{m_s c}\left(1 \pm \sqrt{1-\frac{Q^2{m_s}^2}{4\pi\epsilon {m_Q}^2\hbar c}}\right) \quad (4)$$

We find that r_+ overlaps r_- when, $\frac{Q^2{m_s}^2}{4\pi\epsilon {m_Q}^2\hbar c} = 1$. This implies,

$$m_Q = \pm\sqrt{\frac{\hbar}{4\pi\epsilon c^3}}\frac{Qm_s c}{\hbar} \quad (5)$$

Equation(5) represents threshold electro-mass when, $\epsilon = \epsilon_0$, $m_s = m_{pl}$ and $Q = Q_{pl}$,where,m_{pl} and Q_{pl} are Planck's mass and charge respectively. In this case, we find $m_Q = m_{pl}$ .This is equivalent to the state where, Schwarzschild's radius or Mach's radius equal to Compton length, then the mass should be equal to Planck's mass (threshold mass). Now we can constrain microscopic charge to maximal limit as Planck's charge with radius equal to Compton length. This result can be shown directly by using the electron wave function in the framework of special scale-relativity and below the scale λ (λ is Compton length), the Planck length,l_{pl}, becomes a minimal scale[9], $\acute{\psi} = \exp\left(i\,4\pi\alpha \ln\left(\lambda/l_{pl}\right)\right)\psi$ .Hence,$\acute{\psi} = \psi$ when $\lambda = l_{pl}$ .therefore, the expression of electron wave function can be generalized to describe electro-mass.
Solving the geodesic equation for the line element metric (2), one finds the possible circular orbits for massless particles:

$$r_c = \frac{3}{2}\frac{\hbar}{m_s c}\left(1 \pm \sqrt{1-\frac{2Q^2{m_s}^2}{9\pi\epsilon {m_Q}^2\hbar c}}\right) \approx \frac{3}{2}\frac{\hbar}{m_s c}\left(1 \pm \left(1-\frac{Q^2{m_s}^2}{9\pi\epsilon {m_Q}^2\hbar c}\right)\right) \quad (6)$$

For massive particles on has:

$$r_c = \left[\frac{r_Q^2}{r_s}+\frac{L^2}{r_s}\right] \pm \left(\frac{r_Q^2}{r_s}+\frac{L^2}{r_s}-\frac{3}{2}\left(\frac{r_s}{r_Q^2+L^2}\right)L^2\right) \quad (7)$$

Where, L is the angular momentum per unit mass of the particle.
According to equation (7) there are different regimes(stable and unstable) depending on the angular momentum. The unstable orbits for massive particles are located at the radius r_{c-}:

$$r_{c-} = \frac{3L^2 r_s}{2\left(r_Q^2+L^2\right)} \quad (8)$$

The other stable orbits,r_{c+} , are located at:

$$r_{c+} = 2\left[\frac{r_Q^2}{r_s} + \frac{L^2}{r_s}\right] - \frac{3L^2 r_s}{2\left(r_Q^2 + L^2\right)} \quad (9)$$

Equation(9),from atomic view point, represents the possible atomic orbits. The smallest radius (Bohr radius) occurs when, $(L = 0)$.
In the case of uncharged nucleus ,$Q = 0$, the unstable orbit is

$$r_{c-} = \frac{3\hbar}{m_s c} \quad (10)$$

The stable orbit is

$$r_{c+} = \frac{3\hbar}{m_s c}\left(\frac{4L^2}{3{r_s}^2} - 1\right) \quad (11)$$

At this limit the stable orbit becomes larger than the unstable one, which approaches, r_s , (behaves like massless).The orbit doesn't vanish when $Q = 0$ .Hence, the existence of the orbit does not depend on electrical force, as well as gravitational force.

3. Generalization of special relativity:

In an attempt to generalize the special relativity to includes the gravitational potential [10], one has to redefine the transformation γ in terms of proper time τ.

As we considered above, in the radial motion, $\theta$and$\emptyset$ are constants.

