Imaginary Physics

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Abstract

The ubiquity of complex numbers throughout fundamental physics has never been satisfactorily explained. Moreover, the mathematical primacy of complex and imaginary numbers suggests the primacy of complex and imaginary structures in Nature, while further implying the existence of imaginary spatial dimensions preceding real dimensions. On this basis a consistent cosmological framework is erected, guided by a direct reading of the empirical and theoretical evidence, embracing essential principles of quantum theory, relativity theory, and string/M-theory.

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Introduction

The Problem of Complexity

The mystery of imaginary numbers can be appreciated by mathematicians and nonspecialists alike. The eminent mathematical physicist Sir Roger Penrose introduces what he calls "the magic number i", the square root of minus one, as follows: [1]

> How is it that –1 can have a square root? The square of a positive number is always positive, and the square of a negative number is again positive (and the square of 0 is just 0 again, so that is hardly of use to us here). It seems impossible that we can find a number whose square is actually negative.

We can appreciate the early mathematicians calling such numbers "impossible" and simply disregarding them when they appeared. While real numbers can represent some notion of *quantity* in our physical (3+1) spacetime – our three spatial dimensions and one time dimension each being measured in real units – there can be no such interpretation of imaginary numbers. Complex numbers (combining both real and imaginary numbers) abound in fundamental physical theory and are renowned for their powerful and "magical" properties, yet no satisfactory ontology of complex and imaginary numbers has come forth. Nevertheless, so long as a calculation yields real numbers, one doesn't need to question how imaginary quantities can so powerfully model physical phenomena – one might just imagine them occupying some abstract mathematical realm. This position became somewhat untenable with the advent of quantum mechanics, however, wherein the primary entity of a quantum system, the *wavefunction* or *state vector*, is itself complex. John G. Cramer, the originator of the Transactional Interpretation of quantum mechanics (TI), sums up the problem of *complexity* in quantum mechanics as follows: [2]

> One of the serious objections to Schrödinger's early semiclassical interpretation of the SV [state vector]… is that the SV is a complex quantity. Complex functions are also found in classical physics, but are invariably interpreted either (1) as an indication that the solution is unphysical, as in the case of the Lorentz transformations with $v > c$, or (2) as a shorthand way of dealing with two independent and equally valid solutions of the equations, one real and one imaginary, as in the case of complex electrical impedance. In the latter case the complex algebra is essentially a mathematical device for avoiding trigonometry, and the physical variables of interest are ultimately extracted as the real (or imaginary) part of the complex variables. Never in classical physics is the full complex function "swallowed whole" as it is in quantum mechanics. This is the problem of complexity.

This dichotomy cannot be overstated. In a nutshell, modern physical theory is built upon nonphysical numbers, quantities having no representation in our 3+1 spacetime. While working physicists apply complex numbers on a daily basis without a second thought, the more thoughtful stop to ponder. To set the stage I wish to quote Roger Penrose at some length, writing here in conclusion to his masterful tome *The Road to Reality*, the passage appearing just ten pages from the end of the book. Keep in mind that this represents the sober reflections of an expert mathematical physicist, reaching towards fundamental "questions of principle" from his deep and broad perspective on modern physics. [3]

> I should like to single out just two particular aspects of the mathematics that underlies our understanding of the workings of the world… for I believe that they may hint at important but largely unaddressed questions of principle in our physical theory. The first is the role of the *complex-number* system, which we find to be so fundamental to the operations of quantum mechanics – as opposed to the real-number system, which had provided the foundation of all successful previous theories. The second is the role of *symmetry*, which has a central importance in virtually all 20th-century theories, particularly in relation to the gauge-theory formulation of physical interactions.
>
> First, consider the complex numbers. It has been a recurring theme of this book that there is not only a special

> magic in the mathematics of these numbers, but that Nature herself appears to harness this magic in weaving her universe at its deepest levels. Yet we may well question whether this is really a true feature of our world, or whether it is merely the mathematical utility of these numbers that has led to their extensive use in physical theory. Many physicists would, I believe, lean towards this second view. But, to them, there is still something of a mystery – needing some kind of explanation – as to why the role of these numbers should appear to be so universal in the framework of quantum theory…. To such physicists, the real numbers seem 'natural' and the complex numbers 'mysterious'. But from a purely mathematical standpoint, there is nothing especially more 'natural' about the real numbers than the complex numbers. Indeed, in view of the somewhat magical mathematical status of the complex numbers, one might well take the opposite view and regard them as being distinctly more 'natural' or 'God-given' than the reals.
>
> From my own peculiar standpoint, the importance of complex numbers… in the basis of physics is indeed to be viewed as a 'natural' thing, and the puzzle is indeed perhaps the other way around. How is it that real structures seem to play such an important part in physics?

The final sentence sets the theme for what follows. Penrose makes the crucial point that the complex numbers are more fundamental than the real numbers – they *precede* the real numbers, are more "God-given". Pure imaginary numbers are also in some sense more fundamental than real numbers. While going from the real numbers to the imaginaries requires taking the root of a negative, real numbers emerge from the product or quotient of any two imaginary numbers, or by raising to any even power. No construction of real numbers can yield an imaginary number. Further, the complex numbers are *complete* – any operation on complex numbers will land you back in the complex numbers – while the real numbers are not.

The mathematical primacy of complex and imaginary numbers forms the foundation stone for the current work. We take the view that, if mathematical truths indeed reflect physical truths (and vice versa), then complex and imaginary structures precede real structures in Nature. On this basis we set out to erect a consistent cosmological framework embracing essential principles of geometry, quantum mechanics, relativity theory, and string/M-theory.

The argument set forth in this paper does not pretend to be physics in the canonical sense. The reader may consider it philosophy, direct inference and logic, built upon the ground of physical and mathematical facts and principles while assuming as little as possible. Wherever practical, experts are called upon to present a current understanding of the relevant physics. As a general rule, the most direct reading of the facts is taken.

Part 1

Real and Imaginary Dimensions

With the discovery that universal numerical patterns are embodied in geometric forms, the Greek geometers understood numbers as emerging from the properties of space, so called Euclidean 3-space, having three real dimensions. Extending this concept to the imaginary domain, just as the real numbers provide a measure of real space, imaginary numbers provide a measure of imaginary space. Herein lies our foundational conjecture:

- The complex functions and symmetries describing fundamental properties of Nature imply complex degrees of freedom, interpreted directly as real and imaginary spatial dimensions.

What exactly is an imaginary spatial dimension? For present purposes, let us simply contemplate a dimension of space having imaginary coordinates – that is, the dimension is measured in imaginary units. While imaginary dimensions are common in mathematics, note that here we are talking about imaginary dimensions existing objectively in Nature.

The empirical evidence suggests that our physical space has three real dimensions and no more. Where are these imaginary dimensions? Why don't they appear to us? We will be addressing these questions in due course, but first let us consider imaginary dimensions from the perspective of *symmetry* as it manifests in fundamental physics.

1.1 Symmetry and Complexity

Three key symmetry groups appear in the standard model of particle physics, known as gauge symmetries, with each involving transformations of complex numbers:

$U(1)$ – The unitary group of transformations of one complex variable.
$SU(2)$ – The special unitary group of transformations of two complex variables.
$SU(3)$ – The special unitary group of transformations of three complex variables.

For present purposes we will focus on the $SU(2)$ symmetry group, which is associated with the quantum attribute of *spin*. For expert testimony I am indebted to mathematician Peter Woit for his accessible introduction to symmetry groups and representation theory, appearing in his string theory critique *Not Even Wrong*. In the following excerpt Woit refers to the mathematician Herman Weyl, a key figure in the development of group theory and representation theory as subsequently applied to physics. [1]

> Weyl's theory applies to many different kinds of groups of transformations in higher dimensions, but the simplest cases are those where one takes the coordinates of these higher dimensions to be complex numbers. The case of the plane is the case of one complex number, the next case involves two complex numbers. Now one is in a situation where it is no longer possible to visualise what is going on. Two complex numbers correspond to four real numbers (two real parts and two imaginary parts), so these transformations are happening in four (real) dimensions.

Note Woit's assumption that the real and imaginary parts each correspond to real dimensions, reflecting the common procedure in classical physics whereby the real and imaginary parts of a complex number are extracted as real variables, as explained by John Cramer above. As Cramer points out, however, this procedure fails in the case of the wavefunction, which "swallows" the complex function whole, just as it fails in quantum mechanics generally. So we are brought to ask if this canonical interpretation is tenable. It boils down to this: in the context of quantum mechanics and symmetry groups, can an imaginary number represent a real quantity or dimension? Woit continues: [2]

> To make visualisation even trickier, there is an additional subtle piece of geometrical structure that does not exist when one is just dealing with real numbers. In the complex plane, multiplication by the imaginary unit corresponded geometrically to a 90-degree counterclockwise rotation. In four dimensions, there is not just one possible axis of rotation as in two dimensions, but an infinity of axes one could imagine rotating about counterclockwise by 90 degrees. The identification of the four dimensions with two complex numbers picks out these axes: it is the axis one rotates about when one multiplies the two complex numbers by the imaginary unit. So, two complex dimensions have both one more real dimension than one can visualize, and in addition have an unvisualisable extra structure.

This excerpt will deepen the reader's appreciation of the mechanisms underlying symmetry groups involving transformations of complex variables. Moreover, Woit's interpretation is confirmed that all four dimensions are real, though we do see the appearance of an "unvisualisable extra structure", something within or beyond the four "real" dimensions. This particular symmetry group came to be known as $SU(2)$, the special unitary group of transformations of two complex variables, which Woit further elucidates as follows: [3]

> The symmetry group $SU(2)$ has a very special role that brought it into play from the earliest days of quantum mechanics. It turns out, for not at all obvious reasons (but then, not much about the geometry of pairs of complex numbers is obvious), that the group $SU(2)$ is very closely related to the group of rotations in three real dimensions. Rotations of three dimensions are extremely important since they correspond to symmetry transformations of the three space dimensions of our real world, so physical systems can provide representations of this group. Mathematicians call the group of rotations of three-dimensional space the group of special orthogonal transformations of three (real) variables and denote this group $SO(3)$. The precise relation between $SO(3)$ and $SU(2)$ is that each rotation in three dimensions corresponds to two distinct elements of $SU(2)$, so $SU(2)$ is in some sense a doubled version of $SO(3)$.

Here we make contact with our "real world", our three-dimensional space in which rotations come under the law of $SO(3)$, which is in turn intimately connected to $SU(2)$. While previously we may have been considering abstract mathematical spaces, here we are talking about *our* space, the space of our universe. Since $SO(3)$ rules over rotations in our three-dimensional space, we infer that each of the three real variables corresponds to a real spatial dimension in Nature. By analogy, the most direct inference is that $SU(2)$, being "in some sense a doubled version of $SO(3)$", also corresponds to spatial dimensions in Nature.

Corroboration of this conjecture can be found in observed quantum spin phenomena. The $SU(2)$ symmetry group precisely describes such phenomena and clearly explains why a spin one-half particle (fermion) must rotate twice to return to its original state. Being an empirical fact, we can deduce that the $SU(2)$ symmetry group is not just an abstract mathematical structure, but corresponds to spatial dimensions objectively present in Nature. Certainly, a fermion is aware of (responds to) imaginary dimensions.

In the following excerpt Woit extends his discussion to spinors, being mathematical objects represented by two complex numbers: [4]

> From the point of view of representation theory, Weyl spinors are the fundamental representations that occur when one studies the representations of rotations in four-dimensional space-time. Recall that in three dimensions the group of rotations is called $SO(3)$, and that spin one-half particles are representations not of this group, but of a doubled version of it, which turns out to be the group $SU(2)$ of transformations on two complex variables. Three-dimensional geometry thus has a subtle and non-obvious aspect, since to really understand it one must study not just the obvious three-dimensional vectors, but also pairs of complex numbers. These pairs of complex numbers, or spinors, are in some sense more fundamental than vectors. One can construct vectors out of them, but can't construct spinors just using vectors.

Once again we are reminded that the complex numbers are more fundamental than the real numbers. The key point, however, is that *three-dimensional geometry has a subtle and non-obvious aspect*. Since this conclusion is forced upon us by both the theoretical and empirical evidence, we may assume that what Woit calls three-dimensional geometry corresponds directly to our objective physical 3-space. Physics has yet to come to terms with this fact. As Woit suggests, the notion of our 3-space somehow including two complex dimensions is counter-intuitive and unvisualizable. So we are faced with a conundrum: Where are these complex dimensions? Why don't we observe them directly? More generally, why do various physical phenomena appear to respond to specific symmetry groups and therefore to specific spatial configurations?

Note that the real and complex aspects of space are considered distinct entities ("...one must study not just the obvious three-dimensional vectors, but also pairs of complex numbers"), implying that the complex aspect is in *addition* to the real aspect. Beyond this, canonical physics offers no coherent picture. But there is an alternate interpretation of the facts: rather than being distinct entities, the real and complex aspects of space could (in general) represent various configurations of the same dimensions.

How is this possible? To build such a picture we must turn to fundamentals.

1.2 Elements of Space

Each of the following postulates will be vindicated in due course:

- Spatial dimensions can be either real or imaginary, and of positive or negative polarity.
- Just as imaginary numbers are more fundamental than the real numbers, imaginary dimensions are more fundamental than real dimensions. All spatial dimensions are fundamentally imaginary.
- A real dimension is constructed (projected) from two imaginary dimensions, in some sense as a *product*. It follows that imaginary dimensions of the same polarity project a negative real dimension, while imaginary dimensions of opposite polarity project a positive real dimension.
- Our universe is constructed from a total of ten imaginary dimensions, five positive and five negative.
- Our physical 3-space is constructed from four imaginary dimensions, one positive and three negative.

According to this model, a particular spatial dimension can be either real or imaginary, but never both. For a spatial dimension to be both real and imaginary (complex) would be as impossible as drawing a line **i** centimeters long in our real space, or a line one centimeter long in an imaginary space. Real and imaginary numbers reflect Nature only in the context of corresponding dimensions. Complex numbers abound in physics because, from our perspective occupying a real space, fundamental phenomena generally mix the real numbers (dimensions) with the imaginary dimensions. Accordingly, we add a concluding postulate:

- While no spatial dimension can be both real and imaginary (complex), a real dimension can combine with an imaginary dimension to yield a composite complex dimension.

The composite nature of complex dimensions is clearly reflected in the structure of complex numbers. Whenever the term "complex dimension" appears within the current framework, conjoined real and imaginary dimensions are implied.

The foregoing principles are encoded in Figure 1, demonstrating the graphical convention appearing throughout this paper in pursuit of conceptual clarity.

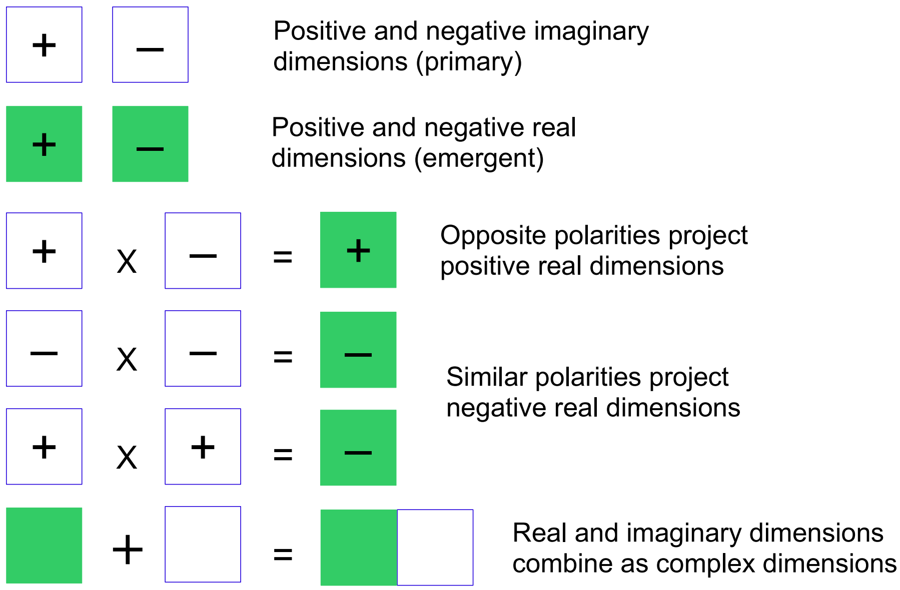

Figure 1: Fundamental algebra of space

The essential paradigm shift implicit in this model is that *real space is not fundamental*; rather, we arrive at the somewhat radical conclusion that real spatial dimensions emerge from the interaction (product) of imaginary dimensions. Technical readers might understand real dimensions as emerging from the *cross product* of orthogonal imaginary bases, while the spatial polarities might in turn be understood in terms of *handedness*.

In three real dimensions, the cross product yields a vector perpendicular to the vectors being multiplied, with magnitude determined by the positive area of a parallelogram having sides defined by the two vectors. Figure 2 illustrates this principle when applied to orthogonal vectors in imaginary space. Since the area of the rectangle is a product of two imaginary numbers, the magnitude of the cross product is real. Extending this principle to space itself, real spatial dimensions are each projected from two orthogonal imaginary dimensions, as described mathematically by the cross product.

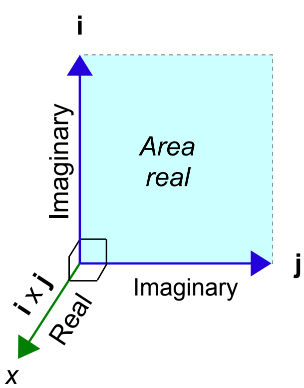

Figure 2: Cross product of vectors in imaginary space

1.3 The Structure of Real 3-Space

In 1843 the Irish mathematician Sir William Rowan Hamilton was focused on the problem of extending the complex plane to three-dimensional space when famously the solution came to him during a walk in Dublin. In an event celebrated annually to this day, he carved the following into the stonework of Brougham Bridge:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

The formula represents the multiplicative rules for what Hamilton called *quaternions*, which he studied and taught for the rest of his days [5]. Quaternions quickly found many applications in physics with great success, before being largely replaced by alternate, more intuitive mathematical methods. Subsequently quaternions remained out of vogue until being revived in the late twentieth century for their utility in calculating orientation and rotations in three-dimensional space. Due to their computational efficiency and immunity to gimbal lock, today quaternions play an essential role in applications such as computer graphics, computer games and spacecraft control software. While there is no question that quaternions provide an elegant and precise model of orientation and rotation in physical space, mathematicians have yet to fully come to terms with them. Roger Penrose offers the following perspective: [6]

> [Quaternions have] a very beautiful algebraic structure and, apparently, the potential for a wonderful calculus finely tuned to the treatment of the physics and geometry of our 3-dimensional physical space. Indeed, Hamilton himself devoted the remaining 22 years of his life attempting to develop such a calculus. However, from our present perspective, as we look back over the 19th and 20th centuries, we must still regard these heroic efforts as having resulted in relative failure.

What was it about quaternions that so captivated Hamilton? And why have they not lived up to their mathematical promise? Hamilton described his discovery in a letter to a friend: [7]

> And here there dawned on me the notion that we must admit, in some sense, a fourth dimension of space for the purpose of calculating with triples… An electric circuit seemed to close, and a spark flashed forth.

Hamilton's key insight was that when the complex plane is generalized to three-dimensional space, the resulting mathematical object is four-dimensional. The *general quaternion*, q, is written as

$$\boldsymbol{q} = t + u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

where **i**, **j**, **k** are each an independent "square root of minus one", and t, u, v, w are real numbers.

What is this formula telling us? It may be tempting to assign the three imaginary parts to our three spatial dimensions and the real part t to time, but this approach fails on technical grounds [8]. Rather, the clue is that *our three spatial dimensions are represented as imaginary*. Why is this so? And what does the fourth part represent, and why is it real? How could operations in one real and three imaginary dimensions possibly reflect properties of our real three-dimensional space?

