On the Mass-Energy and Charge-Energy Equivalences

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Abstract

As the energies associated with particles are finite, they cannot originate from point like entities. This leads us to present the following postulate: "The smallest unit of charge or mass should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential (ϕ) becomes the limiting value, set by the Planck scale".

Using this postulate, we can derive a general proof for both mass-energy and charge-energy equivalences ($E = mc^2$ and $E = qV_{planck}$ respectively) and derive their relativistic energy-momentum, relativistic-energy and relativistic-momentum relations. The results are in accordance with special relativity.

We then discuss the non-covariance nature of the present classical electrodynamics and show how the proposed postulate makes it a fully covariant theorem with the rest of the classical electrodynamics.

1 Introduction

Einstein proposed mass-energy equivalence in 1905 [1], in his paper entitled: "Does the inertia of a body depends upon its energy content?". He concluded that the mass of a body is a measure of its energy content. That is, if the energy changes by L, the mass changes in the same sense by L/c^2. This equivalence can be summarized in the famous equation:

$$E = mc^2 \tag{1}$$

However, we emphasize that the mass-energy equivalence stated above in equation (1) is strictly applicable to indivisible mass particles only. if one were to find the total relativistic mass M for a collection of mass particles, one should take into account the energy-momentum relation as shown below, where the velocities u_i of each particle m_i are obtained with relative to the center-of-momentum of the mass body M.

$$(Mc^2)^2 = \sum_{i=1}^{i=n}(\gamma_i m_i u_i c)^2 + \sum_{i=1}^{i=n}(m_i c^2)^2 \tag{2}$$

On the other hand, in classical electrostatics, energy of a charge q is given by:

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$$E = (k_d)\frac{1}{4\pi\epsilon_0}\frac{q^2}{r} \tag{3}$$

where, (k_d) is a scalar factor, which depends on how the charge is distributed. When the charge is assumed to be distributed with a constant density, the factor (k_d) becomes $(\frac{3}{5})$ whereas if the charge is on the surface, the factor (k_d) becomes $(\frac{1}{2})$. Further, these relations are derived by bringing in infinitesimal amounts of charge from infinity and constructing their corresponding charge configurations.

We observe that the energy content of a charge q in equation (3) depends on the structure of the charge, i.e. configuration and the radius of its charge distribution. Further, it suggests that, when the radius of the charge configuration goes to zero, its energy content becomes infinite. In contrast, the energy content of a mass particle in equation (1) is independent of both configuration and radius. Further, it represents the energy of an indivisible mass particle.

From these observations, we state that there must also exist an indivisible charge, a charge particle whose energy content does not depend on how its charge content is distributed or configured.

We then argue that, similar to what Einstein concluded for mass, the total charge of a charge particle must represent a measure of its energy content, which is bounded. This leads us to introduce a new postulate which states that: "**A charge particle or a mass particle should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential ϕ becomes the limiting value, set by the Planck scale".** This postulate leads us to derive both the charge-energy $E = qV_{planck}$ and mass-energy $E = mc^2$ equivalences. We then derive the energy-momentum relation for both charge and mass particles in motion and show that both mass-energy and charge-energy equivalences are relativistically covariant. Further, we show that the momentum associated with both charge and mass particles in motion are covariant as well. This makes the classical electrodynamics a fully covariant theorem.

2 Energy equivalence, relativistic-energy and relativistic-momentum of charge and mass particles in motion

From classical interpretation, we can derive the following generalized relation for $(\frac{dE}{dp})$, from force (F), energy (E), momentum (p) and velocity (u).

$$dE = F \cdot dx = (\frac{dp}{dt})dx = (u)dp \tag{4}$$

$$\frac{dE}{dp} = u \tag{5}$$

The electromagnetic vector potential A is defined as given below, where (J) current density, (q) charge, (r) distance from the charge, (u) velocity of the charge, (ρ) charge density, (ϕ_E) electrical scalar-potential and $\mu_0\epsilon_0 = \frac{1}{c^2}$.

