On the Fibonacci numbers, the Koide formula, and the distribution of primes

J. S. Markovitch
P.O. Box 2411
West Brattleboro, VT 05303[*]
(Dated: November 5, 2012)

The Koide formula from physics is modified for use with the reciprocals of primes found in the intervals defined by the Fibonacci numbers. This formula's resultant values are found to alternate lower, higher, lower, higher, etc. from the interval $(5, 8]$ to the interval $(514\,229, 832\,040]$. This pattern, inverted, is also shown to occur when the corresponding results are computed for non-primes.

[*]Electronic address: jsmarkovitch@gmail.com

Contents

In the 1980s the Koide formula [1–3] was introduced to relate the charged lepton masses. According to this formula

$$\left(\sqrt{x_1}+\sqrt{x_2}+\sqrt{x_3}\right)^2/\left(x_1+x_2+x_3\right)=3/2 \quad ,$$

where x_1, x_2, and x_3 represent these masses. Here the *Koide sum*

$$\left(\sqrt{x_1}+\sqrt{x_2}+\sqrt{x_3}\right)^2/\left(x_1+x_2+x_3\right) \quad ,$$

along with intervals defined either by the Fibonacci numbers or golden ratio ϕ, are adapted to explore the distribution of primes. These results follow up the author's earlier explorations of the distribution of primes also using intervals defined by the Fibonacci numbers [4].

I. NORMALIZED KOIDE SUMS FOR PRIME RECIPROCALS

Consider the following:

$$\frac{\left(\sqrt{\frac{1}{2}}\right)^2}{1\left(\frac{1}{2}\right)}=1.000000000 \tag{1.1}$$

$$\frac{\left(\sqrt{\frac{1}{3}}\right)^2}{1\left(\frac{1}{3}\right)}=1.000000000 \tag{1.2}$$

$$\frac{\left(\sqrt{\frac{1}{5}}\right)^2}{1\left(\frac{1}{5}\right)}=1.000000000 \tag{1.3}$$

$$\frac{\left(\sqrt{\frac{1}{7}}\right)^2}{1\left(\frac{1}{7}\right)}=1.000000000 \tag{1.4}$$

$$\frac{\left(\sqrt{\frac{1}{11}}+\sqrt{\frac{1}{13}}\right)^2}{2\left(\frac{1}{11}+\frac{1}{13}\right)}\approx 0.9982608643\blacktriangledown \tag{1.5}$$

$$\frac{\left(\sqrt{\frac{1}{17}}+\sqrt{\frac{1}{19}}\right)^2}{2\left(\frac{1}{17}+\frac{1}{19}\right)}\approx 0.9992277988\blacktriangle \tag{1.6}$$

$$\frac{\left(\sqrt{\frac{1}{23}}+\sqrt{\frac{1}{29}}+\sqrt{\frac{1}{31}}\right)^2}{3\left(\frac{1}{23}+\frac{1}{29}+\frac{1}{31}\right)}\approx 0.9957855589\blacktriangledown \tag{1.7}$$

$$\frac{\left(\sqrt{\frac{1}{37}}+\sqrt{\frac{1}{41}}+\sqrt{\frac{1}{43}}+\sqrt{\frac{1}{47}}+\sqrt{\frac{1}{53}}\right)^2}{5\left(\frac{1}{37}+\frac{1}{41}+\frac{1}{43}+\frac{1}{47}+\frac{1}{53}\right)}\approx 0.9963366895\blacktriangle \tag{1.8}$$

$$\frac{\left(\sqrt{\frac{1}{59}}+\sqrt{\frac{1}{61}}+\sqrt{\frac{1}{67}}+\sqrt{\frac{1}{71}}+\sqrt{\frac{1}{73}}+\sqrt{\frac{1}{79}}+\sqrt{\frac{1}{83}}+\sqrt{\frac{1}{89}}\right)^2}{8\left(\frac{1}{59}+\frac{1}{61}+\frac{1}{67}+\frac{1}{71}+\frac{1}{73}+\frac{1}{79}+\frac{1}{83}+\frac{1}{89}\right)}\approx 0.9954458134\blacktriangledown \tag{1.9}$$

$$\frac{\left(\sqrt{\frac{1}{97}}+\sqrt{\frac{1}{101}}+\sqrt{\frac{1}{103}}+\sqrt{\frac{1}{107}}+\sqrt{\frac{1}{109}}+\sqrt{\frac{1}{113}}+\sqrt{\frac{1}{127}}+\sqrt{\frac{1}{131}}+\sqrt{\frac{1}{137}}+\sqrt{\frac{1}{139}}\right)^2}{10\left(\frac{1}{97}+\frac{1}{101}+\frac{1}{103}+\frac{1}{107}+\frac{1}{109}+\frac{1}{113}+\frac{1}{127}+\frac{1}{131}+\frac{1}{137}+\frac{1}{139}\right)}\approx 0.9960870652\blacktriangle \tag{1.10}$$

Above, the reciprocals of primes in the intervals defined by the Fibonacci numbers are substituted for the charged lepton masses used by Koide. More specifically, the reciprocals of the primes in the intervals (1,2], (2,3], (3,5], (5,8], etc. are successively subjected to the Koide sum, but whereas the Koide sum makes no adjustment for the number of masses summed, here each sum is "normalized" by dividing by the number of terms summed, thereby producing *normalized Koide sums* taking the form

$$\frac{(\sqrt{x_1}\,)^2}{1\,(x_1)}\,,\frac{(\sqrt{x_1}+\sqrt{x_2}\,)^2}{2\,(x_1+x_2)}\,,\,\cdots\,,\frac{(\sqrt{x_1}+\sqrt{x_2}+\cdots+\sqrt{x_n}\,)^2}{n\,(x_1+x_2+\cdots+x_n)}\quad.$$

This assures that a number close to one is produced for all of the sums that follow.

For Eqs. (1.1)–(1.4) these sums all equal one. Thereafter, the sums alternate lower, higher, lower, higher, etc., up to and including Eq. (1.28) (see below). Note that blue triangles (▲▼) are used to signal whether a sum is higher (▲) or lower (▼) than the one previous. Beyond Eq. (1.28) this "lower-higher" pattern ceases and no obvious pattern replaces it. This behavior from Eq. (1.4) to Eq. (1.28) is remarkable: Why should these normalized Koide sums alternately fall and rise in value across twenty-five terms, and what does this say about the distribution of primes?

$$\frac{\left(\sqrt{\frac{1}{149}}+\cdots+\sqrt{\frac{1}{233}}\right)^2}{17\left(\frac{1}{149}+\cdots+\frac{1}{233}\right)}\approx\frac{1.547956446}{17\times0.09152394935}\approx0.9948899955\blacktriangledown \tag{1.11}$$

$$\frac{\left(\sqrt{\frac{1}{239}}+\cdots+\sqrt{\frac{1}{373}}\right)^2}{23\left(\frac{1}{239}+\cdots+\frac{1}{373}\right)}\approx\frac{1.765385316}{23\times0.07711579174}\approx0.9953328827\blacktriangle \tag{1.12}$$

$$\frac{\left(\sqrt{\frac{1}{379}}+\cdots+\sqrt{\frac{1}{607}}\right)^2}{37\left(\frac{1}{379}+\cdots+\frac{1}{607}\right)}\approx\frac{2.831426275}{37\times0.07691694613}\approx0.9949047422\blacktriangledown \tag{1.13}$$

$$\frac{\left(\sqrt{\frac{1}{613}}+\cdots+\sqrt{\frac{1}{983}}\right)^2}{55\left(\frac{1}{613}+\cdots+\frac{1}{983}\right)}\approx\frac{3.869862583}{55\times0.07071363812}\approx0.9950151024\blacktriangle \tag{1.14}$$

$$\frac{\left(\sqrt{\frac{1}{991}}+\cdots+\sqrt{\frac{1}{1597}}\right)^2}{85\left(\frac{1}{991}+\cdots+\frac{1}{1597}\right)}\approx\frac{5.689695267}{85\times0.06727671181}\approx0.9949593192\blacktriangledown \tag{1.15}$$

$$\frac{\left(\sqrt{\frac{1}{1601}}+\cdots+\sqrt{\frac{1}{2579}}\right)^2}{125\left(\frac{1}{1601}+\cdots+\frac{1}{2579}\right)}\approx\frac{7.619663604}{125\times0.06124997659}\approx0.9952217491\blacktriangle \tag{1.16}$$

$$\frac{\left(\sqrt{\frac{1}{2591}}+\cdots+\sqrt{\frac{1}{4177}}\right)^2}{198\left(\frac{1}{2591}+\cdots+\frac{1}{4177}\right)}\approx\frac{11.79551323}{198\times0.05986884934}\approx0.9950633729\blacktriangledown \tag{1.17}$$

$$\frac{\left(\sqrt{\frac{1}{4201}}+\cdots+\sqrt{\frac{1}{6763}}\right)^2}{297\left(\frac{1}{4201}+\cdots+\frac{1}{6763}\right)}\approx\frac{16.35856085}{297\times0.05534211683}\approx0.9952515844\blacktriangle \tag{1.18}$$

$$\frac{\left(\sqrt{\frac{1}{6779}}+\cdots+\sqrt{\frac{1}{10939}}\right)^2}{458\left(\frac{1}{6779}+\cdots+\frac{1}{10939}\right)}\approx\frac{24.07563225}{458\times0.05281775322}\approx0.9952502602\blacktriangledown \tag{1.19}$$

$$\frac{\left(\sqrt{\frac{1}{10949}}+\cdots+\sqrt{\frac{1}{17707}}\right)^2}{704\left(\frac{1}{10949}+\cdots+\frac{1}{17707}\right)}\approx\frac{35.13788752}{704\times0.05014944539}\approx0.9952606984\blacktriangle \tag{1.20}$$

$$\frac{\left(\sqrt{\frac{1}{17713}}+\cdots+\sqrt{\frac{1}{28657}}\right)^2}{1088\left(\frac{1}{17713}+\cdots+\frac{1}{28657}\right)}\approx\frac{51.86444923}{1088\times0.04789936892}\approx0.9952016407\blacktriangledown \tag{1.21}$$

$$\frac{\left(\sqrt{\frac{1}{28661}}+\cdots+\sqrt{\frac{1}{46351}}\right)^2}{1673\left(\frac{1}{28661}+\cdots+\frac{1}{46351}\right)}\approx\frac{75.83065591}{1673\times0.04554247683}\approx0.9952500898\blacktriangle \tag{1.22}$$

$$\frac{\left(\sqrt{\frac{1}{46381}}+\cdots+\sqrt{\frac{1}{75017}}\right)^2}{2602\left(\frac{1}{46381}+\cdots+\frac{1}{75017}\right)}\approx\frac{113.3704128}{2602\times0.04378101576}\approx0.9951913724\blacktriangledown \tag{1.23}$$

$$\frac{\left(\sqrt{\frac{1}{75029}}+\cdots+\sqrt{\frac{1}{121379}}\right)^2}{4029\left(\frac{1}{75029}+\cdots+\frac{1}{121379}\right)}\approx\frac{167.9441326}{4029\times0.04188514827}\approx0.9951934550\blacktriangle \tag{1.24}$$

$$\frac{\left(\sqrt{\frac{1}{121403}}+\cdots+\sqrt{\frac{1}{196387}}\right)^2}{6263\left(\frac{1}{121403}+\cdots+\frac{1}{196387}\right)}\approx\frac{250.8087539}{6263\times0.04023971920}\approx0.9951884735\blacktriangledown \tag{1.25}$$

$$\frac{\left(\sqrt{\frac{1}{196429}}+\cdots+\sqrt{\frac{1}{317797}}\right)^2}{9738\left(\frac{1}{196429}+\cdots+\frac{1}{317797}\right)}\approx\frac{374.6967511}{9738\times0.03866375428}\approx0.9951903020\blacktriangle \tag{1.26}$$

$$\frac{\left(\sqrt{\frac{1}{317827}}+\cdots+\sqrt{\frac{1}{514229}}\right)^2}{15187\left(\frac{1}{317827}+\cdots+\frac{1}{514229}\right)}\approx\frac{563.2532737}{15187\times0.03726738545}\approx0.9951826742\blacktriangledown \tag{1.27}$$

$$\frac{\left(\sqrt{\frac{1}{514243}}+\cdots+\sqrt{\frac{1}{832003}}\right)^2}{23704\left(\frac{1}{514243}+\cdots+\frac{1}{832003}\right)}\approx\frac{847.7374371}{23704\times0.03593634360}\approx0.9951896187\blacktriangle \tag{1.28}$$

The “lower-higher” pattern ends above, with no obvious pattern replacing it.

$$\frac{\left(\sqrt{\frac{1}{832063}}+\cdots+\sqrt{\frac{1}{1346249}}\right)^2}{36981\left(\frac{1}{832063}+\cdots+\frac{1}{1346249}\right)}\approx\frac{1275.263999}{36981\times0.03465047163}\approx0.9952044260\blacktriangle \tag{1.29}$$

$$\frac{\left(\sqrt{\frac{1}{1346273}}+\cdots+\sqrt{\frac{1}{2178283}}\right)^2}{57909\left(\frac{1}{1346273}+\cdots+\frac{1}{2178283}\right)}\approx\frac{1932.524189}{57909\times0.03353279168}\approx0.9951972387\blacktriangledown \tag{1.30}$$

$$\frac{\left(\sqrt{\frac{1}{2178313}}+\cdots+\sqrt{\frac{1}{3524569}}\right)^2}{90550\left(\frac{1}{2178313}+\cdots+\frac{1}{3524569}\right)}\approx\frac{2920.394553}{90550\times0.03240696706}\approx0.9952098996\blacktriangle \tag{1.31}$$

$$\frac{\left(\sqrt{\frac{1}{3524603}}+\cdots+\sqrt{\frac{1}{5702867}}\right)^2}{142033\left(\frac{1}{3524603}+\cdots+\frac{1}{5702867}\right)}\approx\frac{4441.052625}{142033\times0.03141849018}\approx0.9952022473\blacktriangledown \tag{1.32}$$

$$\frac{\left(\sqrt{\frac{1}{5702897}}+\cdots+\sqrt{\frac{1}{9227443}}\right)^2}{222855\left(\frac{1}{5702897}+\cdots+\frac{1}{9227443}\right)}\approx\frac{6756.338371}{222855\times0.03046340059}\approx0.9952005969\blacktriangledown \tag{1.33}$$

$$\frac{\left(\sqrt{\frac{1}{9227479}}+\cdots+\sqrt{\frac{1}{14930341}}\right)^2}{349862\left(\frac{1}{9227479}+\cdots+\frac{1}{14930341}\right)}\approx\frac{10290.98204}{349862\times0.02955621472}\approx0.9952019836\blacktriangle \tag{1.34}$$

$$\frac{\left(\sqrt{\frac{1}{14930387}}+\cdots+\sqrt{\frac{1}{24157811}}\right)^2}{549903\left(\frac{1}{14930387}+\cdots+\frac{1}{24157811}\right)}\approx\frac{15712.38081}{549903\times0.02871078874}\approx0.9952009543\blacktriangledown \tag{1.35}$$

$$\frac{\left(\sqrt{\frac{1}{24157823}}+\cdots+\sqrt{\frac{1}{39088157}}\right)^2}{865019\left(\frac{1}{24157823}+\cdots+\frac{1}{39088157}\right)}\approx\frac{24027.90776}{865019\times0.02791131162}\approx0.9951993045\blacktriangledown \tag{1.36}$$

$$\frac{\left(\sqrt{\frac{1}{39088193}}+\cdots+\sqrt{\frac{1}{63245971}}\right)^2}{1361581\left(\frac{1}{39088193}+\cdots+\frac{1}{63245971}\right)}\approx\frac{36792.34527}{1361581\times0.02715211358}\approx0.9951999556\blacktriangle \tag{1.37}$$

$$\frac{\left(\sqrt{\frac{1}{63245989}}+\cdots+\sqrt{\frac{1}{102334123}}\right)^2}{2145191\left(\frac{1}{63245989}+\cdots+\frac{1}{102334123}\right)}\approx\frac{56441.13758}{2145191\times 0.02643745907}\approx 0.9951993411\blacktriangledown \tag{1.38}$$

$$\frac{\left(\sqrt{\frac{1}{102334157}}+\cdots+\sqrt{\frac{1}{165580123}}\right)^2}{3381318\left(\frac{1}{102334157}+\cdots+\frac{1}{165580123}\right)}\approx\frac{86663.71496}{3381318\times 0.02575376413}\approx 0.9952004676\blacktriangle \tag{1.39}$$

$$\frac{\left(\sqrt{\frac{1}{165580147}}+\cdots+\sqrt{\frac{1}{267914279}}\right)^2}{5334509\left(\frac{1}{165580147}+\cdots+\frac{1}{267914279}\right)}\approx\frac{133307.2687}{5334509\times 0.02511013451}\approx 0.9951999353\blacktriangledown \tag{1.40}$$

$$\frac{\left(\sqrt{\frac{1}{267914303}}+\cdots+\sqrt{\frac{1}{433494437}}\right)^2}{8419528\left(\frac{1}{267914303}+\cdots+\frac{1}{433494437}\right)}\approx\frac{205229.1884}{8419528\times 0.02449292014}\approx 0.9952010271\blacktriangle \tag{1.41}$$

II. NORMALIZED KOIDE SUMS FOR *NON*-PRIME RECIPROCALS

Interestingly, if one calculates the normalized Koide sums for the *non*-prime reciprocals in the above Fibonacci intervals starting from $(3,5]$, one gets essentially the same lower-higher pattern as earlier, ending at the same point. A key difference, however, is that this new pattern is *inverted*, going lower where the other went higher, etc.

$$\frac{\left(\sqrt{\frac{1}{4}}\right)^2}{1\left(\frac{1}{4}\right)}=1.000000000 \tag{2.1}$$

$$\frac{\left(\sqrt{\frac{1}{6}}+\sqrt{\frac{1}{8}}\right)^2}{2\left(\frac{1}{6}+\frac{1}{8}\right)}\approx 0.9948716593\blacktriangledown \tag{2.2}$$

$$\frac{\left(\sqrt{\frac{1}{9}}+\sqrt{\frac{1}{10}}+\sqrt{\frac{1}{12}}\right)^2}{3\left(\frac{1}{9}+\frac{1}{10}+\frac{1}{12}\right)}\approx 0.9965515819\blacktriangle \tag{2.3}$$

$$\frac{\left(\sqrt{\frac{1}{14}}+\sqrt{\frac{1}{15}}+\sqrt{\frac{1}{16}}+\sqrt{\frac{1}{18}}+\sqrt{\frac{1}{20}}+\sqrt{\frac{1}{21}}\right)^2}{6\left(\frac{1}{14}+\frac{1}{15}+\frac{1}{16}+\frac{1}{18}+\frac{1}{20}+\frac{1}{21}\right)}\approx 0.9946068547\blacktriangledown \tag{2.4}$$

$$\frac{\left(\sqrt{\frac{1}{22}}+\sqrt{\frac{1}{24}}+\sqrt{\frac{1}{25}}+\sqrt{\frac{1}{26}}+\sqrt{\frac{1}{27}}+\sqrt{\frac{1}{28}}+\sqrt{\frac{1}{30}}+\sqrt{\frac{1}{32}}+\sqrt{\frac{1}{33}}+\sqrt{\frac{1}{34}}\right)^2}{10\left(\frac{1}{22}+\frac{1}{24}+\frac{1}{25}+\frac{1}{26}+\frac{1}{27}+\frac{1}{28}+\frac{1}{30}+\frac{1}{32}+\frac{1}{33}+\frac{1}{34}\right)}\approx 0.9952899149\blacktriangle \tag{2.5}$$

$$\frac{\left(\sqrt{\frac{1}{35}}+\cdots+\sqrt{\frac{1}{55}}\right)^2}{16\left(\frac{1}{35}+\cdots+\frac{1}{55}\right)}\approx\frac{5.740696260}{16\times 0.3605846163}\approx 0.9950327884\blacktriangledown \tag{2.6}$$

$$\frac{\left(\sqrt{\frac{1}{56}}+\cdots+\sqrt{\frac{1}{88}}\right)^2}{26\left(\frac{1}{56}+\cdots+\frac{1}{88}\right)}\approx\frac{9.466761842}{26\times 0.3658537987}\approx 0.9952232997\blacktriangle \tag{2.7}$$

$$\frac{\left(\sqrt{\frac{1}{90}}+\cdots+\sqrt{\frac{1}{144}}\right)^2}{45\left(\frac{1}{90}+\cdots+\frac{1}{144}\right)}\approx\frac{17.54160290}{45\times 0.3917478878}\approx 0.9950619002\blacktriangledown \tag{2.8}$$

$$\frac{\left(\sqrt{\frac{1}{145}}+\cdots+\sqrt{\frac{1}{232}}\right)^2}{72\left(\frac{1}{145}+\cdots+\frac{1}{232}\right)}\approx\frac{27.83203857}{72\times 0.3883773889}\approx 0.9953104951\blacktriangle \tag{2.9}$$

$$\frac{\left(\sqrt{\frac{1}{234}}+\cdots+\sqrt{\frac{1}{377}}\right)^2}{121\left(\frac{1}{234}+\cdots+\frac{1}{377}\right)}\approx\frac{48.56171828}{121\times 0.4032722280}\approx 0.9951999836\blacktriangledown \tag{2.10}$$

$$\frac{\left(\sqrt{\frac{1}{378}}+\cdots+\sqrt{\frac{1}{610}}\right)^2}{196\left(\frac{1}{378}+\cdots+\frac{1}{610}\right)}\approx\frac{78.76903731}{196\times0.4037905981}\approx0.9952753860\blacktriangle \tag{2.11}$$

$$\frac{\left(\sqrt{\frac{1}{611}}+\cdots+\sqrt{\frac{1}{987}}\right)^2}{322\left(\frac{1}{611}+\cdots+\frac{1}{987}\right)}\approx\frac{131.4515381}{322\times0.4101844960}\approx0.9952462717\blacktriangledown \tag{2.12}$$

$$\frac{\left(\sqrt{\frac{1}{988}}+\cdots+\sqrt{\frac{1}{1596}}\right)^2}{525\left(\frac{1}{988}+\cdots+\frac{1}{1596}\right)}\approx\frac{216.1823717}{525\times0.4137419513}\approx0.9952482335\blacktriangle \tag{2.13}$$

$$\frac{\left(\sqrt{\frac{1}{1598}}+\cdots+\sqrt{\frac{1}{2584}}\right)^2}{862\left(\frac{1}{1598}+\cdots+\frac{1}{2584}\right)}\approx\frac{360.1685191}{862\times0.4198421717}\approx0.9952047141\blacktriangledown \tag{2.14}$$

$$\frac{\left(\sqrt{\frac{1}{2585}}+\cdots+\sqrt{\frac{1}{4181}}\right)^2}{1399\left(\frac{1}{2585}+\cdots+\frac{1}{4181}\right)}\approx\frac{586.5416572}{1399\times0.4212691150}\approx0.9952255732\blacktriangle \tag{2.15}$$

$$\frac{\left(\sqrt{\frac{1}{4182}}+\cdots+\sqrt{\frac{1}{6765}}\right)^2}{2287\left(\frac{1}{4182}+\cdots+\frac{1}{6765}\right)}\approx\frac{969.1833375}{2287\times0.4258240166}\approx0.9951982927\blacktriangledown \tag{2.16}$$

$$\frac{\left(\sqrt{\frac{1}{6766}}+\cdots+\sqrt{\frac{1}{10946}}\right)^2}{3723\left(\frac{1}{6766}+\cdots+\frac{1}{10946}\right)}\approx\frac{1587.148608}{3723\times0.4283658480}\approx0.9951985106\blacktriangle \tag{2.17}$$

$$\frac{\left(\sqrt{\frac{1}{10947}}+\cdots+\sqrt{\frac{1}{17711}}\right)^2}{6061\left(\frac{1}{10947}+\cdots+\frac{1}{17711}\right)}\approx\frac{2600.015978}{6061\times0.4310449301}\approx0.9951973057\blacktriangledown \tag{2.18}$$

$$\frac{\left(\sqrt{\frac{1}{17712}}+\cdots+\sqrt{\frac{1}{28656}}\right)^2}{9858\left(\frac{1}{17712}+\cdots+\frac{1}{28656}\right)}\approx\frac{4251.001868}{9858\times0.4333016739}\approx0.9952040053\blacktriangle \tag{2.19}$$

$$\frac{\left(\sqrt{\frac{1}{28658}}+\cdots+\sqrt{\frac{1}{46368}}\right)^2}{16038\left(\frac{1}{28658}+\cdots+\frac{1}{46368}\right)}\approx\frac{6953.611968}{16038\times0.4356626835}\approx0.9951988849\blacktriangledown \tag{2.20}$$

$$\frac{\left(\sqrt{\frac{1}{46369}}+\cdots+\sqrt{\frac{1}{75025}}\right)^2}{26055\left(\frac{1}{46369}+\cdots+\frac{1}{75025}\right)}\approx\frac{11342.50208}{26055\times0.4374266906}\approx0.9952049122\blacktriangle \tag{2.21}$$

$$\frac{\left(\sqrt{\frac{1}{75026}}+\cdots+\sqrt{\frac{1}{121393}}\right)^2}{42339\left(\frac{1}{75026}+\cdots+\frac{1}{121393}\right)}\approx\frac{18511.34710}{42339\times0.4393241312}\approx0.9952045873\blacktriangledown \tag{2.22}$$

$$\frac{\left(\sqrt{\frac{1}{121394}}+\cdots+\sqrt{\frac{1}{196418}}\right)^2}{68762\left(\frac{1}{121394}+\cdots+\frac{1}{196418}\right)}\approx\frac{30176.62094}{68762\times0.4409705326}\approx0.9952049749\blacktriangle \tag{2.23}$$

$$\frac{\left(\sqrt{\frac{1}{196419}}+\cdots+\sqrt{\frac{1}{317811}}\right)^2}{111655\left(\frac{1}{196419}+\cdots+\frac{1}{317811}\right)}\approx\frac{49175.64961}{111655\times0.4425470984}\approx0.9952047314\blacktriangledown \tag{2.24}$$

$$\frac{\left(\sqrt{\frac{1}{317812}}+\cdots+\sqrt{\frac{1}{514228}}\right)^2}{181231\left(\frac{1}{317812}+\cdots+\frac{1}{514228}\right)}\approx\frac{80070.62298}{181231\times0.4439438387}\approx0.9952053172\blacktriangle \tag{2.25}$$

$$\frac{\left(\sqrt{\frac{1}{514230}}+\cdots+\sqrt{\frac{1}{832040}}\right)^2}{294107\left(\frac{1}{514230}+\cdots+\frac{1}{832040}\right)}\approx\frac{130330.5361}{294107\times0.4452751101}\approx0.9952046601\blacktriangledown \tag{2.26}$$

The “lower-higher” pattern ends above, at the same interval as before.

$$\frac{\left(\sqrt{\frac{1}{832041}}+\cdots+\sqrt{\frac{1}{1346269}}\right)^2}{477248\left(\frac{1}{832041}+\cdots+\frac{1}{1346269}\right)}\approx\frac{212098.1638}{477248\times0.4465611239}\approx0.9952034650\blacktriangledown \tag{2.27}$$

$$\frac{\left(\sqrt{\frac{1}{1346270}}+\cdots+\sqrt{\frac{1}{2178309}}\right)^2}{774131\left(\frac{1}{1346270}+\cdots+\frac{1}{2178309}\right)}\approx\frac{344899.9961}{774131\times0.4476788915}\approx0.9952040001\blacktriangle \tag{2.28}$$

$$\frac{\left(\sqrt{\frac{1}{2178310}}+\cdots+\sqrt{\frac{1}{3524578}}\right)^2}{1255719\left(\frac{1}{2178310}+\cdots+\frac{1}{3524578}\right)}\approx\frac{560869.2581}{1255719\times0.4488047703}\approx0.9952030689\blacktriangledown \tag{2.29}$$

$$\frac{\left(\sqrt{\frac{1}{3524579}}+\cdots+\sqrt{\frac{1}{5702887}}\right)^2}{2036276\left(\frac{1}{3524579}+\cdots+\frac{1}{5702887}\right)}\approx\frac{911510.2424}{2036276\times0.4497932807}\approx0.9952036201\blacktriangle \tag{2.30}$$

$$\frac{\left(\sqrt{\frac{1}{5702888}}+\cdots+\sqrt{\frac{1}{9227465}}\right)^2}{3301723\left(\frac{1}{5702888}+\cdots+\frac{1}{9227465}\right)}\approx\frac{1481108.286}{3301723\times0.4507483910}\approx0.9952037216\blacktriangle \tag{2.31}$$

$$\frac{\left(\sqrt{\frac{1}{9227466}}+\cdots+\sqrt{\frac{1}{14930352}}\right)^2}{5353025\left(\frac{1}{9227466}+\cdots+\frac{1}{14930352}\right)}\approx\frac{2406127.346}{5353025\times0.4516555896}\approx0.9952036220\blacktriangledown \tag{2.32}$$

$$\frac{\left(\sqrt{\frac{1}{14930353}}+\cdots+\sqrt{\frac{1}{24157817}}\right)^2}{8677562\left(\frac{1}{14930353}+\cdots+\frac{1}{24157817}\right)}\approx\frac{3907772.440}{8677562\times0.4525010235}\approx0.9952036828\blacktriangle \tag{2.33}$$

$$\frac{\left(\sqrt{\frac{1}{24157818}}+\cdots+\sqrt{\frac{1}{39088169}}\right)^2}{14065333\left(\frac{1}{24157818}+\cdots+\frac{1}{39088169}\right)}\approx\frac{6345242.695}{14065333\times0.4533005055}\approx0.9952037773\blacktriangle \tag{2.34}$$

$$\frac{\left(\sqrt{\frac{1}{39088170}}+\cdots+\sqrt{\frac{1}{63245986}}\right)^2}{22796236\left(\frac{1}{39088170}+\cdots+\frac{1}{63245986}\right)}\approx\frac{10301206.74}{22796236\times0.4540597066}\approx0.9952037297\blacktriangledown \tag{2.35}$$

$$\frac{\left(\sqrt{\frac{1}{63245987}}+\cdots+\sqrt{\frac{1}{102334155}}\right)^2}{36942978\left(\frac{1}{63245987}+\cdots+\frac{1}{102334155}\right)}\approx\frac{16720138.96}{36942978\times0.4547743630}\approx0.9952037575\blacktriangle \tag{2.36}$$

$$\frac{\left(\sqrt{\frac{1}{102334156}}+\cdots+\sqrt{\frac{1}{165580141}}\right)^2}{59864668\left(\frac{1}{102334156}+\cdots+\frac{1}{165580141}\right)}\approx\frac{27135069.93}{59864668\times0.4554580591}\approx0.9952036859\blacktriangledown \tag{2.37}$$

$$\frac{\left(\sqrt{\frac{1}{165580142}}+\cdots+\sqrt{\frac{1}{267914296}}\right)^2}{96999646\left(\frac{1}{165580142}+\cdots+\frac{1}{267914296}\right)}\approx\frac{44029506.35}{96999646\times0.4561016894}\approx0.9952037094\blacktriangle \tag{2.38}$$

$$\frac{\left(\sqrt{\frac{1}{267914297}}+\cdots+\sqrt{\frac{1}{433494436}}\right)^2}{157160613\left(\frac{1}{267914297}+\cdots+\frac{1}{433494436}\right)}\approx\frac{71433949.08}{157160613\times0.4567189042}\approx0.9952036444\blacktriangledown \tag{2.39}$$

III. THE FORMAT OF THE APPENDICES

Appendix A provides a summary of the results for both of the above sections—i.e., for reciprocals of both primes and non-primes. The format for the rows of Appendix A (and for all 25 appendices) is:

0) (5, 8] → 1.6000000
1) ▼▲▼ (8, 13] → 1.6250000
2) ▲▼▲ (13, 21] → 1.6153846
3) ▼▲▼ (21, 34] → 1.6190476

where:

- The leftmost column displays row number.
- The first column of blue triangles governs the change, compared to the previous row, in the value of the normalized Koide sum *for prime reciprocals* (▲=larger; ▼=smaller). This column of blue triangles turns grey (▲▼) once the lower-higher pattern described earlier is broken (which is to say, it remains blue only until two successive triangles occur that face in the same direction).
- The second column of blue triangles governs the change, compared to the previous row, in the value of the normalized Koide sum *for non-prime reciprocals* (▲=larger; ▼=smaller). This column of blue triangles likewise turns grey (▲▼) once the lower-higher pattern described earlier is broken.
- The third column of triangles is red (▲▼) if the two columns of blue/grey triangles match direction (e.g., ▲▲ or ▼▼), and green (▲▼) when they do not (e.g., ▲▼ or ▼▲), making it easier to spot the (sometimes rare) instances of a pair of triangles facing the same direction. (The direction of this red/green triangle is of no particular interest, as it merely faces the same way as the first blue/grey triangle.)
- The pair of integers that follow — e.g., (5, 8] — specifies the interval providing the terms used in the normalized Koide sum.
- The rightmost column gives the ratio of the higher of these integers to the lower (e.g., 1.600 0000).