First, in non-relativistic case: $v \ll c$, the metric in time-like signature is:

$$d\tau^2 = \left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right) dt^2 - \frac{1}{c^2}\left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right)^{-1} dr^2 \quad (12)$$

Therefore, the transformation is

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} + g_{rr}\frac{v^2}{c^2}} \quad (13)$$

With, $g_{00} = \left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right)$;$g_{rr} = -\left(1 + \frac{2\hbar}{m_s cr} - \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right)$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00}} = \sqrt{1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}} \quad (14)$$

$$\gamma^{-1} = 1 + \frac{\hbar}{m_s cr} - \frac{\hbar Q_m^2}{8\pi\epsilon c^3 {m_Q}^2 r^2} \quad (15)$$

Second, in relativistic case $v = c$, the metric in space-like signature:

$$g_{00} = -\left(1 - \frac{2\hbar}{m_s cr} + \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right) \quad ; \quad g_{rr} = \left(1 + \frac{2\hbar}{m_s cr} - \frac{\hbar Q^2}{4\pi\epsilon c^3 {m_Q}^2 r^2}\right)$$

$$\gamma = \frac{d\tau}{dt} = \sqrt{g_{00} + g_{rr}} = \sqrt{\frac{4\hbar}{m_s cr} - \frac{\hbar Q^2}{2\pi\epsilon c^3 {m_Q}^2 r^2}} \quad (16)$$

4. Some applications of the model:

1. **Mass defect and beta decay**

In non-relativistic case the observable mass can be obtained according to equation(15)

$$m = m_0\left(1 + \frac{\hbar}{m_s cr} - \frac{\hbar Q^2}{8\pi\epsilon c^3 {m_Q}^2 r^2}\right) \quad (17)$$

If the proton is assumed to be the nucleus origin, then the observable neutron mass is

$$m_n = m_{n0} + m_{n0}\left(\frac{\hbar}{m_{p0} cr} - \frac{\hbar e^2}{8\pi\epsilon c^3 {m_{Qp}}^2 r^2}\right) \quad (18)$$

Where m_{Qp} , stands for proton electro-mass.m_{p0} and m_{n0} are the proton and neutron rest mass respectively.

According to equation (18), the neutron mass does not altered when the second term in R.H.S vanished. This implies that,

$$m_Q = \pm\sqrt{\frac{m_s}{8\pi\epsilon c^2 r}}\ Q \tag{19}$$

On the other hand, if the neutron is assumed to be the nucleus origin ($Q = 0$), then the observable proton mass is

$$m_p = m_{po} + m_{po}\frac{\hbar}{m_{n0}cr} \tag{20}$$

Equation (18) and (20) can be formed as:

$$n \to n_0 + n_0\left(\frac{\hbar}{p_0 c r_p} - \frac{\hbar e^2}{8\pi\epsilon c^3 {p_0}^2 {r_p}^2}\right) \tag{21}$$

$$p \to p_0 + p_0\frac{\hbar}{n_0 c r_n} \tag{22}$$

Where $p_0$refers to proton rest mass and n_0 refers to neutron rest mass.
By adding, equation(21) to (22), one finds,

$$n + p \to (n_0 + p_0) + \left(n_0\frac{\hbar}{p_0 c r_p} + p_0\frac{\hbar}{n_0 c r_n}\right) - n_0\frac{\hbar e^2}{8\pi\epsilon c^3 {m_{Qp}}^2 {r_p}^2} \tag{23}$$

On the other hand, beta decay is given by the β^- and β^+ equations,

$$n \to p + e^- + \bar{\upsilon}_e \tag{24}$$

$$p + \frac{E}{c^2} \to n + e^+ + \upsilon_e \tag{25}$$

Adding equation (24) to equation (25), one has

$$n + p \to (n + p) + (e^- + e^+) + (\bar{\upsilon}_e + \upsilon_e) - \frac{E}{c^2} \tag{26}$$