Recall our postulate that all spatial dimensions are fundamentally imaginary. This implies that each of our three real dimensions is a projection of two imaginary dimensions. We propose that the general quaternion has four terms because our three real dimensions are constructed from four imaginary dimensions, the real part t representing an *intrinsic* imaginary dimension – intrinsic to each of our three real dimensions. The (positive) intrinsic dimension appears in the quaternion as a real number because it is hidden from view – it never appears directly in Nature, but makes its presence known only by its interaction with three negative imaginary dimensions, together projecting into existence three orthogonal positive real dimensions, as illustrated in Figure 3. Note that this schematic representation illustrates the dimensional configuration only and in no way reflects actual spatial geometry! The intrinsic imaginary dimension is black to indicate its special (hidden) status: it binds the three negative dimensions into a positive real isotropic manifold, denoted by the green triangle. The arrows represent interactions between the positive intrinsic dimension and the three negative dimensions, as described mathematically by the cross product, projecting into objective existence our real 3-manifold.

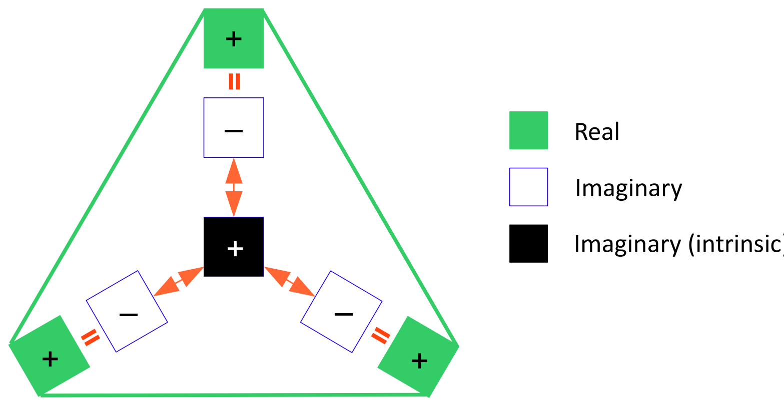

Figure 3: The quaternionic structure of real 3-space

In retrospect, one could argue on strictly logical grounds that three independent dimensions are in themselves insufficient to constitute our 3-space, that a fourth unifying element is required to subsume them into an isotropic manifold. Might this interpretation of quaternions point the way to their further development and application in physics and mathematics? This question is left for specialists.

The scientific and philosophical consequences of this model are profound and far-reaching, of course. It means that our empirical space, and therefore our physical universe, is erected upon a nonphysical, imaginary substratum. Note that the word *physical* is applied here in a specific sense – "encompassed by our 3+1 spacetime" – in contradistinction to the common scientific nuance of "causally related to empirical phenomena". Since our three spatial dimensions and one time dimension are real – measured in real units – imaginary phenomena have no place in 3+1 spacetime. Quite simply, our "real world" is built upon, and is utterly dependent upon, abstract layers of space *outside* or *beneath* our 3+1 spacetime.

One wonders if William Rowan Hamilton entertained such thoughts while working deep into the night. Did he conceive of our physical space as being founded upon imaginary spatial dimensions? Personally I feel there is no doubt that he would have considered this interpretation of his cherished quaternions; whether he took such thoughts seriously is another matter. Hamilton was living at a time of peak materiality, when the materialized Newtonian philosophy of a clockwork (deterministic) universe was in full swing. Whatever his personal thoughts and leanings, to express the view that our physical universe is dependent upon a nonphysical substratum would likely have constituted professional suicide.

For the sake of simplicity, subsequent schematics will generally depict our three real spatial dimensions as in Figure 4. The green (real) boxes are connected to denote their cohesion into an isotropic 3-manifold. Wherever these three joined boxes appear, keep in mind that you are really looking at Figure 3.

Figure 4: Simplified schematic for real 3-space

Part 2

Quantum Mechanics and Minkowski 4-Space

Armed with our newfound appreciation for complex and imaginary numbers, we can now look squarely at some essential facts of quantum mechanics and relativity theory while demonstrating that nonphysical theoretical models can provide consistent explanations for previously unexplained empirical phenomena.

2.1 What is the Quantum Wavefunction?

As pointed out by John Cramer (page 5), complex numbers appear at the very heart of quantum mechanics, in the mathematical description of the primary quantum entity known as the *wavefunction, state vector,* or *quantum state*. Ever since the arrival of quantum mechanics the ontology of the wavefunction has been a subject of great controversy. While some physicists have considered the wavefunction an objective entity, extended in physical space, others argue that it is no more than an abstract mathematical artifact, having no objective existence whatsoever. Further obscuring the problem, there are several different formulations of quantum mechanics, each mathematically equivalent while emphasizing different aspects. Modern formulations tend to value computational utility over conceptual transparency, placing quantum mechanics in an abstract, complex space called *Hilbert space (*each dimension being complex), being analogous to a *configuration space* in classical mechanics while bearing no resemblance to the space of our objective universe. Consequently, physicists commonly apply the formalism of quantum mechanics without a coherent picture of what that formalism might objectively represent, if anything at all.

Once again, Roger Penrose brings us important insights. Figure 5 illustrates schematically what is known as a *momentum state*, representing a particle wavefunction having a clearly defined momentum, and adapted from illustrations by Penrose [1]. Momentum states are the most "wavelike" of all wavefunctions, taking the form of a helix, the major axis of which is shown oriented in some direction x in ordinary physical space (the y and z dimensions are missing), while the u and v directions form a complex plane orthogonal to the x direction, the v direction being imaginary. The phase of the helical wavefunction at any point along the wave (x) is given by the complex number $u + \mathbf{i}v$. While a general wavefunction will be considerably less regular than a pure momentum state, the pertinent point here is that, according to the quantum formalism, wavefunctions are complex waves, of this particular dimensionality.

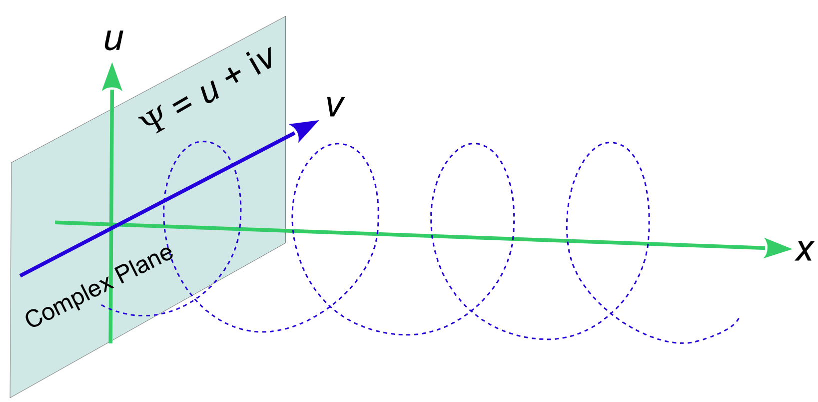

Figure 5: The abstract wavefunction (momentum state)

From this perspective we can understand John Cramer's statement that complex functions are "swallowed whole" by quantum theory (the problem of complexity), since the primary entity of a quantum system is itself a complex function. Accordingly, the wavefunction is shown here as a dashed line to emphasize that it is generally considered an abstraction, a purely mathematical construct, often called a *probability wave* representing "knowledge of the system". Penrose explains the illustration as follows: [2]

> The *x* direction in my picture corresponds to some actual direction in ordinary space, but the *u* and *v* directions are not ordinary spatial directions; they are put in to represent the complex plane of possible values of the wavefunction… To get the *full* picture of these waves, we should have to try to imagine that this is going on in all the three dimensions of space at once, which is hard to do, because we would need two extra dimensions (five in all) in order to fit in the complex plane as well as the spatial dimensions!

Like Peter Woit in the context of symmetry groups, Penrose clearly does not consider the wavefunction's complex plane to represent objective spatial dimensions, leaving us with a rather obscure entity having a particular location and orientation in physical space but no objective phase, hence no objective existence – in accordance with the widely accepted Copenhagen interpretation of quantum mechanics. The reasoning is clear enough: since canonical physics allows only real spatial dimensions in Nature, a complex entity cannot exist objectively in space.

Today, however, the "probability wave" interpretation of the wavefunction is facing an unexpected challenge. In November 2011, English physicists Matthew F. Pusey, Jonathan Barret and Terry Rudolph dropped a bombshell on quantum foundations research by turning this interpretation on its head. In a paper titled *The quantum state cannot be interpreted statistically* they derive a theorem (given mild assumptions) establishing that the abstract "probability wave" interpretation is inconsistent with quantum theory. According to their reasoning, for quantum mechanics to be consistent the wavefunction must be an objective entity or, as written in the original abstract, a "physically distinct state": [3]

> Here we show that, given only very mild assumptions, the statistical interpretation of the quantum state is inconsistent with the predictions of quantum theory. This result holds even in the presence of small amounts of experimental noise, and is therefore amenable to experimental test using present or near-future technology. If the predictions of quantum theory are confirmed, such a test would show that distinct quantum states must correspond to physically distinct states of reality.

Response to the paper has been mixed. While some hailed it as the most important result since Bell's theorem (more on this in a moment), others have written it off as pseudo-science. As one might expect when discussing the primary entity underlying physical reality, reactions can be religious in flavor. It is revealing that some physicists misinterpreted the title to mean that the wavefunction cannot yield stochastic predictions of quantum phenomena (the Born rule), which is clearly not what the authors wrote or intended. Taking heed, in early 2012 the authors published a revised version of their paper, unambiguously titled *On the reality of the quantum state*. The abstract of the revised paper strikes a direct and defiant note: [4]

> Quantum states are the key mathematical objects in quantum theory. It is therefore surprising that physicists have been unable to agree on what a quantum state represents. One possibility is that a pure quantum state corresponds directly to reality. But there is a long history of suggestions that a quantum state (even a pure state) represents only knowledge or information of some kind. Here we show that any model in which a quantum state represents mere information about an underlying physical state of the system must make predictions which contradict those of quantum theory.

Since the predictions of quantum theory have never been contradicted, the importance of this result cannot be overstated. A stunned silence has ensued. We have arrived at a cathartic moment for canonical physics. How can we reconcile a complex wave with an objective entity, a "physically distinct state"?

Let us suppose that Pusey's theorem is correct, that the wavefunction is an objective wave, extended in space. This would imply that the complex plane of the wavefunction, as illustrated in Figure 5 (page 14), corresponds to objective spatial dimensions. But we immediately encounter a problem, of course: if our 3+1 spacetime was to include an imaginary spatial dimension we would be living in a complex space and the geometry and physics of our world would differ dramatically from what we observe. Since the wavefunction simply won't fit into our physical space we are left with just one alternative: the complex wavefunction must occupy a higher-dimensional space, *outside* our 3+1 spacetime.

2.2 The Enigma of Quantum Nonlocality

Deepening the mystery of the objective, higher-dimensional wavefunction, experiments have demonstrated the property of *nonlocality*, what Einstein so mistrusted as "spooky action at a distance", which is related to a phenomenon called *entanglement*. According to the quantum formalism, a quantum state can span the universe while behaving holistically, as one thing, as if fully present at one location in space. That is, the wavefunction appears not to abide by the rules of Einstein's special theory of relativity, which has been consistently verified in all other arenas of physics and which limits all signals to light speed. In 1964 the Irish physicist John Stewart Bell published a famous paper establishing what is now known as *Bell's theorem*, providing a theoretical basis for subsequent experiments demonstrating nonlocal effects. The first decisive experimental test of Bell's theorem was published in 1982 by Alain Aspect and his collaborators, establishing nonlocality as a fact of Nature, as elaborated here by philosopher of physics Tim Maudlin: [5]

> Aspect's experiment and other such experiments have produced observable data which cannot be predicted by any theory which disallows influence of the career of one particle on the behavior of the other once they separate. Somehow the particles must remain in communication, the observable behavior of one being determined, in part, by the nature of observations carried out on its twin. After being created together the pair of particles remain interconnected.

Aspect et al. created pairs of entangled photons and separated them to opposite wings of the experiment before measuring correlations between the pair. The *choice* of a particular measurement on one photon was shown to influence the state of the other, the experiment being carefully contrived so that no luminal or subluminal signal could account for the results. With all loopholes closed, physicists and philosophers are still coming to terms with what is arguably the most philosophically important scientific result in history. Exactly what are Bell's theorem and the Aspect experiment telling us? In order to proceed it is vital that we come to terms with this question. Maudlin offers his studied analysis as follows: [6]

> There are at least three features of the quantum connection which deserve our close attention. All of them are, to some extent, surprising. The first two prevent our assimilation of these quantum effects to those of a force like gravitation. The last presents problems for reconciling the results of experiments like that of Aspect with the rest of our physical picture.
>
> 1. The quantum connection is unattenuated….
>
> The quantum connection [in contrast to a force like gravity] appears to be unaffected by distance. Quantum theory predicts exactly the same correlations will continue unchanged no matter how far apart the two wings of the experiment are. If Aspect had put one wing of his experiment on the moon he would have obtained precisely the same results. No classical force displays this behavior.
>
> 2. The quantum connection is discriminating….
>
> Gravitational forces affect similarly situated objects in the same way. The quantum connection, however, is a private arrangement between our two photons. When one is measured its twin is affected, but no other

> particle in the universe need be…. The quantum connection depends on history. Only particles which have interacted with each other in the past seem to retain this power of private communication. No classical force exhibits this kind of exclusivity.
>
> 3. The quantum connection is faster than light (instantaneous)….
>
> The Special Theory [of Relativity] confers upon light, or rather upon the speed of light in a vacuum, a unique role in the space-time structure. It is often said that this speed constitutes an absolute physical limit which cannot be broached. If so, then no relativistic theory can permit instantaneous effects or causal processes. We must therefore regard with grave suspicion anything thought to outpace light. The quantum connection appears to violate this fundamental law….
>
> It is surprising that the communication between particles is unattenuated and discriminating, but often our best counsel is simply to accept the surprising things our theories tell us. The speed of the communication is another matter. We cannot simply accept the pronouncements of our best theories, no matter how strange, if those pronouncements contradict each other. The two foundation stones of modern physics, Relativity and quantum theory, appear to be telling us quite different things about the world.

The reader will appreciate why many consider the foundations of quantum theory to be the most important and challenging problem in science. Physics and philosophy both stand bewildered before the facts. To find our way forward, let us take a closer look at Special Relativity, which appears to contradict the theoretical and experimental facts presented by quantum mechanics.

2.3 Special Relativity and Minkowski Spacetime

Einstein's special theory of relativity provides a classic example of the power of inference, taking the most direct reading of the facts despite the philosophical consequences. The theory is derived from just two empirical facts: physics is the same in any inertial frame of reference (known as *Lorentz symmetry*), and the speed of light c is constant in any reference frame, regardless of relative motions. While the resulting mathematical framework was already worked out in the late 19th century, notably by Henri Poincaré and Hendrik Lorentz (who derived what are known as the *Lorentz transformations*, so central to Special Relativity), it took Einstein's conceptual genius to place the mathematics in the correct context.

Just three years after Einstein published his seminal paper on Special Relativity (inconspicuously titled *On the electrodynamics of moving objects*), Hermann Minkowski presented an elegant reformulation showing how space and time are inextricably linked, and that time could be treated almost like another spatial dimension – but not quite. Figure 6 depicts what has become known as *Minkowski spacetime* [7]. Remarkably, all of Special Relativity emerges from this simple diagram. While Minkowski spacetime has three spatial dimensions and one time dimension, there is no way to depict all four dimensions graphically. The 3D graphic is missing the third spatial dimension, of course, while the 2D version shows just one spatial dimension (x), which could be pointing anywhere in physical space. Each point in Minkowski spacetime represents an event at a particular location in physical space and time.

For the sake of simplicity and transparency it is convenient to choose units so that the speed of light c equals one. For instance, we could measure time in *seconds* and distance in *light-seconds*. Consequently, the diagonal lines in Figure 6 represent the paths of photons moving at the speed of light, forming what is called a *light cone* or *null cone*. The cones converge at the present moment. As stated by Maudlin, above, it is often said that light-speed represents an absolute physical limit. If you place yourself at the origin – being your "here and now" – the past cone defines the totality of spacetime wherein past events could possibly influence you now, while the future cone defines the full extent of spacetime you could possibly influence in the future by some action in the present.

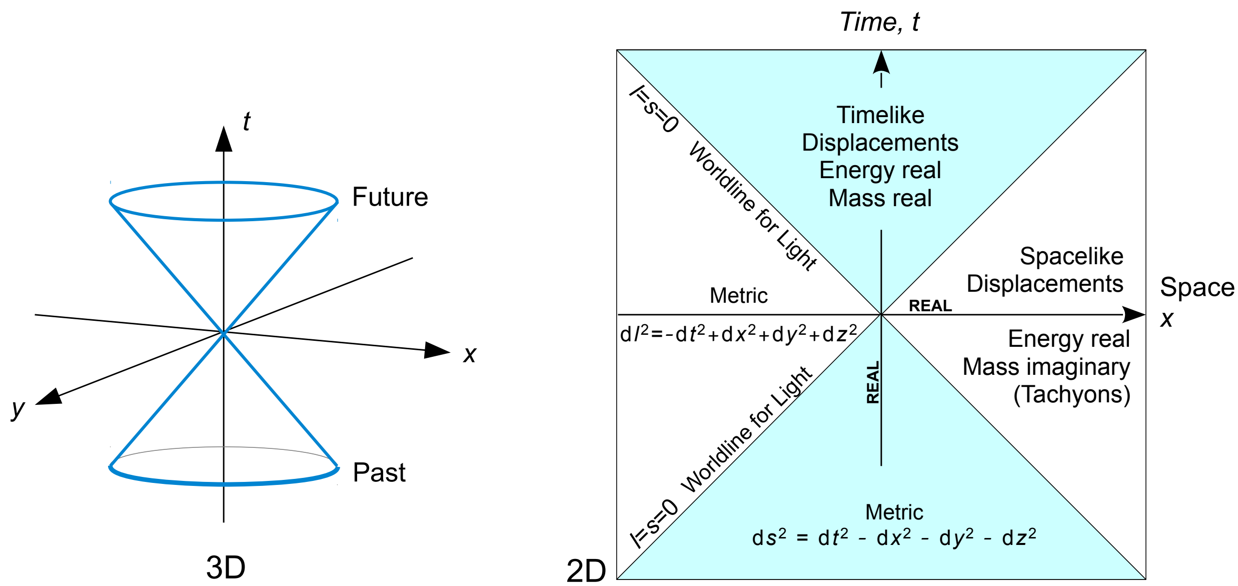

Figure 6: Minkowski spacetime

Minkowski spacetime connects our three spatial dimensions and one time dimension by way of the *Minkowski metric*, being the rule defining *displacement* in Minkowski spacetime. The metric is defined in alternate ways, the most "physical" formulation being as follows (measuring from the origin):

$$s^2 = t^2 - x^2 - y^2 - z^2$$

where t represents time, and x, y, and z are the three spatial dimensions. The displacement s is interpreted as the time experienced (or measured by an ideal clock) while traversing that particular worldline, or path through spacetime. Displacements are real (s^2 is positive) only within the past and future light cones, these regions being known as *timelike*. An alternative formulation of the Minkowski metric yields positive (real) displacements for *spacelike* regions, being those outside the light cones, denoted l and expressed as follows:

$$l^2 = -t^2 + x^2 + y^2 + z^2$$

Displacements *on* the light cones are said to be *lightlike*, where both metrics become zero – hence the term *null* cone. On the null cone, the time contribution to the metric is equal and opposite to the resultant space contribution, yielding zero net displacement in Minkowski spacetime. Hence, time experienced becomes zero at the speed of light, implying that photons do not experience time. Further, the spacelike displacement l also becomes zero at the speed of light, implying that a photon does not experience space in its direction of motion – or rather, there is no distance between any two points on its worldline. Hence Maudlin's statement that Special Relativity confers upon the speed of light a unique role in the structure of spacetime.