IV. ANALYSIS OF THE DATA IN THE APPENDICES

All 25 appendices are computer-generated and differ only in the sequences they employ to determine the intervals involved. So the *sequence initiators* in Appendix A are

$$\begin{aligned} g_0 &= 5 \\ g_1 &= 8 \end{aligned} \quad ,$$

whereas in Appendix B they are

$$\begin{aligned} g_0 &= 4 \\ g_1 &= 7 \end{aligned} \quad .$$

For both, the remaining terms of each sequence determined by

$$g_{n+2} = g_n + g_{n+1} \quad .$$

(See the Contents for the sequence initiators g_0 and g_1 used in each appendix.)

Accordingly, in Appendix A, the Fibonacci numbers $0, 1, 1, 2, 3, \mathbf{5, 8}, 13, 21, \ldots$, which derive from

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_{n+2} &= F_n + F_{n+1} \end{aligned} \quad ,$$

supply the integers that define the successive intervals used, where the sequence initiators of Appendix A's first row are the two integers in boldface above (5 and 8). And in Appendix B, the Lucas numbers $2, 1, 3, \mathbf{4, 7}, 11, 18, 29, 47, \ldots$, which derive from

$$\begin{aligned} L_0 &= 2 \\ L_1 &= 1 \\ L_{n+2} &= L_n + L_{n+1} \quad , \end{aligned}$$

supply the integers that define the successive intervals used, where the sequence initiators of Appendix B's first row likewise appear in boldface above (4 and 7). Note that one reason the early numbers in each sequence are skipped is that the intervals used must be of sufficient width to avoid undefined values for the normalized Koide sums.

An examination of the details of Appendix A should make the above format clearer still. Here the key results from earlier are restated more compactly. The intervals used are again defined by the successive Fibonacci numbers, where the *initiating interval* is $(5, 8]$, and the final interval is, as earlier, $(267\,914\,296, 433\,494\,437]$. The two columns of blue triangles (governing primes and non-primes, respectively) match the earlier two sets of blue triangles. But note that here, following the convention introduced above, the blue triangles *turn gray* after the interval $(514\,229,\ 832\,040]$, marking (in the same place as earlier) the end of the lower-higher pattern noted at the outset.

One advantage this form has over that used earlier is that it is now easily seen that the blue triangles in each row always face in opposite directions, clearly showing that the lower-higher pattern is inverted for primes versus non-primes, at least for the intervals covered by Appendix A (hence, the total absence of red triangles in this appendix). This total absence of red triangles *almost* applies to Appendix B as well; its green triangles are continuous except for rows 2 and 6. (But note that the Lucas numbers produce very few blue triangles, as its lower-higher pattern stops early on.)

The first question that arises is: *Which initiating intervals should one expect to give green triangles—and which red—triangles?*

This issue is partially solvable by exploring Appendices C though L, whose initiating intervals take the form $(N^p, N^{p+1}]$, where $N = 3, 4, 5, \ldots 12$, and $p = 2, 3, 4,$ and 5. In all instances red triangles occupy the first four (or more) rows. However, in most instances these triangles eventually turn green, remaining so almost continuously. Moreover, this transition appears to occur when and if the ratio in the rightmost column gets close enough to the golden ratio ϕ (i.e., at roughly 1.618, where $\phi = (\sqrt{5} + 1)/2 \approx 1.618\,033\,9887$). It is therefore logical to expect that green triangles might dominate the results from the very first row if the sequence initiators used formed a ratio approximating ϕ; which is to say, if

$$\frac{g_1}{g_0} \approx \phi \quad .$$

Appendices M through V use initiating intervals of $(N^p, \lfloor N^p \phi \rfloor]$, where $N = 3, 4, 5, \ldots 12$, and $p = 2, 3, 4,$ and 5. Here, twenty-one of forty rows—about half—have no red triangles at all, supporting the above conjecture.

Appendix W uses initiating intervals of $(P, \lfloor P\phi \rfloor]$, where P is a prime in the interval $[7, 59]$. Here, six of fourteen rows—again, about half—have no red triangles at all, again supporting the above conjecture.

And Appendices X and Y each use a set of initiating intervals whose ratios g_1/g_0 form a progression that passes close to ϕ. In both appendices the red triangles decline as these ratios approach ϕ. Moreover, when the sequence initiators form ratios of exactly 1.61 and 1.618, respectively, the red triangles disappear completely, again suggesting that ϕ plays a key role in the suppression of red triangles.

As it turns out, in all 25 appendices the values in the rightmost column zigzag ever closer to ϕ, closing in on it from both sides. It appears that when a point is reached where the intervals successively grow in size by a factor sufficiently close to ϕ few red triangles are produced. But it does not necessarily follow that ϕ is unique in this regard. In fact, a preliminary examination of successive intervals of the form

$$\begin{aligned} (nk^0, nk^1] &\rightarrow\ k \\ (nk^1, nk^2] &\rightarrow\ k \\ (nk^2, nk^3] &\rightarrow\ k \\ \ldots \text{ etc.} & \end{aligned}$$

suggests that there are values for k different from ϕ that will work roughly as well in suppressing red triangles (e.g., 1.9, 2.0, and 2.1). So it may be that it is *the uniformity of increase* in interval size, rather than anything special about ϕ, that suppresses the red triangles.

The second question that arises is: *Do the Fibonacci numbers have a special relationship with primes?*

It appears they do. In Appendix A the Fibonacci numbers produce 24 consecutive rows of oppositely-facing pairs of blue triangles. This is excellent evidence of a specific link between the Fibonacci numbers and the distribution of primes. Moreover, this lower-higher pattern is not entirely unexpected, given that the author earlier showed that the *number* of primes in the intervals defined by the Fibonacci numbers produces a related pattern [4].

It is initially plausible to attribute these lower-higher patterns to the zigzag convergence on ϕ that is characteristic of the rightmost column in all appendices; but a closer look suggests that the value ϕ is converged on too rapidly to have the needed effect on interval size. Still another possibility is that a dense sequence of primes tends to be followed by a less dense sequence, which in turn is followed by a more dense sequence, etc., and that this tendency replicates itself on ever larger scales governed by the Fibonacci numbers. This explanation is at least plausible for the *number* of primes found in the Fibonacci intervals (given that the number of primes might very well reflect prime density); but it is not immediately clear that the normalized Koide sums of this article in any way reflect prime density.

APPENDIX A: USING 5 AND 8 AS INITIATORS

0) $(5, 8] \rightarrow 1.6000000$
1) ▼▲▼$(8, 13] \rightarrow 1.6250000$
2) ▲▼▲$(13, 21] \rightarrow 1.6153846$
3) ▼▲▼$(21, 34] \rightarrow 1.6190476$
4) ▲▼▲$(34, 55] \rightarrow 1.6176471$
5) ▼▲▼$(55, 89] \rightarrow 1.6181818$
6) ▲▼▲$(89, 144] \rightarrow 1.6179775$
7) ▼▲▼$(144, 233] \rightarrow 1.6180556$
8) ▲▼▲$(233, 377] \rightarrow 1.6180258$
9) ▼▲▼$(377, 610] \rightarrow 1.6180371$
10) ▲▼▲$(610, 987] \rightarrow 1.6180328$
11) ▼▲▼$(987, 1597] \rightarrow 1.6180344$
12) ▲▼▲$(1597, 2584] \rightarrow 1.6180338$
13) ▼▲▼$(2584, 4181] \rightarrow 1.6180341$
14) ▲▼▲$(4181, 6765] \rightarrow 1.6180340$
15) ▼▲▼$(6765, 10946] \rightarrow 1.6180340$
16) ▲▼▲$(10946, 17711] \rightarrow 1.6180340$
17) ▼▲▼$(17711, 28657] \rightarrow 1.6180340$
18) ▲▼▲$(28657, 46368] \rightarrow 1.6180340$
19) ▼▲▼$(46368, 75025] \rightarrow 1.6180340$
20) ▲▼▲$(75025, 121393] \rightarrow 1.6180340$
21) ▼▲▼$(121393, 196418] \rightarrow 1.6180340$
22) ▲▼▲$(196418, 317811] \rightarrow 1.6180340$
23) ▼▲▼$(317811, 514229] \rightarrow 1.6180340$
24) ▲▼▲$(514229, 832040] \rightarrow 1.6180340$
25) ▲▼▲$(832040, 1346269] \rightarrow 1.6180340$
26) ▼▲▼$(1346269, 2178309] \rightarrow 1.6180340$
27) ▲▼▲$(2178309, 3524578] \rightarrow 1.6180340$
28) ▼▲▼$(3524578, 5702887] \rightarrow 1.6180340$
29) ▼▲▼$(5702887, 9227465] \rightarrow 1.6180340$
30) ▲▼▲$(9227465, 14930352] \rightarrow 1.6180340$
31) ▼▲▼$(14930352, 24157817] \rightarrow 1.6180340$
32) ▼▲▼$(24157817, 39088169] \rightarrow 1.6180340$
33) ▲▼▲$(39088169, 63245986] \rightarrow 1.6180340$
34) ▼▲▼$(63245986, 102334155] \rightarrow 1.6180340$
35) ▲▼▲$(102334155, 165580141] \rightarrow 1.6180340$
36) ▼▲▼$(165580141, 267914296] \rightarrow 1.6180340$
37) ▲▼▲$(267914296, 433494437] \rightarrow 1.6180340$

APPENDIX B: USING 4 AND 7 AS INITIATORS

0) $(4, 7] \rightarrow 1.7500000$
1) $\blacktriangle\blacktriangledown\blacktriangle(7, 11] \rightarrow 1.5714286$
2) $\blacktriangledown\blacktriangledown\blacktriangledown(11, 18] \rightarrow 1.6363636$
3) $\blacktriangledown\blacktriangle\blacktriangledown(18, 29] \rightarrow 1.6111111$
4) $\blacktriangle\blacktriangledown\blacktriangle(29, 47] \rightarrow 1.6206897$
5) $\blacktriangle\blacktriangledown\blacktriangle(47, 76] \rightarrow 1.6170213$
6) $\blacktriangledown\blacktriangledown\blacktriangledown(76, 123] \rightarrow 1.6184211$
7) $\blacktriangledown\blacktriangle\blacktriangledown(123, 199] \rightarrow 1.6178862$
8) $\blacktriangle\blacktriangledown\blacktriangle(199, 322] \rightarrow 1.6180905$
9) $\blacktriangledown\blacktriangle\blacktriangledown(322, 521] \rightarrow 1.6180124$
10) $\blacktriangledown\blacktriangle\blacktriangledown(521, 843] \rightarrow 1.6180422$
11) $\blacktriangledown\blacktriangle\blacktriangledown(843, 1364] \rightarrow 1.6180308$
12) $\blacktriangledown\blacktriangle\blacktriangledown(1364, 2207] \rightarrow 1.6180352$
13) $\blacktriangledown\blacktriangle\blacktriangledown(2207, 3571] \rightarrow 1.6180335$
14) $\blacktriangle\blacktriangledown\blacktriangle(3571, 5778] \rightarrow 1.6180342$
15) $\blacktriangledown\blacktriangle\blacktriangledown(5778, 9349] \rightarrow 1.6180339$
16) $\blacktriangle\blacktriangledown\blacktriangle(9349, 15127] \rightarrow 1.6180340$
17) $\blacktriangledown\blacktriangle\blacktriangledown(15127, 24476] \rightarrow 1.6180340$
18) $\blacktriangle\blacktriangledown\blacktriangle(24476, 39603] \rightarrow 1.6180340$
19) $\blacktriangledown\blacktriangle\blacktriangledown(39603, 64079] \rightarrow 1.6180340$
20) $\blacktriangle\blacktriangledown\blacktriangle(64079, 103682] \rightarrow 1.6180340$
21) $\blacktriangledown\blacktriangle\blacktriangledown(103682, 167761] \rightarrow 1.6180340$
22) $\blacktriangledown\blacktriangle\blacktriangledown(167761, 271443] \rightarrow 1.6180340$
23) $\blacktriangle\blacktriangledown\blacktriangle(271443, 439204] \rightarrow 1.6180340$
24) $\blacktriangledown\blacktriangle\blacktriangledown(439204, 710647] \rightarrow 1.6180340$
25) $\blacktriangle\blacktriangledown\blacktriangle(710647, 1149851] \rightarrow 1.6180340$
26) $\blacktriangledown\blacktriangle\blacktriangledown(1149851, 1860498] \rightarrow 1.6180340$
27) $\blacktriangle\blacktriangledown\blacktriangle(1860498, 3010349] \rightarrow 1.6180340$
28) $\blacktriangle\blacktriangledown\blacktriangle(3010349, 4870847] \rightarrow 1.6180340$
29) $\blacktriangledown\blacktriangle\blacktriangledown(4870847, 7881196] \rightarrow 1.6180340$
30) $\blacktriangle\blacktriangledown\blacktriangle(7881196, 12752043] \rightarrow 1.6180340$
31) $\blacktriangle\blacktriangledown\blacktriangle(12752043, 20633239] \rightarrow 1.6180340$
32) $\blacktriangledown\blacktriangle\blacktriangledown(20633239, 33385282] \rightarrow 1.6180340$
33) $\blacktriangle\blacktriangledown\blacktriangle(33385282, 54018521] \rightarrow 1.6180340$
34) $\blacktriangledown\blacktriangle\blacktriangledown(54018521, 87403803] \rightarrow 1.6180340$
35) $\blacktriangledown\blacktriangle\blacktriangledown(87403803, 141422324] \rightarrow 1.6180340$
36) $\blacktriangledown\blacktriangle\blacktriangledown(141422324, 228826127] \rightarrow 1.6180340$
37) $\blacktriangle\blacktriangledown\blacktriangle(228826127, 370248451] \rightarrow 1.6180340$

APPENDIX C: USING 3^p AND 3^{p+1} AS INITIATORS

0)	$(9, 27] \rightarrow 3.0000000$	$(27, 81] \rightarrow 3.0000000$
1)	▲▲▲$(27, 36] \rightarrow 1.3333333$	▲▲▲$(81, 108] \rightarrow 1.3333333$
2)	▼▼▼$(36, 63] \rightarrow 1.7500000$	▼▼▼$(108, 189] \rightarrow 1.7500000$
3)	▲▲▲$(63, 99] \rightarrow 1.5714286$	▲▲▲$(189, 297] \rightarrow 1.5714286$
4)	▼▼▼$(99, 162] \rightarrow 1.6363636$	▼▼▼$(297, 486] \rightarrow 1.6363636$
5)	▲▲▲$(162, 261] \rightarrow 1.6111111$	▼▲▼$(486, 783] \rightarrow 1.6111111$
6)	▼▼▼$(261, 423] \rightarrow 1.6206897$	▼▼▼$(783, 1269] \rightarrow 1.6206897$
7)	▲▼▲$(423, 684] \rightarrow 1.6170213$	▼▲▼$(1269, 2052] \rightarrow 1.6170213$
8)	▲▼▲$(684, 1107] \rightarrow 1.6184211$	▲▼▲$(2052, 3321] \rightarrow 1.6184211$
9)	▼▲▼$(1107, 1791] \rightarrow 1.6178862$	▲▲▲$(3321, 5373] \rightarrow 1.6178862$
10)	▲▼▲$(1791, 2898] \rightarrow 1.6180905$	▼▲▼$(5373, 8694] \rightarrow 1.6180905$
11)	▲▼▲$(2898, 4689] \rightarrow 1.6180124$	▲▼▲$(8694, 14067] \rightarrow 1.6180124$
12)	▼▲▼$(4689, 7587] \rightarrow 1.6180422$	▲▼▲$(14067, 22761] \rightarrow 1.6180422$
13)	▲▲▲$(7587, 12276] \rightarrow 1.6180308$	▲▼▲$(22761, 36828] \rightarrow 1.6180308$
14)	▼▲▼$(12276, 19863] \rightarrow 1.6180352$	▼▲▼$(36828, 59589] \rightarrow 1.6180352$
15)	▲▼▲$(19863, 32139] \rightarrow 1.6180335$	▲▼▲$(59589, 96417] \rightarrow 1.6180335$
16)	▼▲▼$(32139, 52002] \rightarrow 1.6180342$	▼▲▼$(96417, 156006] \rightarrow 1.6180342$
17)	▲▼▲$(52002, 84141] \rightarrow 1.6180339$	▲▼▲$(156006, 252423] \rightarrow 1.6180339$
18)	▼▼▼$(84141, 136143] \rightarrow 1.6180340$	▲▼▲$(252423, 408429] \rightarrow 1.6180340$
19)	▼▲▼$(136143, 220284] \rightarrow 1.6180340$	▼▲▼$(408429, 660852] \rightarrow 1.6180340$
20)	▲▼▲$(220284, 356427] \rightarrow 1.6180340$	▲▼▲$(660852, 1069281] \rightarrow 1.6180340$
21)	▲▼▲$(356427, 576711] \rightarrow 1.6180340$	▼▲▼$(1069281, 1730133] \rightarrow 1.6180340$
22)	▼▲▼$(576711, 933138] \rightarrow 1.6180340$	▲▼▲$(1730133, 2799414] \rightarrow 1.6180340$
23)	▲▼▲$(933138, 1509849] \rightarrow 1.6180340$	▼▲▼$(2799414, 4529547] \rightarrow 1.6180340$
24)	▲▼▲$(1509849, 2442987] \rightarrow 1.6180340$	▲▼▲$(4529547, 7328961] \rightarrow 1.6180340$
25)	▼▲▼$(2442987, 3952836] \rightarrow 1.6180340$	▲▼▲$(7328961, 11858508] \rightarrow 1.6180340$
26)	▲▼▲$(3952836, 6395823] \rightarrow 1.6180340$	▼▲▼$(11858508, 19187469] \rightarrow 1.6180340$
27)	▼▲▼$(6395823, 10348659] \rightarrow 1.6180340$	▼▲▼$(19187469, 31045977] \rightarrow 1.6180340$
28)	▼▲▼$(10348659, 16744482] \rightarrow 1.6180340$	▲▼▲$(31045977, 50233446] \rightarrow 1.6180340$
29)	▲▼▲$(16744482, 27093141] \rightarrow 1.6180340$	▼▲▼$(50233446, 81279423] \rightarrow 1.6180340$
30)	▲▼▲$(27093141, 43837623] \rightarrow 1.6180340$	▲▼▲$(81279423, 131512869] \rightarrow 1.6180340$
31)	▼▲▼$(43837623, 70930764] \rightarrow 1.6180340$	▲▼▲$(131512869, 212792292] \rightarrow 1.6180340$
32)	▲▼▲$(70930764, 114768387] \rightarrow 1.6180340$	▼▲▼$(212792292, 344305161] \rightarrow 1.6180340$

0)	$(81, 243] \rightarrow 3.0000000$	$(243, 729] \rightarrow 3.0000000$
1)	▲▲▲$(243, 324] \rightarrow 1.3333333$	▲▲▲$(729, 972] \rightarrow 1.3333333$
2)	▼▼▼$(324, 567] \rightarrow 1.7500000$	▼▼▼$(972, 1701] \rightarrow 1.7500000$
3)	▲▲▲$(567, 891] \rightarrow 1.5714286$	▲▲▲$(1701, 2673] \rightarrow 1.5714286$
4)	▼▼▼$(891, 1458] \rightarrow 1.6363636$	▼▼▼$(2673, 4374] \rightarrow 1.6363636$
5)	▼▲▼$(1458, 2349] \rightarrow 1.6111111$	▲▲▲$(4374, 7047] \rightarrow 1.6111111$
6)	▼▼▼$(2349, 3807] \rightarrow 1.6206897$	▲▼▲$(7047, 11421] \rightarrow 1.6206897$
7)	▲▲▲$(3807, 6156] \rightarrow 1.6170213$	▼▲▼$(11421, 18468] \rightarrow 1.6170213$
8)	▼▼▼$(6156, 9963] \rightarrow 1.6184211$	▲▼▲$(18468, 29889] \rightarrow 1.6184211$
9)	▲▼▲$(9963, 16119] \rightarrow 1.6178862$	▼▲▼$(29889, 48357] \rightarrow 1.6178862$
10)	▲▼▲$(16119, 26082] \rightarrow 1.6180905$	▼▲▼$(48357, 78246] \rightarrow 1.6180905$
11)	▼▲▼$(26082, 42201] \rightarrow 1.6180124$	▲▼▲$(78246, 126603] \rightarrow 1.6180124$
12)	▲▼▲$(42201, 68283] \rightarrow 1.6180422$	▲▼▲$(126603, 204849] \rightarrow 1.6180422$
13)	▲▼▲$(68283, 110484] \rightarrow 1.6180308$	▼▲▼$(204849, 331452] \rightarrow 1.6180308$
14)	▼▲▼$(110484, 178767] \rightarrow 1.6180352$	▲▼▲$(331452, 536301] \rightarrow 1.6180352$
15)	▼▲▼$(178767, 289251] \rightarrow 1.6180335$	▼▲▼$(536301, 867753] \rightarrow 1.6180335$
16)	▲▼▲$(289251, 468018] \rightarrow 1.6180342$	▲▼▲$(867753, 1404054] \rightarrow 1.6180342$
17)	▼▲▼$(468018, 757269] \rightarrow 1.6180339$	▼▲▼$(1404054, 2271807] \rightarrow 1.6180339$
18)	▼▲▼$(757269, 1225287] \rightarrow 1.6180340$	▲▼▲$(2271807, 3675861] \rightarrow 1.6180340$
19)	▲▼▲$(1225287, 1982556] \rightarrow 1.6180340$	▼▲▼$(3675861, 5947668] \rightarrow 1.6180340$
20)	▼▲▼$(1982556, 3207843] \rightarrow 1.6180340$	▲▼▲$(5947668, 9623529] \rightarrow 1.6180340$
21)	▲▼▲$(3207843, 5190399] \rightarrow 1.6180340$	▲▼▲$(9623529, 15571197] \rightarrow 1.6180340$
22)	▼▼▼$(5190399, 8398242] \rightarrow 1.6180340$	▼▲▼$(15571197, 25194726] \rightarrow 1.6180340$
23)	▲▼▲$(8398242, 13588641] \rightarrow 1.6180340$	▼▲▼$(25194726, 40765923] \rightarrow 1.6180340$
24)	▼▲▼$(13588641, 21986883] \rightarrow 1.6180340$	▲▼▲$(40765923, 65960649] \rightarrow 1.6180340$
25)	▲▼▲$(21986883, 35575524] \rightarrow 1.6180340$	▼▲▼$(65960649, 106726572] \rightarrow 1.6180340$
26)	▼▲▼$(35575524, 57562407] \rightarrow 1.6180340$	▲▼▲$(106726572, 172687221] \rightarrow 1.6180340$
27)	▲▼▲$(57562407, 93137931] \rightarrow 1.6180340$	▲▼▲$(172687221, 279413793] \rightarrow 1.6180340$
28)	▼▲▼$(93137931, 150700338] \rightarrow 1.6180340$	▼▼▼$(279413793, 452101014] \rightarrow 1.6180340$

APPENDIX D: USING 4^p AND 4^{p+1} AS INITIATORS

0)	$(16, 64] \rightarrow 4.0000000$	$(64, 256] \rightarrow 4.0000000$
1)	▲▲▲$(64, 80] \rightarrow 1.2500000$	▲▲▲$(256, 320] \rightarrow 1.2500000$
2)	▼▼▼$(80, 144] \rightarrow 1.8000000$	▼▼▼$(320, 576] \rightarrow 1.8000000$
3)	▲▲▲$(144, 224] \rightarrow 1.5555556$	▲▲▲$(576, 896] \rightarrow 1.5555556$
4)	▼▼▼$(224, 368] \rightarrow 1.6428571$	▼▼▼$(896, 1472] \rightarrow 1.6428571$
5)	▲▲▲$(368, 592] \rightarrow 1.6086957$	▲▲▲$(1472, 2368] \rightarrow 1.6086957$
6)	▼▼▼$(592, 960] \rightarrow 1.6216216$	▼▼▼$(2368, 3840] \rightarrow 1.6216216$
7)	▲▲▲$(960, 1552] \rightarrow 1.6166667$	▲▲▲$(3840, 6208] \rightarrow 1.6166667$
8)	▲▼▲$(1552, 2512] \rightarrow 1.6185567$	▼▼▼$(6208, 10048] \rightarrow 1.6185567$
9)	▲▼▲$(2512, 4064] \rightarrow 1.6178344$	▲▲▲$(10048, 16256] \rightarrow 1.6178344$
10)	▲▼▲$(4064, 6576] \rightarrow 1.6181102$	▲▼▲$(16256, 26304] \rightarrow 1.6181102$
11)	▲▲▲$(6576, 10640] \rightarrow 1.6180049$	▼▲▼$(26304, 42560] \rightarrow 1.6180049$
12)	▲▼▲$(10640, 17216] \rightarrow 1.6180451$	▲▼▲$(42560, 68864] \rightarrow 1.6180451$
13)	▼▲▼$(17216, 27856] \rightarrow 1.6180297$	▼▲▼$(68864, 111424] \rightarrow 1.6180297$
14)	▲▼▲$(27856, 45072] \rightarrow 1.6180356$	▼▲▼$(111424, 180288] \rightarrow 1.6180356$
15)	▼▲▼$(45072, 72928] \rightarrow 1.6180334$	▲▼▲$(180288, 291712] \rightarrow 1.6180334$
16)	▼▲▼$(72928, 118000] \rightarrow 1.6180342$	▼▲▼$(291712, 472000] \rightarrow 1.6180342$
17)	▼▲▼$(118000, 190928] \rightarrow 1.6180339$	▼▲▼$(472000, 763712] \rightarrow 1.6180339$
18)	▲▼▲$(190928, 308928] \rightarrow 1.6180340$	▼▲▼$(763712, 1235712] \rightarrow 1.6180340$
19)	▼▲▼$(308928, 499856] \rightarrow 1.6180340$	▲▼▲$(1235712, 1999424] \rightarrow 1.6180340$
20)	▲▼▲$(499856, 808784] \rightarrow 1.6180340$	▼▲▼$(1999424, 3235136] \rightarrow 1.6180340$
21)	▼▲▼$(808784, 1308640] \rightarrow 1.6180340$	▲▼▲$(3235136, 5234560] \rightarrow 1.6180340$
22)	▲▼▲$(1308640, 2117424] \rightarrow 1.6180340$	▲▼▲$(5234560, 8469696] \rightarrow 1.6180340$
23)	▲▼▲$(2117424, 3426064] \rightarrow 1.6180340$	▼▲▼$(8469696, 13704256] \rightarrow 1.6180340$
24)	▲▼▲$(3426064, 5543488] \rightarrow 1.6180340$	▲▼▲$(13704256, 22173952] \rightarrow 1.6180340$
25)	▼▲▼$(5543488, 8969552] \rightarrow 1.6180340$	▲▼▲$(22173952, 35878208] \rightarrow 1.6180340$
26)	▼▲▼$(8969552, 14513040] \rightarrow 1.6180340$	▼▲▼$(35878208, 58052160] \rightarrow 1.6180340$
27)	▼▲▼$(14513040, 23482592] \rightarrow 1.6180340$	▲▼▲$(58052160, 93930368] \rightarrow 1.6180340$
28)	▲▼▲$(23482592, 37995632] \rightarrow 1.6180340$	▼▲▼$(93930368, 151982528] \rightarrow 1.6180340$
29)	▼▲▼$(37995632, 61478224] \rightarrow 1.6180340$	▼▼▼$(151982528, 245912896] \rightarrow 1.6180340$
30)	▲▼▲$(61478224, 99473856] \rightarrow 1.6180340$	▲▼▲$(245912896, 397895424] \rightarrow 1.6180340$