$$A = \frac{\mu_0}{4\pi}\int_{vol}(\frac{J}{r})dv = \frac{1}{(c^2 4\pi\epsilon_0)}\int_{vol}\frac{(\rho u)}{r}dv = \frac{u}{c^2}\int_{vol}\frac{\rho}{4\pi\epsilon_0 r}dv = \frac{u}{c^2}\phi_E \tag{6}$$

$$A = \frac{u}{c^2}\phi_E \tag{7}$$

$$\phi_E = \frac{q}{4\pi\epsilon_0 r} \tag{8}$$

The electromagnetic momentum p_q of a charge q with velocity u is defined as (qA):

$$p_q = qA = q(\frac{u}{c^2}\phi_E) \tag{9}$$

by combining equations (5) and (9):

$$\frac{dE}{dp} = u = \frac{p_q c^2}{q\phi_E} \tag{10}$$

The energy of a single point-charge particle interacting with its own field, known as self-energy, tends to go to infinity as the radius of the particle goes to zero, $r \to 0$. The self-energy of a charge particle q can be worked out as:

$$E = q\phi_E = q\left(\frac{q}{4\pi\epsilon_0 r}\right) \tag{11}$$

The scalar-potential ϕ_E becomes infinite as $r \to 0$, during which the corresponding self-energy of the charge particle becomes infinite as well. However, as particles cannot possess infinite energies, we argue that the self-energy of a charge particle must be bounded and finite. Therefore, the scalar-potential ϕ_E which tends to go to infinity as $r \to 0$ must be finite, so that the total energy of a charge particle becomes finite and bounded. That is, as the physical size of a charge particle goes to zero ($r \to 0$), the scalar-potential must reach a maximum limit. Thus, we introduce the following postulate: "**A charge particle should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential ϕ_E becomes the limiting value, set by the Planck scale"**. The postulate presented above leads us to find the maximum scalar-potential of a self-interacting charge particle and thereby to derive its charge-energy equivalence.

$$(\phi_E)_{max} = \left(\frac{q}{4\pi\epsilon_0 r}\right)_{max} = (voltage)_{planck} = V_{planck} \tag{12}$$

$$(\phi_E)_{max} = V_{planck} \tag{13}$$

by substituting in equation (11):

$$E = q(\phi_E)_{max} = qV_{planck} \tag{14}$$

Equation (14) gives us the total self-energy or the charge-energy equivalence of a charge particle. We then use the same postulate presented above to derive the self-energy of a mass particle (mass-energy equivalence).

$$\phi_G = \frac{Gm}{r} \tag{15}$$

$$E = m\phi_G = m(\frac{Gm}{r}) \tag{16}$$

The Self-energy of a mass particle becomes infinite (∞) as $r \to 0$. As for the postulate presented earlier, the gravitational scalar-potential (ϕ_G) must also be finite and bound by the Planck scale.

$$(\phi_G)_{max} = (\frac{Gm}{r})_{max} = (velocity)^2_{planck} = c^2 \tag{17}$$

$$(\phi_G)_{max} = c^2 \tag{18}$$

$$E = m(\phi_G)_{max} = mc^2 \tag{19}$$

Equation (19) is the mass-energy equivalence relation ($E = mc^2$) which was affirmed by Einstein in about eighteen different presentations. However, he was not able to provide a conclusive general proof of this seminal hypothesis from first principles [2][3]. The same mass-energy equivalence is proved above, from first principles, using the postulate we presented in this paper. Further, by using the same postulate and a similar set of procedures led us to obtain a general proof for the charge-energy equivalence relation given in equation (14).

Now let us derive the momentum-energy relations for both charge and mass particles, where $\phi_E = V_{planck}$ and $\phi_G = c^2$ are their self-interacting scalar-potentials, respectively.

