On the Energy-Mass and Energy-Charge Equivalences

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Abstract

As the energies associated with charge and mass particles are finite, they cannot originate from point like entities. This leads us to present the following conjecture: "The smallest unit of charge or mass should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential (ϕ) becomes the limiting value, set by the Planck scale".

Using this new conjecture, we can derive a general proof for both energy-mass and energy-charge equivalences ($E = mc^2$ and $E = qV_{planck}$ respectively) and derive their relativistic energy-momentum, relativistic-energy and relativistic-momentum relations. The results are in accordance with special relativity.

We then discuss the non-covariance nature of the present classical electrodynamics and show how our work makes it a fully covariant theorem with the rest of the classical electrodynamics.

1 Introduction

Einstein proposed mass-energy equivalence in 1905 [4], in his paper entitled: "Does the inertia of a body depends upon its energy content?". He concluded that the mass of a body is a measure of its energy content. That is, if the energy changes by L, the mass changes in the same sense by L/c^2. This equivalence can be summarized in the famous equation:

$$E = mc^2 \tag{1}$$

However, we emphasize that the energy-mass equivalence stated above in equation (1) is strictly applicable to indivisible mass particles only. if one were to find the total relativistic mass M for a collection of mass particles, one should take into account the energy-momentum relation as shown below, where the velocities u_i of each particle m_i are obtained with relative to the center-of-momentum of the mass body M.

$$(Mc^2)^2 = \sum_{i=1}^{i=n}(\gamma_i m_i u_i c)^2 + \sum_{i=1}^{i=n}(m_i c^2)^2 \tag{2}$$

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On the other hand, in classical electrostatics, energy of a charge q is given by:

$$E = (k_d)\frac{1}{4\pi\epsilon_0}\frac{q^2}{r} \tag{3}$$

where, (k_d) is a scalar factor, which depends on the distribution (configuration) of the charge. When the charge is assumed to be distributed with a constant density, the factor becomes $(\frac{3}{5})$ whereas if the charge is on the surface, the factor becomes $(\frac{1}{2})$. Further, these relation are derived by bringing in infinitesimal amounts of charge from infinity and constructing their corresponding charge configurations.

We observe that the energy content of a charge q in equation (3) is dependent on the structure of the charge, i.e. configuration and the radius of its charge distribution. Further, it suggests that, when the radius of the charge configuration goes to zero, its energy content becomes infinite. In contrast, the energy content of a mass particle in equation (1) is independent of both configuration and radius. Further, it represents the energy of an indivisible mass particle.

From these observations, it is implied that there must also exist an indivisible charge, a charge particle whose energy content must not be a function of how its charge content is distributed or configured.

We then argue that, similar to what Einstein concluded for a mass particle, the total charge of a charge particle must represent a measure of its energy content, which is bounded. This leads us to introduce a new conjecture which states that: **"A charge (or a mass) particle should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential ϕ_E becomes the limiting value, set by the Planck scale".** This conjecture leads us to derive both energy-mass $E = (mc^2)$ and energy-charge $E = (qV_{planck})$ equivalences. We then derive the energy-momentum relation for both charge and mass bodies in motion and show that both energy-mass and energy-charge equivalences are covariant. Further, we show that the momenta of both charge and mass bodies in motion are covariant. This makes the classical electrodynamics a fully covariant theorem.