0)	$(256, 1024] \rightarrow 4.0000000$	$(1024, 4096] \rightarrow 4.0000000$
1)	▲▲▲$(1024, 1280] \rightarrow 1.2500000$	▲▲▲$(4096, 5120] \rightarrow 1.2500000$
2)	▼▼▼$(1280, 2304] \rightarrow 1.8000000$	▼▼▼$(5120, 9216] \rightarrow 1.8000000$
3)	▲▲▲$(2304, 3584] \rightarrow 1.5555556$	▲▲▲$(9216, 14336] \rightarrow 1.5555556$
4)	▼▼▼$(3584, 5888] \rightarrow 1.6428571$	▼▼▼$(14336, 23552] \rightarrow 1.6428571$
5)	▲▲▲$(5888, 9472] \rightarrow 1.6086957$	▲▲▲$(23552, 37888] \rightarrow 1.6086957$
6)	▼▼▼$(9472, 15360] \rightarrow 1.6216216$	▼▼▼$(37888, 61440] \rightarrow 1.6216216$
7)	▲▲▲$(15360, 24832] \rightarrow 1.6166667$	▲▲▲$(61440, 99328] \rightarrow 1.6166667$
8)	▼▼▼$(24832, 40192] \rightarrow 1.6185567$	▼▼▼$(99328, 160768] \rightarrow 1.6185567$
9)	▲▲▲$(40192, 65024] \rightarrow 1.6178344$	▲▲▲$(160768, 260096] \rightarrow 1.6178344$
10)	▼▲▼$(65024, 105216] \rightarrow 1.6181102$	▼▼▼$(260096, 420864] \rightarrow 1.6181102$
11)	▲▼▲$(105216, 170240] \rightarrow 1.6180049$	▼▲▼$(420864, 680960] \rightarrow 1.6180049$
12)	▼▲▼$(170240, 275456] \rightarrow 1.6180451$	▼▼▼$(680960, 1101824] \rightarrow 1.6180451$
13)	▲▼▲$(275456, 445696] \rightarrow 1.6180297$	▼▲▼$(1101824, 1782784] \rightarrow 1.6180297$
14)	▲▼▲$(445696, 721152] \rightarrow 1.6180356$	▼▲▼$(1782784, 2884608] \rightarrow 1.6180356$
15)	▼▲▼$(721152, 1166848] \rightarrow 1.6180334$	▲▼▲$(2884608, 4667392] \rightarrow 1.6180334$
16)	▲▼▲$(1166848, 1888000] \rightarrow 1.6180342$	▼▲▼$(4667392, 7552000] \rightarrow 1.6180342$
17)	▲▼▲$(1888000, 3054848] \rightarrow 1.6180339$	▲▼▲$(7552000, 12219392] \rightarrow 1.6180339$
18)	▼▲▼$(3054848, 4942848] \rightarrow 1.6180340$	▲▼▲$(12219392, 19771392] \rightarrow 1.6180340$
19)	▲▼▲$(4942848, 7997696] \rightarrow 1.6180340$	▼▼▼$(19771392, 31990784] \rightarrow 1.6180340$
20)	▲▼▲$(7997696, 12940544] \rightarrow 1.6180340$	▲▼▲$(31990784, 51762176] \rightarrow 1.6180340$
21)	▲▼▲$(12940544, 20938240] \rightarrow 1.6180340$	▲▼▲$(51762176, 83752960] \rightarrow 1.6180340$
22)	▼▲▼$(20938240, 33878784] \rightarrow 1.6180340$	▼▲▼$(83752960, 135515136] \rightarrow 1.6180340$
23)	▲▼▲$(33878784, 54817024] \rightarrow 1.6180340$	▲▼▲$(135515136, 219268096] \rightarrow 1.6180340$
24)	▼▲▼$(54817024, 88695808] \rightarrow 1.6180340$	▼▲▼$(219268096, 354783232] \rightarrow 1.6180340$

APPENDIX E: USING 5^p AND 5^{p+1} AS INITIATORS

0)	$(25, 125] \rightarrow 5.0000000$	$(125, 625] \rightarrow 5.0000000$
1)	▲▲▲$(125, 150] \rightarrow 1.2000000$	▲▲▲$(625, 750] \rightarrow 1.2000000$
2)	▼▼▼$(150, 275] \rightarrow 1.8333333$	▼▼▼$(750, 1375] \rightarrow 1.8333333$
3)	▲▲▲$(275, 425] \rightarrow 1.5454545$	▲▲▲$(1375, 2125] \rightarrow 1.5454545$
4)	▼▼▼$(425, 700] \rightarrow 1.6470588$	▼▼▼$(2125, 3500] \rightarrow 1.6470588$
5)	▲▲▲$(700, 1125] \rightarrow 1.6071429$	▲▲▲$(3500, 5625] \rightarrow 1.6071429$
6)	▲▼▲$(1125, 1825] \rightarrow 1.6222222$	▼▼▼$(5625, 9125] \rightarrow 1.6222222$
7)	▼▲▼$(1825, 2950] \rightarrow 1.6164384$	▲▲▲$(9125, 14750] \rightarrow 1.6164384$
8)	▲▼▲$(2950, 4775] \rightarrow 1.6186441$	▼▼▼$(14750, 23875] \rightarrow 1.6186441$
9)	▼▲▼$(4775, 7725] \rightarrow 1.6178010$	▲▼▲$(23875, 38625] \rightarrow 1.6178010$
10)	▲▼▲$(7725, 12500] \rightarrow 1.6181230$	▼▲▼$(38625, 62500] \rightarrow 1.6181230$
11)	▼▲▼$(12500, 20225] \rightarrow 1.6180000$	▲▼▲$(62500, 101125] \rightarrow 1.6180000$
12)	▲▼▲$(20225, 32725] \rightarrow 1.6180470$	▼▲▼$(101125, 163625] \rightarrow 1.6180470$
13)	▲▼▲$(32725, 52950] \rightarrow 1.6180290$	▲▼▲$(163625, 264750] \rightarrow 1.6180290$
14)	▼▲▼$(52950, 85675] \rightarrow 1.6180359$	▼▲▼$(264750, 428375] \rightarrow 1.6180359$
15)	▲▼▲$(85675, 138625] \rightarrow 1.6180333$	▼▲▼$(428375, 693125] \rightarrow 1.6180333$
16)	▲▼▲$(138625, 224300] \rightarrow 1.6180343$	▼▲▼$(693125, 1121500] \rightarrow 1.6180343$
17)	▲▼▲$(224300, 362925] \rightarrow 1.6180339$	▲▼▲$(1121500, 1814625] \rightarrow 1.6180339$
18)	▼▲▼$(362925, 587225] \rightarrow 1.6180340$	▼▲▼$(1814625, 2936125] \rightarrow 1.6180340$
19)	▲▼▲$(587225, 950150] \rightarrow 1.6180340$	▲▼▲$(2936125, 4750750] \rightarrow 1.6180340$
20)	▼▲▼$(950150, 1537375] \rightarrow 1.6180340$	▲▼▲$(4750750, 7686875] \rightarrow 1.6180340$
21)	▲▼▲$(1537375, 2487525] \rightarrow 1.6180340$	▲▼▲$(7686875, 12437625] \rightarrow 1.6180340$
22)	▼▲▼$(2487525, 4024900] \rightarrow 1.6180340$	▲▼▲$(12437625, 20124500] \rightarrow 1.6180340$
23)	▲▼▲$(4024900, 6512425] \rightarrow 1.6180340$	▼▲▼$(20124500, 32562125] \rightarrow 1.6180340$
24)	▼▲▼$(6512425, 10537325] \rightarrow 1.6180340$	▲▼▲$(32562125, 52686625] \rightarrow 1.6180340$
25)	▼▲▼$(10537325, 17049750] \rightarrow 1.6180340$	▲▼▲$(52686625, 85248750] \rightarrow 1.6180340$
26)	▲▼▲$(17049750, 27587075] \rightarrow 1.6180340$	▼▲▼$(85248750, 137935375] \rightarrow 1.6180340$
27)	▲▼▲$(27587075, 44636825] \rightarrow 1.6180340$	▲▼▲$(137935375, 223184125] \rightarrow 1.6180340$
28)	▼▲▼$(44636825, 72223900] \rightarrow 1.6180340$	▲▼▲$(223184125, 361119500] \rightarrow 1.6180340$

0)	$(625, 3125] \to 5.0000000$	$(3125, 15625] \to 5.0000000$
1)	▲▲▲$(3125, 3750] \to 1.2000000$	▲▲▲$(15625, 18750] \to 1.2000000$
2)	▼▼▼$(3750, 6875] \to 1.8333333$	▼▼▼$(18750, 34375] \to 1.8333333$
3)	▲▲▲$(6875, 10625] \to 1.5454545$	▲▲▲$(34375, 53125] \to 1.5454545$
4)	▼▼▼$(10625, 17500] \to 1.6470588$	▼▼▼$(53125, 87500] \to 1.6470588$
5)	▲▲▲$(17500, 28125] \to 1.6071429$	▲▲▲$(87500, 140625] \to 1.6071429$
6)	▼▼▼$(28125, 45625] \to 1.6222222$	▼▼▼$(140625, 228125] \to 1.6222222$
7)	▲▲▲$(45625, 73750] \to 1.6164384$	▲▲▲$(228125, 368750] \to 1.6164384$
8)	▼▼▼$(73750, 119375] \to 1.6186441$	▼▼▼$(368750, 596875] \to 1.6186441$
9)	▼▲▼$(119375, 193125] \to 1.6178010$	▲▲▲$(596875, 965625] \to 1.6178010$
10)	▲▼▲$(193125, 312500] \to 1.6181230$	▼▼▼$(965625, 1562500] \to 1.6181230$
11)	▼▲▼$(312500, 505625] \to 1.6180000$	▲▲▲$(1562500, 2528125] \to 1.6180000$
12)	▲▼▲$(505625, 818125] \to 1.6180470$	▼▲▼$(2528125, 4090625] \to 1.6180470$
13)	▼▲▼$(818125, 1323750] \to 1.6180290$	▲▼▲$(4090625, 6618750] \to 1.6180290$
14)	▲▼▲$(1323750, 2141875] \to 1.6180359$	▼▲▼$(6618750, 10709375] \to 1.6180359$
15)	▼▲▼$(2141875, 3465625] \to 1.6180333$	▲▼▲$(10709375, 17328125] \to 1.6180333$
16)	▲▼▲$(3465625, 5607500] \to 1.6180343$	▲▼▲$(17328125, 28037500] \to 1.6180343$
17)	▼▲▼$(5607500, 9073125] \to 1.6180339$	▲▼▲$(28037500, 45365625] \to 1.6180339$
18)	▼▲▼$(9073125, 14680625] \to 1.6180340$	▲▼▲$(45365625, 73403125] \to 1.6180340$
19)	▲▼▲$(14680625, 23753750] \to 1.6180340$	▲▼▲$(73403125, 118768750] \to 1.6180340$
20)	▼▲▼$(23753750, 38434375] \to 1.6180340$	▲▼▲$(118768750, 192171875] \to 1.6180340$
21)	▲▼▲$(38434375, 62188125] \to 1.6180340$	▼▲▼$(192171875, 310940625] \to 1.6180340$

APPENDIX F: USING 6^p AND 6^{p+1} AS INITIATORS

0)	$(36, 216] \rightarrow 6.0000000$	$(216, 1296] \rightarrow 6.0000000$
1)	▲▲▲$(216, 252] \rightarrow 1.1666667$	▲▲▲$(1296, 1512] \rightarrow 1.1666667$
2)	▼▼▼$(252, 468] \rightarrow 1.8571429$	▼▼▼$(1512, 2808] \rightarrow 1.8571429$
3)	▲▲▲$(468, 720] \rightarrow 1.5384615$	▲▲▲$(2808, 4320] \rightarrow 1.5384615$
4)	▼▼▼$(720, 1188] \rightarrow 1.6500000$	▼▼▼$(4320, 7128] \rightarrow 1.6500000$
5)	▲▲▲$(1188, 1908] \rightarrow 1.6060606$	▲▲▲$(7128, 11448] \rightarrow 1.6060606$
6)	▼▼▼$(1908, 3096] \rightarrow 1.6226415$	▼▼▼$(11448, 18576] \rightarrow 1.6226415$
7)	▲▲▲$(3096, 5004] \rightarrow 1.6162791$	▲▲▲$(18576, 30024] \rightarrow 1.6162791$
8)	▼▼▼$(5004, 8100] \rightarrow 1.6187050$	▼▼▼$(30024, 48600] \rightarrow 1.6187050$
9)	▽▲▽$(8100, 13104] \rightarrow 1.6177778$	▽▲▽$(48600, 78624] \rightarrow 1.6177778$
10)	△▼△$(13104, 21204] \rightarrow 1.6181319$	△▼△$(78624, 127224] \rightarrow 1.6181319$
11)	▽▲▽$(21204, 34308] \rightarrow 1.6179966$	▽▲▽$(127224, 205848] \rightarrow 1.6179966$
12)	△▼△$(34308, 55512] \rightarrow 1.6180483$	▽△▽$(205848, 333072] \rightarrow 1.6180483$
13)	▽▲▽$(55512, 89820] \rightarrow 1.6180285$	△▽△$(333072, 538920] \rightarrow 1.6180285$
14)	△▼△$(89820, 145332] \rightarrow 1.6180361$	△▽△$(538920, 871992] \rightarrow 1.6180361$
15)	▽▲▽$(145332, 235152] \rightarrow 1.6180332$	▽△▽$(871992, 1410912] \rightarrow 1.6180332$
16)	△▼△$(235152, 380484] \rightarrow 1.6180343$	▽△▽$(1410912, 2282904] \rightarrow 1.6180343$
17)	▽▲▽$(380484, 615636] \rightarrow 1.6180339$	△▽△$(2282904, 3693816] \rightarrow 1.6180339$
18)	△▼△$(615636, 996120] \rightarrow 1.6180340$	▽△▽$(3693816, 5976720] \rightarrow 1.6180340$
19)	▽▲▽$(996120, 1611756] \rightarrow 1.6180340$	▽△▽$(5976720, 9670536] \rightarrow 1.6180340$
20)	△▼△$(1611756, 2607876] \rightarrow 1.6180340$	△▽△$(9670536, 15647256] \rightarrow 1.6180340$
21)	▽▲▽$(2607876, 4219632] \rightarrow 1.6180340$	▽△▽$(15647256, 25317792] \rightarrow 1.6180340$
22)	△▼△$(4219632, 6827508] \rightarrow 1.6180340$	▽△▽$(25317792, 40965048] \rightarrow 1.6180340$
23)	▽▲▽$(6827508, 11047140] \rightarrow 1.6180340$	△▽△$(40965048, 66282840] \rightarrow 1.6180340$
24)	△▼△$(11047140, 17874648] \rightarrow 1.6180340$	▽△▽$(66282840, 107247888] \rightarrow 1.6180340$
25)	△▽△$(17874648, 28921788] \rightarrow 1.6180340$	△▽△$(107247888, 173530728] \rightarrow 1.6180340$
26)	▽△▽$(28921788, 46796436] \rightarrow 1.6180340$	△▽△$(173530728, 280778616] \rightarrow 1.6180340$
27)	△▽△$(46796436, 75718224] \rightarrow 1.6180340$	△▽△$(280778616, 454309344] \rightarrow 1.6180340$

0)	$(1296, 7776] \rightarrow 6.0000000$	$(7776, 46656] \rightarrow 6.0000000$
1)	▲▲▲$(7776, 9072] \rightarrow 1.1666667$	▲▲▲$(46656, 54432] \rightarrow 1.1666667$
2)	▼▼▼$(9072, 16848] \rightarrow 1.8571429$	▼▼▼$(54432, 101088] \rightarrow 1.8571429$
3)	▲▲▲$(16848, 25920] \rightarrow 1.5384615$	▲▲▲$(101088, 155520] \rightarrow 1.5384615$
4)	▼▼▼$(25920, 42768] \rightarrow 1.6500000$	▼▼▼$(155520, 256608] \rightarrow 1.6500000$
5)	▲▲▲$(42768, 68688] \rightarrow 1.6060606$	▲▲▲$(256608, 412128] \rightarrow 1.6060606$
6)	▼▼▼$(68688, 111456] \rightarrow 1.6226415$	▼▼▼$(412128, 668736] \rightarrow 1.6226415$
7)	▲▲▲$(111456, 180144] \rightarrow 1.6162791$	▲▲▲$(668736, 1080864] \rightarrow 1.6162791$
8)	▼▼▼$(180144, 291600] \rightarrow 1.6187050$	▼▼▼$(1080864, 1749600] \rightarrow 1.6187050$
9)	▲▲▲$(291600, 471744] \rightarrow 1.6177778$	▲▲▲$(1749600, 2830464] \rightarrow 1.6177778$
10)	▼▼▼$(471744, 763344] \rightarrow 1.6181319$	▼▼▼$(2830464, 4580064] \rightarrow 1.6181319$
11)	▼▲▼$(763344, 1235088] \rightarrow 1.6179966$	▲▲▲$(4580064, 7410528] \rightarrow 1.6179966$
12)	▲▼▲$(1235088, 1998432] \rightarrow 1.6180483$	▼▼▼$(7410528, 11990592] \rightarrow 1.6180483$
13)	▲▲▲$(1998432, 3233520] \rightarrow 1.6180285$	▲▲▲$(11990592, 19401120] \rightarrow 1.6180285$
14)	▲▼▲$(3233520, 5231952] \rightarrow 1.6180361$	▼▼▼$(19401120, 31391712] \rightarrow 1.6180361$
15)	▲▼▲$(5231952, 8465472] \rightarrow 1.6180332$	▲▼▲$(31391712, 50792832] \rightarrow 1.6180332$
16)	▼▲▼$(8465472, 13697424] \rightarrow 1.6180343$	▼▼▼$(50792832, 82184544] \rightarrow 1.6180343$
17)	▲▼▲$(13697424, 22162896] \rightarrow 1.6180339$	▲▼▲$(82184544, 132977376] \rightarrow 1.6180339$
18)	▲▼▲$(22162896, 35860320] \rightarrow 1.6180340$	▼▲▼$(132977376, 215161920] \rightarrow 1.6180340$
19)	▼▲▼$(35860320, 58023216] \rightarrow 1.6180340$	▲▼▲$(215161920, 348139296] \rightarrow 1.6180340$

APPENDIX G: USING 7^p AND 7^{p+1} AS INITIATORS

0)	$(49, 343] \rightarrow 7.0000000$	$(343, 2401] \rightarrow 7.0000000$
1)	▲▲▲$(343, 392] \rightarrow 1.1428571$	▲▲▲$(2401, 2744] \rightarrow 1.1428571$
2)	▼▼▼$(392, 735] \rightarrow 1.8750000$	▼▼▼$(2744, 5145] \rightarrow 1.8750000$
3)	▲▲▲$(735, 1127] \rightarrow 1.5333333$	▲▲▲$(5145, 7889] \rightarrow 1.5333333$
4)	▼▼▼$(1127, 1862] \rightarrow 1.6521739$	▼▼▼$(7889, 13034] \rightarrow 1.6521739$
5)	▲▲▲$(1862, 2989] \rightarrow 1.6052632$	▲▲▲$(13034, 20923] \rightarrow 1.6052632$
6)	▼▼▼$(2989, 4851] \rightarrow 1.6229508$	▼▼▼$(20923, 33957] \rightarrow 1.6229508$
7)	▼▲▼$(4851, 7840] \rightarrow 1.6161616$	▲▲▲$(33957, 54880] \rightarrow 1.6161616$
8)	▼▼▼$(7840, 12691] \rightarrow 1.6187500$	▼▼▼$(54880, 88837] \rightarrow 1.6187500$
9)	▲▲▲$(12691, 20531] \rightarrow 1.6177606$	▼▲▼$(88837, 143717] \rightarrow 1.6177606$
10)	▼▲▼$(20531, 33222] \rightarrow 1.6181384$	▲▼▲$(143717, 232554] \rightarrow 1.6181384$
11)	▲▼▲$(33222, 53753] \rightarrow 1.6179941$	▲▼▲$(232554, 376271] \rightarrow 1.6179941$
12)	▼▼▼$(53753, 86975] \rightarrow 1.6180492$	▼▲▼$(376271, 608825] \rightarrow 1.6180492$
13)	▼▲▼$(86975, 140728] \rightarrow 1.6180282$	▲▼▲$(608825, 985096] \rightarrow 1.6180282$
14)	▲▼▲$(140728, 227703] \rightarrow 1.6180362$	▼▲▼$(985096, 1593921] \rightarrow 1.6180362$
15)	▲▼▲$(227703, 368431] \rightarrow 1.6180331$	▼▲▼$(1593921, 2579017] \rightarrow 1.6180331$
16)	▲▼▲$(368431, 596134] \rightarrow 1.6180343$	▲▼▲$(2579017, 4172938] \rightarrow 1.6180343$
17)	▼▲▼$(596134, 964565] \rightarrow 1.6180339$	▲▼▲$(4172938, 6751955] \rightarrow 1.6180339$
18)	▼▲▼$(964565, 1560699] \rightarrow 1.6180340$	▼▲▼$(6751955, 10924893] \rightarrow 1.6180340$
19)	▲▼▲$(1560699, 2525264] \rightarrow 1.6180340$	▲▼▲$(10924893, 17676848] \rightarrow 1.6180340$
20)	▼▲▼$(2525264, 4085963] \rightarrow 1.6180340$	▲▼▲$(17676848, 28601741] \rightarrow 1.6180340$
21)	▲▼▲$(4085963, 6611227] \rightarrow 1.6180340$	▼▲▼$(28601741, 46278589] \rightarrow 1.6180340$
22)	▼▲▼$(6611227, 10697190] \rightarrow 1.6180340$	▲▼▲$(46278589, 74880330] \rightarrow 1.6180340$
23)	▲▼▲$(10697190, 17308417] \rightarrow 1.6180340$	▲▼▲$(74880330, 121158919] \rightarrow 1.6180340$
24)	▼▲▼$(17308417, 28005607] \rightarrow 1.6180340$	▼▲▼$(121158919, 196039249] \rightarrow 1.6180340$
25)	▲▼▲$(28005607, 45314024] \rightarrow 1.6180340$	▲▼▲$(196039249, 317198168] \rightarrow 1.6180340$

0)	$(2401, 16807] \to 7.0000000$	$(16807, 117649] \to 7.0000000$
1)	▲▲▲$(16807, 19208] \to 1.1428571$	▲▲▲$(117649, 134456] \to 1.1428571$
2)	▼▼▼$(19208, 36015] \to 1.8750000$	▼▼▼$(134456, 252105] \to 1.8750000$
3)	▲▲▲$(36015, 55223] \to 1.5333333$	▲▲▲$(252105, 386561] \to 1.5333333$
4)	▼▼▼$(55223, 91238] \to 1.6521739$	▼▼▼$(386561, 638666] \to 1.6521739$
5)	▲▲▲$(91238, 146461] \to 1.6052632$	▲▲▲$(638666, 1025227] \to 1.6052632$
6)	▼▼▼$(146461, 237699] \to 1.6229508$	▼▼▼$(1025227, 1663893] \to 1.6229508$
7)	▲▲▲$(237699, 384160] \to 1.6161616$	▲▲▲$(1663893, 2689120] \to 1.6161616$
8)	▼▼▼$(384160, 621859] \to 1.6187500$	▼▼▼$(2689120, 4353013] \to 1.6187500$
9)	▲▲▲$(621859, 1006019] \to 1.6177606$	▲▲▲$(4353013, 7042133] \to 1.6177606$
10)	▼▼▼$(1006019, 1627878] \to 1.6181384$	▲▼▲$(7042133, 11395146] \to 1.6181384$
11)	▲▲▲$(1627878, 2633897] \to 1.6179941$	▼▲▼$(11395146, 18437279] \to 1.6179941$
12)	▼▼▼$(2633897, 4261775] \to 1.6180492$	▲▼▲$(18437279, 29832425] \to 1.6180492$
13)	▼▲▼$(4261775, 6895672] \to 1.6180282$	▼▲▼$(29832425, 48269704] \to 1.6180282$
14)	▲▼▲$(6895672, 11157447] \to 1.6180362$	▲▼▲$(48269704, 78102129] \to 1.6180362$
15)	▼▲▼$(11157447, 18053119] \to 1.6180331$	▲▲▲$(78102129, 126371833] \to 1.6180331$
16)	▲▼▲$(18053119, 29210566] \to 1.6180343$	▲▼▲$(126371833, 204473962] \to 1.6180343$
17)	▼▲▼$(29210566, 47263685] \to 1.6180339$	▲▼▲$(204473962, 330845795] \to 1.6180339$

APPENDIX H: USING 8^p AND 8^{p+1} AS INITIATORS

0)	$(64, 512] \rightarrow 8.0000000$	$(512, 4096] \rightarrow 8.0000000$
1)	▲▲▲$(512, 576] \rightarrow 1.1250000$	▲▲▲$(4096, 4608] \rightarrow 1.1250000$
2)	▼▼▼$(576, 1088] \rightarrow 1.8888889$	▼▼▼$(4608, 8704] \rightarrow 1.8888889$
3)	▲▲▲$(1088, 1664] \rightarrow 1.5294118$	▲▲▲$(8704, 13312] \rightarrow 1.5294118$
4)	▼▼▼$(1664, 2752] \rightarrow 1.6538462$	▼▼▼$(13312, 22016] \rightarrow 1.6538462$
5)	▲▲▲$(2752, 4416] \rightarrow 1.6046512$	▲▲▲$(22016, 35328] \rightarrow 1.6046512$
6)	▼▼▼$(4416, 7168] \rightarrow 1.6231884$	▼▼▼$(35328, 57344] \rightarrow 1.6231884$
7)	▲▲▲$(7168, 11584] \rightarrow 1.6160714$	▲▲▲$(57344, 92672] \rightarrow 1.6160714$
8)	▼▼▼$(11584, 18752] \rightarrow 1.6187845$	▼▼▼$(92672, 150016] \rightarrow 1.6187845$
9)	▲▲▲$(18752, 30336] \rightarrow 1.6177474$	▲▲▲$(150016, 242688] \rightarrow 1.6177474$
10)	▼▲▼$(30336, 49088] \rightarrow 1.6181435$	▼▼▼$(242688, 392704] \rightarrow 1.6181435$
11)	▼▲▼$(49088, 79424] \rightarrow 1.6179922$	▲▲▲$(392704, 635392] \rightarrow 1.6179922$
12)	▲▼▲$(79424, 128512] \rightarrow 1.6180500$	▼▼▼$(635392, 1028096] \rightarrow 1.6180500$
13)	▲▼▲$(128512, 207936] \rightarrow 1.6180279$	▲▲▲$(1028096, 1663488] \rightarrow 1.6180279$
14)	▼▲▼$(207936, 336448] \rightarrow 1.6180363$	▼▲▼$(1663488, 2691584] \rightarrow 1.6180363$
15)	▲▼▲$(336448, 544384] \rightarrow 1.6180331$	▲▼▲$(2691584, 4355072] \rightarrow 1.6180331$
16)	▼▲▼$(544384, 880832] \rightarrow 1.6180343$	▼▲▼$(4355072, 7046656] \rightarrow 1.6180343$
17)	▲▼▲$(880832, 1425216] \rightarrow 1.6180339$	▲▼▲$(7046656, 11401728] \rightarrow 1.6180339$
18)	▼▲▼$(1425216, 2306048] \rightarrow 1.6180340$	▼▲▼$(11401728, 18448384] \rightarrow 1.6180340$
19)	▲▼▲$(2306048, 3731264] \rightarrow 1.6180340$	▲▼▲$(18448384, 29850112] \rightarrow 1.6180340$
20)	▲▼▲$(3731264, 6037312] \rightarrow 1.6180340$	▼▲▼$(29850112, 48298496] \rightarrow 1.6180340$
21)	▼▲▼$(6037312, 9768576] \rightarrow 1.6180340$	▲▼▲$(48298496, 78148608] \rightarrow 1.6180340$
22)	▲▼▲$(9768576, 15805888] \rightarrow 1.6180340$	▲▼▲$(78148608, 126447104] \rightarrow 1.6180340$
23)	▼▲▼$(15805888, 25574464] \rightarrow 1.6180340$	▲▼▲$(126447104, 204595712] \rightarrow 1.6180340$
24)	▼▲▼$(25574464, 41380352] \rightarrow 1.6180340$	▼▼▼$(204595712, 331042816] \rightarrow 1.6180340$

0)	$(4096, 32768] \rightarrow 8.0000000$	$(32768, 262144] \rightarrow 8.0000000$
1)	▲▲▲$(32768, 36864] \rightarrow 1.1250000$	▲▲▲$(262144, 294912] \rightarrow 1.1250000$
2)	▼▼▼$(36864, 69632] \rightarrow 1.8888889$	▼▼▼$(294912, 557056] \rightarrow 1.8888889$
3)	▲▲▲$(69632, 106496] \rightarrow 1.5294118$	▲▲▲$(557056, 851968] \rightarrow 1.5294118$
4)	▼▼▼$(106496, 176128] \rightarrow 1.6538462$	▼▼▼$(851968, 1409024] \rightarrow 1.6538462$
5)	▲▲▲$(176128, 282624] \rightarrow 1.6046512$	▲▲▲$(1409024, 2260992] \rightarrow 1.6046512$
6)	▼▼▼$(282624, 458752] \rightarrow 1.6231884$	▼▼▼$(2260992, 3670016] \rightarrow 1.6231884$
7)	▲▲▲$(458752, 741376] \rightarrow 1.6160714$	▲▲▲$(3670016, 5931008] \rightarrow 1.6160714$
8)	▼▼▼$(741376, 1200128] \rightarrow 1.6187845$	▼▼▼$(5931008, 9601024] \rightarrow 1.6187845$
9)	▲▲▲$(1200128, 1941504] \rightarrow 1.6177474$	▲▲▲$(9601024, 15532032] \rightarrow 1.6177474$
10)	▼▼▼$(1941504, 3141632] \rightarrow 1.6181435$	▼▼▼$(15532032, 25133056] \rightarrow 1.6181435$
11)	▼▲▼$(3141632, 5083136] \rightarrow 1.6179922$	▲▲▲$(25133056, 40665088] \rightarrow 1.6179922$
12)	▲▼▲$(5083136, 8224768] \rightarrow 1.6180500$	▲▼▲$(40665088, 65798144] \rightarrow 1.6180500$
13)	▼▲▼$(8224768, 13307904] \rightarrow 1.6180279$	▼▲▼$(65798144, 106463232] \rightarrow 1.6180279$
14)	▼▼▼$(13307904, 21532672] \rightarrow 1.6180363$	▲▼▲$(106463232, 172261376] \rightarrow 1.6180363$
15)	▲▼▲$(21532672, 34840576] \rightarrow 1.6180331$	▲▼▲$(172261376, 278724608] \rightarrow 1.6180331$
16)	▼▲▼$(34840576, 56373248] \rightarrow 1.6180343$	▲▼▲$(278724608, 450985984] \rightarrow 1.6180343$