2.4 Euclidean Spacetime and Imaginary Time

Readers will note the resemblance between both Minkowski metrics and the theorem of Pythagoras in four dimensions, only the signs being different. When extended to four dimensions the Pythagorean theorem is known as the distance metric in Euclidean 4-space. This resemblance inspired early relativity theorists to complete the analogy by taking the time coordinate t to be imaginary, according to $t = \mathbf{i}w$.

(As elsewhere, imaginary quantities are in bold.) The alternate formulation of the Minkowski metric then becomes

$$s^2 = \boldsymbol{w}^2 + x^2 + y^2 + z^2$$

corresponding to the distance metric for Euclidean 4-space. Since $\boldsymbol{w}$ is imaginary, the first term becomes negative when squared, yielding the spacelike metric for Minkowski spacetime. This procedure later led to the so called *Euclideanization* of spacetime. According to this scheme, the time coordinate is "rotated" on the complex plane into $\boldsymbol{\tau} = \mathbf{i}t$, where $\boldsymbol{\tau}$ is known as "imaginary time" [8]. This scheme has been particularly successful in providing consistent solutions within the context of Richard Feynman's "sum over histories" or "path integral" formulation of quantum mechanics, as Stephen Hawking explains: [9]

> To avoid the technical difficulties with Feynman's sum over histories, one must use imaginary time. That is to say, for the purposes of the calculation one must measure time using imaginary numbers, rather than real ones. This has an interesting effect on space-time: the distinction between time and space disappears completely. A space-time in which events have imaginary values of the time coordinate is said to be Euclidean, after the ancient Greek Euclid, who founded the study of the geometry of two-dimensional surfaces. What we now call Euclidean space-time is very similar except that it has four dimensions instead of two. In Euclidean space-time there is no difference between the time direction and directions in space…. As far as everyday quantum mechanics is concerned, we may regard our use of imaginary time and Euclidean space-time as merely a mathematical device (or trick) to calculate answers about real space-time.

Imaginary time has found many powerful applications in modern physical theory. In 1983 Hawking and James Hartle invoked the concept in a cosmological model known as the "no boundary" proposal [10], while relativity theorists and quantum field theorists routinely pass into imaginary time to simplify their calculations. Tellingly, not all of physics is captured in imaginary time, while no satisfactory ontology of imaginary time has come forth. What is "imaginary time", and why does it appear in fundamental physics?

2.5 The Domain of the Wavefunction

Let us take stock. On a strictly logical basis (Pusey's theorem) we have surmised that the wavefunction is an objective entity present in space. We have also learned (from Roger Penrose) that the wavefunction is a complex wave extended in real and imaginary dimensions, implying that it is cannot be objectively present in our physical space, which has no such imaginary dimension. Further, we have learned (Bell's theorem) that the wavefunction appears not to respect the laws of Special Relativity, which rule over displacements in our 3+1 spacetime. Finally, we have discovered that Euclidean spacetime, having three real spatial dimensions and one imaginary time dimension, has many important applications in physics, and that "the distinction between time and space disappears completely" in Euclidean spacetime (quoting Hawking, above).

According to the current framework, since the wavefunction is an objective entity, it follows that its complex plane corresponds to objective spatial dimensions, as illustrated in Figure 7. Moreover, it is clear that an objective entity extended in three real dimensions and one imaginary dimension must occupy a space with at least that many dimensions. That is, the wavefunction simply cannot fit inside our 3+1 spacetime, but occupies another space, another world, imperfectly and incompletely described by "Euclidean spacetime".

- So called Euclidean spacetime is not a spacetime at all, but a 4-space, having three real spatial dimensions and one imaginary spatial dimension, being the domain of the complex wavefunction.

It is important to think this through carefully. The wavefunction, constituting the basis of physical manifestation, is required to occupy a higher-dimensional space. While a three-dimensional *representation* or *projection* of the wavefunction may exist in our 3+1 spacetime, the complete entity (as formulated by quantum mechanics today) is extended in four spatial dimensions, one of which is imaginary. Note that the abstract complex plane of Figure 5 is replaced by the real direction y and the imaginary direction $\boldsymbol{w}$, each corresponding to spatial dimensions present in Nature. Just two real dimensions are depicted, of course, there being no way to show the third (z) real dimension.

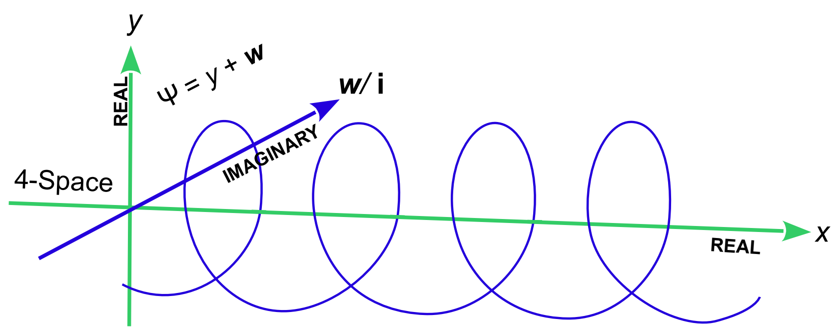

Figure 7: The objective wavefunction

Since the wavefunction is spatially correlated with physical phenomena in our 3+1 spacetime, the higher-dimensional space is required to interpenetrate (permeate) our 3-space, as depicted in Figure 8. While the spaces are delineated vertically for clarity, keep in mind that the two real manifolds are in fact superimposed. That is, the three real dimensions of each space coincide – they are the same dimensions manifesting in distinct spatial manifolds.

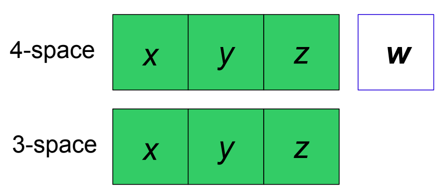

Figure 8: The spatial domain of quantum mechanics

2.6 Braneworlds

The concept of *branes* (more precisely, *D-branes*) provides an apt metaphor and a suitable mechanism for these interpenetrating spaces [11]. Emerging unambiguously from the mathematics of string theory, D-branes are essentially subspaces of a higher-dimensional space called the *bulk*. The bulk includes a total of ten spatial dimensions (nine plus a tenth more subtle dimension), while a 3-brane includes three spatial dimensions, a 4-brane four dimensions, and so on, up to a maximum of nine. According to string theory, branes confine all fields except gravity. That is, matter fields (therefore matter) on branes cannot leak into higher dimensions or into other branes, while gravity can travel freely through the bulk.

The mathematical prediction of branes has led theorists to speculate that our universe could in fact be a 3-brane. So called *braneworld* scenarios typically picture our 3-brane as one of many (more or less similar) 3-branes floating in a higher-dimensional space, as string theorist Brian Greene explains in his book *The Hidden Reality*: [12]

> [A three-brane that is enormous, perhaps infinitely big], would *fill* the space we occupy, like water filling a huge fish tank. Such ubiquity suggests that rather than think of the three-brane as an object that happens to be situated within our three spatial dimensions, we should envision it as the very substrate of space itself. Just as fish inhabit the water, we would inhabit a space-filling three-brane. Space, at least the space we directly inhabit, would be far more corporeal that generally imagined. Space would be a thing, an object, an entity – a three-brane. As we run and walk, as we live and breathe, we move in and through a three-brane. String theorists call this the *braneworld scenario*....
>
> In string theory there are more than just three spatial dimensions. And a higher-dimensional expanse offers ample room for accommodating more than one three-brane. Starting conservatively, imagine that there are two enormous three-branes. You may find it difficult to picture this. I certainly do. Evolution has prepared us to identify objects, those presenting opportunity as well as danger, that sit squarely *within* three-dimensional space. Consequently, although we can easily picture two ordinary three-dimensional objects inhabiting a region of space, few of us can picture two coexisting but separate three-dimensional entities, each of which could fully fill three-dimensional space.

Greene makes the crucial point that two 3-branes could theoretically occupy the same 3-space whilst remaining separate on a higher dimension. Consequently, since branes are transparent to gravity, we have the fascinating specter of two materially isolated worlds seeing the same gravitational field. This picture closely reflects our model of interpenetrating spaces, as demanded by the complex wavefunction – the difference being, of course, that the interpenetrating brane is required to be of higher dimension.

Without further ado, this not being the moment to discuss the merits of string theory or the objective existence of branes, we simply adopt the working hypothesis that our physical universe can be represented by a 3-brane, sharing the same three-dimensional space as a 4-brane, the fourth dimension of which is imaginary.

2.7 The Wavefunction is a Wave of What?

Having surmised that the wavefunction is an objective entity occupying an interpenetrating 4-brane, we are brought to a foundational question: What exactly is waving? Just as an ocean wave is an oscillation of sea water, and an electromagnetic wave is an oscillation of the electromagnetic field, the objective wavefunction can only be an oscillation of some objective field or medium. To approach this question we recall that real quantities or properties cannot be represented on an imaginary dimension, or vice versa, which further implies that objective fields or media must be either real or imaginary – they cannot be both. Yet we know that the wavefunction is a complex wave, meaning that it is extended in both real and imaginary dimensions. What is this telling us?

There is a more direct logical route to this foundational question. Imagine for a moment that the wavefunction is a material wave, an oscillation of some substantial field or medium. Since quantum mechanics is the (non-relativistic) mathematical theory of energy and matter (as we know them), and since the wavefunction is the fundamental mathematical entity representing any quantum system (thus matter and energy), we would be forced to conclude that the wavefunction is an oscillation of some entity which is in turn reliant on a wavefunction. *Reductio ad absurdum* – being logically inconsistent, it cannot be true. Logically, the wavefunction can only be an oscillation of some entity *preceding* matter. The most obvious candidate is an oscillation of space itself – a gravitational wave.

Within the framework there is a further reason why the wavefunction can only be a gravitational wave. We have learned that the wavefunction occupies a 4-brane while somehow interacting with our 3-brane, and only gravity, the geometry of space itself, can pass freely through and between branes. According to Einstein's general theory of relativity, a gravitational wave can be of any frequency and travels at the speed of light. On this basis, the following conjectures will prove of central importance to the framework:

- The complex wavefunction represents an objective gravitational wave extended in three real dimensions plus one imaginary dimension, occupying a 4-brane consisting of three real dimensions and one imaginary dimension (not including time).
- The 4-brane is superimposed upon (interpenetrates) our 3-brane, the three real dimensions of each space being coincident – they are the same three dimensions manifesting in each brane.
- The three real dimensions of the 3-brane and 4-brane see the same (real) gravitational field and waves. All other fields (matter fields) are confined to a particular brane.
- Just as an imaginary quantity cannot be represented in a real space, a real quantity cannot be represented in an imaginary space. Accordingly, a real gravitational field cannot "leak" into imaginary dimensions.

Recall our foundational postulate that all spatial dimensions are fundamentally imaginary. Because our real 3-space is constructed from four imaginary dimensions (section 1.3), and because the wavefunction is an oscillation of space itself, the wavefunction also is fundamentally imaginary, even while it *appears* to evolve through both real and imaginary dimensions.

2.8 Quantum Nonlocality Unveiled

The apparent incompatibility of quantum mechanics and Special Relativity is particularly disquieting for physicists because Einstein's theory is regarded as a singularly beautiful theory, erected upon the bare minimum of physical principles, elegant and symmetrical, conceptually whole and logically complete in itself. To touch it would be like taking a knife to the Mona Lisa. Can Einstein's special theory be saved? It turns out that indeed Special Relativity will live on. When extended to higher dimensions, Special Relativity, far from contradicting quantum nonlocality, in fact provides the *mechanism* for quantum nonlocality.

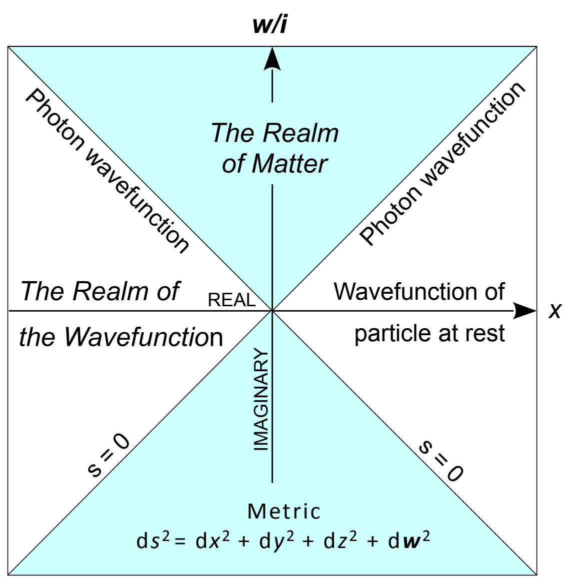

Figure 9: Minkowski 4-Space

Figure 9 depicts what I will call *Minkowski 4-space*. (While the metric for Euclidean spacetime may be considered "Euclidean", a space having an imaginary dimension certainly could not.) Experts will recognize it essentially as Euclidean spacetime, but with all four dimensions interpreted as spatial. There is no time dimension in Minkowski 4-space.

Following the Euclideanization procedure, time is rotated on the complex plane according to:

$$\boldsymbol{w} = \mathbf{i}t$$

where $\boldsymbol{w}$ is the fourth spatial dimension, which is imaginary. As in the depiction of the wavefunction in Figure 7, the imaginary dimension $\boldsymbol{w}$ is divided by $\mathbf{i}$ to render it real, since imaginary values cannot be depicted directly on the graphic! Minkowski 4-space is home for the complex quantum wavefunction, which the reader can lucidly illustrate by imagining Figure 7 overlaid on Figure 9 with the respective dimensions aligned.

Technical readers who are familiar with Minkowski diagrams depicting Minkowski spacetime and Euclidean spacetime may have to retrain themselves to properly interpret Figure 9. In particular, note the following:

- First and foremost, keep in mind that all four dimensions are spatial. Since the wavefunction evolves in time, clearly an additional time dimension is required, yielding a 4+1 spacetime. This shortcoming will be addressed in due course; first we must investigate the properties of Minkowski 4-space itself.
- While the metric is unchanged from that of Euclidean spacetime, all four terms are now spatial, with the important consequence that the displacement s can only be interpreted spatially.
- Since all four dimensions are spatial, the orientation of a vector in Minkowski 4-space relates not to velocity as it does in spacetime, but to a particular geometrical *orientation* or *direction* relative to the real and imaginary dimensions.
- While the null cone is defined by $s = 0$, just as it is in the Minkowski and Euclidean spacetimes, in Minkowski 4-space the displacement s refers not to time experienced, as in Minkowski spacetime, but to *spatial distance*. It follows that there is zero spatial distance between any two points on the null cone. That is, despite appearances in our 3-space, every point on the null cone represents the *same location* in Minkowski 4-space, defined by the origin.
- Just as a light wave is confined to its light cone in Minkowski spacetime, the photon's wavefunction, being a gravitational wave traveling at the speed of light in our 3+1 spacetime, is confined to its null cone in Minkowski 4-space. It follows that, while a wavefunction is extended in our real 3-space, it occupies a single location in Minkowski 4-space, defined by its null cone.

In the above few statements lies the key to one of the great mysteries of physics – a consistent understanding of the holistic wavefunction, and therefore of quantum nonlocality. The reader might wish to reread these statements, since a clear understanding requires reorienting the mind from old patterns to new. When the logic presents itself, everything follows easily.

Having established a logical basis for the holistic wavefunction, how are we to conceptually interpret this? Geometrically, it means that the length of any 4-vector in Minkowski 4-space is dependent on its orientation relative to the real and imaginary dimensions. When the real and imaginary components correspond, the vector has no length at all: hence it exists at just one location in Minkowski 4-space. In effect, the imaginary dimension reduces the distance between any two points in three real dimensions, depending on the vector's *orientation* relative to the real and imaginary dimensions.

Conceptually, how is this possible? How could an additional spatial dimension *reduce* spatial distance?

Recall that the three real dimensions are projected from four imaginary dimensions, which, along with the additional imaginary dimension in the 4-brane, form the arrangement represented in Figure 10. The fourth (imaginary) dimension of the 4-brane would appear to directly access the imaginary dimensions underpinning our real 3-manifold, effectively providing a "short cut" through the three real dimensions. Whatever the actual mechanism, the metric clearly infers that orientation on the fourth (imaginary) dimension reduces distance in Minkowski 4-space, even to no distance at all.

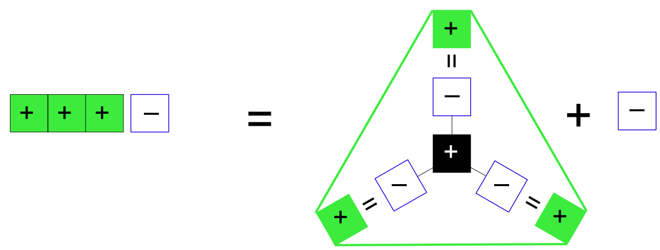

Figure 10: The inner structure of Minkowski 4-space

Experts will note that the preceding explanation of the holistic wavefunction appears limited to the lightlike displacements of massless particles. The propagation speed of the wavefunction is expressed as

$$wv = c^2$$

where w is the *phase velocity,* corresponding to the velocity of the wavefunction; and v is the *group velocity,* corresponding to the velocity of the associated particle [13]. Hence, the photon and the photon wavefunction each propagate at speed c (in accord with relativity theory), while the wavefunction of a particle at rest propagates at infinite speed (action at a distance). Consequently, wavefunctions are confined to the "real" regions of Minkowski 4-space, on or outside the null cone, meaning they cannot travel slower than light speed (relative to our 3-space), while implying that only lightlike wavefunctions (those associated with massless particles) adhere to the null cone.

This limitation will be addressed in Part 3 as we deepen our understanding of higher-dimensional space, time and relativity. For now, let us celebrate the fact that, at least for the case of lightlike phenomena, Special Relativity and quantum mechanics stand unblemished and reconciled.

2.9 The Transactional Interpretation of Quantum Mechanics

During the 1980s physicist John G. Cramer introduced a startling interpretation of quantum mechanics, what he called the *transactional interpretation* (TI) [14]. While appearing like science fiction and having consistency problems of its own (which, as we shall see, are potentially resolvable under this framework), TI has the curious distinction of resolving many paradoxes in quantum mechanics that other interpretations cannot touch. Cramer was inspired by the *absorber* theory of John Archibald Wheeler and Richard Feynman, describing electromagnetic interaction as a time-symmetric process; the electromagnetic wave equation has two solutions, known as *retarded* and *advanced*, which correspond to electromagnetic waves traveling forward and backward in time. It turns out that the relativistic version of the Schrödinger equation (which governs the evolution of the wavefunction in time) also has advanced and retarded solutions, suggesting that the wavefunction can travel both forwards and backwards in time.

According to TI, each quantum event involves a *transaction* between an *emitter* and an *absorber*. The emitter sends out an "offer wave", which at some time in the future is received by any number of absorbers, each of which in turn sends a "confirmation wave" *back in time* to the emitter. This means that the emitter receives the confirmation waves at the same instant that it emits the offer wave! Cramer describes the interaction as a "handshake" between the emitter and absorber, occurring in what he calls "pseudo-time". The offer wave is analogous to the wavefunction, while the confirmation wave is an attenuated conjugate. Glossing over details, when certain criteria are met, a particular transaction is completed between the emitter and an absorber and the wavefunction collapses, manifesting the associated event. (Note that we will be discussing wavefunction collapse in Part 4; for now the reader needs only understand that the transaction occurs across both space and time.)

Despite its paradox-resolving powers, reaction to Cramer's theory has been muted. While a few brave physicists and philosophers recognize the theory's merits and push it forward, most turn a blind eye [15]. It is just too weird, violating common sense. What is this "pseudo-time", and how can anything travel back in time? Indeed, I would suggest that *philosophical* issues present the greatest obstacle to TI being taken seriously by the physics community – physics simply cannot provide a philosophical or cosmological context for it. Theoretical physicist Anthony Zee touches upon this question in his delightful book *Fearful Symmetry*: [16]

> Physicists are careful to say that their knowledge is limited to the physical world. The realization that the world may be divided into the physical and, for lack of a better term, the nonphysical surely ranks as a major turning point in intellectual history, and one that has made possible the advent of Western science. But eventually we will have to cross the dividing line. I believe that a deep understanding of time reversal will take us across that line.