2 Energy equivalence, relativistic-energy and relativistic-momentum of charge and mass bodies in motion

From classical interpretation, we can derive the following generalized relation for $(\frac{dE}{dp})$, from force (F), energy (E), momentum (p) and velocity (u).

$$dE = F.dx = (\frac{dp}{dt})dx = (u)dp \tag{4}$$

$$\frac{dE}{dp} = u \tag{5}$$

The electromagnetic vector potential A is defined as given below, where (J) current density, (q) charge, (r) distance from the charge, (u) velocity of the charge, (ρ) charge density, (ϕ_E) electrical scalar-potential and $\mu_0\epsilon_0 = \frac{1}{c^2}$.

$$A = \frac{\mu_0}{4\pi}\int_{vol}(\frac{J}{r})dv = \frac{1}{(c^2 4\pi\epsilon_0)}\int_{vol}\frac{(\rho u)}{r}dv = \frac{u}{c^2}\int_{vol}\frac{\rho}{4\pi\epsilon_0 r}dv = \frac{u}{c^2}\phi_E \tag{6}$$

$$A = \frac{u}{c^2}\phi_E \tag{7}$$

$$\phi_E = \frac{q}{4\pi\epsilon_0 r} \tag{8}$$

The electromagnetic momentum p_q of a charge q with velocity u is defined as (qA).

$$p_q = qA = q(\frac{u}{c^2}\phi_E) \tag{9}$$

$$\frac{dE}{dp} = u = \frac{p_q c^2}{q\phi_E} \tag{10}$$

The energy of a single point-charge particle interacting with its own field tends to go to infinity as the radius of the particle goes to zero, $r \to 0$. The self-energy of a charge particle q_0 can be worked out as:

$$E_q = q_0\phi_E = q_0\left(\frac{q_0}{4\pi\epsilon_0 r}\right) \tag{11}$$

The scalar-potential ϕ_E becomes infinite as $r \to 0$ and in that process the corresponding self-energy of a charge particle becomes infinite as well. However, we argue that the self-energy of a charge particle cannot be infinite. The scalar-potential ϕ_E which tends to go to infinity as $r \to 0$ must be finite, so that the total energy of a charge particle becomes finite. We then postulate that **"**As the physical size of a charge particle (or a mass particle) goes to zero ($r \to 0$), the scalar-potential will reach a maximum limit set by the Planck scale". This leads us to present the following conjecture: "**A charge (or a mass) particle should possess a physical boundary and cannot originate from point like entities. At this boundary, the scalar-potential ϕ_E becomes the limiting value, set by the Planck scale"**. The conjecture presented above leads us to find the maximum scalar-potential of a self-interacting charge particle and thereby to derive its charge-energy equivalence.

$$(\phi_E)_{max} = \left(\frac{q_0}{4\pi\epsilon_0 r}\right)_{max} = (voltage)_{planck} = V_{planck} \tag{12}$$

$$(\phi_E)_{planck} = V_{planck} \tag{13}$$

$$E_0 = q_0(\phi_E)_{planck} = q_0 V_{planck} \tag{14}$$

Equation (14) gives us the total self-energy or the charge-energy equivalence of a charge particle. We then use the same conjecture presented above to derive the self-energy of a mass particle (mass-energy equivalence).

$$\phi_G = \frac{G_n m_0}{r} \tag{15}$$

$$E_0 = m_0\phi_G = m_0(\frac{G_n m_0}{r}) \tag{16}$$

The Self-energy of a mass particle becomes infinite (∞) as $r \to 0$. As for the conjecture presented earlier, the gravitational scalar-potential (ϕ_G) must also be finite and bound by the Planck scale.

$$(\phi_G)_{max} = (\frac{G_n m_0}{r})_{max} = (velocity)^2_{planck} = c^2 \tag{17}$$

$$(\phi_G)_{planck} = c^2 \tag{18}$$

$$E_0 = m_0(\phi_G)_{max} = m_0 c^2 \tag{19}$$

Equation (19) is the mass-energy equivalence relation $(E = mc^2)$ which was affirmed by Einstein in about eighteen different presentations. However, he was not able to provide a conclusive general proof of this seminal hypothesis from first principles [9][8]. The same mass-energy equivalence is proved above, from first principles, using our conjecture. Using the same conjecture and a similar set of procedures led us to obtain a general proof for the charge-energy equivalence relation given in equation (14).

