A rewriting system applied to the simplest algebraic identities

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A rewriting system applied to the simplest algebraic identities is shown to yield second- and third-order equations that share a property associated with 137.036.

I. TWO SYMMETRIC IDENTITIES

The symmetry of this *second*-order identity

$$M^2 = M^2$$

and this *third*-order identity

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2$$

will be "broken" by making the substitution

$$M \to M - y$$

on their left hand sides, and the substitution

$$M^n \to M^n - x^p$$

on their right hand sides, where p equals the order of each identity. Above, y and x are variables such that

$$0 < y \leq 0.1 \tag{1a}$$

$$0 < x \leq 0.1 \quad , \tag{1b}$$

whereas M and N are positive integer constants fulfilling

$$M = \frac{N^3}{3} + 1 \tag{1c}$$

so that necessarily

$$M \geq 10 \quad . \tag{1d}$$

The reason for altering these identities using the above *rewriting system* (an admittedly unusual thing to do) is to change them from related *identities* that are true for *all* values of M and N, into slightly asymmetric *conditional equations* that are true for *some* values of M and N. It will be shown that the resultant equations share an interesting property involving their derivatives, where this property is associated with 137.036.

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II. TWO CONDITIONAL EQUATIONS

Begin with the second-order identity

$$M^2 = M^2$$

and break its symmetry by making the substitution

$$M \to M - y$$

on its left hand side, and the substitution

$$M^n \to M^n - x^p$$

on its right hand side, where $p = 2$, the identity's order. This produces

$$(M - y)^2 = M^2 - x^2 \quad . \tag{2a}$$

Similarly, for the third-order identity

$$\left(\frac{M}{N}\right)^3 + M^2 = \left(\frac{M}{N}\right)^3 + M^2$$

apply the same substitutions, where $p = 3$, to get

$$\left(\frac{M - y}{N}\right)^3 + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad . \tag{2b}$$

III. THEIR SHARED PROPERTY

Theorem 1 will show that for Eq. (2a)

$$\frac{dy}{dx} \approx \frac{x}{M} \quad ,$$

whereas Theorem 2 will show that for Eq. (2b)

$$\frac{dy}{dx} \approx \frac{x^2}{M} \quad .$$

Accordingly, for *both* equations

$$\frac{dy}{dx} \approx \frac{1}{M^p} \tag{3}$$

at

$$x = \frac{1}{M} \quad ,$$

where p equals the order of each equation.

Theorem 1. *Assume Eq.* (2a)

$$(M-y)^2 = M^2 - x^2 \quad .$$

Then

$$\frac{dy}{dx} \approx \frac{x}{M} \quad . \tag{4a}$$

Proof. Equation (2a)

$$(M-y)^2 = M^2 - x^2$$

expands and simplifies to

$$2My - y^2 = x^2 \quad .$$

It follows that

$$2Mdy - 2ydy = 2xdx \quad .$$

But by Eq. (1a) $y \leq 0.1$ and by Eq. (1d) $M \geq 10$, so that $2ydy$ is small compared to $2Mdy$. Hence,

$$\frac{dy}{dx} \approx \frac{x}{M}$$

holds. □

Theorem 2. *Assume Eq.* (2b)

$$\left(\frac{M-y}{N}\right)^3 + (M-y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad .$$

Then

$$\frac{dy}{dx} \approx \frac{x^2}{M} \quad . \tag{4b}$$

Proof. Equation (2b)

$$\frac{(M-y)^3}{N^3} + (M-y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3$$

expands and simplifies to

$$-\frac{3M^2y}{N^3} + \frac{3My^2}{N^3} - \frac{y^3}{N^3} - 2My + y^2 = -\frac{x^3}{N^3} - x^3 \quad ,$$

or

$$3M^2y - 3My^2 + y^3 + 2MN^3y - N^3y^2 = (N^3+1)x^3 \quad .$$

It follows that

$$(3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y)dy = 3(N^3+1)x^2dx$$

so that

$$\frac{dy}{dx} = \frac{3(N^3+1)x^2}{3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y} \quad .$$

We now want to identify and remove the smallest terms from the denominator. As Eq. (1c) requires that

$$N^3 = 3M - 3 \quad ,$$

substituting for N^3 gives

$$\begin{aligned}
\frac{dy}{dx} &= \frac{3(3M-3+1)x^2}{3M^2 - 6My + 3y^2 + 2M(3M-3) - 2(3M-3)y} \\
&= \frac{3(3M-2)x^2}{3M^2 - 6My + 3y^2 + 6M^2 - 6M - 6My + 6y} \\
&= \frac{3(3M-2)x^2}{9M^2 - 12My + 3y^2 - 6M + 6y} \\
&= \frac{(3M-2)x^2}{3M^2 - 4My + y^2 - 2M + 2y} \\
&= \frac{(3M-2)x^2}{(3M-2)M - 4My + y^2 + 2y} \quad .
\end{aligned}$$

But by Eq. (1a) $y \leq 0.1$ and by Eq. (1d) $M \geq 10$, so $4My$, y^2, and $2y$ are small compared to $(3M-2)M$. So,

$$\frac{dy}{dx} \approx \frac{3M-2}{3M-2} \times \frac{x^2}{M} \quad .$$

Accordingly, the approximation

$$\frac{dy}{dx} \approx \frac{x^2}{M}$$

holds. □

Remark 1. When comparing Eq. (4a) of Theorem 1 against Eq. (4b) of Theorem 2, we see that for both

$$\frac{dy}{dx} \approx \frac{x^{p-1}}{M} \quad , \tag{4c}$$

with only their values for p differing (2 and 3, respectively). At $x = 1/M$ this gives Eq. (3), the shared property introduced in the previous section.

IV. THE MINIMAL CASE AND 137.036

As promised at the beginning of Section (III), Theorems 1 and 2 show that for Eqs. (2a) and (2b)

$$\frac{dy}{dx} \approx \frac{1}{M^p} \quad \text{at} \quad x = \frac{1}{M} \quad , \tag{5}$$

where p equals the order of each equation. But we know that $M = N^3/3 + 1$ for Eq. (2b), and that the smallest positive integers (M, N) fulfilling this condition are $(10, 3)$. So for this *minimal case* the right hand side of Eq. (2b) at $x = 1/M$ gives

$$\begin{aligned}
\frac{M^3 - x^3}{N^3} + M^2 - x^3 &= \frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} \\
&= \frac{999.999}{3^3} + 99.999 \\
&= 137.036 \quad .
\end{aligned}$$

This makes 137.036 the smallest value at which the third-order equation behaves like the second-order equation in fulfilling Eq. (5), which makes 137.036 a fundamental constant for Eqs. (2a) and (2b).