$mass$	$charge$
$p_m = mu$	$p_q = qA$
$p_m = (\frac{mu}{c^2})\phi_G$	$p_q = (\frac{qu}{c^2})\phi_E$
$p_m = (m\phi_G)\frac{u}{c^2}$	$p_q = (q\phi_E)\frac{u}{c^2}$
$\frac{dE_m}{dp_m} = u = \frac{p_m c^2}{m\phi_G}$	$\frac{dE_q}{dp_q} = u = \frac{p_q c^2}{q\phi_E}$
$(m\phi_G)dE_m = (p_m c^2)dp_m$	$(q\phi_E)dE_q = (p_q c^2)dp_q$
$(mc^2)dE_m = (p_m c^2)dp_m$	$(qV_{planck})dE_q = (p_q c^2)dp_q$
$E_m dE_m = (p_m c^2)dp_m$	$E_q dE_q = (p_q c^2)dp_q$
$\frac{E_m^2}{2} = \frac{(p_m c)^2}{2} + k_m$	$\frac{E_q^2}{2} = \frac{(p_q c)^2}{2} + k_q$

By introducing boundary conditions, where the energies become rest frame energies, m_0c^2 and q_0V_{planck} when the velocity becomes zero ($u = 0$), we can obtain the following results.

$$E_m^2 = (p_m c)^2 + E_m^2(0) \Longleftrightarrow E_q^2 = (p_q c)^2 + E_q^2(0) \tag{20}$$

$$E^2 = (p_m c)^2 + (m_0c^2)^2 \Longleftrightarrow E^2 = (p_q c)^2 + (q_0V_{planck})^2 \tag{21}$$

$$(mc^2)^2 = (muc)^2 + (m_0c^2)^2 \Longleftrightarrow (qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0V_{planck})^2 \tag{22}$$

Equation (22) gives us the relativistic energy-momentum relations for both charge and mass particles in motion. In Einstein's Special Relativity, only the relativistic momentum-energy relation for mass bodies in motion was derived. On the other hand, by observing the finiteness of energies associated with a given amount of charge (or mass), and by introducing our new postulate led us to derive the relativistic energy-momentum relation for both mass and charge particles in motion.

Further, using charge-momentum $p_q = qA = q(\frac{u\phi_E}{c^2})$ and the self-interacting scalar-potential $\phi_E = V_{planck}$, we can show that the energy of a charge particle is covariant (similar to that of mass particles).

$mass$	$charge$
$(mc^2)^2 = (pc)^2 + (m_0c^2)^2$	$(qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0V_{Planck})^2$
$(mc^2)^2 = (muc)^2 + (m_0c^2)^2$	$(qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0V_{planck})^2$
$(mc^2)^2(1 - \frac{u^2}{c^2}) = (m_0c^2)^2$	$q^2V_{planck}^2(1 - \frac{u^2}{c^2}) = q_0^2V_{planck}^2$

$$mc^2 = \gamma m_0c^2 \Longleftrightarrow qV_{planck} = \gamma q_0V_{planck} \tag{23}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{24}$$

This shows that the mass-energy equivalence ($E = mc^2$) and the charge-energy equivalence ($E = qV_{planck}$) are both relativistically covariant. In like manner, we can show that mass-momentum (mu) and charge-momentum (qA) are relativistically covariant as well (for $p \neq 0$, i.e. $u \neq 0$).

$mass$	$charge$
$m^2c^4 = m^2u^2c^2 + m_0^2c^4$	$q^2V_{planck}^2 = q^2u^2(\frac{V_{planck}}{c})^2 + q_0^2V_{planck}^2$
$c^4(m^2u^2 - m^2\frac{u^4}{c^2}) = (c^4)m_0^2u^2$	$(V_{planck}^2)(q^2u^2 - q^2\frac{u^4}{c^2}) = (V_{planck}^2)(q_0^2u^2)$
$(mu)^2(1 - \frac{u^2}{c^2}) = m_0^2u^2$	$(qu)^2(1 - \frac{u^2}{c^2}) = q_0^2u^2$
$mu = \frac{m_0u}{\sqrt{1-\frac{u^2}{c^2}}}$	$qu(\frac{V_{planck}}{c^2}) = \frac{q_0u}{\sqrt{1-\frac{u^2}{c^2}}}(\frac{V_{planck}}{c^2})$

$$mu = \gamma m_0 u \Longleftrightarrow qu(\frac{V_{planck}}{c^2}) = \gamma q_0 u(\frac{V_{planck}}{c^2}) \tag{25}$$