Now let us derive the momentum-energy relations for both charge and mass particles, where $\phi_E = V_{planck}$ and $\phi_G = c^2$ are their self-interacting scalar-potentials, respectively.

$mass$	$charge$
$p_m = mu$	$p_q = qA$
$p_m = (\frac{mu}{c^2})\phi_G$	$p_q = (\frac{qu}{c^2})\phi_E$
$p_m = (m\phi_G)\frac{u}{c^2}$	$p_q = (q\phi_E)\frac{u}{c^2}$
$\frac{dE_m}{dp_m} = u = \frac{p_m c^2}{m\phi_G}$	$\frac{dE_q}{dp_q} = u = \frac{p_q c^2}{q\phi_E}$
$(m\phi_G)dE_m = (p_m c^2)dp_m$	$(q\phi_E)dE_q = (p_q c^2)dp_q$
$(mc^2)dE_m = (p_m c^2)dp_m$	$(qV_{planck})dE_q = (p_q c^2)dp_q$
$E_m dE_m = (p_m c^2)dp_m$	$E_q dE_q = (p_q c^2)dp_q$
$\frac{E_m^2}{2} = \frac{(p_m c)^2}{2} + k_m$	$\frac{E_q^2}{2} = \frac{(p_q c)^2}{2} + k_q$

By introducing boundary conditions, where the energies become rest frame energies, when the velocity becomes zero $(u = 0)$, we can obtain the following results.

$$E_m^2 = (p_m c)^2 + E_m^2(0) \Longleftrightarrow E_q^2 = (p_q c)^2 + E_q^2(0) \tag{20}$$

$$E^2 = (p_m c)^2 + (m_0 c^2)^2 \Longleftrightarrow E^2 = (p_q c)^2 + (q_0 V_{planck})^2 \tag{21}$$

$$(mc^2)^2 = (muc)^2 + (m_0 c^2)^2 \Longleftrightarrow (qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0 V_{planck})^2 \tag{22}$$

Equation (22) represents relativistic energy-momentum equations for both charge and mass particles in motion. In Einstein's Special Relativity, only the relativistic momentum-energy relation for mass bodies in motion is derived. On the other hand, by observing the finiteness of energies associated with a given amount of charge (or mass), and by introducing our new conjecture led us to derive the relativistic momentum-energy relation for both mass and charge particles in motion.

Now using charge-momentum $p_q = qA = q(\frac{u\phi_E}{c^2})$ and the self-interacting scalar-potential $\phi_E = V_{planck}$, we can show that the energy of a charge particle is covariant (similar to that of mass particles).

$mass$	$charge$
$(mc^2)^2 = (p_m c)^2 + (m_0 c^2)^2$	$(qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0 V_{Planck})^2$
$(mc^2)^2 = (muc)^2 + (m_0 c^2)^2$	$(qV_{planck})^2 = (qu)^2(\frac{V_{planck}}{c})^2 + (q_0 V_{planck})^2$
$(mc^2)^2(1 - \frac{u^2}{c^2}) = (m_0 c^2)^2$	$q^2 V_{planck}^2(1 - \frac{u^2}{c^2}) = q_0^2 V_{planck}^2$

$$mc^2 = \gamma m_0 c^2 \Longleftrightarrow qV_{planck} = \gamma q_0 V_{planck} \tag{23}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{24}$$

This shows that the mass-energy equivalence $(E = mc^2)$ and the charge-energy equivalence $(E = qV_{planck})$ are both relativistically covariant. In like manner, we can show that mass-momentum (mu) and charge-momentum (qA) are covariant as well (for $p \neq 0$, i.e. $u \neq 0$).