The dividing line demarcates our physical space from nonphysical spaces, and as Zee points out, time reversal takes us across that line. The alert reader will know where this is headed. Cramer's "pseudo-time" is, of course, the imaginary *spatial* dimension of Minkowski 4-space.

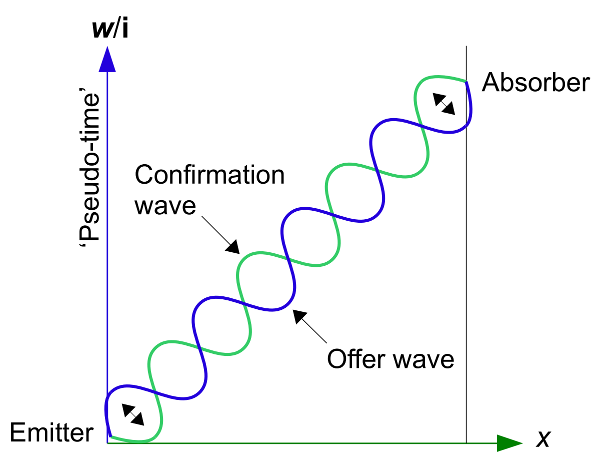

Figure 11: The wavefunction as a standing wave in Minkowski 4-space

Cramer points out that the offer and confirmation waves can be represented as a 4-vector standing wave, as illustrated in Figure 11, adapted from a drawing in his paper reviewing TI [17]. Keep in mind that this two-dimensional depiction does not reveal the deeper complex nature of these waves, each spiraling through real and imaginary dimensions. Moreover, the correct picture is to see these standing waves in *motion*, each oscillating back and forth as indicated by the small arrows. It follows that this entire picture, encompassing the space and "pseudo-time" dimensions shown in the figure, is changing *in time* (though clearly not physical time). Expressed another way, according to the standing wave representation of TI, pseudo-time is really a dimension of space, $\boldsymbol{w}$. Following this line of reasoning, it could be argued that TI *requires* the existence of Minkowski 4-space.

2.10 TI and the Aspect Experiment

Following our previous argument suggesting the presence of a 4-brane interpenetrating our 3-brane, Cramer's independent line of reasoning has brought us to the same conclusion from the perspective of quantum phenomena rather than the quantum formalism. A 4+1 spacetime is required to exist *within* our 3+1 spacetime, *coincident* with our spacetime. To test out these ideas, let us see what we can make of the Aspect experiment (and similar experiments) in the context of TI and Minkowski 4-space. For the sake of expediency just the bare facts are presented here; readers seeking a more detailed understanding of Bell's theorem and the Aspect experiment are directed to the references [18].

Entangled photons (produced at the same time by the same source) are known to always share the same *polarization*, meaning the light waves take the same preferred axis normal to the axis of propagation. Aspect sent pairs of entangled photons in opposite directions through polarizers to detectors situated some twelve meters apart. By cleverly measuring the polarization of the photon pairs at opposite wings of the experiment, Aspect demonstrated that *Bell's inequality was violated* – tech talk for establishing quantum nonlocality as an empirical fact of Nature.

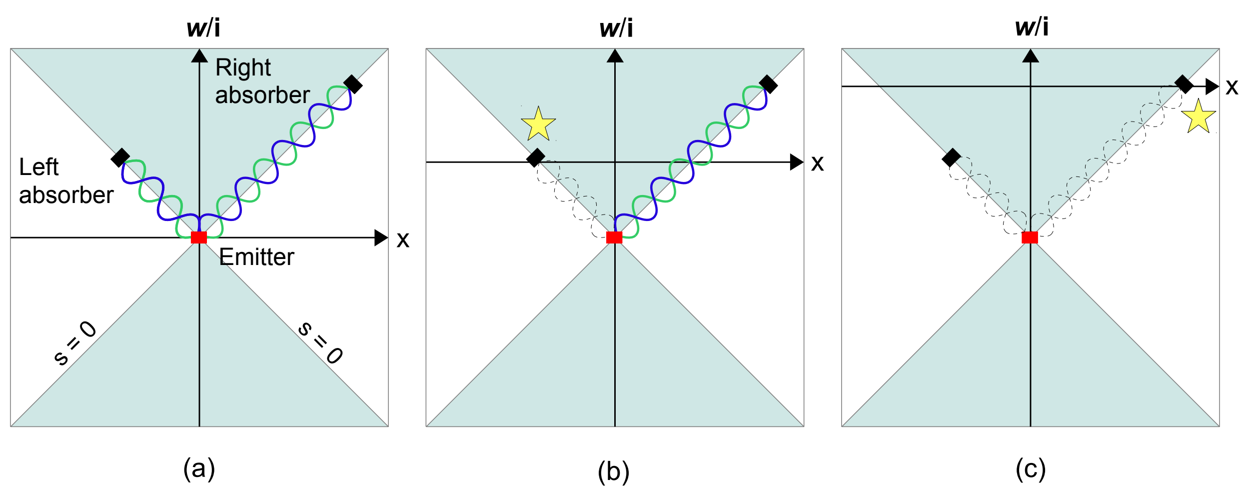

Figure 12: The Aspect experiment in Minkowski 4-space

Figure 12 presents a minimalist depiction of the Aspect experiment in the context of TI and Minkowski 4-space. Three stages of the experiment are shown, advancing in time from left to right. Note that the two wings are of different lengths to emphasize that one photon will always be absorbed before the other.

(a) The left diagram illustrates the moment in time when the photon pair are created. Since spatial distance on the null cone is zero, the offer waves and confirmation waves occupy the same location in Minkowski 4-space. This picture therefore manifests spontaneously, with both wings constituting one holistic wavefunction technically located at the null cone's origin. Until collapse occurs, the entire wavefunction is confined to its null cone (birth cone) in Minkowski 4-space.

(b) The center diagram illustrates the moment when the first photon is absorbed. Since time has passed, the imaginary spatial dimension has moved downwards (the *x* axis, corresponding to the moment, has moved up). Empirically, it is known that the wavefunction either adjusts its polarization to match the polarizer axis and passes through to be absorbed by the detector, or else it adjusts its polarization to the *normal* of the polarizer axis and is absorbed by the polarizer. The mechanism underlying this adjustment process has no direct relevance to the general mechanism of quantum nonlocality presented here, the important point being that the "adjustment" occurs *across both space and time* (from our perspective in the 3-brane), spontaneously throughout the holistic spatio-temporal wavefunction, the totality of which occupies just one location in Minkowski 4-space. Upon absorption, just this one wing of the wavefunction spontaneously collapses (its energy is transferred to the detected photon, being the process of state reduction, to be addressed in Part 4).

(c) The third diagram illustrates the moment in time when the second photon is absorbed. The same process occurs as for the first photon, with the exception that the polarization of the second photon has already been determined across time by the first measurement. Therefore, the measured polarization of the pair will always correlate.

Let us take stock of how our understanding is measuring up to the three features of the quantum connection which "deserve our close attention", as presented by Tim Maudlin:

1. *The quantum connection is unattenuated*. In Minkowski 4-space there is no distance between any two points on a null cone – therefore the complete wavefunction occupies just one location in Minkowski 4-space. Since the quantum connection has no distance to travel, it is unattenuated.

2. *The quantum connection is discriminating*. The complete wavefunction lives on a null cone in Minkowski 4-space, and everything on the null cone is entangled with (occupies the same location as) everything else. In general the wavefunction cannot interact nonlocally with wavefunctions on other null cones, since they are spatially separated in Minkowski 4-space. (As we shall discover later, however, wavefunctions separated in the 4-brane can occupy the same location in a higher dimensional space, representing a deeper mechanism of entanglement.)

3. *The quantum connection is faster than light (instantaneous)*. Since the entire wavefunction adheres to its null cone, there is no distance for the quantum connection to travel. Therefore it operates instantaneously.

We conclude that the Aspect experiment is consistent with the transactional interpretation in the context of an interpenetrating 4-brane with local spatial geometry represented by Minkowski 4-space. Having made progress, however, there is more to be done. We have yet to address the mysterious mechanism of time.

Part 3

Extra Dimensions, Symmetry, and Time

Having come to terms with deep mysteries concerning space and the quantum wavefunction, the reader may be feeling that we have found the holy grail, that potentially all of physics can be explained on the basis of a 4+1 spacetime interpenetrating our physical universe. Since Special Relativity and quantum mechanics meld so naturally within this framework, surely this interpenetrating space can explain our physical world? Alas, a direct reading of theoretical and mathematical facts reveals that a 4-brane permeating our 3-brane is still not enough to contain all of physics, nor to explain the nature and origin of time.

3.1 Kaluza's 5-dimensional Einstein-Maxwell theory

In 1919 the German-Polish mathematician Theodor Kaluza made a remarkable discovery. By formulating Einstein's General Relativity theory in five dimensions (four real spatial dimensions plus one time dimension) he derived two sets of field equations, one being Einstein's gravitational field equations, the other being Maxwell's equations of electromagnetism. In a nutshell, both gravity and electromagnetism were seen to emerge from the geometry of "empty" spacetime of a higher dimension. Legend has it that the normally reserved Kaluza danced about like an ebullient schoolboy, convinced he had unified physics. Later he wrote that his mathematical result revealed "virtually unsurpassed formal unity... which could not amount to the mere alluring play of a capricious accident." [1]

In those days gravity and electromagnetism were the only forces known to physics, and Einstein's success in describing gravity in purely geometric terms inspired efforts to integrate electromagnetism into a similar framework. Kaluza sent his work to Einstein, who was impressed but for one glaring detail: our physical universe appears to have three spatial dimensions – where is the fourth? This anomaly troubled Einstein and only two years later did he recommend publication of Kaluza's paper, which appeared in 1921.

In 1926 Oscar Klein suggested that the fourth spatial dimension is curled up in a tiny circle (compactified), and therefore undetectable to us. Theorists embraced this idea while adding more compactified dimensions in efforts to incorporate the strong and weak nuclear forces into the scheme, but these efforts encountered internal inconsistencies and predictions in conflict with experiment. Meanwhile, physicists have suggested other ways to hide the unobserved spatial dimensions, beyond the traditional compactified schemes – *projective* theories in which the extra dimensions are not physically real, and *non-compactified* theories in which dimensions are not necessarily lengthlike or compact. Nevertheless, while many so-called Kaluza-Klein theories have been devised over the years, none has been fully successful [2]. In fact, the problem of fitting Kaluza's model to physical reality has turned out to be so intractable that not all physicists are convinced the theory is anything more than a mathematical curiosity. Roger Penrose, for instance, opines the following: [3]

> Elegant as it is, the Kaluza-Klein perspective on Einstein-Maxwell theory does not provide us with a compelling picture of reality. There is certainly no strong motivation from physical directions to adopt it.

All higher-dimensional theories face the common problem of explaining why our physical universe appears in every way to have just three spatial dimensions, and further, why gravity does not spread into the extra dimensions. The empirical fact that a gravitational field diminishes with the square of distance leads to the conclusion that gravity spreads in just three dimensions (the surface area of a sphere increasing with the square of its radius). Moreover, it has been shown theoretically that physics as we know it would not be possible in spaces having other than three real dimensions – planetary orbits and chemical atoms would be unstable in a four-dimensional space, for instance. So we are faced with a mystery. While the elegance and economy of Kaluza's theory are undeniable, nobody has been able to make it work in the real world. Nine decades after its discovery, this beautiful mathematical result still has not found its place in physics.

Kaluza's 5-dimensional Einstein-Maxwell theory is essentially General Relativity formulated in a 4+1 spacetime (having four real spatial dimensions and one real time dimension), which yields both gravity and electromagnetism *in 3+1 spacetime*. Accordingly, the fourth spatial dimension, while real, is treated differently from the first three dimensions. In a comprehensive review of Kaluza-Klein Gravity, physicists J. M. Overduin and P. S. Wesson explain this distinction as follows: [4]

> Kaluza's achievement was to show that five-dimensional general relativity contains both Einstein's four-dimensional theory of gravity and Maxwell's theory of electromagnetism. He however imposed a somewhat artificial restriction (the cylinder condition) on the coordinates, essentially barring the fifth one a priori from making a direct appearance in the laws of physics.

As a result of this mathematical sleight of hand, all fields (including gravity) are confined to the first three dimensions. Roger Penrose, following a technical discussion about constraints upon the fourth spatial dimension required by Kaluza's scheme, and the need for a *Killing vector* to impose $U(1)$ symmetry on the fourth dimension, adds: [5]

> All that one needs, in addition, is that the Killing vector have a constant non-zero (in fact negative) norm. This eliminates an unwanted scalar field, and the exact 4-dimensional [3+1] Einstein-Maxwell theory is thereby expressed!

Let us pause to consider the profundity of this result. We are not talking about the prediction of phenomena, but the derivation of fundamental physical law! Indeed, as Kaluza observed, the result displays "unsurpassed formal unity", and we take the view, with Kaluza, that it could not amount to a "capricious accident". Accordingly, rather than trying to shoehorn Kaluza's theory into our physical world, where clearly it does not belong, we acknowledge that it must apply to some other space, having properties suggested by the theory itself. We know it cannot apply to our 3+1 spacetime, nor can it apply to the 4-brane, since in Kaluza's theory the fourth spatial dimension is real, in contrast to the imaginary fourth dimension of Minkowski 4-space.

The "cylinder condition" imposed on the fourth spatial dimension – which singles it out as special, different from the other three – has led to criticism that Kaluza's theory is arbitrary and contrived, there being no rational justification for preventing the fourth dimension from appearing directly in the physics of 3+1 spacetime. After all, what makes one real dimension different from any other? An important clue lies in the negative amplitude of the Killing vector, as described by Penrose. From this we learn that the fourth spatial dimension is required to be in some sense *negative* in relation to the first three dimensions.

Figure 13 illustrates how such a space may be constructed. Along with the imaginary dimension included in Minkowski 4-space, this higher space includes a second imaginary dimension having the same (negative) polarity as the first, in addition to the same three (positive) real dimensions. Due to their like polarity, these two imaginary dimensions together project a *negative* real dimension (depicted by the green rectangle).

The proposal is that Kaluza's theory applies to "physics" in a higher-dimensional brane, which is superimposed on both the 4-brane and our 3-brane. One may consider this higher-dimensional brane to be a 5-space, having three real and two imaginary dimensions, or as a 4-space, having four real dimensions, the fourth being negative. The cylinder condition reflects the negative polarity of this fourth (real) spatial dimension. Kaluza's theory, placed in this context, is telling us that processes in this higher-dimensional brane generate fundamental physical law underpinning physics in our 3+1 spacetime.

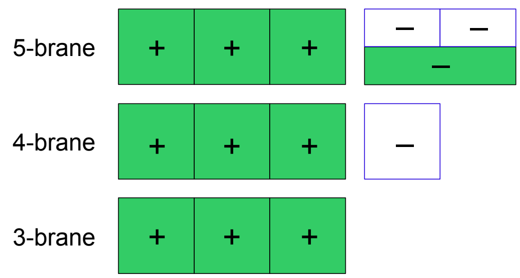

Figure 13: Spatial dimensions of the lower three branes

Note that the first three dimensions of each brane form coincident manifolds – all three manifolds see the same gravitational field and waves. While not appearing directly in the physics of 3+1 spacetime, the fourth (negative real) dimension is *everywhere present* in the 5-brane, as are the two imaginary dimensions on a more fundamental level. As a powerful consequence of this model, Kaluza's mathematical treatment demonstrates how positive real fields (including gravitational fields) are confined to the three positive real dimensions. Moreover, as previously discussed, a real field cannot leak into an imaginary dimension – they are of a different order. In each of these three branes, therefore, the need for compactification disappears.

The alert reader will be asking a crucial question. By deriving Einstein's gravitational field equations in the 5-brane, General Relativity spontaneously manifests in our 3-brane, since each real manifold shares the same space and therefore the same gravity. Electromagnetism is another story, however, since fields other than gravity are confined to a particular brane. So we have derived an electromagnetic field ruled over by Maxwell's equations in the 5-brane, while apparently having no contact with our 3-brane. What good is a derivation of electromagnetism that is confined to another world? Nature indeed has an elegant answer to this question, to be addressed as we take another spiral into the depths of natural law.

3.2 Extra Dimensions

Superstring theory, incorporating a symmetry framework known as *supersymmetry* (SUSY), fixes the number of spatial dimensions at nine, while M-theory reveals a tenth spatial dimension hidden in the mathematics. Note that these numbers are *fixed* by the mathematics; within the current string formalism they cannot be more or less [6].

We have determined that at least three interpenetrating spaces are required to account for known physics:

1. Our physical universe of three real spatial dimensions, understood as a 3-brane. This constitutes everything we can empirically know.
2. A 4-brane, consisting of three real dimensions coinciding with our physical 3-space, plus one imaginary spatial dimension. This is home to the wavefunction (as currently formulated in four spatial dimensions).
3. A 5-brane, consisting of three real dimensions coinciding with our physical space, plus two negative

imaginary dimensions. The two imaginary dimensions combine to project a negative real dimension, providing a natural environment for Kaluza's five-dimensional Einstein-Maxwell theory,

In a nutshell, a 5-brane and a 4-brane interpenetrate our 3-brane. Notice that a simple pattern has emerged: each brane includes just one more imaginary dimension than the brane below it. This brings us to present a key conjecture inspired by the aesthetics of order and symmetry. Let us suppose that this same pattern continues into the higher dimensions. To account for the spatial dimensions as fixed by string theory requires seven interpenetrating spaces, the seventh being a 9-brane.

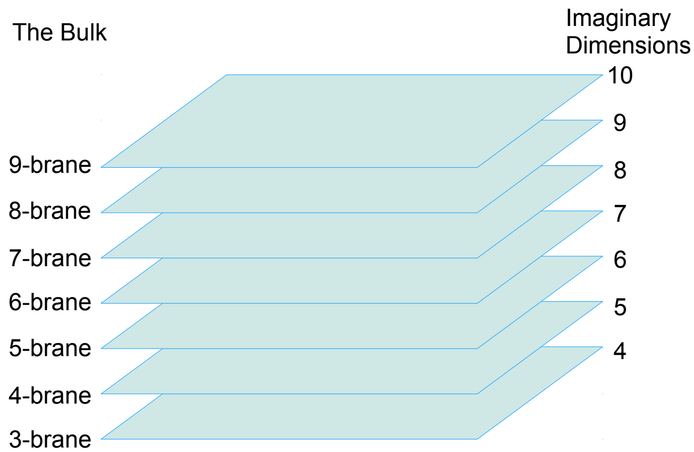

Figure 14: Interpenetrating spaces

According to this model, the seven branes occupy the same higher-dimensional space (the bulk), with each brane excluding spatial dimensions beyond its own particular dimensionality. Most importantly, rather than being stacked as depicted in Figure 14, the seven branes are superimposed. It follows that corresponding manifolds in different branes see the same gravitational field, while all other fields are confined to a particular brane. Consequently, the seven branes are *materially isolated* – there can be no interaction between the "matter" of the various branes, meaning in principle that we cannot empirically observe any brane other than the 3-brane (our physical universe). While everywhere present, the higher branes are forever *inaccessible* to our physical senses and instruments. Only gravity (the geometry of space itself) is shared by branes.

3.3 The Tenth Dimension

While superstring theory requires nine spatial dimensions, the undisputed intellectual leader of string theory, Edward Witten, found a tenth spatial dimension locked up in his advanced mathematics which had previously gone undetected by (approximate) perturbative methods. But this tenth dimension is not like the other nine. While strings are required to vibrate in nine dimensions, Witten found that under certain conditions a particular type of string called the *Heterotic-E* could itself become extended in a *tenth* dimension. Brian Greene makes this distinction as follows: [7]

> [The constraint of nine spatial dimensions] arises from counting the number of independent directions in which a string can vibrate, and requiring that this number be just right to ensure that quantum-mechanical probabilities have sensible values. The new dimension we have just uncovered is *not* one in which the Heterotic-E string can vibrate, since it is a dimension that is locked up within the structure of the "strings" themselves.