APPENDIX I: USING 9^p AND 9^{p+1} AS INITIATORS

0)	$(81, 729] \rightarrow 9.0000000$	$(729, 6561] \rightarrow 9.0000000$
1)	▲▲▲$(729, 810] \rightarrow 1.1111111$	▲▲▲$(6561, 7290] \rightarrow 1.1111111$
2)	▼▼▼$(810, 1539] \rightarrow 1.9000000$	▼▼▼$(7290, 13851] \rightarrow 1.9000000$
3)	▲▲▲$(1539, 2349] \rightarrow 1.5263158$	▲▲▲$(13851, 21141] \rightarrow 1.5263158$
4)	▼▼▼$(2349, 3888] \rightarrow 1.6551724$	▼▼▼$(21141, 34992] \rightarrow 1.6551724$
5)	▲▲▲$(3888, 6237] \rightarrow 1.6041667$	▲▲▲$(34992, 56133] \rightarrow 1.6041667$
6)	▼▼▼$(6237, 10125] \rightarrow 1.6233766$	▼▼▼$(56133, 91125] \rightarrow 1.6233766$
7)	▲▲▲$(10125, 16362] \rightarrow 1.6160000$	▲▲▲$(91125, 147258] \rightarrow 1.6160000$
8)	▼▼▼$(16362, 26487] \rightarrow 1.6188119$	▼▼▼$(147258, 238383] \rightarrow 1.6188119$
9)	▼▲▼$(26487, 42849] \rightarrow 1.6177370$	▲▲▲$(238383, 385641] \rightarrow 1.6177370$
10)	▲▼▲$(42849, 69336] \rightarrow 1.6181474$	▼▼▼$(385641, 624024] \rightarrow 1.6181474$
11)	▼▲▼$(69336, 112185] \rightarrow 1.6179907$	▲▲▲$(624024, 1009665] \rightarrow 1.6179907$
12)	▲▼▲$(112185, 181521] \rightarrow 1.6180505$	▼▼▼$(1009665, 1633689] \rightarrow 1.6180505$
13)	▼▲▼$(181521, 293706] \rightarrow 1.6180277$	▼▲▼$(1633689, 2643354] \rightarrow 1.6180277$
14)	▼▲▼$(293706, 475227] \rightarrow 1.6180364$	▼▲▼$(2643354, 4277043] \rightarrow 1.6180364$
15)	▼▲▼$(475227, 768933] \rightarrow 1.6180331$	▼▲▼$(4277043, 6920397] \rightarrow 1.6180331$
16)	▲▼▲$(768933, 1244160] \rightarrow 1.6180343$	▲▼▲$(6920397, 11197440] \rightarrow 1.6180343$
17)	▼▲▼$(1244160, 2013093] \rightarrow 1.6180339$	▲▼▲$(11197440, 18117837] \rightarrow 1.6180339$
18)	▲▼▲$(2013093, 3257253] \rightarrow 1.6180340$	▲▼▲$(18117837, 29315277] \rightarrow 1.6180340$
19)	▲▼▲$(3257253, 5270346] \rightarrow 1.6180340$	▼▲▼$(29315277, 47433114] \rightarrow 1.6180340$
20)	▲▼▲$(5270346, 8527599] \rightarrow 1.6180340$	▲▼▲$(47433114, 76748391] \rightarrow 1.6180340$
21)	▼▲▼$(8527599, 13797945] \rightarrow 1.6180340$	▼▲▼$(76748391, 124181505] \rightarrow 1.6180340$
22)	▲▼▲$(13797945, 22325544] \rightarrow 1.6180340$	▼▲▼$(124181505, 200929896] \rightarrow 1.6180340$
23)	▲▼▲$(22325544, 36123489] \rightarrow 1.6180340$	▲▼▲$(200929896, 325111401] \rightarrow 1.6180340$

0)	$(6561, 59049] \rightarrow 9.0000000$	$(59049, 531441] \rightarrow 9.0000000$
1)	▲▲▲$(59049, 65610] \rightarrow 1.1111111$	▲▲▲$(531441, 590490] \rightarrow 1.1111111$
2)	▼▼▼$(65610, 124659] \rightarrow 1.9000000$	▼▼▼$(590490, 1121931] \rightarrow 1.9000000$
3)	▲▲▲$(124659, 190269] \rightarrow 1.5263158$	▲▲▲$(1121931, 1712421] \rightarrow 1.5263158$
4)	▼▼▼$(190269, 314928] \rightarrow 1.6551724$	▼▼▼$(1712421, 2834352] \rightarrow 1.6551724$
5)	▲▲▲$(314928, 505197] \rightarrow 1.6041667$	▲▲▲$(2834352, 4546773] \rightarrow 1.6041667$
6)	▼▼▼$(505197, 820125] \rightarrow 1.6233766$	▼▼▼$(4546773, 7381125] \rightarrow 1.6233766$
7)	▲▲▲$(820125, 1325322] \rightarrow 1.6160000$	▲▲▲$(7381125, 11927898] \rightarrow 1.6160000$
8)	▼▼▼$(1325322, 2145447] \rightarrow 1.6188119$	▼▼▼$(11927898, 19309023] \rightarrow 1.6188119$
9)	▲▲▲$(2145447, 3470769] \rightarrow 1.6177370$	▲▲▲$(19309023, 31236921] \rightarrow 1.6177370$
10)	▼▼▼$(3470769, 5616216] \rightarrow 1.6181474$	▼▼▼$(31236921, 50545944] \rightarrow 1.6181474$
11)	▼▲▼$(5616216, 9086985] \rightarrow 1.6179907$	▲▲▲$(50545944, 81782865] \rightarrow 1.6179907$
12)	▼▼▼$(9086985, 14703201] \rightarrow 1.6180505$	▼▼▼$(81782865, 132328809] \rightarrow 1.6180505$
13)	▼▲▼$(14703201, 23790186] \rightarrow 1.6180277$	▲▲▲$(132328809, 214111674] \rightarrow 1.6180277$
14)	▼▼▼$(23790186, 38493387] \rightarrow 1.6180364$	▼▼▼$(214111674, 346440483] \rightarrow 1.6180364$

APPENDIX J: USING 10^p AND 10^{p+1} AS INITIATORS

0)	$(100, 1000] \rightarrow 10.0000000$	$(1000, 10000] \rightarrow 10.0000000$
1)	▲▲▲$(1000, 1100] \rightarrow 1.1000000$	▲▲▲$(10000, 11000] \rightarrow 1.1000000$
2)	▼▼▼$(1100, 2100] \rightarrow 1.9090909$	▼▼▼$(11000, 21000] \rightarrow 1.9090909$
3)	▲▲▲$(2100, 3200] \rightarrow 1.5238095$	▲▲▲$(21000, 32000] \rightarrow 1.5238095$
4)	▼▼▼$(3200, 5300] \rightarrow 1.6562500$	▼▼▼$(32000, 53000] \rightarrow 1.6562500$
5)	▲▲▲$(5300, 8500] \rightarrow 1.6037736$	▲▲▲$(53000, 85000] \rightarrow 1.6037736$
6)	▼▼▼$(8500, 13800] \rightarrow 1.6235294$	▼▼▼$(85000, 138000] \rightarrow 1.6235294$
7)	▲▲▲$(13800, 22300] \rightarrow 1.6159420$	▲▲▲$(138000, 223000] \rightarrow 1.6159420$
8)	▼▼▼$(22300, 36100] \rightarrow 1.6188341$	▼▼▼$(223000, 361000] \rightarrow 1.6188341$
9)	▼▲▼$(36100, 58400] \rightarrow 1.6177285$	▲▲▲$(361000, 584000] \rightarrow 1.6177285$
10)	▲▼▲$(58400, 94500] \rightarrow 1.6181507$	▼▼▼$(584000, 945000] \rightarrow 1.6181507$
11)	▼▲▼$(94500, 152900] \rightarrow 1.6179894$	▲▲▲$(945000, 1529000] \rightarrow 1.6179894$
12)	▲▼▲$(152900, 247400] \rightarrow 1.6180510$	▲▼▲$(1529000, 2474000] \rightarrow 1.6180510$
13)	▼▲▼$(247400, 400300] \rightarrow 1.6180275$	▼▲▼$(2474000, 4003000] \rightarrow 1.6180275$
14)	▲▼▲$(400300, 647700] \rightarrow 1.6180365$	▲▼▲$(4003000, 6477000] \rightarrow 1.6180365$
15)	▼▲▼$(647700, 1048000] \rightarrow 1.6180330$	▼▲▼$(6477000, 10480000] \rightarrow 1.6180330$
16)	▲▼▲$(1048000, 1695700] \rightarrow 1.6180344$	▼▼▼$(10480000, 16957000] \rightarrow 1.6180344$
17)	▼▲▼$(1695700, 2743700] \rightarrow 1.6180339$	▲▼▲$(16957000, 27437000] \rightarrow 1.6180339$
18)	▲▼▲$(2743700, 4439400] \rightarrow 1.6180340$	▲▼▲$(27437000, 44394000] \rightarrow 1.6180340$
19)	▼▲▼$(4439400, 7183100] \rightarrow 1.6180340$	▼▲▼$(44394000, 71831000] \rightarrow 1.6180340$
20)	▲▼▲$(7183100, 11622500] \rightarrow 1.6180340$	▲▼▲$(71831000, 116225000] \rightarrow 1.6180340$
21)	▲▼▲$(11622500, 18805600] \rightarrow 1.6180340$	▲▼▲$(116225000, 188056000] \rightarrow 1.6180340$
22)	▼▲▼$(18805600, 30428100] \rightarrow 1.6180340$	▲▼▲$(188056000, 304281000] \rightarrow 1.6180340$
23)	▲▼▲$(30428100, 49233700] \rightarrow 1.6180340$	▼▲▼$(304281000, 492337000] \rightarrow 1.6180340$

0)	$(10000, 100000] \to 10.0000000$	$(100000, 1000000] \to 10.0000000$
1)	▲▲▲$(100000, 110000] \to 1.1000000$	▲▲▲$(1000000, 1100000] \to 1.1000000$
2)	▼▼▼$(110000, 210000] \to 1.9090909$	▼▼▼$(1100000, 2100000] \to 1.9090909$
3)	▲▲▲$(210000, 320000] \to 1.5238095$	▲▲▲$(2100000, 3200000] \to 1.5238095$
4)	▼▼▼$(320000, 530000] \to 1.6562500$	▼▼▼$(3200000, 5300000] \to 1.6562500$
5)	▲▲▲$(530000, 850000] \to 1.6037736$	▲▲▲$(5300000, 8500000] \to 1.6037736$
6)	▼▼▼$(850000, 1380000] \to 1.6235294$	▼▼▼$(8500000, 13800000] \to 1.6235294$
7)	▲▲▲$(1380000, 2230000] \to 1.6159420$	▲▲▲$(13800000, 22300000] \to 1.6159420$
8)	▼▼▼$(2230000, 3610000] \to 1.6188341$	▼▼▼$(22300000, 36100000] \to 1.6188341$
9)	▲▲▲$(3610000, 5840000] \to 1.6177285$	▲▲▲$(36100000, 58400000] \to 1.6177285$
10)	▼▼▼$(5840000, 9450000] \to 1.6181507$	▼▼▼$(58400000, 94500000] \to 1.6181507$
11)	▲▲▲$(9450000, 15290000] \to 1.6179894$	▲▲▲$(94500000, 152900000] \to 1.6179894$
12)	▼▼▼$(15290000, 24740000] \to 1.6180510$	▼▼▼$(152900000, 247400000] \to 1.6180510$
13)	▲▲▲$(24740000, 40030000] \to 1.6180275$	▲▲▲$(247400000, 400300000] \to 1.6180275$

APPENDIX K: USING 11^p AND 11^{p+1} AS INITIATORS

0)	$(121, 1331] \rightarrow 11.0000000$	$(1331, 14641] \rightarrow 11.0000000$
1)	$\blacktriangle\blacktriangle\blacktriangle(1331, 1452] \rightarrow 1.0909091$	$\blacktriangle\blacktriangle\blacktriangle(14641, 15972] \rightarrow 1.0909091$
2)	$\blacktriangledown\blacktriangledown\blacktriangledown(1452, 2783] \rightarrow 1.9166667$	$\blacktriangledown\blacktriangledown\blacktriangledown(15972, 30613] \rightarrow 1.9166667$
3)	$\blacktriangle\blacktriangle\blacktriangle(2783, 4235] \rightarrow 1.5217391$	$\blacktriangle\blacktriangle\blacktriangle(30613, 46585] \rightarrow 1.5217391$
4)	$\blacktriangledown\blacktriangledown\blacktriangledown(4235, 7018] \rightarrow 1.6571429$	$\blacktriangledown\blacktriangledown\blacktriangledown(46585, 77198] \rightarrow 1.6571429$
5)	$\blacktriangle\blacktriangle\blacktriangle(7018, 11253] \rightarrow 1.6034483$	$\blacktriangle\blacktriangle\blacktriangle(77198, 123783] \rightarrow 1.6034483$
6)	$\blacktriangledown\blacktriangledown\blacktriangledown(11253, 18271] \rightarrow 1.6236559$	$\blacktriangledown\blacktriangledown\blacktriangledown(123783, 200981] \rightarrow 1.6236559$
7)	$\blacktriangle\blacktriangle\blacktriangle(18271, 29524] \rightarrow 1.6158940$	$\blacktriangle\blacktriangle\blacktriangle(200981, 324764] \rightarrow 1.6158940$
8)	$\blacktriangledown\blacktriangledown\blacktriangledown(29524, 47795] \rightarrow 1.6188525$	$\blacktriangledown\blacktriangledown\blacktriangledown(324764, 525745] \rightarrow 1.6188525$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(47795, 77319] \rightarrow 1.6177215$	$\blacktriangle\blacktriangle\blacktriangle(525745, 850509] \rightarrow 1.6177215$
10)	$\blacktriangledown\blacktriangledown\blacktriangledown(77319, 125114] \rightarrow 1.6181534$	$\blacktriangledown\blacktriangledown\blacktriangledown(850509, 1376254] \rightarrow 1.6181534$
11)	$\blacktriangle\blacktriangle\blacktriangle(125114, 202433] \rightarrow 1.6179884$	$\blacktriangledown\blacktriangle\blacktriangledown(1376254, 2226763] \rightarrow 1.6179884$
12)	$\blacktriangledown\blacktriangledown\blacktriangledown(202433, 327547] \rightarrow 1.6180514$	$\blacktriangle\blacktriangledown\blacktriangle(2226763, 3603017] \rightarrow 1.6180514$
13)	$\blacktriangle\blacktriangledown\blacktriangle(327547, 529980] \rightarrow 1.6180273$	$\blacktriangledown\blacktriangle\blacktriangledown(3603017, 5829780] \rightarrow 1.6180273$
14)	$\blacktriangle\blacktriangledown\blacktriangle(529980, 857527] \rightarrow 1.6180365$	$\blacktriangledown\blacktriangle\blacktriangledown(5829780, 9432797] \rightarrow 1.6180365$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(857527, 1387507] \rightarrow 1.6180330$	$\blacktriangle\blacktriangledown\blacktriangle(9432797, 15262577] \rightarrow 1.6180330$
16)	$\blacktriangle\blacktriangledown\blacktriangle(1387507, 2245034] \rightarrow 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(15262577, 24695374] \rightarrow 1.6180344$
17)	$\blacktriangle\blacktriangledown\blacktriangle(2245034, 3632541] \rightarrow 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(24695374, 39957951] \rightarrow 1.6180338$
18)	$\blacktriangle\blacktriangledown\blacktriangle(3632541, 5877575] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(39957951, 64653325] \rightarrow 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(5877575, 9510116] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(64653325, 104611276] \rightarrow 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(9510116, 15387691] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(104611276, 169264601] \rightarrow 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(15387691, 24897807] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(169264601, 273875877] \rightarrow 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(24897807, 40285498] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(273875877, 443140478] \rightarrow 1.6180340$

0)	$(14641, 161051] \to 11.0000000$	$(161051, 1771561] \to 11.0000000$
1)	▲▲▲$(161051, 175692] \to 1.0909091$	▲▲▲$(1771561, 1932612] \to 1.0909091$
2)	▼▼▼$(175692, 336743] \to 1.9166667$	▼▼▼$(1932612, 3704173] \to 1.9166667$
3)	▲▲▲$(336743, 512435] \to 1.5217391$	▲▲▲$(3704173, 5636785] \to 1.5217391$
4)	▼▼▼$(512435, 849178] \to 1.6571429$	▼▼▼$(5636785, 9340958] \to 1.6571429$
5)	▲▲▲$(849178, 1361613] \to 1.6034483$	▲▲▲$(9340958, 14977743] \to 1.6034483$
6)	▼▼▼$(1361613, 2210791] \to 1.6236559$	▼▼▼$(14977743, 24318701] \to 1.6236559$
7)	▲▲▲$(2210791, 3572404] \to 1.6158940$	▲▲▲$(24318701, 39296444] \to 1.6158940$
8)	▼▼▼$(3572404, 5783195] \to 1.6188525$	▼▼▼$(39296444, 63615145] \to 1.6188525$
9)	▲▲▲$(5783195, 9355599] \to 1.6177215$	▲▲▲$(63615145, 102911589] \to 1.6177215$
10)	▼▼▼$(9355599, 15138794] \to 1.6181534$	▼▼▼$(102911589, 166526734] \to 1.6181534$
11)	▼▲▼$(15138794, 24494393] \to 1.6179884$	▲▲▲$(166526734, 269438323] \to 1.6179884$
12)	▼▼▼$(24494393, 39633187] \to 1.6180514$	▲▼▲$(269438323, 435965057] \to 1.6180514$

APPENDIX L: USING 12^p AND 12^{p+1} AS INITIATORS

0)	$(144, 1728] \rightarrow 12.0000000$	$(1728, 20736] \rightarrow 12.0000000$
1)	▲▲▲$(1728, 1872] \rightarrow 1.0833333$	▲▲▲$(20736, 22464] \rightarrow 1.0833333$
2)	▼▼▼$(1872, 3600] \rightarrow 1.9230769$	▼▼▼$(22464, 43200] \rightarrow 1.9230769$
3)	▲▲▲$(3600, 5472] \rightarrow 1.5200000$	▲▲▲$(43200, 65664] \rightarrow 1.5200000$
4)	▼▼▼$(5472, 9072] \rightarrow 1.6578947$	▼▼▼$(65664, 108864] \rightarrow 1.6578947$
5)	▲▲▲$(9072, 14544] \rightarrow 1.6031746$	▲▲▲$(108864, 174528] \rightarrow 1.6031746$
6)	▼▼▼$(14544, 23616] \rightarrow 1.6237624$	▼▼▼$(174528, 283392] \rightarrow 1.6237624$
7)	▲▲▲$(23616, 38160] \rightarrow 1.6158537$	▲▲▲$(283392, 457920] \rightarrow 1.6158537$
8)	▼▼▼$(38160, 61776] \rightarrow 1.6188679$	▼▼▼$(457920, 741312] \rightarrow 1.6188679$
9)	▲▲▲$(61776, 99936] \rightarrow 1.6177156$	▲▲▲$(741312, 1199232] \rightarrow 1.6177156$
10)	▼▼▼$(99936, 161712] \rightarrow 1.6181556$	▲▼▲$(1199232, 1940544] \rightarrow 1.6181556$
11)	▼▲▼$(161712, 261648] \rightarrow 1.6179875$	▲▲▲$(1940544, 3139776] \rightarrow 1.6179875$
12)	▼▲▼$(261648, 423360] \rightarrow 1.6180517$	▼▼▼$(3139776, 5080320] \rightarrow 1.6180517$
13)	▼▲▼$(423360, 685008] \rightarrow 1.6180272$	▲▼▲$(5080320, 8220096] \rightarrow 1.6180272$
14)	▼▲▼$(685008, 1108368] \rightarrow 1.6180366$	▼▼▼$(8220096, 13300416] \rightarrow 1.6180366$
15)	▲▼▲$(1108368, 1793376] \rightarrow 1.6180330$	▼▲▼$(13300416, 21520512] \rightarrow 1.6180330$
16)	▼▲▼$(1793376, 2901744] \rightarrow 1.6180344$	▲▼▲$(21520512, 34820928] \rightarrow 1.6180344$
17)	▲▼▲$(2901744, 4695120] \rightarrow 1.6180338$	▼▲▼$(34820928, 56341440] \rightarrow 1.6180338$
18)	▼▲▼$(4695120, 7596864] \rightarrow 1.6180340$	▲▼▲$(56341440, 91162368] \rightarrow 1.6180340$
19)	▲▼▲$(7596864, 12291984] \rightarrow 1.6180340$	▼▲▼$(91162368, 147503808] \rightarrow 1.6180340$
20)	▲▼▲$(12291984, 19888848] \rightarrow 1.6180340$	▲▼▲$(147503808, 238666176] \rightarrow 1.6180340$
21)	▲▼▲$(19888848, 32180832] \rightarrow 1.6180340$	▼▼▼$(238666176, 386169984] \rightarrow 1.6180340$

0)	$(20736, 248832] \to 12.0000000$	$(248832, 2985984] \to 12.0000000$
1)	▲▲▲$(248832, 269568] \to 1.0833333$	▲▲▲$(2985984, 3234816] \to 1.0833333$
2)	▼▼▼$(269568, 518400] \to 1.9230769$	▼▼▼$(3234816, 6220800] \to 1.9230769$
3)	▲▲▲$(518400, 787968] \to 1.5200000$	▲▲▲$(6220800, 9455616] \to 1.5200000$
4)	▼▼▼$(787968, 1306368] \to 1.6578947$	▼▼▼$(9455616, 15676416] \to 1.6578947$
5)	▲▲▲$(1306368, 2094336] \to 1.6031746$	▲▲▲$(15676416, 25132032] \to 1.6031746$
6)	▼▼▼$(2094336, 3400704] \to 1.6237624$	▼▼▼$(25132032, 40808448] \to 1.6237624$
7)	▲▲▲$(3400704, 5495040] \to 1.6158537$	▲▲▲$(40808448, 65940480] \to 1.6158537$
8)	▼▼▼$(5495040, 8895744] \to 1.6188679$	▼▼▼$(65940480, 106748928] \to 1.6188679$
9)	▲▲▲$(8895744, 14390784] \to 1.6177156$	▲▲▲$(106748928, 172689408] \to 1.6177156$
10)	▼▼▼$(14390784, 23286528] \to 1.6181556$	▼▼▼$(172689408, 279438336] \to 1.6181556$
11)	▲▲▲$(23286528, 37677312] \to 1.6179875$	▲▲▲$(279438336, 452127744] \to 1.6179875$

APPENDIX M: USING 3^p AND $\lfloor 3^p\phi \rfloor$ AS INITIATORS

0)	$(9, 14] \rightarrow 1.5555556$	$(27, 43] \rightarrow 1.5925926$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(14, 23] \rightarrow 1.6428571$	$\blacktriangle\blacktriangledown\blacktriangle(43, 70] \rightarrow 1.6279070$
2)	$\blacktriangle\blacktriangle\blacktriangle(23, 37] \rightarrow 1.6086957$	$\blacktriangledown\blacktriangle\blacktriangledown(70, 113] \rightarrow 1.6142857$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(37, 60] \rightarrow 1.6216216$	$\blacktriangle\blacktriangledown\blacktriangle(113, 183] \rightarrow 1.6194690$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(60, 97] \rightarrow 1.6166667$	$\blacktriangledown\blacktriangle\blacktriangledown(183, 296] \rightarrow 1.6174863$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(97, 157] \rightarrow 1.6185567$	$\blacktriangledown\blacktriangledown\blacktriangledown(296, 479] \rightarrow 1.6182432$
6)	$\blacktriangle\blacktriangledown\blacktriangle(157, 254] \rightarrow 1.6178344$	$\blacktriangledown\blacktriangledown\blacktriangledown(479, 775] \rightarrow 1.6179541$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(254, 411] \rightarrow 1.6181102$	$\blacktriangle\blacktriangledown\blacktriangle(775, 1254] \rightarrow 1.6180645$
8)	$\blacktriangledown\blacktriangledown\blacktriangledown(411, 665] \rightarrow 1.6180049$	$\blacktriangledown\blacktriangle\blacktriangledown(1254, 2029] \rightarrow 1.6180223$
9)	$\blacktriangle\blacktriangledown\blacktriangle(665, 1076] \rightarrow 1.6180451$	$\blacktriangle\blacktriangledown\blacktriangle(2029, 3283] \rightarrow 1.6180384$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(1076, 1741] \rightarrow 1.6180297$	$\blacktriangledown\blacktriangle\blacktriangledown(3283, 5312] \rightarrow 1.6180323$
11)	$\blacktriangle\blacktriangledown\blacktriangle(1741, 2817] \rightarrow 1.6180356$	$\blacktriangle\blacktriangledown\blacktriangle(5312, 8595] \rightarrow 1.6180346$
12)	$\blacktriangle\blacktriangledown\blacktriangle(2817, 4558] \rightarrow 1.6180334$	$\blacktriangledown\blacktriangle\blacktriangledown(8595, 13907] \rightarrow 1.6180337$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(4558, 7375] \rightarrow 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(13907, 22502] \rightarrow 1.6180341$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(7375, 11933] \rightarrow 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(22502, 36409] \rightarrow 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(11933, 19308] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(36409, 58911] \rightarrow 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(19308, 31241] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(58911, 95320] \rightarrow 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(31241, 50549] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(95320, 154231] \rightarrow 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(50549, 81790] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(154231, 249551] \rightarrow 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(81790, 132339] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(249551, 403782] \rightarrow 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(132339, 214129] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(403782, 653333] \rightarrow 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(214129, 346468] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(653333, 1057115] \rightarrow 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(346468, 560597] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1057115, 1710448] \rightarrow 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(560597, 907065] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1710448, 2767563] \rightarrow 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(907065, 1467662] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2767563, 4478011] \rightarrow 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(1467662, 2374727] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4478011, 7245574] \rightarrow 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(2374727, 3842389] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7245574, 11723585] \rightarrow 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(3842389, 6217116] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11723585, 18969159] \rightarrow 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(6217116, 10059505] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(18969159, 30692744] \rightarrow 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(10059505, 16276621] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(30692744, 49661903] \rightarrow 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(16276621, 26336126] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(49661903, 80354647] \rightarrow 1.6180340$
31)	$\blacktriangledown\blacktriangle\blacktriangledown(26336126, 42612747] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(80354647, 130016550] \rightarrow 1.6180340$
32)	$\blacktriangle\blacktriangledown\blacktriangle(42612747, 68948873] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(130016550, 210371197] \rightarrow 1.6180340$
33)	$\blacktriangledown\blacktriangle\blacktriangledown(68948873, 111561620] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(210371197, 340387747] \rightarrow 1.6180340$

0)	$(81, 131] \to 1.6172840$	$(243, 393] \to 1.6172840$
1)	$\blacktriangle\blacktriangledown\blacktriangle(131, 212] \to 1.6183206$	$\blacktriangledown\blacktriangle\blacktriangledown(393, 636] \to 1.6183206$
2)	$\blacktriangle\blacktriangledown\blacktriangle(212, 343] \to 1.6179245$	$\blacktriangledown\blacktriangle\blacktriangledown(636, 1029] \to 1.6179245$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(343, 555] \to 1.6180758$	$\blacktriangledown\blacktriangle\blacktriangledown(1029, 1665] \to 1.6180758$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(555, 898] \to 1.6180180$	$\blacktriangle\blacktriangledown\blacktriangle(1665, 2694] \to 1.6180180$
5)	$\blacktriangle\blacktriangledown\blacktriangle(898, 1453] \to 1.6180401$	$\blacktriangledown\blacktriangle\blacktriangledown(2694, 4359] \to 1.6180401$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1453, 2351] \to 1.6180317$	$\blacktriangle\blacktriangledown\blacktriangle(4359, 7053] \to 1.6180317$
7)	$\blacktriangle\blacktriangledown\blacktriangle(2351, 3804] \to 1.6180349$	$\blacktriangle\blacktriangledown\blacktriangle(7053, 11412] \to 1.6180349$
8)	$\blacktriangle\blacktriangledown\blacktriangle(3804, 6155] \to 1.6180336$	$\blacktriangledown\blacktriangle\blacktriangledown(11412, 18465] \to 1.6180336$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(6155, 9959] \to 1.6180341$	$\blacktriangle\blacktriangledown\blacktriangle(18465, 29877] \to 1.6180341$
10)	$\blacktriangle\blacktriangledown\blacktriangle(9959, 16114] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(29877, 48342] \to 1.6180339$
11)	$\blacktriangle\blacktriangledown\blacktriangle(16114, 26073] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(48342, 78219] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(26073, 42187] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(78219, 126561] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(42187, 68260] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(126561, 204780] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(68260, 110447] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(204780, 331341] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(110447, 178707] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(331341, 536121] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(178707, 289154] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(536121, 867462] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(289154, 467861] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(867462, 1403583] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(467861, 757015] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1403583, 2271045] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(757015, 1224876] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2271045, 3674628] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1224876, 1981891] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3674628, 5945673] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(1981891, 3206767] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(5945673, 9620301] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(3206767, 5188658] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(9620301, 15565974] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(5188658, 8395425] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(15565974, 25186275] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(8395425, 13584083] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(25186275, 40752249] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(13584083, 21979508] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(40752249, 65938524] \to 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(21979508, 35563591] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(65938524, 106690773] \to 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(35563591, 57543099] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(106690773, 172629297] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(57543099, 93106690] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(172629297, 279320070] \to 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(93106690, 150649789] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(279320070, 451949367] \to 1.6180340$