$mass$	$charge$
$m^2c^4 = m^2u^2c^2 + m_0^2c^4$	$q^2V_{planck}^2 = q^2u^2(\frac{V_{planck}}{c})^2 + q_0^2V_{planck}^2$
$c^4(m^2u^2 - m^2\frac{u^4}{c^2}) = (c^4)m_0^2u^2$	$(V_{planck}^2)(q^2u^2 - q^2\frac{u^4}{c^2}) = (V_{planck}^2)(q_0^2u^2)$
$(mu)^2(1 - \frac{u^2}{c^2}) = m_0^2u^2$	$(qu)^2(1 - \frac{u^2}{c^2}) = q_0^2u^2$
$mu = \frac{m_0u}{\sqrt{1-\frac{u^2}{c^2}}}$	$qu(\frac{V_{planck}}{c^2}) = \frac{q_0u}{\sqrt{1-\frac{u^2}{c^2}}}(\frac{V_{planck}}{c^2})$

$$mu = \gamma m_0 u \Longleftrightarrow qu(\frac{V_{planck}}{c^2}) = \gamma q_0 u(\frac{V_{planck}}{c^2}) \tag{25}$$

Note that, all the above derivations were based on the assumption that there exist an indivisible quanta of charge (and mass). One can then define the notion of a body with charge Q (or with mass M) as a collection of many such particles. Thus:

$$(Mc^2)^2 = \sum_{i=1}^{i=n}(\gamma_i m_i u_i c)^2 + \sum_{i=1}^{i=n}(m_i c^2)^2 \tag{26}$$

$$(QV_{planck})^2 = \sum_{i=1}^{i=n}(\gamma_i q_i u_i \frac{V_{planck}}{c})^2 + \sum_{i=1}^{i=n}(q_i V_{planck})^2 \tag{27}$$

We would like to emphasize some facts regarding the rest-mass and the relativistic-mass concepts. The relativistic-mass is derived from the relativistic energy or relativistic momentum of the system and thus it is argued that relativistic-mass (γm_0) is not a good concept. Einstein wrote "It is not good to introduce the concept of the mass $M = \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the rest-mass m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion" [15]. The same set of arguments holds true for the proposed relativistic charge-energy ($\gamma q_0 V_{planck}$) and relativistic charge-momentum $\gamma q_0 u(\frac{V_{planck}}{c^2})$ concepts as well.

3 Classical electron theory and its lack of relativistic covariance

Max Abraham [1] and H.A Lorentz [13], based on Maxwell's theory of electricity and magnetism developed the first set of theories for the classical electron. As for the classical electrostatics, the rest energy U_0 of a spherical charge body with radius r, associated with total charge e, uniformly distributed over its surface is given by:

$$U_0 = \left(\frac{1}{2}\right)\frac{e^2}{4\pi\epsilon_0 r} \tag{28}$$

One can then obtain the relativistic electromagnetic energy U of the moving charge e as for the definition given below [23] [11] [18] [19]:

$$U = \frac{1}{2}\int_{all-space}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}H^2\right)dv \tag{29}$$

$$U = \gamma\frac{e^2}{8\pi\epsilon_0 r}\left(1 + \frac{1}{3}\beta^2\right) \tag{30}$$

and its relativistic electromagnetic momentum P as:

$$P = \epsilon_0\int_{all-space}(E \times B)\,dv \tag{31}$$

$$P = \frac{4}{3}\gamma u \frac{e^2}{8\pi\epsilon_0 r c^2} \tag{32}$$

where u is the velocity of the charge e, and

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{33}$$

$$\beta = \frac{u}{c} \tag{34}$$

However, according to energy-mass equivalence and the theory or relativity, we can find the equivalent electromagnetic invariant mass m_e of an electron as:

$$m_e = \frac{U_0}{c^2} = \frac{e^2}{8\pi\epsilon_0 r c^2} \tag{35}$$

Thus, the equations (30) and (32) can be written in terms of the electromagnetic invariant mass m_e as given below.

$$U = \gamma m_e c^2 (1 + \frac{1}{3}\beta^2) \tag{36}$$

$$P = \frac{4}{3}\gamma m_e u \tag{37}$$

From equations (36) and (37), it is immediately obvious that the terms U and P do not transform properly as an energy-momentum four-vector. Also the relativistic energy-momentum relation $U^2 = (Pc)^2 + (U_0)^2$ is violated, which implies that the terms U and P are neither relativistically covariant nor transformed as an energy-momentum four-vector.