Within our model of seven interpenetrating spaces, the highest space is a 9-brane. Where is the tenth spatial dimension of M-theory? Recall that we are counting our three real dimensions as three, when in fact they consist of four imaginary dimensions. The tenth dimension is none other than the intrinsic dimension hidden within the three real dimensions – right before us, yet so hard to see!

Accordingly, the Universe includes ten imaginary spatial dimensions, but when we count our real space as three dimensions, the total is nine. It follows that each brane includes one more imaginary dimension than its designation would suggest. Nevertheless, to avoid confusion we will continue to count our real 3-space as three dimensions, with the implicit understanding that the 3-manifold is constructed from four imaginary dimensions.

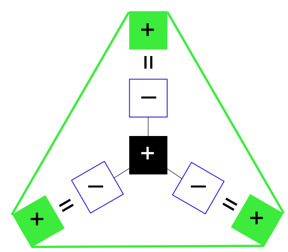

Figure 15: The tenth (intrinsic) dimension within real 3-space

3.4 Foundations of Symmetry

Recall the insightful words of Roger Penrose (quoted in the introduction) suggesting that two great mysteries rise up above all others in the theoretical foundations of physics, these being *complexity* and *symmetry*. Having reduced the problem of complexity to the presence of imaginary spatial dimensions, on the basis of our interpenetrating brane model we are now ready to tackle the problem of symmetry.

Symmetry itself is not the problem, of course – the problem is that we don't understand it. Most of us relate the idea of symmetry to transformations in physical space, such as under reflection in a mirror or by spinning around full circle and returning to our original state. By comparison, the idea of symmetry in modern physics is rather obscure. The symmetry groups appearing in the standard model of particle physics each involve transformations of complex variables, which are not so easy to visualize – nobody knows how to visualize an imaginary dimension, let alone a complex one. Yet we know that these symmetries exist in Nature. The following three symmetry groups are of central importance to the standard model:

- *SU*(2), the special unitary group of two complex variables, which rules over quantum *spin* phenomena.
- *SU*(2) x *U*(1), combining *SU*(2) with the unitary group of one complex variable, under which the electromagnetic and weak nuclear forces are unified as the *electroweak* interaction.
- *SU*(3), the special unitary group of three complex variables, ruling over *quantum chromodynamics*, the theory of strong interactions involving quarks and gluons.

In Part 1 we argued for the existence of imaginary spatial dimensions on the basis of the *SU*(2) symmetry group and its role in objective quantum spin phenomena. Just as the three real dimensions of the *SO*(3) symmetry group correspond directly to our real 3-space, the two complex dimensions of the *SU*(2) symmetry group correspond to an objective space having two complex dimensions. Since elementary particles physically respond to these dimensions (such as a fermion spinning twice on its axis to return to its original state), we can infer that these dimensions exist objectively in Nature. The proposal is generalized as follows:

- Each of the symmetry groups appearing in the standard model corresponds to a particular dimensional configuration of an interpenetrating brane.

3.5 *SU*(2) and the 4-Brane

This means, for instance, that the *SU*(2) symmetry group codifies transformations in an objective space configured as two complex dimensions. Could such a space exist in Nature? Recall that, while a particular spatial dimension can be either real or imaginary (never complex), real and imaginary dimensions may combine to form a composite "complex" dimension. Accordingly, Figure 16 illustrates how the five imaginary dimensions of the 4-brane can present themselves as two complex dimensions. Note that each spatial configuration is constructed from the same five imaginary dimensions, the intrinsic dimension being positive and the others negative. The implication is that quantum spin phenomena respond to a particular *spatial configuration* of the 4-brane, presenting itself as two composite complex dimensions bound together by the common intrinsic dimension, representing a precise spatial context for the *SU*(2) symmetry group.

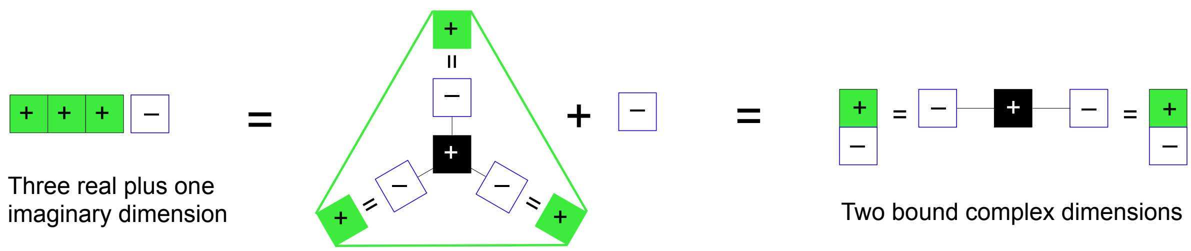

Figure 16: Spatial configurations of the 4-brane

So we have solved a mystery only to be confronted with another. Elementary particles are objective, physical phenomena – they can be empirically observed and measured, implying that they are objectively present in the 3-brane, while at the same time they demonstrate spin properties consistent with a particular spatial configuration of the 4-brane. How can particles behave as if they are in the 4-brane when we know they are objectively present in our 3-brane? We will approach this mystery in due course.

The model seems to suggest (at least so far as spin phenomena are concerned) that the universe favors a particular direction, being that real dimension which is "lost" when the five imaginary dimensions present themselves as two bound complex dimensions. Recent observations of galaxy dynamics and the Cosmic Microwave Background indeed reveal such a "preferred axis" on a cosmic scale [8].

3.6 *SU*(2) x *U*(1) and the 5-Brane

Now we turn our attention to the 5-brane, which includes an additional imaginary dimension, as illustrated in Figure 17. By the simplest accounting, the 5-brane would include two bound complex dimensions (as in the 4-brane) along with a solitary imaginary dimension. Note that this configuration presents just two real dimensions – if the third (lost) real dimension is allowed to align itself with the solitary imaginary dimension (as indicated by the dashed line in Figure 17), a complex dimension results. Thereby it is possible to arrive at a spatial configuration supporting the *SU*(2) x *U*(1) symmetry of electroweak theory.

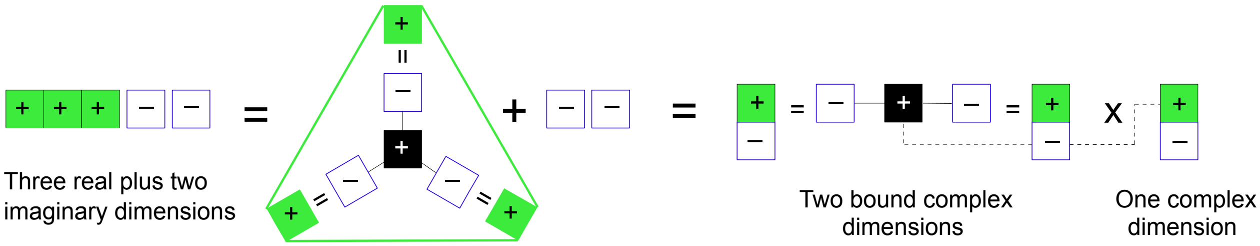

Figure 17: Spatial configurations of the 5-brane

Once again we are faced with the paradox that unification of the electromagnetic and weak forces occurs in a space reflecting that of the 5-brane, yet the resulting phenomena are observed empirically in our 3-brane. Recall that Kaluza's Einstein-Maxwell theory describes gravity and electromagnetism in the 5-brane, rendering this spatial context for the *SU*(2) x *U*(1) symmetry group the second independent argument that electromagnetism has its source in the 5-brane. Further, it is worth noting that Kaluza's theory imposes the *U*(1) symmetry group on the fourth spatial dimension, in accord with this model.

3.7 *SU*(3) and the 6-Brane

The 6-brane introduces a third imaginary dimension, as shown in Figure 18, allowing each of the additional imaginary dimensions to align itself with a real dimension. The resulting spatial configuration of three bound complex dimensions is almost perfectly symmetrical, but not quite. On the basis that space includes a total of ten imaginary dimensions – five positive and five negative – all five negative dimensions are already accounted for in the 5-brane. In the 6-brane, therefore, the additional imaginary dimension is required to be positive. It is clear that the *SU*(3) symmetry group, so crucial to the standard model, can in principle be supported by this beautiful spatial configuration of the 6-brane.

Clearly, the spatial structure of the 6-brane represents a radical departure from that of the lower three branes. While the lower branes each include a real 3-manifold, the 6-brane constitutes a *complex* 3-manifold, having three complex dimensions (represented by the blue triangle). This distinction will prove of utmost importance, since it imposes a natural dividing line between the lower three branes (each of which includes a real 3-manifold) and the higher branes (which do not).

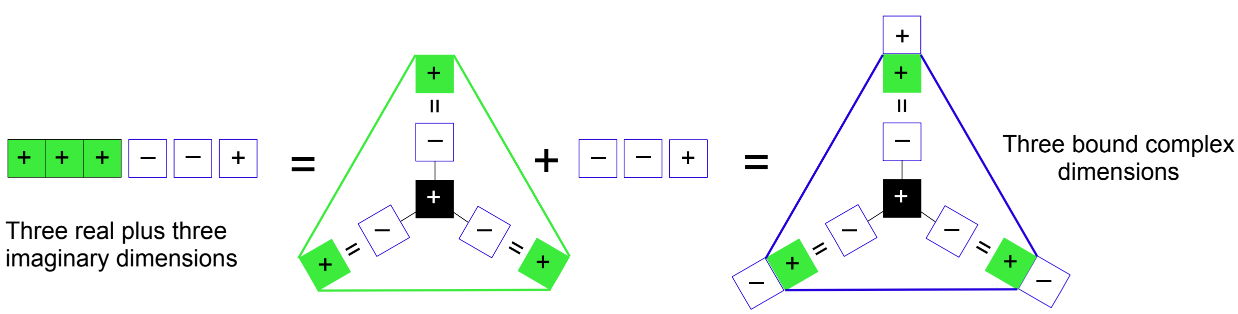

Figure 18: Spatial configuration of the 6-brane

3.8 Spatial Motion and Time

The attentive reader will have noted a glaring omission in the current model. Thus far we have been discussing the *spatial* dimensions of the lower four branes, without regard to *time*. Yet we know that time exists in our 3-brane, while the evolving wavefunction requires time in the 4-brane, and Kaluza's Einstein-Maxwell theory is formulated in 4+1 spacetime, implying time in the 5-brane. Finally we can address this omission: we now have all the pieces we need to approach the puzzle of time.

Within this framework, time emerges as a natural and unambiguous consequence of two factors: the spatial structures of the interpenetrating branes, and the mysterious phenomenon of *spatial motion*. In any particular space (brane), time corresponds to motion of the entire space on (or along) a higher dimension. This principle can be established on logical principles alone. Figure 19 depicts the reduced case of a two-dimensional space (surface) moving along a higher (third) dimension. At whatever coordinate on the higher dimension w the two-dimensional space is located, the entire two-dimensional space is present. On the other hand, at any location in the two-dimensional space, just one location on the w dimension is present. Logically, the w axis is indeed of higher dimension than the two-dimensional space – that is, the entire two-dimensional space is included within the higher dimension and moves as an integral entity relative to it.

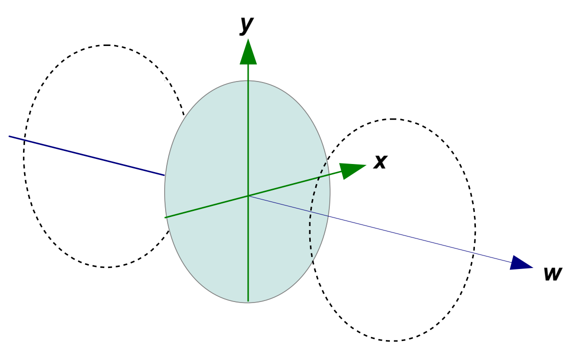

Figure 19: Spatial motion on a higher dimension

Extending this simple model to our physical universe suggests that our 3+1 spacetime consists of a 3-brane in motion relative to a higher (fourth) dimension, which is *not included* in the 3-brane. Since time in the 3-brane is rotated into the imaginary dimension $\boldsymbol{w}$ in the 4-brane, it follows that time results from motion of the $\boldsymbol{w}$ dimension relative to the three real dimensions. One can visualize this spatial motion by picturing the vertical ($\boldsymbol{w}$) axis of Minkowski 4-space (Figure 9) flowing constantly downward, the future above irrevocably becoming the past below. At any moment in physical time (denoted $t3$), our physical universe reflects a "slice" of the 4-brane at a particular coordinate of the imaginary fourth dimension.

Taking this principle to its logical conclusion, time in the 4-brane implies motion relative to the second imaginary ($\boldsymbol{v}$) dimension in the 5-brane, while time in the 5-brane eventuates from motion relative to a still higher dimension in the 6-brane (call it $\boldsymbol{u}$). Where does it end? What is the origin of time? Since a complete answer to this question would take us well outside the realm of "physics", and therefore outside the scope of this paper, the reader is asked to consider the following conjecture:

- Time originates with spatial motions of the two imaginary ($\boldsymbol{w}$ and $\boldsymbol{v}$) dimensions in the 5-brane relative to each other and to the three real dimensions.

The implication is that there is no spatial motion (and therefore no time) in the higher branes (the dimension $\boldsymbol{u}$ is static). This conjecture finds support from technical considerations alone. Recall that

Kaluza's Einstein-Maxwell theory derives Einsteins' field equations in the 5-brane, representing the bedrock of space and time (spacetime) as we know them. Moreover, since real 3-manifolds exist only in the lower three branes, one might assume that the same applies to time.

Figure 20 provides a schematic representation of the process underpinning time and energy in the lower three branes. Note that the 6-brane includes two *positive* imaginary dimensions in its construction (colored black) – the intrinsic *space* dimension and what we may call the intrinsic *time* dimension ***u*** – rendering the 6-brane the *precursor* to spacetime in the lower branes. The rotating circle represents the very heart of the process, the "engine room" of objective manifestation, set into activity by the spatial motions of the imaginary dimensions ***v*** and ***w*** in the 5-brane. For present purposes these spatial motions are taken as primary, *a priori*.

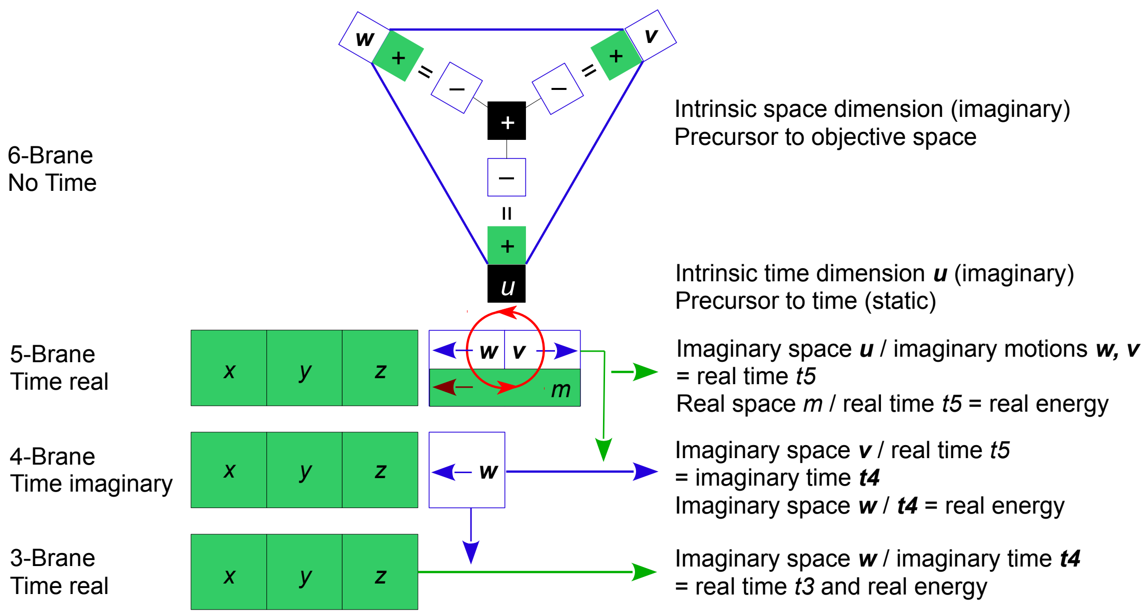

Figure 20: The genesis of energy and time

Motion implies time, of course, and relates directly to *energy*. Upon this foundation, the nature of time and energy in the three lower branes emerges from the algebra of imaginary numbers. First, since Kaluza's Einstein-Maxwell theory requires *real* time in the 5-brane, we surmise that the *motions* of the ***v*** and ***w*** dimensions are *imaginary* – that is, imaginary space over real time (*t5*). These imaginary motions set the 5-brane into motion relative to the (static) imaginary dimension ***u*** in the 6-brane, yielding real time in the 5-brane (imaginary space ***u*** over the imaginary motions ***v*** and/or ***w***) while also projecting the fourth real dimension *m* into motion, manifesting as *real energy* (real space *m* over real time *t5*). The key insight is that real time and real energy emerge from the same imaginary spatial motions in the 5-brane.

Recall that Kaluza's theory derives both gravity and electromagnetism in the 5-brane, which implies the manifestation of *energy*. Corroboration comes from recent developments in string theory, what string theorist Steven Gubser describes as the *gauge/string duality* (also known as the AdS/CFT correspondence, to be discussed later), as applied to heavy ion collisions: [9]

> Subsequent developments seem to indicate that many aspects of heavy ion collisions have close analogies in gravitational systems. The gravitational systems in question always involve an extra dimension. It's not like the extra dimensions of string theory in its theory-of-everything guise. This extra dimension… is not rolled up. It's at

> right angles to our usual ones, and we can't move into it in the usual way. What it describes is energy scale – meaning the characteristic energy of a physical process. By combining the fifth dimension with the ones we know and love, you get a curved five-dimensional spacetime.

Gubser's description closely reflects Kaluza's model in the context of the current framework. A full understanding of Figure 20 promises to explain the miraculous appearance of real energy in the 5-brane while illuminating the mysterious relationship between energy and time, so fundamental to physics.

As the 5-brane is projected into the 4-brane, the imaginary motion of the fifth dimension ***v*** manifests as *imaginary time* in the 4-brane (***t4***). Note that this is not the "imaginary time" of current physics, which is in fact imaginary *space* – the fourth spatial dimension ***w***. Rather, somewhat bizarrely, *4-brane time is imaginary*. Accordingly, in the 4-brane the dimension ***w*** is moving in imaginary time, once again manifesting as *real energy* (***w/t4***).

Following the same pattern, as the 4-brane is projected into the 3-brane, the motion of the fourth dimension ***w*** manifests in the 3-brane as *real time* (imaginary space ***w*** over imaginary time ***t4*** equals real motion, time or energy). It follows that, while we experience just the ***w*** motion (as time), the 3-brane is in fact undergoing a total of three spatial motions, along the ***u***, ***v*** and ***w*** dimensions.

Having illuminated the nature of time on the basis of interpenetrating branes and spatial motions, we are left with a foundational question: What is the origin of these spatial motions? What "moves" the imaginary dimensions ***v*** and ***w***? And what exactly is "imaginary motion"? While a satisfactory answer to these questions would take us into cosmological and philosophical territory beyond the scope of this paper, the reader may wish to contemplate the following words from Plato's dialogue *Timaeus:* [10]

> Now the nature of the ideal being was everlasting, but to bestow this attribute in its fullness upon a creature was impossible. Wherefore he resolved to have a moving image of eternity, and when he set in order the heaven, he made this image eternal but moving according to number, while eternity itself rests in unity; and this image we call time.

3.9 The 5-Dimensional Wavefunction

Having established that each of the lower three branes includes a real 3-manifold and a time dimension, along with real energy, we are brought to a remarkable conclusion. First, given these properties, each of these branes constitutes an *objective world*, containing real three-dimensional forms and processes evolving in time. Further, since branes are transparent to gravity, the wavefunction appears identically in each of the three (coincident) real 3-manifolds, while extending also into the imaginary dimension(s) of the 4-brane and 5-brane. While real fields (including gravitational fields) are confined to the real 3-manifolds, the wavefunction itself extends into all available dimensions, suggesting that the canonical formulation of the wavefunction is incomplete.