APPENDIX N: USING 4^p AND $\lfloor 4^p\phi \rfloor$ AS INITIATORS

0)	$(16, 25] \to 1.5625000$	$(64, 103] \to 1.6093750$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(25, 41] \to 1.6400000$	$\blacktriangle\blacktriangledown\blacktriangle(103, 167] \to 1.6213592$
2)	$\blacktriangle\blacktriangle\blacktriangle(41, 66] \to 1.6097561$	$\blacktriangle\blacktriangledown\blacktriangle(167, 270] \to 1.6167665$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(66, 107] \to 1.6212121$	$\blacktriangledown\blacktriangle\blacktriangledown(270, 437] \to 1.6185185$
4)	$\blacktriangle\blacktriangledown\blacktriangle(107, 173] \to 1.6168224$	$\blacktriangle\blacktriangledown\blacktriangle(437, 707] \to 1.6178490$
5)	$\blacktriangle\blacktriangledown\blacktriangle(173, 280] \to 1.6184971$	$\blacktriangle\blacktriangledown\blacktriangle(707, 1144] \to 1.6181047$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(280, 453] \to 1.6178571$	$\blacktriangle\blacktriangledown\blacktriangle(1144, 1851] \to 1.6180070$
7)	$\blacktriangle\blacktriangledown\blacktriangle(453, 733] \to 1.6181015$	$\blacktriangledown\blacktriangle\blacktriangledown(1851, 2995] \to 1.6180443$
8)	$\blacktriangle\blacktriangledown\blacktriangle(733, 1186] \to 1.6180082$	$\blacktriangle\blacktriangledown\blacktriangle(2995, 4846] \to 1.6180301$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(1186, 1919] \to 1.6180438$	$\blacktriangledown\blacktriangle\blacktriangledown(4846, 7841] \to 1.6180355$
10)	$\blacktriangle\blacktriangledown\blacktriangle(1919, 3105] \to 1.6180302$	$\blacktriangledown\blacktriangle\blacktriangledown(7841, 12687] \to 1.6180334$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(3105, 5024] \to 1.6180354$	$\blacktriangle\blacktriangledown\blacktriangle(12687, 20528] \to 1.6180342$
12)	$\blacktriangle\blacktriangledown\blacktriangle(5024, 8129] \to 1.6180334$	$\blacktriangledown\blacktriangle\blacktriangledown(20528, 33215] \to 1.6180339$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(8129, 13153] \to 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(33215, 53743] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(13153, 21282] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(53743, 86958] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(21282, 34435] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(86958, 140701] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(34435, 55717] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(140701, 227659] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(55717, 90152] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(227659, 368360] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(90152, 145869] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(368360, 596019] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(145869, 236021] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(596019, 964379] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(236021, 381890] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(964379, 1560398] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(381890, 617911] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1560398, 2524777] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(617911, 999801] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2524777, 4085175] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(999801, 1617712] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4085175, 6609952] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(1617712, 2617513] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6609952, 10695127] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(2617513, 4235225] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(10695127, 17305079] \to 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(4235225, 6852738] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(17305079, 28000206] \to 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(6852738, 11087963] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(28000206, 45305285] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(11087963, 17940701] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(45305285, 73305491] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(17940701, 29028664] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(73305491, 118610776] \to 1.6180340$
30)	$\blacktriangledown\blacktriangle\blacktriangledown(29028664, 46969365] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(118610776, 191916267] \to 1.6180340$
31)	$\blacktriangle\blacktriangledown\blacktriangle(46969365, 75998029] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(191916267, 310527043] \to 1.6180340$

0)	$(256, 414] \to 1.6171875$	$(1024, 1656] \to 1.6171875$
1)	$(414, 670] \to 1.6183575$	$(1656, 2680] \to 1.6183575$
2)	$(670, 1084] \to 1.6179104$	$(2680, 4336] \to 1.6179104$
3)	$(1084, 1754] \to 1.6180812$	$(4336, 7016] \to 1.6180812$
4)	$(1754, 2838] \to 1.6180160$	$(7016, 11352] \to 1.6180160$
5)	$(2838, 4592] \to 1.6180409$	$(11352, 18368] \to 1.6180409$
6)	$(4592, 7430] \to 1.6180314$	$(18368, 29720] \to 1.6180314$
7)	$(7430, 12022] \to 1.6180350$	$(29720, 48088] \to 1.6180350$
8)	$(12022, 19452] \to 1.6180336$	$(48088, 77808] \to 1.6180336$
9)	$(19452, 31474] \to 1.6180341$	$(77808, 125896] \to 1.6180341$
10)	$(31474, 50926] \to 1.6180339$	$(125896, 203704] \to 1.6180339$
11)	$(50926, 82400] \to 1.6180340$	$(203704, 329600] \to 1.6180340$
12)	$(82400, 133326] \to 1.6180340$	$(329600, 533304] \to 1.6180340$
13)	$(133326, 215726] \to 1.6180340$	$(533304, 862904] \to 1.6180340$
14)	$(215726, 349052] \to 1.6180340$	$(862904, 1396208] \to 1.6180340$
15)	$(349052, 564778] \to 1.6180340$	$(1396208, 2259112] \to 1.6180340$
16)	$(564778, 913830] \to 1.6180340$	$(2259112, 3655320] \to 1.6180340$
17)	$(913830, 1478608] \to 1.6180340$	$(3655320, 5914432] \to 1.6180340$
18)	$(1478608, 2392438] \to 1.6180340$	$(5914432, 9569752] \to 1.6180340$
19)	$(2392438, 3871046] \to 1.6180340$	$(9569752, 15484184] \to 1.6180340$
20)	$(3871046, 6263484] \to 1.6180340$	$(15484184, 25053936] \to 1.6180340$
21)	$(6263484, 10134530] \to 1.6180340$	$(25053936, 40538120] \to 1.6180340$
22)	$(10134530, 16398014] \to 1.6180340$	$(40538120, 65592056] \to 1.6180340$
23)	$(16398014, 26532544] \to 1.6180340$	$(65592056, 106130176] \to 1.6180340$
24)	$(26532544, 42930558] \to 1.6180340$	$(106130176, 171722232] \to 1.6180340$
25)	$(42930558, 69463102] \to 1.6180340$	$(171722232, 277852408] \to 1.6180340$
26)	$(69463102, 112393660] \to 1.6180340$	$(277852408, 449574640] \to 1.6180340$

APPENDIX O: USING 5^p AND $\lfloor 5^p\phi \rfloor$ AS INITIATORS

0)	$(25, 40] \rightarrow 1.6000000$	$(125, 202] \rightarrow 1.6160000$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(40, 65] \rightarrow 1.6250000$	$\blacktriangle\blacktriangledown\blacktriangle(202, 327] \rightarrow 1.6188119$
2)	$\blacktriangle\blacktriangledown\blacktriangle(65, 105] \rightarrow 1.6153846$	$\blacktriangledown\blacktriangle\blacktriangledown(327, 529] \rightarrow 1.6177370$
3)	$\blacktriangle\blacktriangledown\blacktriangle(105, 170] \rightarrow 1.6190476$	$\blacktriangle\blacktriangledown\blacktriangle(529, 856] \rightarrow 1.6181474$
4)	$\blacktriangle\blacktriangledown\blacktriangle(170, 275] \rightarrow 1.6176471$	$\blacktriangledown\blacktriangle\blacktriangledown(856, 1385] \rightarrow 1.6179907$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(275, 445] \rightarrow 1.6181818$	$\blacktriangledown\blacktriangledown\blacktriangledown(1385, 2241] \rightarrow 1.6180505$
6)	$\blacktriangle\blacktriangledown\blacktriangle(445, 720] \rightarrow 1.6179775$	$\blacktriangledown\blacktriangle\blacktriangledown(2241, 3626] \rightarrow 1.6180277$
7)	$\blacktriangle\blacktriangledown\blacktriangle(720, 1165] \rightarrow 1.6180556$	$\blacktriangle\blacktriangledown\blacktriangle(3626, 5867] \rightarrow 1.6180364$
8)	$\blacktriangledown\blacktriangledown\blacktriangledown(1165, 1885] \rightarrow 1.6180258$	$\blacktriangle\blacktriangledown\blacktriangle(5867, 9493] \rightarrow 1.6180331$
9)	$\blacktriangle\blacktriangledown\blacktriangle(1885, 3050] \rightarrow 1.6180371$	$\blacktriangle\blacktriangledown\blacktriangle(9493, 15360] \rightarrow 1.6180343$
10)	$\blacktriangle\blacktriangledown\blacktriangle(3050, 4935] \rightarrow 1.6180328$	$\blacktriangle\blacktriangledown\blacktriangle(15360, 24853] \rightarrow 1.6180339$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(4935, 7985] \rightarrow 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(24853, 40213] \rightarrow 1.6180340$
12)	$\blacktriangledown\blacktriangledown\blacktriangledown(7985, 12920] \rightarrow 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(40213, 65066] \rightarrow 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(12920, 20905] \rightarrow 1.6180341$	$\blacktriangledown\blacktriangle\blacktriangledown(65066, 105279] \rightarrow 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(20905, 33825] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(105279, 170345] \rightarrow 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(33825, 54730] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(170345, 275624] \rightarrow 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(54730, 88555] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(275624, 445969] \rightarrow 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(88555, 143285] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(445969, 721593] \rightarrow 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(143285, 231840] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(721593, 1167562] \rightarrow 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(231840, 375125] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1167562, 1889155] \rightarrow 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(375125, 606965] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1889155, 3056717] \rightarrow 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(606965, 982090] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3056717, 4945872] \rightarrow 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(982090, 1589055] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4945872, 8002589] \rightarrow 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(1589055, 2571145] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(8002589, 12948461] \rightarrow 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(2571145, 4160200] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12948461, 20951050] \rightarrow 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(4160200, 6731345] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(20951050, 33899511] \rightarrow 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(6731345, 10891545] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(33899511, 54850561] \rightarrow 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(10891545, 17622890] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(54850561, 88750072] \rightarrow 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(17622890, 28514435] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(88750072, 143600633] \rightarrow 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(28514435, 46137325] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(143600633, 232350705] \rightarrow 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(46137325, 74651760] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(232350705, 375951338] \rightarrow 1.6180340$

0)	$(625, 1011] \to 1.6176000$	$(3125, 5056] \to 1.6179200$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(1011, 1636] \to 1.6181998$	$\blacktriangle\blacktriangledown\blacktriangle(5056, 8181] \to 1.6180775$
2)	$\blacktriangle\blacktriangledown\blacktriangle(1636, 2647] \to 1.6179707$	$\blacktriangledown\blacktriangle\blacktriangledown(8181, 13237] \to 1.6180174$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(2647, 4283] \to 1.6180582$	$\blacktriangle\blacktriangledown\blacktriangle(13237, 21418] \to 1.6180403$
4)	$\blacktriangle\blacktriangledown\blacktriangle(4283, 6930] \to 1.6180247$	$\blacktriangledown\blacktriangle\blacktriangledown(21418, 34655] \to 1.6180316$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(6930, 11213] \to 1.6180375$	$\blacktriangle\blacktriangledown\blacktriangle(34655, 56073] \to 1.6180349$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(11213, 18143] \to 1.6180326$	$\blacktriangledown\blacktriangle\blacktriangledown(56073, 90728] \to 1.6180336$
7)	$\blacktriangle\blacktriangledown\blacktriangle(18143, 29356] \to 1.6180345$	$\blacktriangle\blacktriangledown\blacktriangle(90728, 146801] \to 1.6180341$
8)	$\blacktriangle\blacktriangledown\blacktriangle(29356, 47499] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(146801, 237529] \to 1.6180339$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(47499, 76855] \to 1.6180341$	$\blacktriangledown\blacktriangle\blacktriangledown(237529, 384330] \to 1.6180340$
10)	$\blacktriangle\blacktriangledown\blacktriangle(76855, 124354] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(384330, 621859] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(124354, 201209] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(621859, 1006189] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(201209, 325563] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1006189, 1628048] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(325563, 526772] \to 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(1628048, 2634237] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(526772, 852335] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2634237, 4262285] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(852335, 1379107] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4262285, 6896522] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(1379107, 2231442] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(6896522, 11158807] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(2231442, 3610549] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(11158807, 18055329] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(3610549, 5841991] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(18055329, 29214136] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(5841991, 9452540] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(29214136, 47269465] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(9452540, 15294531] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(47269465, 76483601] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(15294531, 24747071] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(76483601, 123753066] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(24747071, 40041602] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(123753066, 200236667] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(40041602, 64788673] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(200236667, 323989733] \to 1.6180340$

APPENDIX P: USING 6^p AND $\lfloor 6^p\phi \rfloor$ AS INITIATORS

0)	$(36, 58] \to$ 1.6111111	$(216, 349] \to$ 1.6157407
1)	▼▼▼$(58, 94] \to$ 1.6206897	▲▼▲$(349, 565] \to$ 1.6189112
2)	▽▲▽$(94, 152] \to$ 1.6170213	▼▲▽$(565, 914] \to$ 1.6176991
3)	△▼△$(152, 246] \to$ 1.6184211	▲▼△$(914, 1479] \to$ 1.6181619
4)	▽▲▽$(246, 398] \to$ 1.6178862	▼▲▽$(1479, 2393] \to$ 1.6179851
5)	▽△▽$(398, 644] \to$ 1.6180905	▲▼△$(2393, 3872] \to$ 1.6180527
6)	△▽△$(644, 1042] \to$ 1.6180124	▼▲▽$(3872, 6265] \to$ 1.6180269
7)	△▽△$(1042, 1686] \to$ 1.6180422	▲▼△$(6265, 10137] \to$ 1.6180367
8)	▽△▽$(1686, 2728] \to$ 1.6180308	△▽△$(10137, 16402] \to$ 1.6180329
9)	△▽△$(2728, 4414] \to$ 1.6180352	△▽△$(16402, 26539] \to$ 1.6180344
10)	▽△▽$(4414, 7142] \to$ 1.6180335	▽△▽$(26539, 42941] \to$ 1.6180338
11)	△▽△$(7142, 11556] \to$ 1.6180342	△▽△$(42941, 69480] \to$ 1.6180340
12)	▽△▽$(11556, 18698] \to$ 1.6180339	▽△▽$(69480, 112421] \to$ 1.6180340
13)	△▽△$(18698, 30254] \to$ 1.6180340	△▽△$(112421, 181901] \to$ 1.6180340
14)	▽△▽$(30254, 48952] \to$ 1.6180340	▽△▽$(181901, 294322] \to$ 1.6180340
15)	▽△▽$(48952, 79206] \to$ 1.6180340	▽△▽$(294322, 476223] \to$ 1.6180340
16)	△▽△$(79206, 128158] \to$ 1.6180340	△▽△$(476223, 770545] \to$ 1.6180340
17)	▽▽▼$(128158, 207364] \to$ 1.6180340	▽△▽$(770545, 1246768] \to$ 1.6180340
18)	▽△▽$(207364, 335522] \to$ 1.6180340	▽△▽$(1246768, 2017313] \to$ 1.6180340
19)	△▽△$(335522, 542886] \to$ 1.6180340	△▽△$(2017313, 3264081] \to$ 1.6180340
20)	▽△▽$(542886, 878408] \to$ 1.6180340	△▽△$(3264081, 5281394] \to$ 1.6180340
21)	△▽△$(878408, 1421294] \to$ 1.6180340	△▽△$(5281394, 8545475] \to$ 1.6180340
22)	▽△▽$(1421294, 2299702] \to$ 1.6180340	▽△▽$(8545475, 13826869] \to$ 1.6180340
23)	△▽△$(2299702, 3720996] \to$ 1.6180340	△▽△$(13826869, 22372344] \to$ 1.6180340
24)	▽△▽$(3720996, 6020698] \to$ 1.6180340	▽△▽$(22372344, 36199213] \to$ 1.6180340
25)	▽△▽$(6020698, 9741694] \to$ 1.6180340	▽△▽$(36199213, 58571557] \to$ 1.6180340
26)	△▽△$(9741694, 15762392] \to$ 1.6180340	△▽△$(58571557, 94770770] \to$ 1.6180340
27)	▽△▽$(15762392, 25504086] \to$ 1.6180340	▽△▽$(94770770, 153342327] \to$ 1.6180340
28)	▽△▽$(25504086, 41266478] \to$ 1.6180340	▽△▽$(153342327, 248113097] \to$ 1.6180340
29)	△▽△$(41266478, 66770564] \to$ 1.6180340	△▽△$(248113097, 401455424] \to$ 1.6180340

0)	$(1296, 2096] \to 1.6172840$	$(7776, 12581] \to 1.6179270$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(2096, 3392] \to 1.6183206$	$\blacktriangledown\blacktriangle\blacktriangledown(12581, 20357] \to 1.6180749$
2)	$\blacktriangle\blacktriangle\blacktriangle(3392, 5488] \to 1.6179245$	$\blacktriangle\blacktriangledown\blacktriangle(20357, 32938] \to 1.6180184$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(5488, 8880] \to 1.6180758$	$\blacktriangledown\blacktriangle\blacktriangledown(32938, 53295] \to 1.6180400$
4)	$\blacktriangle\blacktriangledown\blacktriangle(8880, 14368] \to 1.6180180$	$\blacktriangle\blacktriangledown\blacktriangle(53295, 86233] \to 1.6180317$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(14368, 23248] \to 1.6180401$	$\blacktriangledown\blacktriangle\blacktriangledown(86233, 139528] \to 1.6180349$
6)	$\blacktriangle\blacktriangledown\blacktriangle(23248, 37616] \to 1.6180317$	$\blacktriangle\blacktriangledown\blacktriangle(139528, 225761] \to 1.6180337$
7)	$\blacktriangle\blacktriangledown\blacktriangle(37616, 60864] \to 1.6180349$	$\blacktriangle\blacktriangledown\blacktriangle(225761, 365289] \to 1.6180341$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(60864, 98480] \to 1.6180336$	$\blacktriangle\blacktriangledown\blacktriangle(365289, 591050] \to 1.6180339$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(98480, 159344] \to 1.6180341$	$\blacktriangledown\blacktriangle\blacktriangledown(591050, 956339] \to 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(159344, 257824] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(956339, 1547389] \to 1.6180340$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(257824, 417168] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1547389, 2503728] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(417168, 674992] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2503728, 4051117] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(674992, 1092160] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4051117, 6554845] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(1092160, 1767152] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6554845, 10605962] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(1767152, 2859312] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(10605962, 17160807] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(2859312, 4626464] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(17160807, 27766769] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(4626464, 7485776] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(27766769, 44927576] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(7485776, 12112240] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(44927576, 72694345] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(12112240, 19598016] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(72694345, 117621921] \to 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(19598016, 31710256] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(117621921, 190316266] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(31710256, 51308272] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(190316266, 307938187] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(51308272, 83018528] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(307938187, 498254453] \to 1.6180340$

APPENDIX Q: USING 7^p AND $\lfloor 7^p\phi \rfloor$ AS INITIATORS

0)	$(49, 79] \rightarrow 1.6122449$	$(343, 554] \rightarrow 1.6151603$
1)	▲▼▲$(79, 128] \rightarrow 1.6202532$	▼▲▼$(554, 897] \rightarrow 1.6191336$
2)	▼▲▼$(128, 207] \rightarrow 1.6171875$	▲▼▲$(897, 1451] \rightarrow 1.6176143$
3)	▲▼▲$(207, 335] \rightarrow 1.6183575$	▼▲▼$(1451, 2348] \rightarrow 1.6181943$
4)	▼▲▼$(335, 542] \rightarrow 1.6179104$	▲▼▲$(2348, 3799] \rightarrow 1.6179727$
5)	▼▲▼$(542, 877] \rightarrow 1.6180812$	▲▼▲$(3799, 6147] \rightarrow 1.6180574$
6)	▲▼▲$(877, 1419] \rightarrow 1.6180160$	▼▲▼$(6147, 9946] \rightarrow 1.6180251$
7)	▼▲▼$(1419, 2296] \rightarrow 1.6180409$	▲▼▲$(9946, 16093] \rightarrow 1.6180374$
8)	▲▼▲$(2296, 3715] \rightarrow 1.6180314$	▲▼▲$(16093, 26039] \rightarrow 1.6180327$
9)	▲▼▲$(3715, 6011] \rightarrow 1.6180350$	▼▲▼$(26039, 42132] \rightarrow 1.6180345$
10)	▼▲▼$(6011, 9726] \rightarrow 1.6180336$	▲▼▲$(42132, 68171] \rightarrow 1.6180338$
11)	▲▼▲$(9726, 15737] \rightarrow 1.6180341$	▲▼▲$(68171, 110303] \rightarrow 1.6180341$
12)	▲▼▲$(15737, 25463] \rightarrow 1.6180339$	▼▲▼$(110303, 178474] \rightarrow 1.6180340$
13)	▼▲▼$(25463, 41200] \rightarrow 1.6180340$	▼▲▼$(178474, 288777] \rightarrow 1.6180340$
14)	▼▲▼$(41200, 66663] \rightarrow 1.6180340$	▲▼▲$(288777, 467251] \rightarrow 1.6180340$
15)	▲▼▲$(66663, 107863] \rightarrow 1.6180340$	▼▲▼$(467251, 756028] \rightarrow 1.6180340$
16)	▼▲▼$(107863, 174526] \rightarrow 1.6180340$	▲▼▲$(756028, 1223279] \rightarrow 1.6180340$
17)	▼▲▼$(174526, 282389] \rightarrow 1.6180340$	▲▼▲$(1223279, 1979307] \rightarrow 1.6180340$
18)	▲▼▲$(282389, 456915] \rightarrow 1.6180340$	▼▲▼$(1979307, 3202586] \rightarrow 1.6180340$
19)	▼▲▼$(456915, 739304] \rightarrow 1.6180340$	▲▼▲$(3202586, 5181893] \rightarrow 1.6180340$
20)	▼▲▼$(739304, 1196219] \rightarrow 1.6180340$	▲▼▲$(5181893, 8384479] \rightarrow 1.6180340$
21)	▲▼▲$(1196219, 1935523] \rightarrow 1.6180340$	▲▼▲$(8384479, 13566372] \rightarrow 1.6180340$
22)	▼▲▼$(1935523, 3131742] \rightarrow 1.6180340$	▼▲▼$(13566372, 21950851] \rightarrow 1.6180340$
23)	▼▲▼$(3131742, 5067265] \rightarrow 1.6180340$	▲▼▲$(21950851, 35517223] \rightarrow 1.6180340$
24)	▲▼▲$(5067265, 8199007] \rightarrow 1.6180340$	▼▲▼$(35517223, 57468074] \rightarrow 1.6180340$
25)	▼▲▼$(8199007, 13266272] \rightarrow 1.6180340$	▲▼▲$(57468074, 92985297] \rightarrow 1.6180340$
26)	▼▼▼$(13266272, 21465279] \rightarrow 1.6180340$	▼▲▼$(92985297, 150453371] \rightarrow 1.6180340$
27)	▲▼▲$(21465279, 34731551] \rightarrow 1.6180340$	▲▼▲$(150453371, 243438668] \rightarrow 1.6180340$
28)	▼▲▼$(34731551, 56196830] \rightarrow 1.6180340$	▼▲▼$(243438668, 393892039] \rightarrow 1.6180340$

0)	$(2401, 3884] \to 1.6176593$	$(16807, 27194] \to 1.6180163$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(3884, 6285] \to 1.6181771$	$\blacktriangle\blacktriangledown\blacktriangle(27194, 44001] \to 1.6180407$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(6285, 10169] \to 1.6179793$	$\blacktriangle\blacktriangledown\blacktriangle(44001, 71195] \to 1.6180314$
3)	$\blacktriangle\blacktriangledown\blacktriangle(10169, 16454] \to 1.6180549$	$\blacktriangledown\blacktriangle\blacktriangledown(71195, 115196] \to 1.6180350$
4)	$\blacktriangle\blacktriangledown\blacktriangle(16454, 26623] \to 1.6180260$	$\blacktriangledown\blacktriangle\blacktriangledown(115196, 186391] \to 1.6180336$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(26623, 43077] \to 1.6180370$	$\blacktriangle\blacktriangledown\blacktriangle(186391, 301587] \to 1.6180341$
6)	$\blacktriangle\blacktriangledown\blacktriangle(43077, 69700] \to 1.6180328$	$\blacktriangledown\blacktriangle\blacktriangledown(301587, 487978] \to 1.6180339$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(69700, 112777] \to 1.6180344$	$\blacktriangle\blacktriangledown\blacktriangle(487978, 789565] \to 1.6180340$
8)	$\blacktriangle\blacktriangledown\blacktriangle(112777, 182477] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(789565, 1277543] \to 1.6180340$
9)	$\blacktriangle\blacktriangledown\blacktriangle(182477, 295254] \to 1.6180341$	$\blacktriangle\blacktriangledown\blacktriangle(1277543, 2067108] \to 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(295254, 477731] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2067108, 3344651] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(477731, 772985] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3344651, 5411759] \to 1.6180340$
12)	$\blacktriangle\blacktriangledown\blacktriangle(772985, 1250716] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(5411759, 8756410] \to 1.6180340$
13)	$\blacktriangledown\blacktriangledown\blacktriangledown(1250716, 2023701] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(8756410, 14168169] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(2023701, 3274417] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(14168169, 22924579] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(3274417, 5298118] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(22924579, 37092748] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(5298118, 8572535] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(37092748, 60017327] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(8572535, 13870653] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(60017327, 97110075] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(13870653, 22443188] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(97110075, 157127402] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(22443188, 36313841] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(157127402, 254237477] \to 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(36313841, 58757029] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(254237477, 411364879] \to 1.6180340$

APPENDIX R: USING 8^p AND $\lfloor 8^p\phi \rfloor$ AS INITIATORS

0)	$(64, 103] \to 1.6093750$	$(512, 828] \to 1.6171875$
1)	$\blacktriangle\blacktriangledown\blacktriangle(103, 167] \to 1.6213592$	$\blacktriangledown\blacktriangle\blacktriangledown(828, 1340] \to 1.6183575$
2)	$\blacktriangle\blacktriangledown\blacktriangle(167, 270] \to 1.6167665$	$\blacktriangle\blacktriangledown\blacktriangle(1340, 2168] \to 1.6179104$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(270, 437] \to 1.6185185$	$\blacktriangle\blacktriangledown\blacktriangle(2168, 3508] \to 1.6180812$
4)	$\blacktriangle\blacktriangledown\blacktriangle(437, 707] \to 1.6178490$	$\blacktriangledown\blacktriangle\blacktriangledown(3508, 5676] \to 1.6180160$
5)	$\blacktriangle\blacktriangledown\blacktriangle(707, 1144] \to 1.6181047$	$\blacktriangle\blacktriangledown\blacktriangle(5676, 9184] \to 1.6180409$
6)	$\blacktriangle\blacktriangledown\blacktriangle(1144, 1851] \to 1.6180070$	$\blacktriangledown\blacktriangle\blacktriangledown(9184, 14860] \to 1.6180314$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(1851, 2995] \to 1.6180443$	$\blacktriangle\blacktriangledown\blacktriangle(14860, 24044] \to 1.6180350$
8)	$\blacktriangle\blacktriangledown\blacktriangle(2995, 4846] \to 1.6180301$	$\blacktriangle\blacktriangledown\blacktriangle(24044, 38904] \to 1.6180336$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(4846, 7841] \to 1.6180355$	$\blacktriangledown\blacktriangle\blacktriangledown(38904, 62948] \to 1.6180341$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(7841, 12687] \to 1.6180334$	$\blacktriangledown\blacktriangle\blacktriangledown(62948, 101852] \to 1.6180339$
11)	$\blacktriangle\blacktriangledown\blacktriangle(12687, 20528] \to 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(101852, 164800] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(20528, 33215] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(164800, 266652] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(33215, 53743] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(266652, 431452] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(53743, 86958] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(431452, 698104] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(86958, 140701] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(698104, 1129556] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(140701, 227659] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1129556, 1827660] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(227659, 368360] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1827660, 2957216] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(368360, 596019] \to 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(2957216, 4784876] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(596019, 964379] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4784876, 7742092] \to 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(964379, 1560398] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7742092, 12526968] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(1560398, 2524777] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12526968, 20269060] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(2524777, 4085175] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(20269060, 32796028] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(4085175, 6609952] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(32796028, 53065088] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(6609952, 10695127] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(53065088, 85861116] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(10695127, 17305079] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(85861116, 138926204] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(17305079, 28000206] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(138926204, 224787320] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(28000206, 45305285] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(224787320, 363713524] \to 1.6180340$