On the other hand, if they are to be covariant, they should be of the form:

$$U = \gamma m_e c^2 \tag{38}$$

$$P = \gamma m_e u \tag{39}$$

which satisfies the energy-momentum relation:

$$U^2 = (Pc)^2 + (U_0)^2 \tag{40}$$

and gives rise to a relativistically covariant energy-momentum four-vector.

In our derivations of the relativistic electromagnetic energy and momentum in equations (23) and (25):

$$U = \gamma e V_{planck} \tag{41}$$

$$P = \gamma e u \frac{V_{planck}}{c^2} \tag{42}$$

We can write the above two equations (41) and (42) in terms of the electromagnetic invariant mass m_e:

$$m_e = \frac{U_0}{c^2} = e\frac{V_{planck}}{c^2} \tag{43}$$

$$U = \gamma m_e c^2 \tag{44}$$

$$P = \gamma m_e u \tag{45}$$

From equations (44) and (45), one can observe immediately that the terms U and P are relativistically covariant and they form a relativistically covariant energy-momentum four-vector. Also, the energy-momentum relation is not violated.

$$(\gamma eV_{planck})^2 = (\gamma eu\frac{V_{planck}}{c})^2 + (eV_{planck})^2 \tag{46}$$

$$U^2 = (Pc)^2 + (U_0)^2 \tag{47}$$

Below are a list of notable works, which are supportive of our work presented in this paper.

1. J.W Butler, in his paper titled "On the Trouton-Noble Experiment" published in 1968 [2], showed that the Trouton-Noble experiment's [5] Null result can be explained, if the energy density of an Electromagnetic field is expressed as $\frac{1}{8\pi}(\frac{1}{(1-\beta^2)})(E^2 - H^2)$, where $\beta = \frac{v}{c}$, in Gaussian units in vacuum. However, the conventional Electromagnetic energy density equation $\frac{1}{8\pi}(E^2 + H^2)$ cannot explain the Null result. Similar work has been done by Fermi [6], Wilson [22], Kwal [12] and Rohrlich [20].

2. J.W Butler, in his paper titled "A proposed Electromagnetic Momentum-Energy 4-Vector for charge bodies", published in 1969 [3], argues that "the conventional electromagnetic momentum and energy density expressions are known not to lead to a momentum-energy 4-vector for the fields of charged bodies. Yet the rest of classical electrodynamics is a co-variant theory. This is a most remarkable anomaly." In his paper, he derives a 4-vector to represent the 4-momentum, contained within a volume element (dv) of the electromagnetic field of a charged body with a 4-velocity $u = (\gamma u, \gamma c)$. This leads to a resolution of the famous $(\frac{4}{3})$ problem, and accounts for the energy of a moving charge as $U = \gamma U_0$, where (U_0) is the rest frame energy of the charge and (U) is the energy transformed to the laboratory frame with a Lorentz transform.

In this paper, he further argues that the anomalous values for the energy and the momentum of an electron presented in equations (36) and (37) are usually "explained" by the assumptions of ad-hoc forces (Poincare stresses). But these ad-hoc forces are assumed to be also non-covariant, but in a different way from electromagnetic forces. That is, these ad-hoc forces are "assumed to compensate" for the non-covariance of the electromagnetic force, so that the entire electron system becomes covariant.

Further, it was shown in this paper that the source of the non-covariance of energy and momentum density expressions arise from the procedure used to derive the Poynting's theorem, which is shown not to be covariant in the presence of moving sources. In other words, Poynting's theorem is covariant only in the absence of charges in moving frames. A similar analysis on hidden momentum and electromagnetic mass of a charge has been carried out by V. Hnizdo [10].