Note that, all the above derivations were based on the assumption that there exist an indivisible quanta of charge (or mass). One can then define the notion of a body with total charge Q (or with total mass M) as a collection of many such particles. Thus:

$$(Mc^2)^2 = \sum_{i=1}^{i=n}(\gamma_i m_i u_i c)^2 + \sum_{i=1}^{i=n}(m_i c^2)^2 \tag{26}$$

$$(QV_{planck})^2 = \sum_{i=1}^{i=n}(\gamma_i q_i u_i \frac{V_{planck}}{c})^2 + \sum_{i=1}^{i=n}(q_i V_{planck})^2 \tag{27}$$

We would like to emphasize some facts regarding the rest-mass and the relativistic-mass concepts. The relativistic-mass is derived from the relativistic energy ($\gamma m_0 c^2$) or relativistic momentum ($\gamma m_0 u$) of the system and thus it is argued that relativistic-mass (γm_0) is not a good concept. Einstein wrote "It is not good to introduce the concept of the mass $M = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the rest-mass m_0. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion" [4]. The same set of arguments holds true for the proposed relativistic charge-energy ($\gamma q_0 V_{planck}$) and relativistic charge-momentum $\gamma q_0 u(\frac{V_{planck}}{c^2})$ concepts as well.

3 Classical electron theory and its lack of relativistic covariance

Max Abraham [5] and H.A Lorentz [6], based on Maxwell's theory of electricity and magnetism developed the first set of theories for the classical electron. As for the classical electrostatics, the rest energy U_0 of a spherical charge body with radius r, associated with total charge e, uniformly distributed over its surface is given by:

$$U_0 = \left(\frac{1}{2}\right)\frac{e^2}{4\pi\epsilon_0 r} \tag{28}$$

One can then obtain the relativistic electromagnetic energy U of the moving charge e as for the definition given below [7] [8] [9] [10]:

$$U = \frac{1}{2}\int_{all-space}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}H^2\right)dv \tag{29}$$

$$U = \gamma\frac{e^2}{8\pi\epsilon_0 r}\left(1 + \frac{1}{3}\beta^2\right) \tag{30}$$

and its relativistic electromagnetic momentum P as:

$$P = \epsilon_0 \int_{all-space} (E \times B)\, dv \tag{31}$$

$$P = \frac{4}{3}\gamma u \frac{e^2}{8\pi\epsilon_0 r c^2} \tag{32}$$

where u is the velocity of the charge e, and

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{33}$$

$$\beta = \frac{u}{c} \tag{34}$$

However, according to mass-energy equivalence and the theory or relativity, we can find the equivalent electromagnetic invariant mass m_e of an electron with charge e as:

$$m_e = \frac{U_0}{c^2} = \frac{e^2}{8\pi\epsilon_0 r c^2} \tag{35}$$

Therefore, equations (30) and (32) can be written in terms of the electromagnetic invariant mass m_e as given below.

$$U = \gamma m_e c^2 (1 + \frac{1}{3}\beta^2) \tag{36}$$

$$P = \frac{4}{3}\gamma m_e u \tag{37}$$

From equations (36) and (37), it is immediately obvious that the terms U and P do not transform properly as an energy-momentum four-vector. Also the relativistic energy-momentum relation $U^2 = (Pc)^2 + (U_0)^2$ is violated, which implies that the terms U and P are neither relativistically covariant nor transformed as an energy-momentum four-vector.

On the other hand, if they are to be relativistically covariant, they should be of the form:

$$U = \gamma m_e c^2 \tag{38}$$

$$P = \gamma m_e u \tag{39}$$

which satisfies the energy-momentum relation:

$$U^2 = (Pc)^2 + (U_0)^2 \tag{40}$$

and gives rise to a relativistically covariant energy-momentum four-vector.