0)	$(4096, 6627] \rightarrow 1.6179199$	$(32768, 53019] \rightarrow 1.6180115$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(6627, 10723] \rightarrow 1.6180776$	$\blacktriangle\blacktriangledown\blacktriangle(53019, 85787] \rightarrow 1.6180426$
2)	$\blacktriangle\blacktriangledown\blacktriangle(10723, 17350] \rightarrow 1.6180173$	$\blacktriangledown\blacktriangle\blacktriangledown(85787, 138806] \rightarrow 1.6180307$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(17350, 28073] \rightarrow 1.6180403$	$\blacktriangle\blacktriangledown\blacktriangle(138806, 224593] \rightarrow 1.6180352$
4)	$\blacktriangle\blacktriangledown\blacktriangle(28073, 45423] \rightarrow 1.6180316$	$\blacktriangle\blacktriangle\blacktriangle(224593, 363399] \rightarrow 1.6180335$
5)	$\blacktriangle\blacktriangledown\blacktriangle(45423, 73496] \rightarrow 1.6180349$	$\blacktriangledown\blacktriangle\blacktriangledown(363399, 587992] \rightarrow 1.6180342$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(73496, 118919] \rightarrow 1.6180336$	$\blacktriangle\blacktriangledown\blacktriangle(587992, 951391] \rightarrow 1.6180339$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(118919, 192415] \rightarrow 1.6180341$	$\blacktriangledown\blacktriangle\blacktriangledown(951391, 1539383] \rightarrow 1.6180340$
8)	$\blacktriangle\blacktriangledown\blacktriangle(192415, 311334] \rightarrow 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(1539383, 2490774] \rightarrow 1.6180340$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(311334, 503749] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2490774, 4030157] \rightarrow 1.6180340$
10)	$\blacktriangle\blacktriangledown\blacktriangle(503749, 815083] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4030157, 6520931] \rightarrow 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(815083, 1318832] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6520931, 10551088] \rightarrow 1.6180340$
12)	$\blacktriangle\blacktriangledown\blacktriangle(1318832, 2133915] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(10551088, 17072019] \rightarrow 1.6180340$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(2133915, 3452747] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(17072019, 27623107] \rightarrow 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(3452747, 5586662] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(27623107, 44695126] \rightarrow 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(5586662, 9039409] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(44695126, 72318233] \rightarrow 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(9039409, 14626071] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(72318233, 117013359] \rightarrow 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(14626071, 23665480] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(117013359, 189331592] \rightarrow 1.6180340$
18)	$\blacktriangledown\blacktriangledown\blacktriangledown(23665480, 38291551] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(189331592, 306344951] \rightarrow 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(38291551, 61957031] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(306344951, 495676543] \rightarrow 1.6180340$

APPENDIX S: USING 9^p AND $\lfloor 9^p\phi \rfloor$ AS INITIATORS

0)	$(81, 131] \to 1.6172840$	$(729, 1179] \to 1.6172840$
1)	▲▼▲$(131, 212] \to 1.6183206$	▼▲▼$(1179, 1908] \to 1.6183206$
2)	▲▼▲$(212, 343] \to 1.6179245$	▲▼▲$(1908, 3087] \to 1.6179245$
3)	▼▲▼$(343, 555] \to 1.6180758$	▼▲▼$(3087, 4995] \to 1.6180758$
4)	▼▲▼$(555, 898] \to 1.6180180$	▼▲▼$(4995, 8082] \to 1.6180180$
5)	▲▼▲$(898, 1453] \to 1.6180401$	▼▲▼$(8082, 13077] \to 1.6180401$
6)	▼▲▼$(1453, 2351] \to 1.6180317$	▲▼▲$(13077, 21159] \to 1.6180317$
7)	▲▼▲$(2351, 3804] \to 1.6180349$	▼▲▼$(21159, 34236] \to 1.6180349$
8)	▲▼▲$(3804, 6155] \to 1.6180336$	▲▼▲$(34236, 55395] \to 1.6180336$
9)	▼▲▼$(6155, 9959] \to 1.6180341$	▼▲▼$(55395, 89631] \to 1.6180341$
10)	▲▼▲$(9959, 16114] \to 1.6180339$	▲▼▲$(89631, 145026] \to 1.6180339$
11)	▲▼▲$(16114, 26073] \to 1.6180340$	▲▼▲$(145026, 234657] \to 1.6180340$
12)	▼▲▼$(26073, 42187] \to 1.6180340$	▲▼▲$(234657, 379683] \to 1.6180340$
13)	▲▼▲$(42187, 68260] \to 1.6180340$	▼▲▼$(379683, 614340] \to 1.6180340$
14)	▲▼▲$(68260, 110447] \to 1.6180340$	▲▼▲$(614340, 994023] \to 1.6180340$
15)	▼▲▼$(110447, 178707] \to 1.6180340$	▼▲▼$(994023, 1608363] \to 1.6180340$
16)	▼▲▼$(178707, 289154] \to 1.6180340$	▲▼▲$(1608363, 2602386] \to 1.6180340$
17)	▲▼▲$(289154, 467861] \to 1.6180340$	▼▲▼$(2602386, 4210749] \to 1.6180340$
18)	▼▲▼$(467861, 757015] \to 1.6180340$	▲▼▲$(4210749, 6813135] \to 1.6180340$
19)	▼▲▼$(757015, 1224876] \to 1.6180340$	▼▲▼$(6813135, 11023884] \to 1.6180340$
20)	▲▼▲$(1224876, 1981891] \to 1.6180340$	▲▼▲$(11023884, 17837019] \to 1.6180340$
21)	▼▲▼$(1981891, 3206767] \to 1.6180340$	▲▼▲$(17837019, 28860903] \to 1.6180340$
22)	▲▼▲$(3206767, 5188658] \to 1.6180340$	▼▲▼$(28860903, 46697922] \to 1.6180340$
23)	▲▼▲$(5188658, 8395425] \to 1.6180340$	▲▼▲$(46697922, 75558825] \to 1.6180340$
24)	▲▼▲$(8395425, 13584083] \to 1.6180340$	▲▼▲$(75558825, 122256747] \to 1.6180340$
25)	▼▲▼$(13584083, 21979508] \to 1.6180340$	▼▲▼$(122256747, 197815572] \to 1.6180340$
26)	▲▼▲$(21979508, 35563591] \to 1.6180340$	▼▼▼$(197815572, 320072319] \to 1.6180340$

0)	$(6561, 10615] \to 1.6178936$	$(59049, 95543] \to 1.6180291$
1)	$\blacktriangle\blacktriangledown\blacktriangle(10615, 17176] \to 1.6180876$	$\blacktriangle\blacktriangledown\blacktriangle(95543, 154592] \to 1.6180359$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(17176, 27791] \to 1.6180135$	$\blacktriangle\blacktriangledown\blacktriangle(154592, 250135] \to 1.6180333$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(27791, 44967] \to 1.6180418$	$\blacktriangle\blacktriangledown\blacktriangle(250135, 404727] \to 1.6180343$
4)	$\blacktriangle\blacktriangledown\blacktriangle(44967, 72758] \to 1.6180310$	$\blacktriangledown\blacktriangle\blacktriangledown(404727, 654862] \to 1.6180339$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(72758, 117725] \to 1.6180351$	$\blacktriangle\blacktriangledown\blacktriangle(654862, 1059589] \to 1.6180340$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(117725, 190483] \to 1.6180336$	$\blacktriangledown\blacktriangle\blacktriangledown(1059589, 1714451] \to 1.6180340$
7)	$\blacktriangle\blacktriangledown\blacktriangle(190483, 308208] \to 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(1714451, 2774040] \to 1.6180340$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(308208, 498691] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(2774040, 4488491] \to 1.6180340$
9)	$\blacktriangle\blacktriangledown\blacktriangle(498691, 806899] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4488491, 7262531] \to 1.6180340$
10)	$\blacktriangle\blacktriangledown\blacktriangle(806899, 1305590] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7262531, 11751022] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(1305590, 2112489] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11751022, 19013553] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(2112489, 3418079] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(19013553, 30764575] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(3418079, 5530568] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(30764575, 49778128] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(5530568, 8948647] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(49778128, 80542703] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(8948647, 14479215] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(80542703, 130320831] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(14479215, 23427862] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(130320831, 210863534] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(23427862, 37907077] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(210863534, 341184365] \to 1.6180340$

APPENDIX T: USING 10^p AND $\lfloor 10^p\phi \rfloor$ AS INITIATORS

0)	$(100, 161] \rightarrow 1.6100000$	$(1000, 1618] \rightarrow 1.6180000$
1)	$\blacktriangle\blacktriangledown\blacktriangle(161, 261] \rightarrow 1.6211180$	$\blacktriangle\blacktriangledown\blacktriangle(1618, 2618] \rightarrow 1.6180470$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(261, 422] \rightarrow 1.6168582$	$\blacktriangledown\blacktriangle\blacktriangledown(2618, 4236] \rightarrow 1.6180290$
3)	$\blacktriangle\blacktriangledown\blacktriangle(422, 683] \rightarrow 1.6184834$	$\blacktriangle\blacktriangledown\blacktriangle(4236, 6854] \rightarrow 1.6180359$
4)	$\blacktriangle\blacktriangledown\blacktriangle(683, 1105] \rightarrow 1.6178624$	$\blacktriangle\blacktriangledown\blacktriangle(6854, 11090] \rightarrow 1.6180333$
5)	$\blacktriangle\blacktriangledown\blacktriangle(1105, 1788] \rightarrow 1.6180995$	$\blacktriangledown\blacktriangle\blacktriangledown(11090, 17944] \rightarrow 1.6180343$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1788, 2893] \rightarrow 1.6180089$	$\blacktriangledown\blacktriangle\blacktriangledown(17944, 29034] \rightarrow 1.6180339$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(2893, 4681] \rightarrow 1.6180436$	$\blacktriangle\blacktriangledown\blacktriangle(29034, 46978] \rightarrow 1.6180340$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(4681, 7574] \rightarrow 1.6180303$	$\blacktriangledown\blacktriangle\blacktriangledown(46978, 76012] \rightarrow 1.6180340$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(7574, 12255] \rightarrow 1.6180354$	$\blacktriangle\blacktriangledown\blacktriangle(76012, 122990] \rightarrow 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(12255, 19829] \rightarrow 1.6180335$	$\blacktriangledown\blacktriangle\blacktriangledown(122990, 199002] \rightarrow 1.6180340$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(19829, 32084] \rightarrow 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(199002, 321992] \rightarrow 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(32084, 51913] \rightarrow 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(321992, 520994] \rightarrow 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(51913, 83997] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(520994, 842986] \rightarrow 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(83997, 135910] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(842986, 1363980] \rightarrow 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(135910, 219907] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1363980, 2206966] \rightarrow 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(219907, 355817] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2206966, 3570946] \rightarrow 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(355817, 575724] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3570946, 5777912] \rightarrow 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(575724, 931541] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(5777912, 9348858] \rightarrow 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(931541, 1507265] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(9348858, 15126770] \rightarrow 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1507265, 2438806] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(15126770, 24475628] \rightarrow 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(2438806, 3946071] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(24475628, 39602398] \rightarrow 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(3946071, 6384877] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(39602398, 64078026] \rightarrow 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(6384877, 10330948] \rightarrow 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(64078026, 103680424] \rightarrow 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(10330948, 16715825] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(103680424, 167758450] \rightarrow 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(16715825, 27046773] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(167758450, 271438874] \rightarrow 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(27046773, 43762598] \rightarrow 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(271438874, 439197324] \rightarrow 1.6180340$

0)	$(10000, 16180] \to 1.6180000$	$(100000, 161803] \to 1.6180300$
1)	$\blacktriangle\blacktriangledown\blacktriangle(16180, 26180] \to 1.6180470$	$\blacktriangle\blacktriangledown\blacktriangle(161803, 261803] \to 1.6180355$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(26180, 42360] \to 1.6180290$	$\blacktriangledown\blacktriangle\blacktriangledown(261803, 423606] \to 1.6180334$
3)	$\blacktriangle\blacktriangledown\blacktriangle(42360, 68540] \to 1.6180359$	$\blacktriangledown\blacktriangle\blacktriangledown(423606, 685409] \to 1.6180342$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(68540, 110900] \to 1.6180333$	$\blacktriangledown\blacktriangle\blacktriangledown(685409, 1109015] \to 1.6180339$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(110900, 179440] \to 1.6180343$	$\blacktriangle\blacktriangledown\blacktriangle(1109015, 1794424] \to 1.6180340$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(179440, 290340] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(1794424, 2903439] \to 1.6180340$
7)	$\blacktriangle\blacktriangledown\blacktriangle(290340, 469780] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2903439, 4697863] \to 1.6180340$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(469780, 760120] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4697863, 7601302] \to 1.6180340$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(760120, 1229900] \to 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(7601302, 12299165] \to 1.6180340$
10)	$\blacktriangle\blacktriangledown\blacktriangle(1229900, 1990020] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12299165, 19900467] \to 1.6180340$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(1990020, 3219920] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(19900467, 32199632] \to 1.6180340$
12)	$\blacktriangle\blacktriangledown\blacktriangle(3219920, 5209940] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(32199632, 52100099] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(5209940, 8429860] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(52100099, 84299731] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(8429860, 13639800] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(84299731, 136399830] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(13639800, 22069660] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(136399830, 220699561] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(22069660, 35709460] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(220699561, 357099391] \to 1.6180340$

APPENDIX U: USING 11^p AND $\lfloor 11^p\phi \rfloor$ AS INITIATORS

0)	$(121, 195] \to 1.6115702$	$(1331, 2153] \to 1.6175808$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(195, 316] \to 1.6205128$	$\blacktriangledown\blacktriangledown\blacktriangledown(2153, 3484] \to 1.6182072$
2)	$\blacktriangle\blacktriangle\blacktriangle(316, 511] \to 1.6170886$	$\blacktriangledown\blacktriangle\blacktriangledown(3484, 5637] \to 1.6179679$
3)	$\blacktriangle\blacktriangledown\blacktriangle(511, 827] \to 1.6183953$	$\blacktriangledown\blacktriangle\blacktriangledown(5637, 9121] \to 1.6180593$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(827, 1338] \to 1.6178960$	$\blacktriangle\blacktriangledown\blacktriangle(9121, 14758] \to 1.6180243$
5)	$\blacktriangle\blacktriangledown\blacktriangle(1338, 2165] \to 1.6180867$	$\blacktriangledown\blacktriangledown\blacktriangledown(14758, 23879] \to 1.6180377$
6)	$\blacktriangle\blacktriangledown\blacktriangle(2165, 3503] \to 1.6180139$	$\blacktriangle\blacktriangledown\blacktriangle(23879, 38637] \to 1.6180326$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(3503, 5668] \to 1.6180417$	$\blacktriangledown\blacktriangle\blacktriangledown(38637, 62516] \to 1.6180345$
8)	$\blacktriangle\blacktriangledown\blacktriangle(5668, 9171] \to 1.6180311$	$\blacktriangle\blacktriangledown\blacktriangle(62516, 101153] \to 1.6180338$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(9171, 14839] \to 1.6180351$	$\blacktriangledown\blacktriangle\blacktriangledown(101153, 163669] \to 1.6180341$
10)	$\blacktriangle\blacktriangledown\blacktriangle(14839, 24010] \to 1.6180336$	$\blacktriangle\blacktriangledown\blacktriangle(163669, 264822] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(24010, 38849] \to 1.6180342$	$\blacktriangledown\blacktriangle\blacktriangledown(264822, 428491] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(38849, 62859] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(428491, 693313] \to 1.6180340$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(62859, 101708] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(693313, 1121804] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(101708, 164567] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1121804, 1815117] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(164567, 266275] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1815117, 2936921] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(266275, 430842] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2936921, 4752038] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(430842, 697117] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4752038, 7688959] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(697117, 1127959] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7688959, 12440997] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(1127959, 1825076] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12440997, 20129956] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1825076, 2953035] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(20129956, 32570953] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(2953035, 4778111] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(32570953, 52700909] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(4778111, 7731146] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(52700909, 85271862] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(7731146, 12509257] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(85271862, 137972771] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(12509257, 20240403] \to 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(137972771, 223244633] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(20240403, 32749660] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(223244633, 361217404] \to 1.6180340$

0)	$(14641, 23689] \to 1.6179906$	$(161051, 260585] \to 1.6180278$
1)	$\blacktriangle\blacktriangledown\blacktriangle(23689, 38330] \to 1.6180506$	$\blacktriangledown\blacktriangle\blacktriangledown(260585, 421636] \to 1.6180363$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(38330, 62019] \to 1.6180277$	$\blacktriangledown\blacktriangle\blacktriangledown(421636, 682221] \to 1.6180331$
3)	$\blacktriangle\blacktriangledown\blacktriangle(62019, 100349] \to 1.6180364$	$\blacktriangle\blacktriangledown\blacktriangle(682221, 1103857] \to 1.6180343$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(100349, 162368] \to 1.6180331$	$\blacktriangledown\blacktriangle\blacktriangledown(1103857, 1786078] \to 1.6180339$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(162368, 262717] \to 1.6180343$	$\blacktriangledown\blacktriangle\blacktriangledown(1786078, 2889935] \to 1.6180340$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(262717, 425085] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(2889935, 4676013] \to 1.6180340$
7)	$\blacktriangle\blacktriangledown\blacktriangle(425085, 687802] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4676013, 7565948] \to 1.6180340$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(687802, 1112887] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7565948, 12241961] \to 1.6180340$
9)	$\blacktriangle\blacktriangledown\blacktriangle(1112887, 1800689] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12241961, 19807909] \to 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(1800689, 2913576] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(19807909, 32049870] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(2913576, 4714265] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(32049870, 51857779] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(4714265, 7627841] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(51857779, 83907649] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(7627841, 12342106] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(83907649, 135765428] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(12342106, 19969947] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(135765428, 219673077] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(19969947, 32312053] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(219673077, 355438505] \to 1.6180340$

APPENDIX V: USING 12^p AND $\lfloor 12^p\phi \rfloor$ AS INITIATORS

0)	$(144, 232] \rightarrow$ 1.6111111	$(1728, 2795] \rightarrow$ 1.6174769
1)	▼▼▼$(232, 376] \rightarrow$ 1.6206897	▲▼▲$(2795, 4523] \rightarrow$ 1.6182469
2)	▲▲▲$(376, 608] \rightarrow$ 1.6170213	▲▼▲$(4523, 7318] \rightarrow$ 1.6179527
3)	▲▼▲$(608, 984] \rightarrow$ 1.6184211	▼▲▼$(7318, 11841] \rightarrow$ 1.6180650
4)	▲▼▲$(984, 1592] \rightarrow$ 1.6178862	▼▲▼$(11841, 19159] \rightarrow$ 1.6180221
5)	▲▼▲$(1592, 2576] \rightarrow$ 1.6180905	▼▲▼$(19159, 31000] \rightarrow$ 1.6180385
6)	▼▲▼$(2576, 4168] \rightarrow$ 1.6180124	▼▲▼$(31000, 50159] \rightarrow$ 1.6180323
7)	▲▼▲$(4168, 6744] \rightarrow$ 1.6180422	▲▼▲$(50159, 81159] \rightarrow$ 1.6180346
8)	▼▲▼$(6744, 10912] \rightarrow$ 1.6180308	▲▼▲$(81159, 131318] \rightarrow$ 1.6180337
9)	▲▼▲$(10912, 17656] \rightarrow$ 1.6180352	▼▲▼$(131318, 212477] \rightarrow$ 1.6180341
10)	▼▲▼$(17656, 28568] \rightarrow$ 1.6180335	▲▼▲$(212477, 343795] \rightarrow$ 1.6180340
11)	▼▲▼$(28568, 46224] \rightarrow$ 1.6180342	▲▼▲$(343795, 556272] \rightarrow$ 1.6180340
12)	▼▲▼$(46224, 74792] \rightarrow$ 1.6180339	▼▲▼$(556272, 900067] \rightarrow$ 1.6180340
13)	▲▼▲$(74792, 121016] \rightarrow$ 1.6180340	▲▼▲$(900067, 1456339] \rightarrow$ 1.6180340
14)	▼▲▼$(121016, 195808] \rightarrow$ 1.6180340	▲▼▲$(1456339, 2356406] \rightarrow$ 1.6180340
15)	▲▼▲$(195808, 316824] \rightarrow$ 1.6180340	▼▲▼$(2356406, 3812745] \rightarrow$ 1.6180340
16)	▼▲▼$(316824, 512632] \rightarrow$ 1.6180340	▲▼▲$(3812745, 6169151] \rightarrow$ 1.6180340
17)	▲▼▲$(512632, 829456] \rightarrow$ 1.6180340	▲▼▲$(6169151, 9981896] \rightarrow$ 1.6180340
18)	▲▼▲$(829456, 1342088] \rightarrow$ 1.6180340	▼▲▼$(9981896, 16151047] \rightarrow$ 1.6180340
19)	▼▲▼$(1342088, 2171544] \rightarrow$ 1.6180340	▲▼▲$(16151047, 26132943] \rightarrow$ 1.6180340
20)	▲▼▲$(2171544, 3513632] \rightarrow$ 1.6180340	▲▼▲$(26132943, 42283990] \rightarrow$ 1.6180340
21)	▲▼▲$(3513632, 5685176] \rightarrow$ 1.6180340	▲▼▲$(42283990, 68416933] \rightarrow$ 1.6180340
22)	▼▲▼$(5685176, 9198808] \rightarrow$ 1.6180340	▲▼▲$(68416933, 110700923] \rightarrow$ 1.6180340
23)	▲▼▲$(9198808, 14883984] \rightarrow$ 1.6180340	▲▼▲$(110700923, 179117856] \rightarrow$ 1.6180340
24)	▲▼▲$(14883984, 24082792] \rightarrow$ 1.6180340	▲▼▲$(179117856, 289818779] \rightarrow$ 1.6180340
25)	▼▲▼$(24082792, 38966776] \rightarrow$ 1.6180340	▼▲▼$(289818779, 468936635] \rightarrow$ 1.6180340

0)	$(20736, 33551] \to 1.6180073$	$(248832, 402618] \to 1.6180314$
1)	$\blacktriangle\blacktriangledown\blacktriangle(33551, 54287] \to 1.6180442$	$\blacktriangledown\blacktriangle\blacktriangledown(402618, 651450] \to 1.6180350$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(54287, 87838] \to 1.6180301$	$\blacktriangledown\blacktriangle\blacktriangledown(651450, 1054068] \to 1.6180336$
3)	$\blacktriangle\blacktriangledown\blacktriangle(87838, 142125] \to 1.6180355$	$\blacktriangledown\blacktriangle\blacktriangledown(1054068, 1705518] \to 1.6180341$
4)	$\blacktriangle\blacktriangledown\blacktriangle(142125, 229963] \to 1.6180334$	$\blacktriangle\blacktriangledown\blacktriangle(1705518, 2759586] \to 1.6180339$
5)	$\blacktriangle\blacktriangledown\blacktriangle(229963, 372088] \to 1.6180342$	$\blacktriangledown\blacktriangle\blacktriangledown(2759586, 4465104] \to 1.6180340$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(372088, 602051] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(4465104, 7224690] \to 1.6180340$
7)	$\blacktriangle\blacktriangledown\blacktriangle(602051, 974139] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7224690, 11689794] \to 1.6180340$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(974139, 1576190] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11689794, 18914484] \to 1.6180340$
9)	$\blacktriangle\blacktriangledown\blacktriangle(1576190, 2550329] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(18914484, 30604278] \to 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(2550329, 4126519] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(30604278, 49518762] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(4126519, 6676848] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(49518762, 80123040] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(6676848, 10803367] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(80123040, 129641802] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(10803367, 17480215] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(129641802, 209764842] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(17480215, 28283582] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(209764842, 339406644] \to 1.6180340$

APPENDIX W: USING P AND $\lfloor P\phi \rfloor$ AS INITIATORS, WHERE P IS A PRIME IN THE INTERVAL [7,59]

0)	$(7, 11] \rightarrow$ 1.5714286	$(11, 17] \rightarrow$ 1.5454545
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(11, 18] \rightarrow$ 1.6363636	$\blacktriangle\blacktriangledown\blacktriangle(17, 28] \rightarrow$ 1.6470588
2)	$\blacktriangledown\blacktriangle\blacktriangledown(18, 29] \rightarrow$ 1.6111111	$\blacktriangledown\blacktriangle\blacktriangledown(28, 45] \rightarrow$ 1.6071429
3)	$\blacktriangle\blacktriangledown\blacktriangle(29, 47] \rightarrow$ 1.6206897	$\blacktriangle\blacktriangledown\blacktriangle(45, 73] \rightarrow$ 1.6222222
4)	$\blacktriangle\blacktriangledown\blacktriangle(47, 76] \rightarrow$ 1.6170213	$\blacktriangle\blacktriangledown\blacktriangle(73, 118] \rightarrow$ 1.6164384
5)	$\blacktriangledown\blacktriangledown\blacktriangledown(76, 123] \rightarrow$ 1.6184211	$\blacktriangledown\blacktriangle\blacktriangledown(118, 191] \rightarrow$ 1.6186441
6)	$\blacktriangledown\blacktriangle\blacktriangledown(123, 199] \rightarrow$ 1.6178862	$\blacktriangledown\blacktriangle\blacktriangledown(191, 309] \rightarrow$ 1.6178010
7)	$\blacktriangle\blacktriangledown\blacktriangle(199, 322] \rightarrow$ 1.6180905	$\blacktriangledown\blacktriangle\blacktriangledown(309, 500] \rightarrow$ 1.6181230
8)	$\blacktriangledown\blacktriangle\blacktriangledown(322, 521] \rightarrow$ 1.6180124	$\blacktriangle\blacktriangledown\blacktriangle(500, 809] \rightarrow$ 1.6180000
9)	$\blacktriangledown\blacktriangle\blacktriangledown(521, 843] \rightarrow$ 1.6180422	$\blacktriangledown\blacktriangle\blacktriangledown(809, 1309] \rightarrow$ 1.6180470
10)	$\blacktriangledown\blacktriangle\blacktriangledown(843, 1364] \rightarrow$ 1.6180308	$\blacktriangle\blacktriangledown\blacktriangle(1309, 2118] \rightarrow$ 1.6180290
11)	$\blacktriangledown\blacktriangle\blacktriangledown(1364, 2207] \rightarrow$ 1.6180352	$\blacktriangledown\blacktriangle\blacktriangledown(2118, 3427] \rightarrow$ 1.6180359
12)	$\blacktriangledown\blacktriangle\blacktriangledown(2207, 3571] \rightarrow$ 1.6180335	$\blacktriangledown\blacktriangle\blacktriangledown(3427, 5545] \rightarrow$ 1.6180333
13)	$\blacktriangle\blacktriangledown\blacktriangle(3571, 5778] \rightarrow$ 1.6180342	$\blacktriangle\blacktriangledown\blacktriangle(5545, 8972] \rightarrow$ 1.6180343
14)	$\blacktriangledown\blacktriangle\blacktriangledown(5778, 9349] \rightarrow$ 1.6180339	$\blacktriangle\blacktriangledown\blacktriangle(8972, 14517] \rightarrow$ 1.6180339
15)	$\blacktriangle\blacktriangledown\blacktriangle(9349, 15127] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(14517, 23489] \rightarrow$ 1.6180340
16)	$\blacktriangledown\blacktriangle\blacktriangledown(15127, 24476] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(23489, 38006] \rightarrow$ 1.6180340
17)	$\blacktriangle\blacktriangledown\blacktriangle(24476, 39603] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(38006, 61495] \rightarrow$ 1.6180340
18)	$\blacktriangledown\blacktriangle\blacktriangledown(39603, 64079] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(61495, 99501] \rightarrow$ 1.6180340
19)	$\blacktriangle\blacktriangledown\blacktriangle(64079, 103682] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(99501, 160996] \rightarrow$ 1.6180340
20)	$\blacktriangledown\blacktriangle\blacktriangledown(103682, 167761] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(160996, 260497] \rightarrow$ 1.6180340
21)	$\blacktriangledown\blacktriangle\blacktriangledown(167761, 271443] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(260497, 421493] \rightarrow$ 1.6180340
22)	$\blacktriangle\blacktriangledown\blacktriangle(271443, 439204] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(421493, 681990] \rightarrow$ 1.6180340
23)	$\blacktriangledown\blacktriangle\blacktriangledown(439204, 710647] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(681990, 1103483] \rightarrow$ 1.6180340
24)	$\blacktriangle\blacktriangledown\blacktriangle(710647, 1149851] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(1103483, 1785473] \rightarrow$ 1.6180340
25)	$\blacktriangledown\blacktriangle\blacktriangledown(1149851, 1860498] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(1785473, 2888956] \rightarrow$ 1.6180340
26)	$\blacktriangle\blacktriangledown\blacktriangle(1860498, 3010349] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(2888956, 4674429] \rightarrow$ 1.6180340
27)	$\blacktriangle\blacktriangledown\blacktriangle(3010349, 4870847] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(4674429, 7563385] \rightarrow$ 1.6180340
28)	$\blacktriangledown\blacktriangle\blacktriangledown(4870847, 7881196] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(7563385, 12237814] \rightarrow$ 1.6180340
29)	$\blacktriangle\blacktriangledown\blacktriangle(7881196, 12752043] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(12237814, 19801199] \rightarrow$ 1.6180340
30)	$\blacktriangle\blacktriangledown\blacktriangle(12752043, 20633239] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(19801199, 32039013] \rightarrow$ 1.6180340
31)	$\blacktriangledown\blacktriangle\blacktriangledown(20633239, 33385282] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(32039013, 51840212] \rightarrow$ 1.6180340
32)	$\blacktriangle\blacktriangledown\blacktriangle(33385282, 54018521] \rightarrow$ 1.6180340	$\blacktriangle\blacktriangledown\blacktriangle(51840212, 83879225] \rightarrow$ 1.6180340
33)	$\blacktriangledown\blacktriangle\blacktriangledown(54018521, 87403803] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(83879225, 135719437] \rightarrow$ 1.6180340
34)	$\blacktriangledown\blacktriangle\blacktriangledown(87403803, 141422324] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(135719437, 219598662] \rightarrow$ 1.6180340
35)	$\blacktriangledown\blacktriangle\blacktriangledown(141422324, 228826127] \rightarrow$ 1.6180340	$\blacktriangledown\blacktriangle\blacktriangledown(219598662, 355318099] \rightarrow$ 1.6180340