3. J.A Stratton [21] had pointed out that "the classical interpretation of Poynting's theorem appears to rest to a considerable degree on hypothesis". In other words, the application of Poynting's theorem to a charge body in motion, which gives rise to the non-covariance nature of its energy and momentum relations in classical electrodynamics, should be carefully studied.

4. In relation to energy and momentum of moving charge bodies, W. Pauli [16] had stated that "the Maxwell-Lorentz electrodynamics is quite incompatible with the existence of charges, unless it is supplemented by extraneous theoretical concepts".

5. Energy associated with an electron, as per QED and its renormalization techniques, can be separated into two parts: the energy associated by its interactions with other charge particles and energy associated by interactions with itself. In renormalization, the part that interacts with itself is removed or taken out from the theory. Therefore, after the renormalization, the electron's charge doesn't fly-off or repel itself. Further, the infinities which arise, when the radius of the spherical electron goes to zero, is removed with this treatment. Later, one of the fore-fathers who developed the renormalization techniques in QED, Richard Feynman said that the renormalization was more or less "sweeping the dirt under the rug" [7].

On the other hand, in our derivation of the energy-charge equivalence, we identified a quanta or an indivisible amount of charge associated with a particle which does not fly-off. This means that, in our treatment for infinities arising from energies associated with a charge particle, a given amount of charge contained in a single particle is treated as a whole, and thus, the repelling action arising from the classical picture of a charge particle, where the total charge is sub-divided in to smaller charge quantities, which are repulsive, is removed. Therefore, our treatment is only applicable for indivisible charge particles, to which we could apply $E = q\phi$, where ϕ is the scalar potential associated with its charge q and its radius r. Further, in our treatment to eliminate the infinities arising when radius r reaches zero, we conjectured that the scalar potential ϕ must have a cut-off value at Planck scale scalar potential.

6. Lorentz's electromagnetic momentum of a spherical electron [14] shows that the momentum is given by $p = \gamma m_0 u$, where $m_0 = \frac{e^2}{6\pi\epsilon_0 Rc^2}$, leading to a total energy of $E = m_0c^2 = \frac{e^2}{6\pi\epsilon_0 R}$. This relation has been proven with great accuracy by experiments with beta-rays. However, our present conjecture states that the potential scalars are finite and bound by the Planck scale. By using equation (3):

$$E = m_0c^2 = (\frac{2}{3})e(\frac{e}{4\pi\epsilon_0 R}) = (\frac{2}{3})e\phi_{EM} = (k_d)eV_{planck}$$

$$m_0 = (k_d)e\frac{V_{planck}}{c^2}$$

$$p = \gamma m_0 u = \gamma \left((k_d)e(\frac{V_{planck}}{c^2}) \right) u = (k_d)(\gamma e u \frac{V_{planck}}{c^2})$$

The above work shows that the derivation of electromagnetic momentum $eu(\frac{V_{planck}}{c^2})$ being relativistic in equation (25), is on par with that of the finding of Lorentz.

7. Max Planck, publishing his first memoir on relativity [17], produced an equation for the relativistic momentum of a point-mass, where $p = \gamma mu$, in 1906.

4 Conclusions

In this paper, we proposed a new conjecture to treat the electrical and gravitational potentials, so that they become finite and bounded. This led us to derive a general proof for both mass-energy ($E = mc^2$) and charge-energy ($E = qV_{planck}$) equivalences, from first principles. We then derived the momentum-energy equation for a charge particle in motion. The result of this work showed that charge-energy and charge-momentum are relativistically covariant.

The paper then discussed the non-covariance nature of the present classical electrodynamics, introduced by its definitions of electromagnetic field momentum and electromagnetic field energy of a charge body. However, the conjecture presented in the paper and the results derived there of, show that these components are covariant with the rest of the classical electrodynamics.

The present paper is a call for a revision of the classical electrodynamics to make it a fully covariant system with the rest of the classical physics.

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