The wavefunction's fifth spatial dimension ***v*** we might assume relates specifically to the interface between the 5-brane and 4-brane, perhaps explaining why its presence has not been missed in the 3-brane. While quantum mechanics provides consistent (stochastic) results without considering the ***v*** dimension of the wavefunction, it seems reasonable to speculate that this fifth dimension may provide the "hidden variables" required to determine the outcome of a single quantum event, beyond stochastic predictions over an ensemble.

The presence of the wavefunction on every dimension of the three lower branes grants it profound unifying power, as illustrated in Figure 21. Keep in mind that each depicted wavefunction is *the same wavefunction* appearing in the three branes. From its humble beginnings as an insubstantial "probability wave", the wavefunction has become the cornerstone of unification in and of the three worlds (branes).

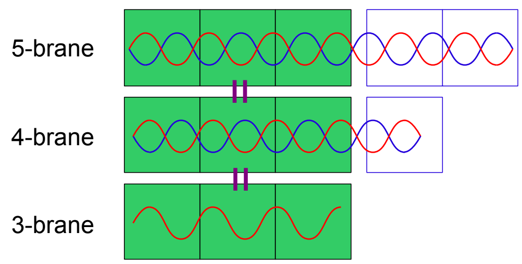

Figure 21: The unifying 5-dimensional wavefunction

Perhaps the most startling consequence of this model is as follows:

- According to quantum theory, every objective entity in our physical universe is fundamentally a wavefunction. By its very nature as a gravitational wave, the wavefunction is always simultaneously present in each of the lower three branes (worlds). Therefore, everything in our 3-brane is represented also in the 4-brane and the 5-brane.

This conclusion carries deep philosophical undertones, of course. It follows that a thorough understanding of physics in our physical universe requires us to consider all three branes – not just in isolation, but as a unified entity.

3.10 Relativity and Nonlocality in the Three Worlds

Since the wavefunction manifests fully in the 5-brane, the 5-brane could be considered the substratum of the 4-brane, which is in turn the substratum of the 3-brane. This should be clear from the fact that quantum mechanics formulates the wavefunction as it manifests in the 4-brane, as a complex wave having one imaginary dimension, while providing the basis for all matter and phenomena in the 3-brane. Being fully extended only in the 5-brane, however, the wavefunction behaves fundamentally as an entity in 5+1 spacetime. It follows that an understanding of spacetime in the 5-brane will deepen our understanding of quantum nonlocality.

Recall our discussion of the Euclidean metric for Minkowski 4-space (Figure 9), being the metric representing *spatial displacement* (distance) in the 4-brane, while ignoring the time dimension.

$$s^2 = x^2 + y^2 + z^2 + \boldsymbol{w}^2$$

From this formula we concluded that locations separated in the three real dimensions may occupy one location in Minkowski 4-space, constituting a null cone where the real and imaginary contributions to the metric correspond, yielding zero displacement (spatial distance) in the 4-brane. Since all four dimensions are spatial, the null cone represents a particular *orientation* or *direction* relative to the real and imaginary dimensions. While appearing as an extended object in our graphical representations in real space, the entire null cone represents *one location* in the complex Minkowski 4-space, given by its origin.

Upon including the second imaginary dimension $\boldsymbol{v}$, the spatial metric for Minkowski 5-space might be expected to appear as follows. Once again, this is a spatial metric only, for calculating distance between points in space, the time dimension being ignored:

$$s^2 = x^2 + y^2 + z^2 + \boldsymbol{w}^2 + \boldsymbol{v}^2$$

It is immediately apparent that whenever the $\boldsymbol{w}$ displacement is of lower magnitude than the resultant real displacement, one can adjust $\boldsymbol{v}$ to yield an interval of zero. That is, given an appropriate displacement or orientation in the $\boldsymbol{v}$ direction, entities separated or extended in Minkowski 4-space can occupy just one location in Minkowski 5-space.

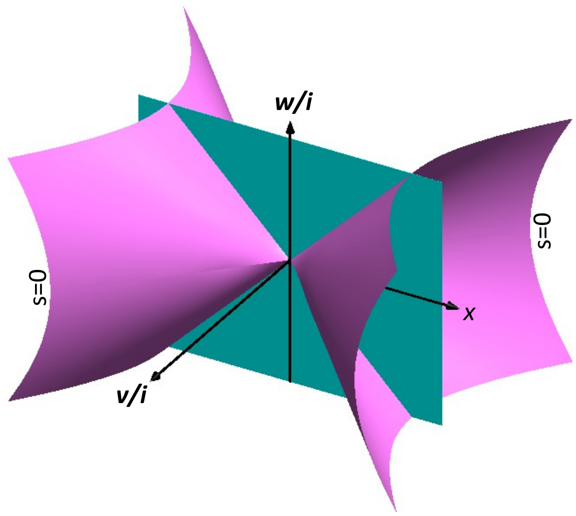

Figure 22: Null surface in Minkowski 5-space

The structure of Minkowski 5-space – representing the local spatial geometry of the 5-brane – is illustrated in Figure 22. Just one real dimension (x) is represented, pointing in some direction in real space, the $\boldsymbol{w}$ and $\boldsymbol{v}$ dimensions being imaginary. The plane corresponds to Minkowski 4-space, which is intersected by the 5-dimensional hyperbolic null surface (where spatial intervals are zero) at the 4-dimensional null cone. Analogous to the null cone in Minkowski 4-space, the null surface represents a single location in Minkowski 5-space. Note the following:

- As a direct consequence of this model, two distinct wavefunctions, separated in Minkowski 4-space, can be located on one null surface in Minkowski 5-space – that is, by being suitably displaced in the $\boldsymbol{v}$ direction they become one entangled wavefunction in the 5-brane. Turning this around, a more accurate picture would be to consider highly unified structures in the 5-brane being projected into greater spatial diversity in the 4-brane, and then into still greater diversity in the 3-brane. On this basis it is proposed that the holistic structures in the 5-brane provide a potential mechanism underpinning causality in the lower branes, while avoiding the causal paradoxes typically dogging faster-than-light schemes.
- While our elucidation of nonlocality has previously focused on lightlike phenomena, the current model provides the basis for nonlocality on a more general level. Recall from section 2.8 that the propagation speed (phase velocity) of a particle wavefunction is given as c^2/v, where v is the velocity of the particle itself. It follows that a lightlike wavefunction adheres to a null cone in Minkowski 4-space, while the wavefunction of a particle with rest mass is confined to regions outside the null cone while traveling faster than light in the 3-brane (the wavefunction of a particle at rest propagates at infinite speed). Accordingly, an appropriate orientation in the $\boldsymbol{v}$ direction places the wavefunction of a massive particle on a null surface in the 5-brane.
- Note that wavefunctions of massive particles are oriented in the $\boldsymbol{v}$ direction (in the 5-brane) while lightlike wavefunctions are not. Given our previous conclusion that real energy emerges in the 5-brane, we find here a clue regarding the origin of mass, along with a possible mechanism connecting mass with curvature of 3+1 spacetime (or the 4-brane), hence connecting quantum mechanics and gravity.

- Thus far we have been discussing just the *spatial* metrics of the three lower branes, while a complete understanding of relativity and nonlocality demands an understanding of the *spacetime* metrics. The key question is: What are the time signatures for the Minkowski 4+1 and 5+1 spacetimes? Answering this question is not as obvious as it may seem. First, consider that the spatial motions of the $\boldsymbol{w}$ and $\boldsymbol{v}$ dimensions have opposite sense to the flow of time. That is, while the $\boldsymbol{w}$ dimension moves in the direction of the past, time in the 3-brane flows towards the future. Similarly, time in the 4-brane (which is imaginary!) has opposite sense to the imaginary motion of the $\boldsymbol{v}$ dimension in the 5-brane. Given these subtleties, a detailed derivation of the spacetime metric signatures is not addressed here.

To sum up, there are two levels of quantum entanglement in our objective universe, and therefore two levels of nonlocality, corresponding to *locality* in the 4-brane and the 5-brane. Moreover, in principle, "time" in a particular brane can be traversed spatially, in either direction, in a higher brane. On the foregoing basis it is proposed that the framework provides the conceptual underpinnings for a rigorous formulation of relativity and quantum nonlocality in the lower three branes, while promising to illuminate the relationships between space, time, and energy (mass) in our physical universe. Further, it is expected that inconsistencies in the transactional interpretation of quantum mechanics, such as the *contingent absorber* challenge raised by Tim Maudlin, will find resolution under this framework [11].

Part 4

Physics in the 3-Brane

Let us take stock of what has been accomplished thus far. First and foremost, the two problems singled out by Roger Penrose as "important but largely unaddressed questions of principle in our physical theory", *complexity* and *symmetry*, have each been elucidated on the basis of imaginary spatial dimensions and interpenetrating higher-dimensional spaces (branes). Following from Pusey's theorem we have surmised that the complex wavefunction is an objective gravitational wave extended in three real dimensions plus two higher (imaginary) dimensions. Accordingly, Special Relativity has been extended into higher dimensions while providing the basis for quantum nonlocality. The $SU(2)$ symmetry group has found a context in a particular configuration of Minkowski 4-space (the 4-brane), while Kaluza's Einstein-Maxwell theory, electroweak unification, and the $SU(2) \times U(1)$ symmetry group have converged in the 5-brane. The $SU(3)$ symmetry group has been shown to reflect the spatial configuration of the 6-brane, being the precursor of spacetime in the lower three branes, while time has been shown to emerge from primary spatial motions in the 5-brane. Note that all of this involves higher dimensions. Beyond the manifestation of physical space and time from imaginary dimensions, we have scarcely addressed physics in *our* world – in the 3-brane.

4.1 Unitary Evolution and State Reduction

The quantum formalism reveals two distinct processes underlying physical manifestation: the *unitary evolution* of the wavefunction, which is deterministic and symmetric in time under the governance of Schrödinger's equation; and the *collapse* of the wavefunction, or *state vector reduction*, a process which is irreversible (time asymmetric) and stochastic, and which occurs only when a *measurement* is performed on the wavefunction. No interpretation of quantum mechanics would be complete without accounting for both of these processes.

While fully present in the 5-brane, the wavefunction is canonically formulated as a *complex* wave (its phase is represented by a complex number), interpreted here as an objective wave extended in the 4-brane. It follows that the unitary evolution of the wavefunction, as currently formulated, takes place in the 4-brane. When a *measurement* takes place, however, observable particles and phenomena appear in the 3-brane. Accordingly, the process of *state reduction* or *wavefunction collapse* could be understood as representing the interface (point of contact) between the 4-brane and our 3-brane. To accomplish this, the most direct reading of the facts suggests that the wavefunction excites *fields* in the 3-brane, as depicted in Figure 23. How many fields are required, and what is their nature? To address this question we begin by taking a look at the current state of the art, known as *quantum field theory*.

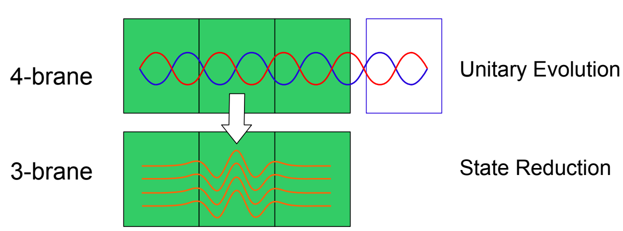

Figure 23: The action of the wavefunction

4.2 The Success and Failure of Quantum Field Theory

When Paul Dirac formulated his relativistic theory of the electron in the 1920s he was led to a field theory, which subsequently evolved into the first quantum field theory, quantum electrodynamics. Since then, quantum field theory (QFT) has become a cornerstone of fundamental physics, as Roger Penrose explains: [1]

> Quantum field theory constitutes the essential background underlying the standard model, as well as practically all other physical theories that attempt to probe the foundations of physical reality….
>
> In fact, QFT appears to underlie virtually all the physical theories that attempt, in a serious way, to provide a picture of the workings of the universe at its deepest levels. Many (and perhaps even most) physicists would take the view that the framework of QFT is 'here to stay', and that the blame for any inconsistencies… lies in the particular scheme to which QFT is being applied, rather than in the framework of QFT itself.

What are these "inconsistencies" faced by QFT? Here we single out three issues which tend to be largely swept under the carpet in the working lives of physicists:

1. Renormalization. QFT is mathematically inconsistent.
2. The grossly wrong vacuum energy (space density) calculation.
3. Field proliferation.

QFT has the curious distinction of producing the most accurate calculation ever in science (the magnetic moment of the electron, correct to ten significant digits) and the worst result in the history of science (the vacuum energy, off by some 120 orders of magnitude) [2]. Meanwhile, the dubious mathematical procedure known as *renormalization*, required to extract finite answers from QFT, remains controversial among physicists and mathematicians alike. Stephen Hawking and Leonard Mlodinow describe the procedure as follows: [3]

> The process of renormalization involves subtracting quantities that are defined to be infinite and negative in such a way that, with careful mathematical accounting, the sum of the negative infinite values and the positive infinite values that arise in the theory almost cancel out, leaving a small remainder, the finite observed values of mass and charge. These manipulations may sound like the sort of things that get you a flunking grade on a school math exam, and renormalization is indeed, as it sounds, mathematically dubious. One consequence is that the values obtained by this method for the mass and charge of the electron can be any finite number. That has the advantage that physicists may choose the negative infinities in a way that gives the right answer, but the disadvantage that the mass and charge of the electron therefore cannot be predicted by the theory.

Note that renormalization places limits on the predictive power of QFT, requiring that mass and charge be inserted into the standard model as field parameters, all contributing to some twenty free parameters in the standard model. In the following two passages Roger Penrose presents a mathematician's perspective: [4]

> Strictly speaking, quantum field theory (at least in most of the fully relevant non-trivial instances of this theory that we know) is *mathematically inconsistent*, and various 'tricks' are needed to provide meaningful calculational operations. It is a very delicate matter of judgment to know whether these tricks are merely stop-gap procedures that enable us to edge forward within a mathematical framework that may perhaps be fundamentally flawed at a deep level, or whether these tricks reflect profound truths that actually have a genuine significance to Nature herself… Some of these appear to be genuinely unravelling some of Nature's secrets. On the other hand, it might well turn out that Nature is a good deal less in sympathy with some of the others!

> Despite the undoubted power and impressive accuracy of quantum field theory (in those few cases where the theory can be fully carried through), one is left with a feeling that deeper understandings are needed before one can be confident of any 'picture of physical reality' that it may seem to lead to.

The "picture of physical reality" presented by the standard model of particle physics includes some sixty fields extended throughout 3+1 spacetime, with each particle species (along with its antiparticle) being understood as excitations of a unique quantum field, in addition to fields of a more substantial nature, as Nobel laureate Frank Wilczek explains in his book *The Lightness of Being:* [5]

> Besides the fluctuating activity of quantum fields, space is filled with several layers of more permanent, substantial stuff. These are ethers in something closer to the original spirit of Aristotle and Descartes – they are materials that fill space. In some cases, we can even identify what they're made of and even produce little samples of it. Physicists usually call these material ethers *condensates*. One could say that [the ethers] condense spontaneously out of empty space as the morning dew or an all-enveloping mist might condense out of moist invisible air.

The most widely known of these "material ethers" is the *Higgs condensate*, while the best understood is known as *chiral symmetry-breaking condensate*. The Michelson-Morley experiment and Einstein's special theory of relativity are widely regarded as having done away with the ether, but Wilczek points out that Einstein later changed his mind on this issue. In fact, Einstein himself claimed that General Relativity is very much an "ethereal" (ether-based) theory of gravitation: [6]

> According to the general theory of relativity space without ether is unthinkable; for in such a space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense.

The "ether" measuring space and time is known as a *metric field*, permitting the notion of *intervals* in space and time. Physicists speak of the metric field giving "rigidity" to space and time, permitting consistent measurements of both. Beyond these "conventional" fields, variations of the standard model (such as those incorporating supersymmetry) add many further fields. Meanwhile, string theory (which incorporates the principles of quantum field theory) does no better, as string theorist Leonard Susskind explains: [7]

> The Laws of Physics are like the "weather of the vacuum", except instead of the temperature, pressure, and humidity, the weather is determined by the values of fields…. String theory has an unexpected answer to the question of how many fields control the local vacuum weather. From the current state of knowledge, it seems that it is in the hundreds or even thousands.

If physics is indeed the pursuit of order and simplicity in Nature, something is clearly wrong. The problem of field proliferation goes beyond the sheer number of fields permeating space and time:

- Many of the fields of the standard model are near-duplicates. For instance, the eight gluons are all similar except for their color charge, yet each requires a unique field, leading to field properties being duplicated many times over. This paints a very uneconomical picture of Nature.
- Discrete states match for the various fields. For instance, multiple particle species might share the same precise electric charge of 1 or spin one-half. The fact that attributes correspond or are consistently related among the various fields implies a deeper order which "informs" each field (be it objective or abstract).

Since the quantum fields of QFT precede matter, they are not considered "physical" but somewhat as effervescent abstractions. Yet they do produce real physical effects, providing the substratum for the creation and annihilation of elementary particles. When the quantum fluctuations for the various fields are accounted for according to Heisenberg's uncertainty principle, the ground state of the vacuum (vacuum energy) is found to be infinite, or some 120 orders of magnitude larger than the observed value in the case of supersymmetric models. The "problem of the cosmological constant" remains a deep embarrassment for theoretical physics.

So let us sum up. Despite its many successes, QFT is known to be mathematically inconsistent. The grossly wrong vacuum energy calculations provide further notice that QFT does not paint a true picture of Nature. Perhaps most telling is the proliferation of fields populating the standard model, many being largely duplicates, surely settling the issue on the basis of economy and aesthetics alone. But there is a deeper reason why QFT does not and cannot correctly reflect Nature: quantum field theory is the logical outcome of bringing together Special Relativity and quantum mechanics *in 3+1 spacetime*.

We have already discovered that the holistic wavefunction occupies a 4-brane and a 5-brane, *outside* our 3+1 spacetime, where it coexists very happily with Special Relativity. It follows that QFT is a mathematical abstraction representing a limiting case of a higher-dimensional process being shoehorned into our physical 3+1 spacetime. We will come across corroboration of this fact from an unexpected source later on. Accordingly, the multitudinous fields of the standard model are no more than mathematical artifacts – they do not exist objectively in space, explaining why their "seething activity" is not observed in Nature. The fields described by quantum field theory are simply not there. Furthermore, despite being a higher-dimensional theory, string theory adopts many of the principles of QFT and so is not immune to QFT's problems.

If the fields of quantum field theory don't exist in Nature, then what does? By taking a direct reading of the quantum measurement process and the collapse of the wavefunction, we find clues about the fields present in Nature, while helping unlock the mysteries of quantum mechanics itself.

4.3 Quantum Attributes and Measurement

The wavefunction evolves continuously and deterministically in time according to the Schrödinger equation until, as physicists are fond of saying, "a miracle happens". Here we take a very direct reading of the essential features of quantum measurement.

According to the quantum formalism, the primary entity of a quantum system is the wavefunction – the *state vector* or *quantum state*. While not itself observable, the wavefunction of an elementary particle has encoded within it everything that can be known about that particle. Each particle is endowed with certain properties, known as *attributes* or *observables*, which manifest objectively when an appropriate measurement takes place. The *static* attributes, such as mass and electric charge, are constant for particles of the same species, whereas the *dynamic* attributes – such as spin direction, momentum, or position – generally vary among particles of the same species.