The formulation of the relativistic electromagnetic energy and momentum in equations (23) and (25):

$$U = \gamma e V_{planck} \tag{41}$$

$$P = \gamma e u \frac{V_{planck}}{c^2} \tag{42}$$

can be re-written in terms of the electromagnetic invariant mass m_e:

$$m_e = \frac{U_0}{c^2} = e\frac{V_{planck}}{c^2} \tag{43}$$

from which we can derive the following relations:

$$U = \gamma m_e c^2 \tag{44}$$

$$P = \gamma m_e u \tag{45}$$

From equations (44) and (45), one can observe immediately that the terms U and P are relativistically covariant and that they form a relativistically covariant energy-momentum four-vector:

$$V^{\mu} = \begin{pmatrix} \frac{E}{c} & p_x & p_y & p_z \end{pmatrix} = \begin{pmatrix} \gamma e\frac{V_{planck}}{c} & \gamma e\frac{V_{planck}}{c^2}u_x & \gamma e\frac{V_{planck}}{c^2}u_y & \gamma e\frac{V_{planck}}{c^2}u_z \end{pmatrix} \tag{46}$$

which gives rise to energy-momentum relation as shown below.

$$V^{\mu}V_{\mu} = \left(\gamma e\frac{V_{planck}}{c}\right)^2 - \sum_{k=1}^{3}\left(\gamma e\frac{V_{planck}}{c^2}u_k\right)^2 = \left(e\frac{V_{planck}}{c}\right)^2 \tag{47}$$

$$(\gamma e V_{planck})^2 = (\gamma e u\frac{V_{planck}}{c})^2 + (eV_{planck})^2 \tag{48}$$

$$U^2 = (Pc)^2 + (U_0)^2 \tag{49}$$

Below are a list of notable works, which are supportive of our work presented in this paper.

1. J.W Butler, in his paper titled "On the Trouton-Noble Experiment" published in 1968 [11], showed that the Trouton-Noble experiment's [12] Null result can be explained, if the energy density of an Electromagnetic field is expressed as $\frac{1}{8\pi}(\frac{1}{(1-\beta^2)})(E^2 - H^2)$, where $\beta = \frac{v}{c}$, in Gaussian units in vacuum. However, the conventional Electromagnetic energy density equation $\frac{1}{8\pi}(E^2 + H^2)$ cannot explain the Null result. Similar work has been done by Fermi [13], Wilson [14], Kwal [15] and Rohrlich [16].

2. J.W Butler, in his paper titled "A proposed Electromagnetic Momentum-Energy 4-Vector for charge bodies", published in 1969 [17], argues that "the conventional electromagnetic momentum and energy density expressions are known not to lead to a momentum-energy 4-vector for the fields of charged bodies. Yet the rest of classical electrodynamics is a co-variant theory. This is a most remarkable anomaly." In his paper, he derives a 4-vector to represent the 4-momentum, contained within a volume element (dv) of the electromagnetic field of a charged body with a 4-velocity $u = (\gamma u, \gamma c)$. This leads to a resolution of the famous $(\frac{4}{3})$ problem, and accounts for the energy of a moving charge as $U = \gamma U_0$, where (U_0) is the rest frame energy of the charge and (U) is the energy transformed to the laboratory frame with a Lorentz transform.

In this paper, he further argues that the anomalous values for the energy and the momentum of an electron presented in equations (36) and (37) are usually "explained" by the assumptions of ad-hoc forces (Poincare stresses). But these ad-hoc forces are assumed to be also non-covariant, but in a different way from electromagnetic forces. That is, these ad-hoc forces are "assumed to compensate" for the non-covariance of the electromagnetic force, so that the entire electron system becomes covariant.

Further, it was shown in this paper that the source of the non-covariance of energy and momentum density expressions arise from the procedure used to derive the Poynting's theorem, which is shown not to be covariant in the presence of moving sources. In other words, Poynting's theorem is covariant only in the absence of charges in moving frames. A similar analysis on hidden momentum and electromagnetic mass of a charge has been carried out by V. Hnizdo [18].

