Neglect of General Covariance

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Abstract

Tensor ranks are not relative. Each tensor type and symmetry denotes its own class of geometric objects, that are not really interpretable as members of another class. Coordinate free geometry is the real theory, so tensor notation with coordinates can only be taken as a distant translation. But there is a perverse tradition in academia to the contrary. Vectors are used to portray physical objects that are plainly nothing of the sort. This only adds confusion to the study of mechanics and electromagnetism. Even in cosmology the wrong tensor ranks and operators are used, invoking a system of compensating errors that eventually go uncompensated on the edge. The two Bianchi identities, with the exterior derivative as the operator, are the correct foundation for the theory. These identities are geometric in their essence, not based in coordinate algebra. By contrast, the cosmological constant is simply a mathematical error. And there is an analytic necessity to deal with - the need to include into the Einstein equation the fictitious source density of the fictitious potential that derives from peculiar noninertial coordinates. The equivalence principle demands this inclusion. Awareness of artifacts while integrating from geometric boundary conditions on the two Bianchi identities is the method to bypass these errors. Coordinate artifacts are a superset of real curvature, and form a background condition that cannot be handled completely at the dynamical equations. [To be clear: the Einstein tensor term on the left of the famous equation may be as adaptable to peculiar coordinates as the term for mechanics on the right. And there are no more difficulties for geometry than mechanics after the correct metric is calculated. But this excludes any salvation for the cosmological constant term.]

General Covariance

General covariance is nothing less than the assertion that coordinate free geometry is the real essence of (classical) things - Einstein-Davis theory that in turn derives from the work of Spinoza.

A lesser implication, that is great fun and important in understanding physics, is that (classical) physics can always be solved by referring to a diagram of the problem, which has the property of retaining its validity even when it is scaled and sparse.

But tensors can be drawn correctly on a scaled and sparse diagram only when they have their proper rank, the one that corresponds to their dimensionality in units of length. Potential momentum is a 1-form, for example, and the Faraday tensor is a 2-form with a dimensionality of L^{-2}. These are covariant tensors - and not contravariant - by reason of the negative power of L. Coordinates are naturally covariant 1-forms with a dimensionality of L^{-1}. Subject to the discipline of geometry, spacetime intervals are the only natural vectors, contravariant with a dimensionality of L^1. (As a source of gravity, and for the sake of the conservation law, kinetic momentum and mass become covariant 1-forms.)

Drawing with the wrong tensor rank leads to gross inconsistencies between diagrams of different scale and sparseness, so that observers no longer have freedom in their choice of units of measurement. Relative motion between observers sets off great paradox.

Cosmological Sanity

The Einstein equation should be thought of as a correspondence principle, an approximate translation between two mathematical systems, with pure geometry on the left and classical mechanics on the right. There is not an enmeshment when the equation is invoked, with one side then becoming incomplete and dependent on the other. Each side has its own independent foundation - mechanics in its laws of conservation, and the geometry of spacetime in the two Bianchi identities.

In this equation, the expression for mechanics on the right side naturally includes the real gravitational potential, and the compensating kinetic momentum of matter and energy subject to that potential, as well as the real source of that potential, in its term for density.

There is also a fictitious potential, that varies merely with location and time, in the Friedmann coordinates. And this fictitious potential also has a fictitious source density that is derivable from the definition of Friedmann coordinates. This fictitious potential, the compensating fictitious momentum, and the source of this fictitious potential are necessarily included in the cosmological equation. And the Einstein tensor term is also necessarily adjusted.

The equivalence principle makes it impossible to discriminate between fictitious and nonfictitious anyway, except where fictitious sources fail to follow geodesics from end to end. There is a unification of the equivalence and general covariance principles here. The usual compensating term in the Einstein equation could be incomplete, since coordinate artifacts are (more than) the full equivalent of real curvature.

These equations are written by tradition in the wrong tensor rank. By considering geometric units, as well as by the use of exterior derivatives to treat the geometry, the fourth rank covariant is mandated.

In this proper fourth tensor rank the cosmological constant becomes a coordinate artifact at best; it breaks the Bianchi identity, and forces nonconservation on mechanics as well.

Integrating from the Bianchi Identities

These Bianchi identities are actually a central feature of physics. They implement the maxim cited in the book GRAVITATION - “The boundary of a boundary is zero.”. Of course, they should be written in proper tensor rank, and also with the use of the proper operator, the exterior derivative. The first identity is of fourth rank, and is arrived at by applying the exterior derivative twice to the metric tensor. The second identity is of the fifth rank, and it stems from applying the exterior derivative and the orthogonal dual before taking the exterior derivative twice.

The operations used to arrive at the identities are geometric in nature, not distorted by weird coordinates. The identities can be used to integrate from boundary conditions that are purely geometric, unlike the usual enmeshment with mechanics in the Einstein equation.

But the geometric boundary conditions must contain the fictitious sources and their effects on the metric as implied by the coordinate system used in the integration. After all, the resulting data as understood within the context of a coordinate system will necessarily contain the fictitious effects of that coordinate system. And the computation starts again where the coordinate behavior exceeds nature.

A big advantage is that the coordinate system is not enmeshed in the integration process, unless the fictitious sources do not follow geodesics end to end. The coordinate system can be static and predefined, a welcome property of general covariance. Doing cosmology in Cartesian coordinates is an eye opener, even as a concept. Certainly the fictitious effects of the Friedmann coordinates are no longer there to confuse us.

Conclusion

Many more examples could be worked or expanded on. But I will just leave this as a word to the wise.