Our task here is to understand conceptually what goes on during the process of measurement, in the hope that it will shine light on the physical reality underlying it. Mathematically, the process is known as *harmonic analysis*. Briefly, waves combine according to the principle of *superposition*, which sums the amplitudes of the constituent waves at every point in space. Many waves can combine into a single superposed wave, which in a sense contains them all. Harmonic analysis is the reverse process of mathematically decomposing a wave into its constituent waves or, more specifically, into a weighted combination of "pure tones" (harmonics). In the case of a momentum measurement, for instance, Roger Penrose describes the process as follows: [8]

> What one does is to apply what is called *harmonic analysis* to the [wavefunction] ψ.... It is closely related to what one does with musical sounds. Any wave form can be split up as a sum of different 'harmonics' (hence the term 'harmonic analysis') which are the pure tones of different pitches (i.e. different pure frequencies). In the case of a wavefunction ψ, the 'pure tones' correspond to the different possible momentum values that the particle might have, and the size of each 'pure tone' contribution to ψ provides the amplitude for that momentum value. The 'pure tones' themselves are referred to as *momentum states*.

The calculated amplitude relates to the probability of an observed particle having that particular momentum value. Attributes other than momentum are "measured" in an analogous fashion, but applying different harmonics, different sets of pure tones. Each dynamic attribute (observable) corresponds to a particular *Hermitian operator*, being a mathematical operation applied to the wavefunction. Each operator represents a unique family of pure tones (known as *eigenstates*, from the German word for *self* or *innate*) which form a complete *basis* set, meaning that any reasonable wavefunction can be represented as a linear superposition (weighted sum) of these pure tones (eigenstates). Each eigenstate is associated with an *eigenvalue*, representing the physical value of the corresponding attribute. Measurement operators in quantum mechanics are generally required to be Hermitian (technically, *self-adjoint*) due to their rather magical property that the eigenstates, while themselves representing complex waves, always have real eigenvalues.

To measure a particular attribute (observable), one applies the corresponding operator to the wavefunction. In practical terms one is doing harmonic analysis, writing out the wavefunction as a weighted sum of operator eigenstates. It is instructive to depict this mathematically as:

$$\psi = c_1\varphi_1 + c_2\varphi_2 + c_3\varphi_3 + \ldots + c_N\varphi_N$$

where ψ represents the wavefunction, $\varphi_1 - \varphi_N$ represent the operator eigenstates (pure tones), and $c_1 - c_N$ are expansion coefficients (amplitudes), which are complex numbers. The *squared modulus* of a particular coefficient (being the sum of the squared real and imaginary parts) is proportional to the probability of that outcome occurring, the result being represented by the associated eigenvalue. This completes the measurement process [9].

4.4 The Wine Glass analogy

For the sake of nonspecialists who may have found the previous section tough going, the following analogy is offered. Since nothing in Nature is perfectly rigid, every object displays certain harmonic properties or "natural frequencies". Hit an object and it will give off its own unique sound according to the harmonics imposed by its geometry and mechanics. If you circle the rim of a wine glass with a wet finger you can get it ringing with a pure tone. Strike it sharply and you will hear a variety of tones of various frequencies. These various harmonic frequencies (wine glass eigenstates) could in principle be represented by an "operator" (call it the wine glass operator).

Now, it is well known that a singer can get a wine glass ringing by matching the voice to the harmonics (eigenstates) of the glass, even causing it to shatter. To find out how a singer's voice was able to break a glass, what do you do? You apply your wine glass operator to the sound wave (hopefully recorded for the purpose) – that is, you do harmonic analysis on the sound wave on the basis of the wine glass eigenstates. As a result you get a weighted sum of wine glass eigenstates, the expansion coefficients reflecting the *amplitudes* of the various eigenstates (wine glass harmonics) required to represent the sound wave.

You now have the information you need to determine how the glass was broken. Since both the glass and the sound wave are real, in this case the amplitudes are real rather than complex. A property of classical waves is that the square of a wave's amplitude is proportional to the *energy carried* by the wave. Hence, if you square these expansion coefficients you get a relative measure of the *excitation energies* exciting each of the wine glass eigenstates. From this information it will become clear which eigenstate (wine glass harmonic), and hence which sound frequency (which note), was responsible for shattering the glass.

- The singer's voice (sound wave) corresponds to the wavefunction.
- The wine glass corresponds to a quantum attribute (observable).

4.5 Operator Eigenstates and State Reduction

While investigations into the ontology of the wavefunction appear frequently in the literature, the ontology of the measurement operators is rarely questioned. According to the conventional view, they are just mathematical operations after all. There is more than one reason why such a view is untenable. Because they form the mathematical basis for quantum phenomena transforming into classical phenomena, the operators deserve our most careful consideration. What does a quantum observable (spin, momentum, etc.) have to do with a waveform? What *are* the operators?

A wavefunction can be considered as a superposition of a particular basis set of eigenstates, the amplitude of each eigenstate determining its contribution to the whole. When a measurement occurs, the wavefunction irreversibly "collapses" to just *one* of these eigenstates, then immediately continues to evolve from the reduced state. Meanwhile, the contributions (amplitudes) of the remaining eigenstates would seem to just "disappear". Once again, we turn to Roger Penrose for his expert translation of the quantum formalism into plain English, while offering his unique personal perspective: [10]

> The jumping of the quantum state to one of the eigenstates… is the process referred to as *state-vector reduction* or *collapse of the wavefunction*. It is one of quantum theory's most puzzling features… I believe that most quantum physicists would not regard state-vector reduction as a *real* action of the physical world, but that it reflects the fact that we should not regard the state vector as describing an 'actual' quantum-level physical reality… Nevertheless, irrespective of whatever attitude we might happen to have about the physical reality of the phenomenon, the way in which quantum mechanics is used in practice is to take the state indeed to jump in this curious way whenever a measurement is deemed to take place. Immediately after the measurement, Schrödinger evolution takes over again – until another measurement is performed on the system, and so on.
>
> I denote Schrödinger evolution by **U** and state reduction by **R**. This alternation between two completely different-looking procedures would appear to be a distinctly odd type of way for a universe to behave! Indeed, we might imagine that, in actuality, this is an approximation to something else, as yet unknown. Perhaps there *is* a more general mathematical equation, or evolution principle of some coherent mathematical kind, which has both **U** and **R** as limiting approximations? My personal opinion is that this kind of change to quantum theory is very likely to be correct – as part of a new 21st century physics, perhaps… However, most physicists appear not to believe that this kind of route is a fruitful one to follow.

Figure 24 is adapted from an accompanying illustration by Penrose, schematically depicting the evolution of a physical system as it alternates between unitary evolution **U** (according to Schrödinger's equation) and state reduction **R**, whereby the wavefunction "jumps" to one of the operator eigenstates. The dashed curves are to remind us that the wavefunction can generally be considered a *superposition* of multiple eigenstates, and can in principle jump to any one of them upon measurement, with the probability of a particular jump being proportional to the squared modulus of that eigenstate's amplitude (contribution to the wavefunction).

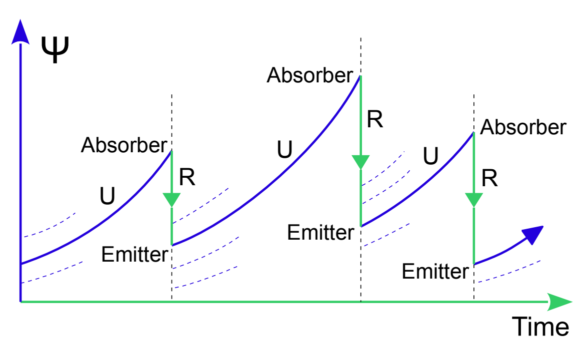

Figure 24: Evolution of a quantum system in time

While state reduction is generally considered to be no more than a mathematical procedure, and hence a purely abstract process, we have learned from Pusey's theorem that the wavefunction is an objective entity, rather more than a figment of the mathematician's imagination. Now we find that this objective entity is irreversibly altered by its encounter with measurement (interaction), in a manner dependent on the attribute measured, implying that state reduction is an *objective process*.

Keep in mind that Figure 24 depicts the process as it appears in physical spacetime, when a more revealing picture can be found in the higher branes. For present purposes let us consider the wavefunction of a massless particle, such as a photon. During its **U** evolution the photon wavefunction remains confined to one location in Minkowski 4-space, defined by its null cone at the moment of emission, while its projection evolves in 3+1 spacetime with the passage of the imaginary dimension ***w***. From the *transactional* perspective, each **U** evolution constitutes a transaction between an emitter and an absorber on the same null cone. When a *measurement* occurs (the transaction is completed), the wavefunction is absorbed; it collapses to a particular eigenstate while *relocating* itself in Minkowski 4-space to the null cone defined by the event in physical spacetime, then proceeds to evolve on the new null cone. Accordingly, Figure 24 presents a picture *opposite* to the more fundamental picture in the 4-brane: **U** evolution occupies *one location* in Minkowski 4-space, whereas **R** evolution constitutes not just reducing but *relocating* the wavefunction in Minkowski 4-space.

We come to a key point: a wavefunction *cannot be relocated* (moved) in Minkowski 4-space. Just as every light ray is inexorably confined to its light cone – it cannot exist off of it – the gravitational wavefunction is bound to its null cone and cannot exist off of it. It follows that the wavefunction passes out of existence to be reborn on a new null cone, defined by the location of the measurement (interaction) in 3+1 spacetime. At this moment when collapse occurs – when the absorbed wavefunction has passed out of existence while the emitted wavefunction has yet to come into being on its own null cone, perhaps far distant in Minkowski 4-space – the wavefunction has ceased to be.

4.6 State Reduction and Wave-Particle Duality

It is this moment, when the wavefunction has passed out of existence, that is of great interest to us. What happens at that moment? What happens to the *energy* of the wavefunction, which gravitational waves must carry? It is clear that an objective (energetic) agent or medium must be involved in this energetic process, while somehow manifesting classical-level phenomena, before generating a wavefunction on a new null cone representing the measured eigenstate. What agent or medium could accomplish this?

We need only return to our analogy of the wine glass. A singer does not have to sing the exact note of a wine glass eigenstate to get it ringing, but perhaps some whole multiple or fraction of the wine glass harmonic. Accordingly, a singer might get a glass ringing on a note lower or higher than her own voice, then stop singing and let the glass continue ringing. This simple analogy reveals the essential physical mechanism underpinning quantum state reduction.

In place of the singer, imagine the wavefunction. In place of the wine glass, imagine a field permeating our physical 3-space which is characterized by the harmonics of a particular quantum attribute (observable). When a measurement (interaction) occurs, the wavefunction excites a particular field harmonic (in the quantum world, just one eigenstate at a time) by transferring its energy to the field. Accordingly, as the wavefunction passes out of existence (having surrendered its energy, it no longer exists), it is replaced by an excited field harmonic, amplifying the process to a classical level. When the transaction is completed through the resulting interaction, the field harmonic collapses, releasing its energy and signature back into the fabric of spacetime through the emitted wavefunction. Just as the wine glass keeps ringing *on its own note* after the singer stops singing (not necessarily the singer's note), the emitted wavefunction reflects the eigenstate and null cone of the measured attribute.

State reduction, then, is a *real physical process* reflecting the excitation of objective, energetic fields by the wavefunction. This same conclusion is suggested by the fact that quantum entities are endowed with just a small set of attributes, when in principle quantum theory permits any attribute (observable) that can be represented as a mathematically appropriate basis, over and above those attributes observed in Nature. Nick Herbert whimsically elaborates in his book *Quantum Reality:* [11]

> According to quantum theory any waveform, no matter how bizarre, corresponds to some dynamic attribute which we could in principle measure… For instance, the "piano" waveform connects to some presently unknown mechanical attribute – call it the piano attribute – which an electron or any other quon is bound to display in a piano measurement situation. Likewise we could test an electron for the size of its tuba attribute, its flute attribute, or its Wurlitzer organ attribute. Physicists have shown little interest in measuring such obscure mechanical properties, but should the need ever arise quantum theory can predict these results as easily as it predicts the results of spin and momentum measurements.

The fact is that nobody has observed the piano attribute. Why? Are we not looking properly, or does the piano attribute simply not exist? Why do we observe just a small set of attributes when in principle quantum mechanics places no limits? Why do the waveforms associated with observed quantum attributes appear to have special status in Nature above and beyond all other possible waveforms? We surmise that the harmonics of the observed quantum attributes relate to *objective realities* in Nature, whereas the harmonics of the piano or tuba attributes do not. Moreover, it is clear that something *substantial* and *energetic* is being excited by the wavefunction, leading to substantial effects. The most direct reading of the facts brings us to the following conclusion:

- Each quantum attribute (static or dynamic) reflects the excitation of an objective field embodying the harmonics (eigenstates) of the associated measurement operator.

According to this model the wavefunction excites a number of energetic fields whose space it shares. The harmonics of each field are described by the corresponding measurement operator, while the calculated probability weightings for a particular measurement correspond to the relative energies exciting the various field eigenstates. Consequently, if an elementary particle can be regarded as the sum of its attributes, it may be thought of as the composite excitation of a number of fields, each manifesting a particular attribute. It follows that the "particle" exists ephemerally, only while the fields are excited, disappearing at the very moment the transaction is completed and the fields return their energy and harmonics back into the fabric of spacetime as the reduced wavefunction.

Here is wave/particle duality laid bare. According to this model, the wavefunction is an objective entity, a gravitational wave fully present in the 5-brane, while the particle is an objective entity appearing ephemerally in our 3+1 spacetime. They both objectively exist, but *never at the same time*. From the moment a wavefunction is emitted until the moment it is absorbed, there *never is* a particle. Likewise, when a particle manifests, there is no wavefunction. This clearly explains why a single photon can appear to pass through both slits of a two-slit experiment, for instance. From the moment the photon is emitted until the moment it is absorbed at the detector, it is *never a photon* – it is a wavefunction, which quite happily goes through both slits at once. The wavefunction never appears directly on a classical level, but precipitates itself into an observable "particle" only when it *interacts* (when it is absorbed, or measured), allowing classical effects (energetic field harmonics) to be realized.

4.7 Quantum Attributive Fields

The concept of attributive fields – fields manifesting quantum attributes rather than particle species – brings with it a profound economy. Rather than having multiple fields identical but for their electric charge, for instance, the harmonics of one field can, in principle, account for the charge of every elementary particle. Accordingly, both static and dynamic attributes are considered excitations of energetic attributive fields (ethers).

How many fields are required to account for the observed attributes? To answer this question we must briefly address the notions of *conjugate waves* and *conjugate attributes*. Mathematically, a wave is transformed into its conjugate wave, and back again, by applying a *Fourier transform* – hence the notion of conjugate waves being "opposite" or reciprocal. Attributes characterized by conjugate waves are known as *conjugate attributes*. Conjugate waves and attributes give rise to Heisenberg's uncertainty principle, the general principles of which are common to all wave phenomena, as described here by mathematical physicist Paul Busch: [12]

> Fourier analysis gives 'uncertainty' relations for any wave propagation phenomenon in that it gives a reciprocal relationship between the widths of the spatial/temporal wave pattern on one hand, and the wave number/frequency distributions on the other.

Consequently, a precise measurement of an attribute precludes measurement of the conjugate attribute. Moreover, in quantum mechanics one member of a conjugate pair constitutes some action of the other with respect to time. These facts can be accounted for by understanding each attribute and its conjugate as excitations of the same attributive field.

It follows that when a theorist "takes a measurement" he or she is simply mimicking Nature by applying harmonic analysis to the wavefunction on the basis of a particular attributive field. The field supporting *position* and *momentum* measurements, for instance, embodies harmonics characterized by the *spatial delta* function (measuring spatial position) and *spatial sine* waveforms (measuring momentum, or mass times velocity, velocity being a function of position with respect to time). This field therefore constitutes a *spatial metric field*: it provides a consistent measure of space everywhere in time (hence conserving momentum).

Spin direction and *spin angular momentum* constitute a further conjugate pair (angular momentum being a function of spatial orientation with respect to time). This field rules over orientation and rotation in space, hence conserving angular momentum. In accordance with the formalism of quantum mechanics, the field supporting the spin attributes is required to embody the *spherical* harmonics passed to the *SU*(2) symmetry group. Having previously noted that the *SU*(2) symmetry reflects a spatial configuration of the 4-brane, we can surmise that this field, while present in the 3-brane, somehow enjoys degrees of freedom representative of the 4-brane. This observation will prove of central importance in what follows.

4.8 Energy and Time

So far so good: we have accounted for four key attributes on the basis of two attributive fields, these being the attributes most generally "measured" by physicists – the dynamic attributes of position/momentum and spin direction/magnitude. From here things become more difficult. There is an important uncertainty relation between *energy* and *time*, the exact interpretation of which remains a point of contention. The relationship between this conjugate pair becomes clear when one considers that energy (or mass) is directly related to temporal frequency by way of Planck's constant, and frequency has units of *reciprocal time*. Planck's constant itself has units of *energy by time* (joule-seconds), further suggesting a deep relationship between these fundamental principles of Nature.

The notion of an *energy operator* enters quantum mechanics in two different contexts. First, there is an operator called the *Hamiltonian*, representing the total "classical" energy of a system, meaning both kinetic energy and potential energy. Of more relevance to us here is the energy operator which acts on the wavefunction, measuring the *full* energy of the system, including both rest energy (mass) and kinetic energy. The energy operator essentially measures the temporal frequency of a wavefunction by applying harmonic analysis on the basis of *temporal sine* waveforms [13].

Time, meanwhile, remains problematic. In an article on the time-energy uncertainty relation, Paul Busch points out that time enters quantum mechanics in at least three different contexts [14]. First there is *external* time or *laboratory* time, forming part of the spatio-temporal environment in which experiments are conducted, while serving as a parameter for input into theoretical models. Then there is *intrinsic* time, wherein time is scaled to suit the temporal scale of the phenomenon – such as a wave-packet having a time unit equivalent to how long it takes to traverse its own length. The third notion of time is of most interest to us here, what Busch calls *observable* time, which essentially means treating time like any other attribute by applying a *time operator* to the wavefunction, a task that has eluded the efforts of the best researchers. In fact, in 1931 Wolfgang Pauli proved a famous theorem showing that there is no self-adjoint time operator. Nonetheless, the conclusion is clear that the current model requires an attributive field supporting the measurement of energy (temporal sine harmonics) and time (temporal delta function). This field constitutes a *temporal metric field*: it provides a consistent measure of time everywhere in space (hence conserving energy).

The energy-time attributive field is of central importance to the current model. Keep in mind that (for a stationary particle) the energy operator measures *rest* mass. It follows that a detailed understanding of the harmonics of this field, in conjunction with imposed symmetries, would in principle explain the particle masses while also providing the theoretical underpinnings for Planck's constant. Further, one might hope that an explanation would emerge for the elementary particles coming in three generations.

4.9 Charge and Flavor

Electric charge is considered a *static* attribute, plugged into quantum mechanics as a free parameter and therefore not generally associated with a measurement operator. In physics today, however, *charge* means much more than just electric charge: there is strong hypercharge and weak hypercharge, plus three strong color charges and two weak color charges, in addition to a variety of *flavor symmetries* with fanciful names such as *strangeness*, *charm*, and *isospin*. At first glance, the prospect of having to economically account for such a diverse collection of charges and flavors might appear to bring down the theory of attributive fields.

But Nature is indeed economical in weaving Her diversity. In his book *The Lightness of Being*, Frank Wilczek explains a unification model achieved in a higher-dimensional (mathematical) space, yielding what he calls the *charge account:* [15]

> In the best case, we might hope that the three distinct symmetry transformations of *SU*(3) x *SU*(2) x *U*(1) are different facets of one larger, master symmetry that includes them all… The master symmetry I find most convincing is based on a group of transformations known as *SO*(10). All the attractive possibilities are minor variants of this one.
>
> Mathematically, *SO*(10) consists of rotations in a ten-dimensional space. I should emphasize that this "space" is purely mathematical. It's not a space that you could move around in, even if you were very small. Rather, the ten-dimensional space of *SO*(10), the master symmetry that absorbs the *SU*(3) x *SU*(2) x *U*(1) of the Core – that unifies, in other words, the strong, weak, and electromagnetic interactions – is a home for concepts. In this

> space, each of the color charges of the Core (red, white, blue, green, and purple) is represented by a separate two-dimensional plane (so there are 5×2 =10 dimensions altogether). Because there are rotations that move any plane into any other, the Core charges and symmetries get unified and expanded in *SO*(10)…
>
> In the Charge Account, all the quarks and leptons appear on an equal footing. Any of them can be transformed into any other. They fall into a very specific pattern, the so-called spinor representation of *SO*(10). When we make separate rotations in the two-dimensional planes, corresponding to the red, white, blue, green, and purple charges, we find in each case that half the particles have a positive unit of charge, half a negative unit… Each possibility for combinations of + and – occurs exactly once, subject to the restriction that the total number of + charges is even.
>
> The electric charges, which within the Core appear to be random decorations, become essential elements in the harmony of unification. They are no longer independent of the other charges. The formula
>
> $Y = -1/3\ (R + W + B) + 1/2\ (G + P)$
>
> expresses electric charge – more precisely hypercharge – in terms of the others. Thus the transformations associated with electric charge rotation turn each of the first three planes through some common angle, and turn the last two through 3/2 as big an angle, in the opposite sense.