0)	$(13, 21] \to 1.6153846$	$(17, 27] \to 1.5882353$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(21, 34] \to 1.6190476$	$\blacktriangledown\blacktriangle\blacktriangledown(27, 44] \to 1.6296296$
2)	$\blacktriangle\blacktriangledown\blacktriangle(34, 55] \to 1.6176471$	$\blacktriangle\blacktriangledown\blacktriangle(44, 71] \to 1.6136364$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(55, 89] \to 1.6181818$	$\blacktriangledown\blacktriangledown\blacktriangledown(71, 115] \to 1.6197183$
4)	$\blacktriangle\blacktriangledown\blacktriangle(89, 144] \to 1.6179775$	$\blacktriangle\blacktriangledown\blacktriangle(115, 186] \to 1.6173913$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(144, 233] \to 1.6180556$	$\blacktriangledown\blacktriangle\blacktriangledown(186, 301] \to 1.6182796$
6)	$\blacktriangle\blacktriangledown\blacktriangle(233, 377] \to 1.6180258$	$\blacktriangledown\blacktriangle\blacktriangledown(301, 487] \to 1.6179402$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(377, 610] \to 1.6180371$	$\blacktriangle\blacktriangledown\blacktriangle(487, 788] \to 1.6180698$
8)	$\blacktriangle\blacktriangledown\blacktriangle(610, 987] \to 1.6180328$	$\blacktriangle\blacktriangledown\blacktriangle(788, 1275] \to 1.6180203$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(987, 1597] \to 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(1275, 2063] \to 1.6180392$
10)	$\blacktriangle\blacktriangledown\blacktriangle(1597, 2584] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(2063, 3338] \to 1.6180320$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(2584, 4181] \to 1.6180341$	$\blacktriangle\blacktriangledown\blacktriangle(3338, 5401] \to 1.6180348$
12)	$\blacktriangle\blacktriangledown\blacktriangle(4181, 6765] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(5401, 8739] \to 1.6180337$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(6765, 10946] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(8739, 14140] \to 1.6180341$
14)	$\blacktriangle\blacktriangledown\blacktriangle(10946, 17711] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(14140, 22879] \to 1.6180339$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(17711, 28657] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(22879, 37019] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(28657, 46368] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(37019, 59898] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(46368, 75025] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(59898, 96917] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(75025, 121393] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(96917, 156815] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(121393, 196418] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(156815, 253732] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(196418, 317811] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(253732, 410547] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(317811, 514229] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(410547, 664279] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(514229, 832040] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(664279, 1074826] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(832040, 1346269] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1074826, 1739105] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(1346269, 2178309] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1739105, 2813931] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(2178309, 3524578] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2813931, 4553036] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(3524578, 5702887] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4553036, 7366967] \to 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(5702887, 9227465] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7366967, 11920003] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(9227465, 14930352] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11920003, 19286970] \to 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(14930352, 24157817] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(19286970, 31206973] \to 1.6180340$
30)	$\blacktriangledown\blacktriangle\blacktriangledown(24157817, 39088169] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(31206973, 50493943] \to 1.6180340$
31)	$\blacktriangle\blacktriangledown\blacktriangle(39088169, 63245986] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(50493943, 81700916] \to 1.6180340$
32)	$\blacktriangledown\blacktriangle\blacktriangledown(63245986, 102334155] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(81700916, 132194859] \to 1.6180340$
33)	$\blacktriangle\blacktriangledown\blacktriangle(102334155, 165580141] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(132194859, 213895775] \to 1.6180340$
34)	$\blacktriangledown\blacktriangle\blacktriangledown(165580141, 267914296] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(213895775, 346090634] \to 1.6180340$

0)	$(19, 30] \to 1.5789474$	$(23, 37] \to 1.6086957$
1)	$(30, 49] \to 1.6333333$	$(37, 60] \to 1.6216216$
2)	$(49, 79] \to 1.6122449$	$(60, 97] \to 1.6166667$
3)	$(79, 128] \to 1.6202532$	$(97, 157] \to 1.6185567$
4)	$(128, 207] \to 1.6171875$	$(157, 254] \to 1.6178344$
5)	$(207, 335] \to 1.6183575$	$(254, 411] \to 1.6181102$
6)	$(335, 542] \to 1.6179104$	$(411, 665] \to 1.6180049$
7)	$(542, 877] \to 1.6180812$	$(665, 1076] \to 1.6180451$
8)	$(877, 1419] \to 1.6180160$	$(1076, 1741] \to 1.6180297$
9)	$(1419, 2296] \to 1.6180409$	$(1741, 2817] \to 1.6180356$
10)	$(2296, 3715] \to 1.6180314$	$(2817, 4558] \to 1.6180334$
11)	$(3715, 6011] \to 1.6180350$	$(4558, 7375] \to 1.6180342$
12)	$(6011, 9726] \to 1.6180336$	$(7375, 11933] \to 1.6180339$
13)	$(9726, 15737] \to 1.6180341$	$(11933, 19308] \to 1.6180340$
14)	$(15737, 25463] \to 1.6180339$	$(19308, 31241] \to 1.6180340$
15)	$(25463, 41200] \to 1.6180340$	$(31241, 50549] \to 1.6180340$
16)	$(41200, 66663] \to 1.6180340$	$(50549, 81790] \to 1.6180340$
17)	$(66663, 107863] \to 1.6180340$	$(81790, 132339] \to 1.6180340$
18)	$(107863, 174526] \to 1.6180340$	$(132339, 214129] \to 1.6180340$
19)	$(174526, 282389] \to 1.6180340$	$(214129, 346468] \to 1.6180340$
20)	$(282389, 456915] \to 1.6180340$	$(346468, 560597] \to 1.6180340$
21)	$(456915, 739304] \to 1.6180340$	$(560597, 907065] \to 1.6180340$
22)	$(739304, 1196219] \to 1.6180340$	$(907065, 1467662] \to 1.6180340$
23)	$(1196219, 1935523] \to 1.6180340$	$(1467662, 2374727] \to 1.6180340$
24)	$(1935523, 3131742] \to 1.6180340$	$(2374727, 3842389] \to 1.6180340$
25)	$(3131742, 5067265] \to 1.6180340$	$(3842389, 6217116] \to 1.6180340$
26)	$(5067265, 8199007] \to 1.6180340$	$(6217116, 10059505] \to 1.6180340$
27)	$(8199007, 13266272] \to 1.6180340$	$(10059505, 16276621] \to 1.6180340$
28)	$(13266272, 21465279] \to 1.6180340$	$(16276621, 26336126] \to 1.6180340$
29)	$(21465279, 34731551] \to 1.6180340$	$(26336126, 42612747] \to 1.6180340$
30)	$(34731551, 56196830] \to 1.6180340$	$(42612747, 68948873] \to 1.6180340$
31)	$(56196830, 90928381] \to 1.6180340$	$(68948873, 111561620] \to 1.6180340$
32)	$(90928381, 147125211] \to 1.6180340$	$(111561620, 180510493] \to 1.6180340$
33)	$(147125211, 238053592] \to 1.6180340$	$(180510493, 292072113] \to 1.6180340$
34)	$(238053592, 385178803] \to 1.6180340$	$(292072113, 472582606] \to 1.6180340$

0)	$(29, 46] \to 1.5862069$	$(31, 50] \to 1.6129032$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(46, 75] \to 1.6304348$	$\blacktriangledown\blacktriangle\blacktriangledown(50, 81] \to 1.6200000$
2)	$\blacktriangle\blacktriangledown\blacktriangle(75, 121] \to 1.6133333$	$\blacktriangledown\blacktriangle\blacktriangledown(81, 131] \to 1.6172840$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(121, 196] \to 1.6198347$	$\blacktriangle\blacktriangledown\blacktriangle(131, 212] \to 1.6183206$
4)	$\blacktriangledown\blacktriangle\blacktriangledown(196, 317] \to 1.6173469$	$\blacktriangle\blacktriangledown\blacktriangle(212, 343] \to 1.6179245$
5)	$\blacktriangle\blacktriangledown\blacktriangle(317, 513] \to 1.6182965$	$\blacktriangledown\blacktriangle\blacktriangledown(343, 555] \to 1.6180758$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(513, 830] \to 1.6179337$	$\blacktriangledown\blacktriangle\blacktriangledown(555, 898] \to 1.6180180$
7)	$\blacktriangledown\blacktriangledown\blacktriangledown(830, 1343] \to 1.6180723$	$\blacktriangle\blacktriangledown\blacktriangle(898, 1453] \to 1.6180401$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(1343, 2173] \to 1.6180194$	$\blacktriangledown\blacktriangle\blacktriangledown(1453, 2351] \to 1.6180317$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(2173, 3516] \to 1.6180396$	$\blacktriangle\blacktriangledown\blacktriangle(2351, 3804] \to 1.6180349$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(3516, 5689] \to 1.6180319$	$\blacktriangle\blacktriangledown\blacktriangle(3804, 6155] \to 1.6180336$
11)	$\blacktriangle\blacktriangledown\blacktriangle(5689, 9205] \to 1.6180348$	$\blacktriangledown\blacktriangle\blacktriangledown(6155, 9959] \to 1.6180341$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(9205, 14894] \to 1.6180337$	$\blacktriangle\blacktriangledown\blacktriangle(9959, 16114] \to 1.6180339$
13)	$\blacktriangle\blacktriangledown\blacktriangle(14894, 24099] \to 1.6180341$	$\blacktriangle\blacktriangledown\blacktriangle(16114, 26073] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(24099, 38993] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(26073, 42187] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(38993, 63092] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(42187, 68260] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(63092, 102085] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(68260, 110447] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(102085, 165177] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(110447, 178707] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(165177, 267262] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(178707, 289154] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(267262, 432439] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(289154, 467861] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(432439, 699701] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(467861, 757015] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(699701, 1132140] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(757015, 1224876] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(1132140, 1831841] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1224876, 1981891] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(1831841, 2963981] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1981891, 3206767] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(2963981, 4795822] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3206767, 5188658] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(4795822, 7759803] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(5188658, 8395425] \to 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(7759803, 12555625] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(8395425, 13584083] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(12555625, 20315428] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(13584083, 21979508] \to 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(20315428, 32871053] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(21979508, 35563591] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(32871053, 53186481] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(35563591, 57543099] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(53186481, 86057534] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(57543099, 93106690] \to 1.6180340$
31)	$\blacktriangledown\blacktriangle\blacktriangledown(86057534, 139244015] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(93106690, 150649789] \to 1.6180340$
32)	$\blacktriangledown\blacktriangle\blacktriangledown(139244015, 225301549] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(150649789, 243756479] \to 1.6180340$
33)	$\blacktriangle\blacktriangledown\blacktriangle(225301549, 364545564] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(243756479, 394406268] \to 1.6180340$

0)	$(37, 59] \to 1.5945946$	$(41, 66] \to 1.6097561$
1)	$(59, 96] \to 1.6271186$	$(66, 107] \to 1.6212121$
2)	$(96, 155] \to 1.6145833$	$(107, 173] \to 1.6168224$
3)	$(155, 251] \to 1.6193548$	$(173, 280] \to 1.6184971$
4)	$(251, 406] \to 1.6175299$	$(280, 453] \to 1.6178571$
5)	$(406, 657] \to 1.6182266$	$(453, 733] \to 1.6181015$
6)	$(657, 1063] \to 1.6179604$	$(733, 1186] \to 1.6180082$
7)	$(1063, 1720] \to 1.6180621$	$(1186, 1919] \to 1.6180438$
8)	$(1720, 2783] \to 1.6180233$	$(1919, 3105] \to 1.6180302$
9)	$(2783, 4503] \to 1.6180381$	$(3105, 5024] \to 1.6180354$
10)	$(4503, 7286] \to 1.6180324$	$(5024, 8129] \to 1.6180334$
11)	$(7286, 11789] \to 1.6180346$	$(8129, 13153] \to 1.6180342$
12)	$(11789, 19075] \to 1.6180338$	$(13153, 21282] \to 1.6180339$
13)	$(19075, 30864] \to 1.6180341$	$(21282, 34435] \to 1.6180340$
14)	$(30864, 49939] \to 1.6180340$	$(34435, 55717] \to 1.6180340$
15)	$(49939, 80803] \to 1.6180340$	$(55717, 90152] \to 1.6180340$
16)	$(80803, 130742] \to 1.6180340$	$(90152, 145869] \to 1.6180340$
17)	$(130742, 211545] \to 1.6180340$	$(145869, 236021] \to 1.6180340$
18)	$(211545, 342287] \to 1.6180340$	$(236021, 381890] \to 1.6180340$
19)	$(342287, 553832] \to 1.6180340$	$(381890, 617911] \to 1.6180340$
20)	$(553832, 896119] \to 1.6180340$	$(617911, 999801] \to 1.6180340$
21)	$(896119, 1449951] \to 1.6180340$	$(999801, 1617712] \to 1.6180340$
22)	$(1449951, 2346070] \to 1.6180340$	$(1617712, 2617513] \to 1.6180340$
23)	$(2346070, 3796021] \to 1.6180340$	$(2617513, 4235225] \to 1.6180340$
24)	$(3796021, 6142091] \to 1.6180340$	$(4235225, 6852738] \to 1.6180340$
25)	$(6142091, 9938112] \to 1.6180340$	$(6852738, 11087963] \to 1.6180340$
26)	$(9938112, 16080203] \to 1.6180340$	$(11087963, 17940701] \to 1.6180340$
27)	$(16080203, 26018315] \to 1.6180340$	$(17940701, 29028664] \to 1.6180340$
28)	$(26018315, 42098518] \to 1.6180340$	$(29028664, 46969365] \to 1.6180340$
29)	$(42098518, 68116833] \to 1.6180340$	$(46969365, 75998029] \to 1.6180340$
30)	$(68116833, 110215351] \to 1.6180340$	$(75998029, 122967394] \to 1.6180340$
31)	$(110215351, 178332184] \to 1.6180340$	$(122967394, 198965423] \to 1.6180340$
32)	$(178332184, 288547535] \to 1.6180340$	$(198965423, 321932817] \to 1.6180340$

0)	$(43, 69] \to 1.6046512$	$(47, 76] \to 1.6170213$
1)	$\blacktriangledown\blacktriangle\blacktriangledown(69, 112] \to 1.6231884$	$\blacktriangledown\blacktriangledown\blacktriangledown(76, 123] \to 1.6184211$
2)	$\blacktriangle\blacktriangledown\blacktriangle(112, 181] \to 1.6160714$	$\blacktriangledown\blacktriangle\blacktriangledown(123, 199] \to 1.6178862$
3)	$\blacktriangle\blacktriangledown\blacktriangle(181, 293] \to 1.6187845$	$\blacktriangle\blacktriangledown\blacktriangle(199, 322] \to 1.6180905$
4)	$\blacktriangle\blacktriangledown\blacktriangle(293, 474] \to 1.6177474$	$\blacktriangledown\blacktriangle\blacktriangledown(322, 521] \to 1.6180124$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(474, 767] \to 1.6181435$	$\blacktriangledown\blacktriangle\blacktriangledown(521, 843] \to 1.6180422$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(767, 1241] \to 1.6179922$	$\blacktriangledown\blacktriangle\blacktriangledown(843, 1364] \to 1.6180308$
7)	$\blacktriangle\blacktriangledown\blacktriangle(1241, 2008] \to 1.6180500$	$\blacktriangledown\blacktriangle\blacktriangledown(1364, 2207] \to 1.6180352$
8)	$\blacktriangle\blacktriangledown\blacktriangle(2008, 3249] \to 1.6180279$	$\blacktriangledown\blacktriangle\blacktriangledown(2207, 3571] \to 1.6180335$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(3249, 5257] \to 1.6180363$	$\blacktriangle\blacktriangledown\blacktriangle(3571, 5778] \to 1.6180342$
10)	$\blacktriangle\blacktriangledown\blacktriangle(5257, 8506] \to 1.6180331$	$\blacktriangledown\blacktriangle\blacktriangledown(5778, 9349] \to 1.6180339$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(8506, 13763] \to 1.6180343$	$\blacktriangle\blacktriangledown\blacktriangle(9349, 15127] \to 1.6180340$
12)	$\blacktriangle\blacktriangledown\blacktriangle(13763, 22269] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(15127, 24476] \to 1.6180340$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(22269, 36032] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(24476, 39603] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(36032, 58301] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(39603, 64079] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(58301, 94333] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(64079, 103682] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(94333, 152634] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(103682, 167761] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(152634, 246967] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(167761, 271443] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(246967, 399601] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(271443, 439204] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(399601, 646568] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(439204, 710647] \to 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(646568, 1046169] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(710647, 1149851] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(1046169, 1692737] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1149851, 1860498] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(1692737, 2738906] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1860498, 3010349] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(2738906, 4431643] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3010349, 4870847] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(4431643, 7170549] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4870847, 7881196] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(7170549, 11602192] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7881196, 12752043] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(11602192, 18772741] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12752043, 20633239] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(18772741, 30374933] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(20633239, 33385282] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(30374933, 49147674] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(33385282, 54018521] \to 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(49147674, 79522607] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(54018521, 87403803] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(79522607, 128670281] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(87403803, 141422324] \to 1.6180340$
31)	$\blacktriangle\blacktriangledown\blacktriangle(128670281, 208192888] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(141422324, 228826127] \to 1.6180340$
32)	$\blacktriangle\blacktriangledown\blacktriangle(208192888, 336863169] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(228826127, 370248451] \to 1.6180340$

0)	$(53, 85] \to 1.6037736$	$(59, 95] \to 1.6101695$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(85, 138] \to 1.6235294$	$\blacktriangledown\blacktriangle\blacktriangledown(95, 154] \to 1.6210526$
2)	$\blacktriangledown\blacktriangle\blacktriangledown(138, 223] \to 1.6159420$	$\blacktriangle\blacktriangledown\blacktriangle(154, 249] \to 1.6168831$
3)	$\blacktriangledown\blacktriangle\blacktriangledown(223, 361] \to 1.6188341$	$\blacktriangledown\blacktriangle\blacktriangledown(249, 403] \to 1.6184739$
4)	$\blacktriangle\blacktriangledown\blacktriangle(361, 584] \to 1.6177285$	$\blacktriangle\blacktriangledown\blacktriangle(403, 652] \to 1.6178660$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(584, 945] \to 1.6181507$	$\blacktriangledown\blacktriangle\blacktriangledown(652, 1055] \to 1.6180982$
6)	$\blacktriangle\blacktriangledown\blacktriangle(945, 1529] \to 1.6179894$	$\blacktriangle\blacktriangledown\blacktriangle(1055, 1707] \to 1.6180095$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(1529, 2474] \to 1.6180510$	$\blacktriangledown\blacktriangle\blacktriangledown(1707, 2762] \to 1.6180434$
8)	$\blacktriangle\blacktriangledown\blacktriangle(2474, 4003] \to 1.6180275$	$\blacktriangle\blacktriangledown\blacktriangle(2762, 4469] \to 1.6180304$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(4003, 6477] \to 1.6180365$	$\blacktriangledown\blacktriangle\blacktriangledown(4469, 7231] \to 1.6180354$
10)	$\blacktriangle\blacktriangledown\blacktriangle(6477, 10480] \to 1.6180330$	$\blacktriangle\blacktriangledown\blacktriangle(7231, 11700] \to 1.6180335$
11)	$\blacktriangle\blacktriangledown\blacktriangle(10480, 16957] \to 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(11700, 18931] \to 1.6180342$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(16957, 27437] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(18931, 30631] \to 1.6180339$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(27437, 44394] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(30631, 49562] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(44394, 71831] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(49562, 80193] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(71831, 116225] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(80193, 129755] \to 1.6180340$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(116225, 188056] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(129755, 209948] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(188056, 304281] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(209948, 339703] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(304281, 492337] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(339703, 549651] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(492337, 796618] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(549651, 889354] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(796618, 1288955] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(889354, 1439005] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(1288955, 2085573] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1439005, 2328359] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(2085573, 3374528] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2328359, 3767364] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(3374528, 5460101] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3767364, 6095723] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(5460101, 8834629] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6095723, 9863087] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(8834629, 14294730] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(9863087, 15958810] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(14294730, 23129359] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(15958810, 25821897] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(23129359, 37424089] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(25821897, 41780707] \to 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(37424089, 60553448] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(41780707, 67602604] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(60553448, 97977537] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(67602604, 109383311] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(97977537, 158530985] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(109383311, 176985915] \to 1.6180340$
31)	$\blacktriangledown\blacktriangle\blacktriangledown(158530985, 256508522] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(176985915, 286369226] \to 1.6180340$
32)	$\blacktriangle\blacktriangledown\blacktriangle(256508522, 415039507] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(286369226, 463355141] \to 1.6180340$

APPENDIX X: USING 100 AND $100+N$ AS INITIATORS, WHERE $N = 11,\ 21,\ 31,\ \ldots\ 101$

0)	$(100, 111] \to 1.1100000$	$(100, 121] \to 1.2100000$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(111, 211] \to 1.9009009$	$\blacktriangledown\blacktriangledown\blacktriangledown(121, 221] \to 1.8264463$
2)	$\blacktriangle\blacktriangle\blacktriangle(211, 322] \to 1.5260664$	$\blacktriangle\blacktriangle\blacktriangle(221, 342] \to 1.5475113$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(322, 533] \to 1.6552795$	$\blacktriangledown\blacktriangledown\blacktriangledown(342, 563] \to 1.6461988$
4)	$\blacktriangle\blacktriangle\blacktriangle(533, 855] \to 1.6041276$	$\blacktriangle\blacktriangle\blacktriangle(563, 905] \to 1.6074600$
5)	$\blacktriangledown\blacktriangledown\blacktriangledown(855, 1388] \to 1.6233918$	$\blacktriangledown\blacktriangledown\blacktriangledown(905, 1468] \to 1.6220994$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1388, 2243] \to 1.6159942$	$\blacktriangledown\blacktriangle\blacktriangledown(1468, 2373] \to 1.6164850$
7)	$\blacktriangledown\blacktriangledown\blacktriangledown(2243, 3631] \to 1.6188141$	$\blacktriangle\blacktriangledown\blacktriangle(2373, 3841] \to 1.6186262$
8)	$\blacktriangle\blacktriangle\blacktriangle(3631, 5874] \to 1.6177362$	$\blacktriangle\blacktriangle\blacktriangle(3841, 6214] \to 1.6178079$
9)	$\blacktriangle\blacktriangledown\blacktriangle(5874, 9505] \to 1.6181478$	$\blacktriangle\blacktriangledown\blacktriangle(6214, 10055] \to 1.6181204$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(9505, 15379] \to 1.6179905$	$\blacktriangledown\blacktriangle\blacktriangledown(10055, 16269] \to 1.6180010$
11)	$\blacktriangle\blacktriangledown\blacktriangle(15379, 24884] \to 1.6180506$	$\blacktriangle\blacktriangledown\blacktriangle(16269, 26324] \to 1.6180466$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(24884, 40263] \to 1.6180276$	$\blacktriangledown\blacktriangle\blacktriangledown(26324, 42593] \to 1.6180292$
13)	$\blacktriangle\blacktriangledown\blacktriangle(40263, 65147] \to 1.6180364$	$\blacktriangle\blacktriangledown\blacktriangle(42593, 68917] \to 1.6180358$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(65147, 105410] \to 1.6180331$	$\blacktriangledown\blacktriangle\blacktriangledown(68917, 111510] \to 1.6180333$
15)	$\blacktriangle\blacktriangledown\blacktriangle(105410, 170557] \to 1.6180343$	$\blacktriangledown\blacktriangle\blacktriangledown(111510, 180427] \to 1.6180343$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(170557, 275967] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(180427, 291937] \to 1.6180339$
17)	$\blacktriangle\blacktriangledown\blacktriangle(275967, 446524] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(291937, 472364] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(446524, 722491] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(472364, 764301] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(722491, 1169015] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(764301, 1236665] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1169015, 1891506] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1236665, 2000966] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(1891506, 3060521] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2000966, 3237631] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(3060521, 4952027] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3237631, 5238597] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(4952027, 8012548] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(5238597, 8476228] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(8012548, 12964575] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(8476228, 13714825] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(12964575, 20977123] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(13714825, 22191053] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(20977123, 33941698] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(22191053, 35905878] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(33941698, 54918821] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(35905878, 58096931] \to 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(54918821, 88860519] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(58096931, 94002809] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(88860519, 143779340] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(94002809, 152099740] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(143779340, 232639859] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(152099740, 246102549] \to 1.6180340$
31)	$\blacktriangledown\blacktriangle\blacktriangledown(232639859, 376419199] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(246102549, 398202289] \to 1.6180340$

0)	$(100, 131] \to 1.3100000$	$(100, 141] \to 1.4100000$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(131, 231] \to 1.7633588$	$\blacktriangledown\blacktriangledown\blacktriangledown(141, 241] \to 1.7092199$
2)	$\blacktriangle\blacktriangle\blacktriangle(231, 362] \to 1.5670996$	$\blacktriangle\blacktriangle\blacktriangle(241, 382] \to 1.5850622$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(362, 593] \to 1.6381215$	$\blacktriangledown\blacktriangledown\blacktriangledown(382, 623] \to 1.6308901$
4)	$\blacktriangle\blacktriangle\blacktriangle(593, 955] \to 1.6104553$	$\blacktriangle\blacktriangle\blacktriangle(623, 1005] \to 1.6131621$
5)	$\blacktriangle\blacktriangledown\blacktriangle(955, 1548] \to 1.6209424$	$\blacktriangledown\blacktriangle\blacktriangledown(1005, 1628] \to 1.6199005$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1548, 2503] \to 1.6169251$	$\blacktriangle\blacktriangledown\blacktriangle(1628, 2633] \to 1.6173219$
7)	$\blacktriangle\blacktriangledown\blacktriangle(2503, 4051] \to 1.6184579$	$\blacktriangledown\blacktriangle\blacktriangledown(2633, 4261] \to 1.6183061$
8)	$\blacktriangle\blacktriangledown\blacktriangle(4051, 6554] \to 1.6178721$	$\blacktriangle\blacktriangledown\blacktriangle(4261, 6894] \to 1.6179301$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(6554, 10605] \to 1.6180958$	$\blacktriangledown\blacktriangle\blacktriangledown(6894, 11155] \to 1.6180737$
10)	$\blacktriangle\blacktriangledown\blacktriangle(10605, 17159] \to 1.6180104$	$\blacktriangledown\blacktriangle\blacktriangledown(11155, 18049] \to 1.6180188$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(17159, 27764] \to 1.6180430$	$\blacktriangle\blacktriangledown\blacktriangle(18049, 29204] \to 1.6180398$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(27764, 44923] \to 1.6180305$	$\blacktriangle\blacktriangledown\blacktriangle(29204, 47253] \to 1.6180318$
13)	$\blacktriangle\blacktriangledown\blacktriangle(44923, 72687] \to 1.6180353$	$\blacktriangledown\blacktriangle\blacktriangledown(47253, 76457] \to 1.6180348$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(72687, 117610] \to 1.6180335$	$\blacktriangle\blacktriangledown\blacktriangle(76457, 123710] \to 1.6180337$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(117610, 190297] \to 1.6180342$	$\blacktriangle\blacktriangledown\blacktriangle(123710, 200167] \to 1.6180341$
16)	$\blacktriangle\blacktriangledown\blacktriangle(190297, 307907] \to 1.6180339$	$\blacktriangledown\blacktriangle\blacktriangledown(200167, 323877] \to 1.6180339$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(307907, 498204] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(323877, 524044] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(498204, 806111] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(524044, 847921] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(806111, 1304315] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(847921, 1371965] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1304315, 2110426] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1371965, 2219886] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(2110426, 3414741] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2219886, 3591851] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(3414741, 5525167] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3591851, 5811737] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(5525167, 8939908] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(5811737, 9403588] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(8939908, 14465075] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(9403588, 15215325] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(14465075, 23404983] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(15215325, 24618913] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(23404983, 37870058] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(24618913, 39834238] \to 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(37870058, 61275041] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(39834238, 64453151] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(61275041, 99145099] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(64453151, 104287389] \to 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(99145099, 160420140] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(104287389, 168740540] \to 1.6180340$
30)	$\blacktriangledown\blacktriangle\blacktriangledown(160420140, 259565239] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(168740540, 273027929] \to 1.6180340$
31)	$\blacktriangle\blacktriangledown\blacktriangle(259565239, 419985379] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(273027929, 441768469] \to 1.6180340$

0)	$(100, 151] \to 1.5100000$	$(100, 161] \to 1.6100000$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(151, 251] \to 1.6622517$	$\blacktriangle\blacktriangledown\blacktriangle(161, 261] \to 1.6211180$
2)	$\blacktriangle\blacktriangle\blacktriangle(251, 402] \to 1.6015936$	$\blacktriangledown\blacktriangle\blacktriangledown(261, 422] \to 1.6168582$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(402, 653] \to 1.6243781$	$\blacktriangle\blacktriangledown\blacktriangle(422, 683] \to 1.6184834$
4)	$\blacktriangle\blacktriangle\blacktriangle(653, 1055] \to 1.6156202$	$\blacktriangle\blacktriangledown\blacktriangle(683, 1105] \to 1.6178624$
5)	$\blacktriangle\blacktriangledown\blacktriangle(1055, 1708] \to 1.6189573$	$\blacktriangle\blacktriangledown\blacktriangle(1105, 1788] \to 1.6180995$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1708, 2763] \to 1.6176815$	$\blacktriangledown\blacktriangle\blacktriangledown(1788, 2893] \to 1.6180089$
7)	$\blacktriangle\blacktriangledown\blacktriangle(2763, 4471] \to 1.6181687$	$\blacktriangledown\blacktriangle\blacktriangledown(2893, 4681] \to 1.6180436$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(4471, 7234] \to 1.6179826$	$\blacktriangledown\blacktriangle\blacktriangledown(4681, 7574] \to 1.6180303$
9)	$\blacktriangle\blacktriangledown\blacktriangle(7234, 11705] \to 1.6180536$	$\blacktriangledown\blacktriangle\blacktriangledown(7574, 12255] \to 1.6180354$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(11705, 18939] \to 1.6180265$	$\blacktriangledown\blacktriangle\blacktriangledown(12255, 19829] \to 1.6180335$
11)	$\blacktriangle\blacktriangledown\blacktriangle(18939, 30644] \to 1.6180369$	$\blacktriangledown\blacktriangle\blacktriangledown(19829, 32084] \to 1.6180342$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(30644, 49583] \to 1.6180329$	$\blacktriangledown\blacktriangle\blacktriangledown(32084, 51913] \to 1.6180339$
13)	$\blacktriangle\blacktriangledown\blacktriangle(49583, 80227] \to 1.6180344$	$\blacktriangle\blacktriangledown\blacktriangle(51913, 83997] \to 1.6180340$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(80227, 129810] \to 1.6180338$	$\blacktriangledown\blacktriangle\blacktriangledown(83997, 135910] \to 1.6180340$
15)	$\blacktriangle\blacktriangledown\blacktriangle(129810, 210037] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(135910, 219907] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(210037, 339847] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(219907, 355817] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(339847, 549884] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(355817, 575724] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(549884, 889731] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(575724, 931541] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(889731, 1439615] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(931541, 1507265] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1439615, 2329346] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1507265, 2438806] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(2329346, 3768961] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2438806, 3946071] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(3768961, 6098307] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3946071, 6384877] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(6098307, 9867268] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6384877, 10330948] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(9867268, 15965575] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(10330948, 16715825] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(15965575, 25832843] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(16715825, 27046773] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(25832843, 41798418] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(27046773, 43762598] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(41798418, 67631261] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(43762598, 70809371] \to 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(67631261, 109429679] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(70809371, 114571969] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(109429679, 177060940] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(114571969, 185381340] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(177060940, 286490619] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(185381340, 299953309] \to 1.6180340$
31)	$\blacktriangledown\blacktriangle\blacktriangledown(286490619, 463551559] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(299953309, 485334649] \to 1.6180340$