3. J.A Stratton [19] had pointed out that "the classical interpretation of Poynting's theorem appears to rest to a considerable degree on hypothesis". In other words, the application of Poynting's theorem to a charge body in motion, which gives rise to the non-covariance nature of its energy and momentum relations in classical electrodynamics, should be carefully studied.

4. In relation to energy and momentum of moving charge bodies, W. Pauli [20] had stated that "the Maxwell-Lorentz electrodynamics is quite incompatible with the existence of charges, unless it is supplemented by extraneous theoretical concepts".

5. Energy associated with an electron, as per QED (Quantum electrodynamics) and its renormalization techniques, can be separated into two parts: the energy associated by its interactions with other charge particles and energy associated by interactions with itself. In renormalization, the part that interacts with itself is removed or taken out from the theory. Therefore, after the renormalization, the electron's charge doesn't fly-off or repel itself. Further, the infinities which arise, when the radius of the spherical electron goes to zero, is removed with this treatment. Later, one of the fore-fathers who developed the renormalization techniques in QED, Richard Feynman said that the renormalization was more or less "sweeping the dirt under the rug" [21].

On the other hand, in our derivation of the charge-energy equivalence, we identified a quanta or an indivisible amount of charge associated with a particle which does not fly-off or repel itself. This means that, in our treatment for infinities arising from energies associated with a charge particle, a given amount of charge contained in a single particle is treated as a whole, and thus, the repelling action arising from the classical picture of a charge particle, where the total charge of the particle is sub-divided in to smaller charge quantities, which are repulsive, is removed. Therefore, our treatment is only applicable for indivisible charge particles, to which we could apply $E = q\phi$, where ϕ is the scalar potential associated with its charge q and its radius r. Further, in our treatment to eliminate the infinities arising when radius r reaches zero, we introduced a postulate, which introduces a cut-off value for the scalar potential ϕ at Planck scale.

6. Lorentz's electromagnetic momentum of a spherical electron [22] shows that the momentum is given by $p = \gamma mu$, where $m = \frac{e^2}{6\pi\epsilon_0 Rc^2}$, leading to a total energy of $E = mc^2 = \frac{e^2}{6\pi\epsilon_0 R}$. This relation has been proven with great accuracy by experiments with beta-rays. However, our present postulate states that the potential scalars are finite and bound by the Planck scale. By using equation (3):

$$E = mc^2 = (\frac{2}{3})e(\frac{e}{4\pi\epsilon_0 R}) = (\frac{2}{3})e\phi_{EM} = (k_d)eV_{planck}$$

$$m = (k_d)e\frac{V_{planck}}{c^2}$$

$$p = \gamma mu = \gamma\left((k_d)e(\frac{V_{planck}}{c^2})\right)u = (k_d)(\gamma eu\frac{V_{planck}}{c^2})$$

The above work shows that the derived electromagnetic momentum $eu(\frac{V_{planck}}{c^2})$ being covariant in equation (25), is on par with that of the finding of Lorentz.

7. Max Planck, publishing his first memoir on relativity [23], produced an equation for the relativistic momentum of a point-mass, where $p = \gamma mu$, in 1906.

4 Conclusions

In this paper, we proposed a new postulate to treat the electric and gravitational scalar potentials, so that they become finite and bounded. This led us to derive a general proof for both mass-energy ($E = mc^2$) and charge-energy ($E = qV_{planck}$) equivalences, from first principles. We then derived the energy-momentum equation for a charge particle in motion. The result of this work showed that charge-energy and charge-momentum are relativistically covariant.

The paper then discussed the non-covariance nature of the present classical electrodynamics, introduced by its definitions of electromagnetic field momentum and electromagnetic field energy, arising from a charge particle in motion. However, the postulate presented in this paper and the results derived there of, show that these components are covariant with the rest of the classical electrodynamics.

The present paper is a call for a revision of the classical electrodynamics to make it a fully covariant system with the rest of the classical physics.

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