Comparingequation (23) with equation (26), the equivalent masses for all of neutron, proton, neutrino and anti-neutrino are obtained. In addition, the required energy for proton decaying,

$$(n + p) \to (n_0 + p_0) \tag{27}$$

$$(e^- + e^+) \to \left(n_0\frac{\hbar}{p_0 c r_p} + p_0\frac{\hbar}{n_0 c r_n}\right) \tag{28}$$

$$(\bar{\upsilon}_e + \upsilon_e) \to 0 \tag{29}$$

$$E \to n_0\frac{\hbar e^2}{8\pi\epsilon c {m_{Qp}}^2 {r_p}^2} \tag{30}$$

In addition one can find a relation between the neutron radius and the proton radius, fromequation (28),

$$r_p = \frac{{m_n}^2}{{m_p}^2} r_n \tag{31}$$

Equation (4) gives the neutron radius,$r_n = 3.981275 \times 10^{-16}\ m$ .Hence, the proton radius is, $r_p = 3.992258 \times 10^{-16}\ m$.
Fromequation (30) we get mass-charge equivalence(electro-mass)relation that agrees with equation (5)

$$m_Q = \pm\sqrt{\frac{\hbar}{8\pi\epsilon c^3}}\frac{Q}{r} \tag{32}$$

According to equation (32) electro-mass tends to W boson masswhen$\frac{Q}{r} = 1$ and$\varepsilon = \varepsilon_0$.
Therefore, the value of ε inside the proton according to equation (32) is given by

$$\varepsilon = \frac{\hbar e^2}{8\pi c^3 m^2 r^2} = 6.2016 \times 10^{-4}\ F\ m^{-1}$$

Where: $r = r_p - r_n$ and $m = m_n - m_p$.
This gives the value of $4\pi\epsilon = 7.793 \times 10^{-3}$, which is equal to the fine structure constant, α_w, at the interaction energies above 80 GeV,[11].

2. bending of light and precession of the perihelion

The successful of GR in explaining bending of light and the precession of the perihelion of mercury provides that, the Newtonian mechanics suffer some information's lack. This can be formed as

$$\delta = \frac{D}{D_f} S(\gamma) \tag{33}$$

Where: D is interaction dimension, D_f is fractal space dimension, $D_f = 2$, [4], $S(\gamma)$ is the information correction due generalized special relativity.
By using equation (16)in relativistic case, the deviation of light near massive microscopic objects is described by the following equation

$S(\gamma) = \gamma^2$ (Open trajectory), $D = 2$

$$\delta = \gamma^2 = g_{00} + g_{rr} = \frac{4\hbar}{m_s c r} - \frac{\hbar Q^2}{2\pi\epsilon c^3 m_Q^2 r^2} \tag{34}$$

Equation (34) tends to $\delta = \frac{4GM}{bc^2}$ in macroscopic objects ($Q = 0$; r ≡ impact parameter b), which is the same equation predicted by solving the null geodesic equation.
Second implication is the precession of the perihelion,

$S(\gamma) = \pi\gamma^2$ (Closed trajectory), $D = 3$.

$$\delta = \frac{3\pi}{2}(g_{00} + g_{rr}) = \frac{6\pi\hbar}{m_s c r} - \frac{3\pi\hbar Q^2}{4\pi\epsilon c^3 m_Q^2 r^2} \tag{35}$$

Similarly, equation (35) construes to that for precession of the perihelion of planets in general relativity $\delta = \frac{6\pi GM}{rc^2}$.

3. Measurement of nucleus radius

To measure the nucleus radius is equivalent to measure the horizon radius. Equation (4) gives the horizon radius. Unfortunately, this value is immeasurable experimentally, because of the light deviation according to equation (33). Hence, the measurable radius is

$$R = \delta.(r_\pm) + \delta_{exp} \tag{36}$$

Where, δ_{exp} is experiment's correction.
Hence, one can estimate the proton radius in Hydrogen atom using equation (36). The result is

$$R_p \geq \delta.(r_\pm) = 8.402 \times 10^{-16}\quad m \tag{37}$$

Equation (37) agrees with the latest estimation of the proton radius, $r_p = 8.4184(67) \times 10^{-16}\ m$,[12].which gives plentiful conclusion.

Conclusion

It is obviously that, the translation of general relativity to microscopic scale through the transformation, $\frac{GM}{c^2} \rightarrow \frac{\hbar}{mc}$, represents new and promising approach in general relativity. Applications of microscopic general relativity in nuclear domain, (mass defect, proton radius and beta decay), approved the validity of the model.

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