0)	$(100, 171] \to 1.7100000$	$(100, 181] \to 1.8100000$
1)	$\blacktriangle\blacktriangle\blacktriangle(171, 271] \to 1.5847953$	$\blacktriangle\blacktriangle\blacktriangle(181, 281] \to 1.5524862$
2)	$\blacktriangle\blacktriangledown\blacktriangle(271, 442] \to 1.6309963$	$\blacktriangledown\blacktriangledown\blacktriangledown(281, 462] \to 1.6441281$
3)	$\blacktriangle\blacktriangle\blacktriangle(442, 713] \to 1.6131222$	$\blacktriangle\blacktriangle\blacktriangle(462, 743] \to 1.6082251$
4)	$\blacktriangle\blacktriangledown\blacktriangle(713, 1155] \to 1.6199158$	$\blacktriangle\blacktriangledown\blacktriangle(743, 1205] \to 1.6218035$
5)	$\blacktriangle\blacktriangledown\blacktriangle(1155, 1868] \to 1.6173160$	$\blacktriangledown\blacktriangle\blacktriangledown(1205, 1948] \to 1.6165975$
6)	$\blacktriangledown\blacktriangle\blacktriangledown(1868, 3023] \to 1.6183084$	$\blacktriangledown\blacktriangledown\blacktriangledown(1948, 3153] \to 1.6185832$
7)	$\blacktriangle\blacktriangledown\blacktriangle(3023, 4891] \to 1.6179292$	$\blacktriangledown\blacktriangle\blacktriangledown(3153, 5101] \to 1.6178243$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(4891, 7914] \to 1.6180740$	$\blacktriangle\blacktriangledown\blacktriangle(5101, 8254] \to 1.6181141$
9)	$\blacktriangle\blacktriangledown\blacktriangle(7914, 12805] \to 1.6180187$	$\blacktriangledown\blacktriangle\blacktriangledown(8254, 13355] \to 1.6180034$
10)	$\blacktriangle\blacktriangledown\blacktriangle(12805, 20719] \to 1.6180398$	$\blacktriangle\blacktriangledown\blacktriangle(13355, 21609] \to 1.6180457$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(20719, 33524] \to 1.6180318$	$\blacktriangledown\blacktriangle\blacktriangledown(21609, 34964] \to 1.6180295$
12)	$\blacktriangle\blacktriangledown\blacktriangle(33524, 54243] \to 1.6180348$	$\blacktriangle\blacktriangledown\blacktriangle(34964, 56573] \to 1.6180357$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(54243, 87767] \to 1.6180337$	$\blacktriangledown\blacktriangle\blacktriangledown(56573, 91537] \to 1.6180333$
14)	$\blacktriangle\blacktriangledown\blacktriangle(87767, 142010] \to 1.6180341$	$\blacktriangle\blacktriangledown\blacktriangle(91537, 148110] \to 1.6180342$
15)	$\blacktriangle\blacktriangledown\blacktriangle(142010, 229777] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(148110, 239647] \to 1.6180339$
16)	$\blacktriangle\blacktriangledown\blacktriangle(229777, 371787] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(239647, 387757] \to 1.6180340$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(371787, 601564] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(387757, 627404] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(601564, 973351] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(627404, 1015161] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(973351, 1574915] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1015161, 1642565] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1574915, 2548266] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1642565, 2657726] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(2548266, 4123181] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2657726, 4300291] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(4123181, 6671447] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4300291, 6958017] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(6671447, 10794628] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(6958017, 11258308] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(10794628, 17466075] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11258308, 18216325] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(17466075, 28260703] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(18216325, 29474633] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(28260703, 45726778] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(29474633, 47690958] \to 1.6180340$
27)	$\blacktriangle\blacktriangledown\blacktriangle(45726778, 73987481] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(47690958, 77165591] \to 1.6180340$
28)	$\blacktriangle\blacktriangledown\blacktriangle(73987481, 119714259] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(77165591, 124856549] \to 1.6180340$
29)	$\blacktriangledown\blacktriangle\blacktriangledown(119714259, 193701740] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(124856549, 202022140] \to 1.6180340$
30)	$\blacktriangle\blacktriangledown\blacktriangle(193701740, 313415999] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(202022140, 326878689] \to 1.6180340$

0)	$(100, 191] \to 1.9100000$	$(100, 201] \to 2.0100000$
1)	$\blacktriangle\blacktriangle\blacktriangle(191, 291] \to 1.5235602$	$\blacktriangle\blacktriangle\blacktriangle(201, 301] \to 1.4975124$
2)	$\blacktriangledown\blacktriangledown\blacktriangledown(291, 482] \to 1.6563574$	$\blacktriangledown\blacktriangledown\blacktriangledown(301, 502] \to 1.6677741$
3)	$\blacktriangle\blacktriangle\blacktriangle(482, 773] \to 1.6037344$	$\blacktriangle\blacktriangle\blacktriangle(502, 803] \to 1.5996016$
4)	$\blacktriangle\blacktriangledown\blacktriangle(773, 1255] \to 1.6235446$	$\blacktriangledown\blacktriangledown\blacktriangledown(803, 1305] \to 1.6251557$
5)	$\blacktriangledown\blacktriangle\blacktriangledown(1255, 2028] \to 1.6159363$	$\blacktriangle\blacktriangle\blacktriangle(1305, 2108] \to 1.6153257$
6)	$\blacktriangle\blacktriangledown\blacktriangle(2028, 3283] \to 1.6188363$	$\blacktriangledown\blacktriangledown\blacktriangledown(2108, 3413] \to 1.6190702$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(3283, 5311] \to 1.6177277$	$\blacktriangledown\blacktriangle\blacktriangledown(3413, 5521] \to 1.6176384$
8)	$\blacktriangle\blacktriangledown\blacktriangle(5311, 8594] \to 1.6181510$	$\blacktriangle\blacktriangledown\blacktriangle(5521, 8934] \to 1.6181851$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(8594, 13905] \to 1.6179893$	$\blacktriangle\blacktriangle\blacktriangle(8934, 14455] \to 1.6179763$
10)	$\blacktriangle\blacktriangledown\blacktriangle(13905, 22499] \to 1.6180511$	$\blacktriangledown\blacktriangle\blacktriangledown(14455, 23389] \to 1.6180560$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(22499, 36404] \to 1.6180275$	$\blacktriangle\blacktriangledown\blacktriangle(23389, 37844] \to 1.6180256$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(36404, 58903] \to 1.6180365$	$\blacktriangle\blacktriangledown\blacktriangle(37844, 61233] \to 1.6180372$
13)	$\blacktriangle\blacktriangledown\blacktriangle(58903, 95307] \to 1.6180330$	$\blacktriangle\blacktriangledown\blacktriangle(61233, 99077] \to 1.6180328$
14)	$\blacktriangle\blacktriangledown\blacktriangle(95307, 154210] \to 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(99077, 160310] \to 1.6180345$
15)	$\blacktriangle\blacktriangledown\blacktriangle(154210, 249517] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(160310, 259387] \to 1.6180338$
16)	$\blacktriangle\blacktriangledown\blacktriangle(249517, 403727] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(259387, 419697] \to 1.6180341$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(403727, 653244] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(419697, 679084] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(653244, 1056971] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(679084, 1098781] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(1056971, 1710215] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(1098781, 1777865] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(1710215, 2767186] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1777865, 2876646] \to 1.6180340$
21)	$\blacktriangledown\blacktriangle\blacktriangledown(2767186, 4477401] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2876646, 4654511] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(4477401, 7244587] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(4654511, 7531157] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(7244587, 11721988] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(7531157, 12185668] \to 1.6180340$
24)	$\blacktriangle\blacktriangledown\blacktriangle(11721988, 18966575] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(12185668, 19716825] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(18966575, 30688563] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(19716825, 31902493] \to 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(30688563, 49655138] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(31902493, 51619318] \to 1.6180340$
27)	$\blacktriangledown\blacktriangle\blacktriangledown(49655138, 80343701] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(51619318, 83521811] \to 1.6180340$
28)	$\blacktriangledown\blacktriangle\blacktriangledown(80343701, 129998839] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(83521811, 135141129] \to 1.6180340$
29)	$\blacktriangle\blacktriangledown\blacktriangle(129998839, 210342540] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(135141129, 218662940] \to 1.6180340$
30)	$\blacktriangledown\blacktriangledown\blacktriangledown(210342540, 340341379] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(218662940, 353804069] \to 1.6180340$

APPENDIX Y: USING 1000 AND $1000 + N$ AS INITIATORS, WHERE $N = 118,\ 218,\ 318,\ \ldots\ 1018$

0)	$(1000, 1118] \to 1.1180000$	$(1000, 1218] \to 1.2180000$
1)	▼▼▼$(1118, 2118] \to 1.8944544$	▼▼▼$(1218, 2218] \to 1.8210181$
2)	▲▲▲$(2118, 3236] \to 1.5278565$	▲▲▲$(2218, 3436] \to 1.5491434$
3)	▼▼▼$(3236, 5354] \to 1.6545117$	▼▼▼$(3436, 5654] \to 1.6455180$
4)	▲▲▲$(5354, 8590] \to 1.6044079$	▲▲▲$(5654, 9090] \to 1.6077114$
5)	▼▼▼$(8590, 13944] \to 1.6232829$	▼▼▼$(9090, 14744] \to 1.6220022$
6)	▲▲▲$(13944, 22534] \to 1.6160356$	▲▲▲$(14744, 23834] \to 1.6165220$
7)	▼▼▼$(22534, 36478] \to 1.6187983$	▲▼▲$(23834, 38578] \to 1.6186121$
8)	▼▲▼$(36478, 59012] \to 1.6177422$	▼▲▼$(38578, 62412] \to 1.6178133$
9)	▲▼▲$(59012, 95490] \to 1.6181455$	▲▼▲$(62412, 100990] \to 1.6181183$
10)	▲▼▲$(95490, 154502] \to 1.6179914$	▼▲▼$(100990, 163402] \to 1.6180018$
11)	▲▼▲$(154502, 249992] \to 1.6180503$	▲▼▲$(163402, 264392] \to 1.6180463$
12)	▲▼▲$(249992, 404494] \to 1.6180278$	▼▲▼$(264392, 427794] \to 1.6180293$
13)	▼▲▼$(404494, 654486] \to 1.6180364$	▼▲▼$(427794, 692186] \to 1.6180358$
14)	▼▲▼$(654486, 1058980] \to 1.6180331$	▲▼▲$(692186, 1119980] \to 1.6180333$
15)	▼▲▼$(1058980, 1713466] \to 1.6180343$	▲▼▲$(1119980, 1812166] \to 1.6180343$
16)	▲▼▲$(1713466, 2772446] \to 1.6180339$	▼▲▼$(1812166, 2932146] \to 1.6180339$
17)	▼▲▼$(2772446, 4485912] \to 1.6180340$	▲▼▲$(2932146, 4744312] \to 1.6180340$
18)	▲▼▲$(4485912, 7258358] \to 1.6180340$	▲▼▲$(4744312, 7676458] \to 1.6180340$
19)	▲▼▲$(7258358, 11744270] \to 1.6180340$	▼▲▼$(7676458, 12420770] \to 1.6180340$
20)	▼▲▼$(11744270, 19002628] \to 1.6180340$	▲▼▲$(12420770, 20097228] \to 1.6180340$
21)	▼▲▼$(19002628, 30746898] \to 1.6180340$	▼▲▼$(20097228, 32517998] \to 1.6180340$
22)	▲▼▲$(30746898, 49749526] \to 1.6180340$	▲▼▲$(32517998, 52615226] \to 1.6180340$
23)	▼▲▼$(49749526, 80496424] \to 1.6180340$	▲▼▲$(52615226, 85133224] \to 1.6180340$
24)	▼▲▼$(80496424, 130245950] \to 1.6180340$	▼▲▼$(85133224, 137748450] \to 1.6180340$
25)	▲▼▲$(130245950, 210742374] \to 1.6180340$	▲▼▲$(137748450, 222881674] \to 1.6180340$
26)	▼▲▼$(210742374, 340988324] \to 1.6180340$	▼▲▼$(222881674, 360630124] \to 1.6180340$

0)	$(1000, 1318] \to 1.3180000$	$(1000, 1418] \to 1.4180000$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(1318, 2318] \to 1.7587253$	$\blacktriangledown\blacktriangledown\blacktriangledown(1418, 2418] \to 1.7052186$
2)	$\blacktriangle\blacktriangle\blacktriangle(2318, 3636] \to 1.5685936$	$\blacktriangle\blacktriangle\blacktriangle(2418, 3836] \to 1.5864351$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(3636, 5954] \to 1.6375138$	$\blacktriangledown\blacktriangledown\blacktriangledown(3836, 6254] \to 1.6303441$
4)	$\blacktriangle\blacktriangle\blacktriangle(5954, 9590] \to 1.6106819$	$\blacktriangle\blacktriangle\blacktriangle(6254, 10090] \to 1.6133674$
5)	$\blacktriangledown\blacktriangledown\blacktriangledown(9590, 15544] \to 1.6208551$	$\blacktriangledown\blacktriangledown\blacktriangledown(10090, 16344] \to 1.6198216$
6)	$\blacktriangle\blacktriangle\blacktriangle(15544, 25134] \to 1.6169583$	$\blacktriangle\blacktriangle\blacktriangle(16344, 26434] \to 1.6173519$
7)	$\blacktriangledown\blacktriangledown\blacktriangledown(25134, 40678] \to 1.6184451$	$\blacktriangledown\blacktriangledown\blacktriangledown(26434, 42778] \to 1.6182946$
8)	$\blacktriangledown\blacktriangle\blacktriangledown(40678, 65812] \to 1.6178770$	$\blacktriangle\blacktriangledown\blacktriangle(42778, 69212] \to 1.6179345$
9)	$\blacktriangle\blacktriangledown\blacktriangle(65812, 106490] \to 1.6180940$	$\blacktriangledown\blacktriangle\blacktriangledown(69212, 111990] \to 1.6180720$
10)	$\blacktriangle\blacktriangle\blacktriangle(106490, 172302] \to 1.6180111$	$\blacktriangle\blacktriangle\blacktriangle(111990, 181202] \to 1.6180195$
11)	$\blacktriangledown\blacktriangle\blacktriangledown(172302, 278792] \to 1.6180427$	$\blacktriangledown\blacktriangle\blacktriangledown(181202, 293192] \to 1.6180395$
12)	$\blacktriangle\blacktriangledown\blacktriangle(278792, 451094] \to 1.6180306$	$\blacktriangle\blacktriangledown\blacktriangle(293192, 474394] \to 1.6180319$
13)	$\blacktriangle\blacktriangledown\blacktriangle(451094, 729886] \to 1.6180353$	$\blacktriangledown\blacktriangle\blacktriangledown(474394, 767586] \to 1.6180348$
14)	$\blacktriangledown\blacktriangle\blacktriangledown(729886, 1180980] \to 1.6180335$	$\blacktriangle\blacktriangledown\blacktriangle(767586, 1241980] \to 1.6180337$
15)	$\blacktriangle\blacktriangledown\blacktriangle(1180980, 1910866] \to 1.6180342$	$\blacktriangledown\blacktriangle\blacktriangledown(1241980, 2009566] \to 1.6180341$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(1910866, 3091846] \to 1.6180339$	$\blacktriangle\blacktriangledown\blacktriangle(2009566, 3251546] \to 1.6180339$
17)	$\blacktriangledown\blacktriangle\blacktriangledown(3091846, 5002712] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(3251546, 5261112] \to 1.6180340$
18)	$\blacktriangle\blacktriangledown\blacktriangle(5002712, 8094558] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(5261112, 8512658] \to 1.6180340$
19)	$\blacktriangledown\blacktriangle\blacktriangledown(8094558, 13097270] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(8512658, 13773770] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(13097270, 21191828] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(13773770, 22286428] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(21191828, 34289098] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(22286428, 36060198] \to 1.6180340$
22)	$\blacktriangle\blacktriangledown\blacktriangle(34289098, 55480926] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(36060198, 58346626] \to 1.6180340$
23)	$\blacktriangledown\blacktriangle\blacktriangledown(55480926, 89770024] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(58346626, 94406824] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(89770024, 145250950] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(94406824, 152753450] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(145250950, 235020974] \to 1.6180340$	$\blacktriangledown\blacktriangledown\blacktriangledown(152753450, 247160274] \to 1.6180340$
26)	$\blacktriangledown\blacktriangle\blacktriangledown(235020974, 380271924] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(247160274, 399913724] \to 1.6180340$

0)	$(1000, 1518] \to 1.5180000$	$(1000, 1618] \to 1.6180000$
1)	$\blacktriangledown\blacktriangledown\blacktriangledown(1518, 2518] \to 1.6587615$	$\blacktriangle\blacktriangledown\blacktriangle(1618, 2618] \to 1.6180470$
2)	$\blacktriangle\blacktriangle\blacktriangle(2518, 4036] \to 1.6028594$	$\blacktriangledown\blacktriangle\blacktriangledown(2618, 4236] \to 1.6180290$
3)	$\blacktriangledown\blacktriangledown\blacktriangledown(4036, 6554] \to 1.6238850$	$\blacktriangle\blacktriangledown\blacktriangle(4236, 6854] \to 1.6180359$
4)	$\blacktriangle\blacktriangle\blacktriangle(6554, 10590] \to 1.6158071$	$\blacktriangle\blacktriangledown\blacktriangle(6854, 11090] \to 1.6180333$
5)	$\blacktriangledown\blacktriangledown\blacktriangledown(10590, 17144] \to 1.6188857$	$\blacktriangledown\blacktriangle\blacktriangledown(11090, 17944] \to 1.6180343$
6)	$\blacktriangle\blacktriangle\blacktriangle(17144, 27734] \to 1.6177088$	$\blacktriangledown\blacktriangle\blacktriangledown(17944, 29034] \to 1.6180339$
7)	$\blacktriangledown\blacktriangle\blacktriangledown(27734, 44878] \to 1.6181582$	$\blacktriangle\blacktriangledown\blacktriangle(29034, 46978] \to 1.6180340$
8)	$\blacktriangle\blacktriangledown\blacktriangle(44878, 72612] \to 1.6179865$	$\blacktriangledown\blacktriangle\blacktriangledown(46978, 76012] \to 1.6180340$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(72612, 117490] \to 1.6180521$	$\blacktriangle\blacktriangledown\blacktriangle(76012, 122990] \to 1.6180340$
10)	$\blacktriangledown\blacktriangle\blacktriangledown(117490, 190102] \to 1.6180271$	$\blacktriangledown\blacktriangle\blacktriangledown(122990, 199002] \to 1.6180340$
11)	$\blacktriangle\blacktriangledown\blacktriangle(190102, 307592] \to 1.6180366$	$\blacktriangle\blacktriangledown\blacktriangle(199002, 321992] \to 1.6180340$
12)	$\blacktriangledown\blacktriangle\blacktriangledown(307592, 497694] \to 1.6180330$	$\blacktriangledown\blacktriangle\blacktriangledown(321992, 520994] \to 1.6180340$
13)	$\blacktriangle\blacktriangledown\blacktriangle(497694, 805286] \to 1.6180344$	$\blacktriangle\blacktriangledown\blacktriangle(520994, 842986] \to 1.6180340$
14)	$\blacktriangle\blacktriangledown\blacktriangle(805286, 1302980] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(842986, 1363980] \to 1.6180340$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(1302980, 2108266] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(1363980, 2206966] \to 1.6180340$
16)	$\blacktriangle\blacktriangledown\blacktriangle(2108266, 3411246] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(2206966, 3570946] \to 1.6180340$
17)	$\blacktriangle\blacktriangledown\blacktriangle(3411246, 5519512] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(3570946, 5777912] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(5519512, 8930758] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(5777912, 9348858] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(8930758, 14450270] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(9348858, 15126770] \to 1.6180340$
20)	$\blacktriangledown\blacktriangle\blacktriangledown(14450270, 23381028] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(15126770, 24475628] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(23381028, 37831298] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(24475628, 39602398] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(37831298, 61212326] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(39602398, 64078026] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(61212326, 99043624] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(64078026, 103680424] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(99043624, 160255950] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(103680424, 167758450] \to 1.6180340$
25)	$\blacktriangledown\blacktriangle\blacktriangledown(160255950, 259299574] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(167758450, 271438874] \to 1.6180340$
26)	$\blacktriangle\blacktriangledown\blacktriangle(259299574, 419555524] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(271438874, 439197324] \to 1.6180340$

0)	$(1000, 1718] \to 1.7180000$	$(1000, 1818] \to 1.8180000$
1)	▲▲▲$(1718, 2718] \to 1.5820722$	▲▲▲$(1818, 2818] \to 1.5500550$
2)	▼▼▼$(2718, 4436] \to 1.6320824$	▼▼▼$(2818, 4636] \to 1.6451384$
3)	▲▲▲$(4436, 7154] \to 1.6127142$	▲▲▲$(4636, 7454] \to 1.6078516$
4)	▲▼▲$(7154, 11590] \to 1.6200727$	▼▼▼$(7454, 12090] \to 1.6219479$
5)	▼▲▼$(11590, 18744] \to 1.6172563$	▲▲▲$(12090, 19544] \to 1.6165426$
6)	▲▼▲$(18744, 30334] \to 1.6183312$	▲▼▲$(19544, 31634] \to 1.6186042$
7)	▼▲▼$(30334, 49078] \to 1.6179205$	▼▲▼$(31634, 51178] \to 1.6178163$
8)	▼▲▼$(49078, 79412] \to 1.6180773$	▼▲▼$(51178, 82812] \to 1.6181172$
9)	▲▲▲$(79412, 128490] \to 1.6180174$	▲▼▲$(82812, 133990] \to 1.6180022$
10)	▲▼▲$(128490, 207902] \to 1.6180403$	▼▲▼$(133990, 216802] \to 1.6180461$
11)	▼▲▼$(207902, 336392] \to 1.6180316$	▼▲▼$(216802, 350792] \to 1.6180294$
12)	▲▼▲$(336392, 544294] \to 1.6180349$	▲▼▲$(350792, 567594] \to 1.6180358$
13)	▲▼▲$(544294, 880686] \to 1.6180336$	▼▲▼$(567594, 918386] \to 1.6180333$
14)	▲▼▲$(880686, 1424980] \to 1.6180341$	▲▼▲$(918386, 1485980] \to 1.6180342$
15)	▼▲▼$(1424980, 2305666] \to 1.6180339$	▼▲▼$(1485980, 2404366] \to 1.6180339$
16)	▲▼▲$(2305666, 3730646] \to 1.6180340$	▼▲▼$(2404366, 3890346] \to 1.6180340$
17)	▲▼▲$(3730646, 6036312] \to 1.6180340$	▼▲▼$(3890346, 6294712] \to 1.6180340$
18)	▼▲▼$(6036312, 9766958] \to 1.6180340$	▲▼▲$(6294712, 10185058] \to 1.6180340$
19)	▲▼▲$(9766958, 15803270] \to 1.6180340$	▼▲▼$(10185058, 16479770] \to 1.6180340$
20)	▼▲▼$(15803270, 25570228] \to 1.6180340$	▼▲▼$(16479770, 26664828] \to 1.6180340$
21)	▼▲▼$(25570228, 41373498] \to 1.6180340$	▲▼▲$(26664828, 43144598] \to 1.6180340$
22)	▲▼▲$(41373498, 66943726] \to 1.6180340$	▼▲▼$(43144598, 69809426] \to 1.6180340$
23)	▼▲▼$(66943726, 108317224] \to 1.6180340$	▲▼▲$(69809426, 112954024] \to 1.6180340$
24)	▲▼▲$(108317224, 175260950] \to 1.6180340$	▲▼▲$(112954024, 182763450] \to 1.6180340$
25)	▲▼▲$(175260950, 283578174] \to 1.6180340$	▼▲▼$(182763450, 295717474] \to 1.6180340$
26)	▲▼▲$(283578174, 458839124] \to 1.6180340$	▼▲▼$(295717474, 478480924] \to 1.6180340$

0)	$(1000, 1918] \to 1.9180000$	$(1000, 2018] \to 2.0180000$
1)	$\blacktriangle\blacktriangle\blacktriangle(1918, 2918] \to 1.5213764$	$\blacktriangle\blacktriangle\blacktriangle(2018, 3018] \to 1.4955401$
2)	$\blacktriangledown\blacktriangledown\blacktriangledown(2918, 4836] \to 1.6572995$	$\blacktriangledown\blacktriangledown\blacktriangledown(3018, 5036] \to 1.6686547$
3)	$\blacktriangle\blacktriangle\blacktriangle(4836, 7754] \to 1.6033912$	$\blacktriangle\blacktriangle\blacktriangle(5036, 8054] \to 1.5992851$
4)	$\blacktriangledown\blacktriangledown\blacktriangledown(7754, 12590] \to 1.6236781$	$\blacktriangledown\blacktriangledown\blacktriangledown(8054, 13090] \to 1.6252794$
5)	$\blacktriangle\blacktriangle\blacktriangle(12590, 20344] \to 1.6158856$	$\blacktriangle\blacktriangle\blacktriangle(13090, 21144] \to 1.6152788$
6)	$\blacktriangledown\blacktriangledown\blacktriangledown(20344, 32934] \to 1.6188557$	$\blacktriangledown\blacktriangledown\blacktriangledown(21144, 34234] \to 1.6190882$
7)	$\blacktriangle\blacktriangle\blacktriangle(32934, 53278] \to 1.6177203$	$\blacktriangle\blacktriangle\blacktriangle(34234, 55378] \to 1.6176316$
8)	$\blacktriangle\blacktriangledown\blacktriangle(53278, 86212] \to 1.6181538$	$\blacktriangledown\blacktriangledown\blacktriangledown(55378, 89612] \to 1.6181877$
9)	$\blacktriangledown\blacktriangle\blacktriangledown(86212, 139490] \to 1.6179882$	$\blacktriangle\blacktriangle\blacktriangle(89612, 144990] \to 1.6179753$
10)	$\blacktriangle\blacktriangledown\blacktriangle(139490, 225702] \to 1.6180515$	$\blacktriangle\blacktriangledown\blacktriangle(144990, 234602] \to 1.6180564$
11)	$\blacktriangle\blacktriangledown\blacktriangle(225702, 365192] \to 1.6180273$	$\blacktriangle\blacktriangledown\blacktriangle(234602, 379592] \to 1.6180254$
12)	$\blacktriangle\blacktriangledown\blacktriangle(365192, 590894] \to 1.6180365$	$\blacktriangledown\blacktriangle\blacktriangledown(379592, 614194] \to 1.6180373$
13)	$\blacktriangledown\blacktriangle\blacktriangledown(590894, 956086] \to 1.6180330$	$\blacktriangle\blacktriangledown\blacktriangle(614194, 993786] \to 1.6180327$
14)	$\blacktriangle\blacktriangledown\blacktriangle(956086, 1546980] \to 1.6180344$	$\blacktriangledown\blacktriangle\blacktriangledown(993786, 1607980] \to 1.6180345$
15)	$\blacktriangledown\blacktriangle\blacktriangledown(1546980, 2503066] \to 1.6180338$	$\blacktriangle\blacktriangledown\blacktriangle(1607980, 2601766] \to 1.6180338$
16)	$\blacktriangledown\blacktriangle\blacktriangledown(2503066, 4050046] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(2601766, 4209746] \to 1.6180341$
17)	$\blacktriangle\blacktriangledown\blacktriangle(4050046, 6553112] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(4209746, 6811512] \to 1.6180340$
18)	$\blacktriangledown\blacktriangle\blacktriangledown(6553112, 10603158] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(6811512, 11021258] \to 1.6180340$
19)	$\blacktriangle\blacktriangledown\blacktriangle(10603158, 17156270] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(11021258, 17832770] \to 1.6180340$
20)	$\blacktriangle\blacktriangledown\blacktriangle(17156270, 27759428] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(17832770, 28854028] \to 1.6180340$
21)	$\blacktriangle\blacktriangledown\blacktriangle(27759428, 44915698] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(28854028, 46686798] \to 1.6180340$
22)	$\blacktriangledown\blacktriangle\blacktriangledown(44915698, 72675126] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(46686798, 75540826] \to 1.6180340$
23)	$\blacktriangle\blacktriangledown\blacktriangle(72675126, 117590824] \to 1.6180340$	$\blacktriangle\blacktriangledown\blacktriangle(75540826, 122227624] \to 1.6180340$
24)	$\blacktriangledown\blacktriangle\blacktriangledown(117590824, 190265950] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(122227624, 197768450] \to 1.6180340$
25)	$\blacktriangle\blacktriangledown\blacktriangle(190265950, 307856774] \to 1.6180340$	$\blacktriangledown\blacktriangle\blacktriangledown(197768450, 319996074] \to 1.6180340$

[1] Y. Koide, "A fermion - boson composite model of quarks and leptons," Phys. Lett. B **120**, 161 (1983).
[2] Y. Koide, "A new view of quark and lepton mass hierarchy," Phys. Rev. D **28**, 252 (1983).
[3] J. S. Markovitch, "The Koide formula and its analogues," (2012) http://vixra.org/abs/1207.0052.
[4] J. S. Markovitch, "On the distribution of prime numbers in the intervals defined by the Fibonacci numbers," (2010) http://vixra.org/abs/1008.0036.