According to Wilczek's "charge account", the various charges of the elementary particles can each be explained by some combination of just five color charges, each either positive or negative, and represented by rotations of two-dimensional planes in a ten-dimensional space. Wilczek indeed has a right to sing the song of unification. The reader will have noted that Wilczek may have been hasty in assuming the ten-dimensional space to be "purely mathematical", however, since the current framework requires the 9-brane to include ten imaginary dimensions. As with our earlier investigation of the *SU*(2) symmetry group and its relation to objective quantum spin phenomena, once again we have to ask if it is reasonable to expect objective phenomena (charges) to be dependent on principles (dimensions) which themselves have no objective reality.

Beyond the five color charges, the various *flavor symmetries* of quantum chromodynamics and electroweak theory have also been shown to be intimately related to each other, and in turn to hypercharge and electric charge. As an example, the flavor quantum numbers have been shown to combine with the baryon quantum number to yield hypercharge, *Y*, as follows: [16]

$$Y = (B + S + C + B' + T)$$

where *B* stands for *baryon number*, *S* is *strangeness*, *C* is *charm*, *B'* is *bottomness*, and *T* is *topness*. Hypercharge is in turn related to *electric charge*, *Q*, and *isospin*, l_3, as follows:

$$Y = 2\ (Q - l_3)$$

A further charge called *weak hypercharge* can be derived in a similar manner, leading us to consider that the diverse charges and flavor quantum numbers of the standard model are all manifestations of a single underlying phenomenon, operating within the parameters of various symmetry groups, each of which has been shown to be supported by a particular *spatial configuration* under the current framework. Accordingly, we ascribe this underlying phenomenon to a single attributive field, supporting *charge* in its broadest sense, along with its conjugate and time-derivative, *magnetic moment,* also in its broadest sense.

The charge attributive field reveals itself as the *electromagnetic field* while transcending our conventional understanding of the electromagnetic field.

4.10 The Four Quantum Attributive Fields

On the foregoing basis it is proposed that the properties (attributes or observables) manifested by elementary particles can in principle be ascribed to the operation of just four quantum attributive fields, as follows:

Field 1	Position *Spatial delta function*	–	Momentum *Spatial sine harmonics*
Field 2	Spin direction *Spherical harmonics*	–	Spin angular momentum *Spherical harmonics*
Field 3	Energy *Temporal sine harmonics*	–	Time *Temporal delta function*
Field 4	Charge ?	–	Magnetic moment ?

These four fields represent the minimum accommodating, in conjunction with various symmetry principles, the diverse range of particle attributes (observables). The four fields account for the four conservation laws: momentum, angular momentum, energy, charge. Note the following:

- The two columns are conjugates of each other. That is, the attributes and harmonics in one column are conjugates of the associated attributes and harmonics in the other column.
- The attributes in the left hand column are independent of time, whereas those in the right hand column are dependent on time.

On the following basis it could be argued that these four attributive fields constitute a complete set. Two fields relate to spatial position:

- The position-momentum field rules over position in real space, in our 3-brane.
- The energy-time field relates to position in imaginary space – on the $\boldsymbol{w}$ dimension in the 4-brane.

The remaining two fields correspond to *orientation* in space:

- The spin field rules over orientation in complex space – $SU(2)$ in the 4-brane – reducing to $SO(3)$ in real space, in both the 4-brane and 3-brane.
- The charge field relates to orientation in complex space – $U(1)$ phase symmetry in the 5-brane.

Einstein admonished us to find the simplest possible solution, but no simpler. According to Anthony Zee, "an unspoken rule among physicists dictates that all things being equal, one goes for the simplest possibility – a rule that has worked remarkably well." [17] Could such a minimal model work in the real world? Could just four quantum attributive fields account for phenomena that the scores of fields required by quantum field theory and the standard model cannot hope to predict?

Part 5

Nature's Fields and the Platonic Polyhedra

Clearly, the four attributive fields are of a very special nature. Not only must they display harmonics reflecting those of the quantum measurement operators, but the entire system of four fields must be *Lorentz invariant* (the laws of physics being the same in any inertial reference frame) as well as *isotropic* (the laws of physics are the same in every direction). Further, the four fields are *energetic* (since they display energetic effects) and objectively present throughout our 3+1 spacetime. What manner of fields might have such properties? In pursuit of this question we approach a fascinating convergence of physics and geometry.

5.1 Quantum Spacetime

A general problem encountered by field theories is that continuous fields yield mathematical inconsistencies. Theorists found early on, for instance, that if the electromagnetic field is considered a continuum, and if an electron is considered a point particle, then the resulting charge density will be infinite and the electric field will be infinite. This same principle has confounded efforts to formulate a consistent theory of quantum gravity. If space is considered a continuum, infinities invariably arise – the mathematics becomes inconsistent. This has led to various theoretical efforts to quantize space (or rather, spacetime), on the basis that the laws of physics, as we understand them, break down at the natural unit of a Planck length, being just 1.6 $\times 10^{-33}$ cm. Theoretical physicist Shahn Majid explains these developments as follows: [1]

> The continuum assumption on space and time seems then to be the root of our problems in quantum gravity. It is tied up with the very idea of point particles, of being able to point to exact positions in space and time and others arbitrarily nearby. But has anyone ever seen an exactly *point* particle, I mean one of truly infinitesimal size? We have argued that the concept is in fact physically meaningless as separations below 10^{-33} cm make no sense. Surely these concepts were invented as a mathematical convenience or idealization valid at everyday scales but as we have seen they are not appropriate as a fundamental structure for space and time. On the other hand, it is extremely hard to think of an alternative, some would say mind-boggling. If spacetime is to consist of 'foam' of size 10^{-33} cm, what are these bits of foam *in* if not a continuum? Or if spacetime is fuzzy due to quantum effects, what is it fuzzy with reference to if not a continuum of possibilities? The fact is that our everyday geometric intuition just is not up to it. How indeed can we have geometry without points in it? We must turn to mathematics to see how better to do geometry itself.

The "geometry without points in it" refers to a rather abstract approach to quantum gravity known as *noncommutative geometry,* devoid of coordinates and therefore of points. Majid describes it as "pre-geometry", from which spacetime emerges.

As Majid suggests, the idea of discrete space can be questioned on strictly logical grounds. If one imagines discrete space as a foamlike structure on a Planck scale, then we have to ask what separates the various "cells" (quanta) of space. More space? Or just "nothing"? Neither answer is consistent. If space divides the quanta of space, then space is continuous. If "nothing" divides the quanta of space, then they are not divided. If something can be spatially divided, this implies a deeper layer of space in which this division occurs.

Noncommutative geometry is claimed to provide such a pre-geometry, a deeper layer of "space" from which quantum spacetime emerges. We note that the term "pre-geometry" would aptly describe the geometry of the imaginary dimensions underpinning our universe, while quaternions, which provide a mathematical description of our physical 3-manifold, are noncommuting under multiplication.

Furthermore, since imaginary quantities cannot be represented in a real space, there can be no real "points" in imaginary space (at least as we understand the term in real space). The suggestion is that noncommutative geometry, when placed in the correct context, may well provide the mathematical machinery to describe the imaginary underpinnings of spacetime.

5.2 Continuous and Discrete Space

We are faced with a dichotomy. While logic dictates that space, in its primary nature, must be continuous, technical considerations insist that it must be discrete. This dichotomy lies at the heart of the problem of quantum gravity, but is resolved very simply under the current framework.

The unitary evolution of the wavefunction, under the rule of Schrödinger's equation, is continuous in space and time. There is nothing at all "quantized" about the wavefunction, implying that the wavefunction is an oscillation of a continuum. Having found that the wavefunction is a gravitational wave, it follows that space itself is a continuum. Discrete phenomena enter into quantum mechanics only upon the *collapse* of the wavefunction, whereupon particles manifest attributes defined by discrete field eigenstates, implying that the attributive fields are themselves discrete. On these grounds the following principles are proposed:

- Space is a continuum.
- The quantum attributive fields are discrete.

The discrete nature of the attributive fields is most obvious for the charge and spin attributes, being confined waveforms, each reflecting *orientation* in space. The remaining attributes constitute the metric fields, each reflecting *position* in space (real and imaginary) – the spatial metric field (measuring position and momentum) and the temporal metric field (measuring time and energy) – which we also take to be discrete. Together the four fields manifest all empirically observable phenomena in our 3-brane, exhausting what we can in principle "measure". That is, from our perspective in the 3-brane, we do not and cannot observe space and time in their more fundamental (continuous) nature; rather, we observe the *effects* of space and time (the gravitational wavefunction and spatial motions in the 4-brane) exciting the quantum attributive fields in our 3-brane. Consequently, since the metric fields are discrete, when we look to the limits of what can in principle be observed in our 3-brane, we will observe both space and time as discrete.

5.3 Spin and the Double Icosahedron

Having surmised that the four attributive fields are objective, discrete structures extending throughout our physical 3-manifold, presumably at a Planck scale, it follows that these structures will each be characterized by some definitive geometry. We find an important clue in the case of the spin attributes, which are measured on the basis of the spherical harmonics passed to $SU(2)$. Crucially, the quantum spin harmonics have been mathematically correlated with the harmonics of the *double* or *binary* icosahedron – more technically known as the icosahedral symmetry group passed to its unitary description under $SU(2)$ [2]. This typically has been accomplished in the context of icosahedral molecules called *fullerenes*, apparently without suspicion that the binary icosahedron might relate to more fundamental phenomena. Further, in a paper titled *Spherical harmonics and the icosahedron*, mathematician Nigel Hitchen proves a theorem establishing a fascinating lower-dimensional case (hence easier to visualize) – he shows that the vertices of two icosahedra, suitably superimposed, will always intersect the nodal lines representing the spherical harmonics of degree three [3].

The implication, of course, is that the spin field is in some sense characterized by the icosahedron.

Extending this idea, it is natural to ask if each of the attributive fields might be characterized by a particular Platonic solid. The demands of isotropy and symmetry in Nature would indeed suggest that the geometry of the attributive fields must be regular, the five Platonic solids being the only regular (convex) polyhedra in three real dimensions.

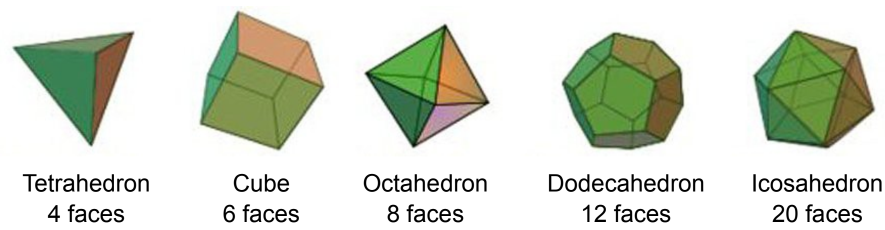

Figure 25: The five Platonic solids

5.4 Field Dualities

Geometers have long recognized various *dualities* among the Platonic polyhedra. In simple terms, two polyhedra can nest together to form a (concave) regular polyhedron only if one polyhedron has the same number of vertices as the other has faces. Accordingly, the tetrahedron is dual to itself, the cube is dual to the octahedron, and the icosahedron is dual to the dodecahedron.

We begin by noting that there are five regular polyhedra and just four attributive fields. Recall the crucial fact, however, that the spin field comes under the $SU(2)$ symmetry group, reflecting a spatial configuration of the 4-brane. That is, in some respects the spin field enjoys the spatial degrees of freedom of the 4-brane, suggesting that the spin field is somehow connected, or *dual*, to the 4-brane. Having ascribed the icosahedron to the spin field, the implication is that the dodecahedron (being dual to the icosahedron) in some sense characterizes the 4-brane. The proposal is generalized as follows:

- Each regular (Platonic) polyhedron in some sense characterizes the geometry and harmonics of an objective field extended throughout real 3-space.
- Dualities between polyhedra correspond to dualities between fields.

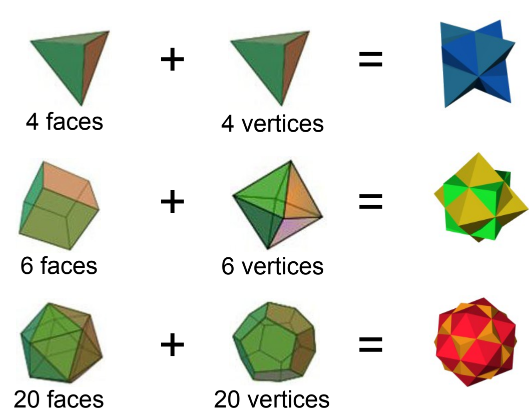

Figure 26: Dualities among the Platonic solids

Naturally, since the Platonic solids are extended in three real dimensions, they are constrained to a space of three real dimensions. Since real 3-manifolds exist only on the lower three branes, we count a total of six real fields underpinning Nature – the four attributive fields in the 3-brane, plus one field each in the 4-brane and 5-brane. Fortuitously, since the tetrahedron is dual to itself, we thus have a total of six polyhedra to account for six fields, with the six fields forming dual pairs as shown in Figure 26.

The geometry of the Platonic solids cannot be applied literally to the fields, of course, since only the cube uniformly tiles Euclidean 3-space, forming what is known as the *cubic honeycomb*. There are also two quasi-regular tilings incorporating two regular polyhedra, known as the *tetrahedral-octahedral honeycomb* and the *gyrated tetrahedral-octahedral honeycomb*. Intriguingly, however, any finite uniform polytope can be projected to its circumsphere to form a uniform honeycomb in *spherical* space, suggesting a rather more abstract relation between the Platonic solids and the attributive fields [4].

Note that the attributive fields cannot be considered "material" in the physical sense – rather, in their unexcited (ground) state they *precede* physical matter, while in their excited states they conspire to manifest or *become* physical matter and phenomena. While extended in three real dimensions, the fields are required to have complex degrees of freedom, further hinting at their ethereal, pre-material nature.

No further hypothesis will be made here regarding the general nature of these fields – this being a task for geometers and theorists. For present purposes the key proposal is that each of the fields can in some sense be characterized by a Platonic solid, and that dualities between polyhedra correspond to dualities between fields. Further, from mathematical evidence we have surmised that the spin attributive field is characterized by the icosahedron, being dual to the 4-brane field which is characterized by the dodecahedron. To proceed further we are faced with the task of identifying the relationships between the remaining fields and polyhedra.

5.5 The Metric Fields

Perhaps surprisingly, the relationships between the fields and polyhedra can be identified on logical grounds, all on the basis of one simple assumption. We begin with the observation that the cube displays a particular symmetry reflecting that of our 3-manifold, suggesting both technically and intuitively that the cube characterizes the spatial metric field measuring position and momentum in real 3-space.

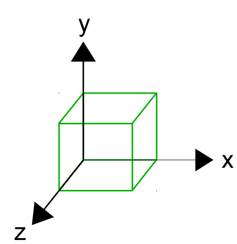

Figure 27: The cube representing a spatial metric field

Space and time in our 3-brane cannot be considered in isolation, of course, but as *spacetime*, united under the laws of relativity, suggesting that the temporal metric field (measuring time and energy) is characterized by the octahedron, being dual to the cube. Consequently, the space and time metric fields are inextricably connected by the duality between the cube and octahedron, providing a geometrical mechanism underpinning Lorentz symmetry in the 3-brane.

5.6 Charge and the Tetrahedron

We are left with just one polyhedron and one attributive field – charge and the tetrahedron – implying, of course, that the charge field is characterized by the tetrahedron. Being dual to itself, a second tetrahedron is ascribed to the sole remaining field, extended throughout the 3-manifold in the 5-brane.

The attentive reader will have noticed a jewel falling into our laps. In section 3.1 we concluded that Kaluza's Einstein-Maxwell theory is a description of physics in the 5-brane, while leaving unanswered the question of how electromagnetism in the 5-brane can manifest in the 3-brane. Now, through an independent line of reasoning we have arrived at the conclusion that both the 5-brane and the charge field are characterized by the tetrahedron. Analogous to the previous dualities, it is proposed that electromagnetism manifests in the 3-brane as a direct result of this geometric duality between the charge field and the 5-brane field.

5.7 Dual Symmetries

We began our discussion of field dualities with the observation that quantum spin phenomena, as manifested by the spin field in our 3-brane, appear to respond to a spatial configuration of the 4-brane corresponding to the $SU(2)$ symmetry group. The implication, of course, is that dual fields share certain properties, or perhaps more accurately, the four attributive fields *inherit* certain properties from their higher-dimensional counterparts. In particular, the quantum attributive fields appear to inherit the spatial degrees of freedom of their higher duals.

It follows that the charge field inherits the degrees of freedom of its dual, the 5-brane field. Having previously noted that the 5-brane underpins the $SU(2) \times U(1)$ symmetry of electroweak theory, we surmise that both the 5-brane field and the charge field are endowed with degrees of freedom representative of the $SU(2) \times U(1)$ symmetry group.

The remaining dual pair – the metric fields – diverge from this pattern since they are each *attributive* fields, present in the 3-brane. Just as the spin and charge fields require complex degrees of freedom, spacetime physics demands that the fields supporting energy-time and position-momentum also enjoy complex degrees of freedom, suggesting a dual relationship with a higher-dimensional field in the 6-brane, being the *precursor* to spacetime (see Figure 20). A field in the 6-brane cannot be characterized by a Platonic solid (there being no real manifold in the 6-brane), but logically by a higher-dimensional polytope extended in three complex dimensions. It follows that the space and time metric fields (cube and octahedron), being dual to each other, are in turn dual to the complex 6-brane field, with each inheriting the spatial degrees of freedom of the 6-brane as reflected in the $SU(3)$ symmetry group.

The following anomalies present themselves:

- We have proposed that all three charges (electric, strong, weak) are manifestations of the one charge field, which inherits the $SU(2) \times U(1)$ symmetry from the 5-brane field. Contrarily, the strong force is governed by the $SU(3)$ symmetry group, reflecting the spatial geometry of the 6-brane. The strong force appears to reflect properties of both the 5-brane and the 6-brane.
- The octahedral (energy-time) field is responsible for measuring time and energy in the 3-brane, while being dual to the 6-brane field. Contrarily, we have previously concluded that real energy emerges from the product of two imaginary motions in the 5-brane. It would seem that the energy-time field *meters* energy (hence mass), while real energy itself originates in the 5-brane and hence is fundamentally electromagnetic in nature.

Given that each of these anomalies requires some mechanism connecting the 5-brane and 6-brane fields, it is fortuitous that such a connection exists within the present geometrical framework. When two tetrahedrons are superimposed as in Figure 26 their intersection forms an octahedron, suggesting a subtle duality between the *dual* tetrahedral fields and the octahedral energy-time field. Subtle indeed, it is difficult to imagine that Nature would not make use of this purely geometrical device.

Figure 28 brings together various elements of our discussion thus far, illustrating the characteristic geometries, dualities, and inherited symmetries among the seven fundamental fields underpinning Nature. Note that the four attributive fields are arranged in a sequence reflecting that of their higher duals. Note also that the Platonic solids are exhausted under this model – each regular polyhedron and duality is fully utilized.

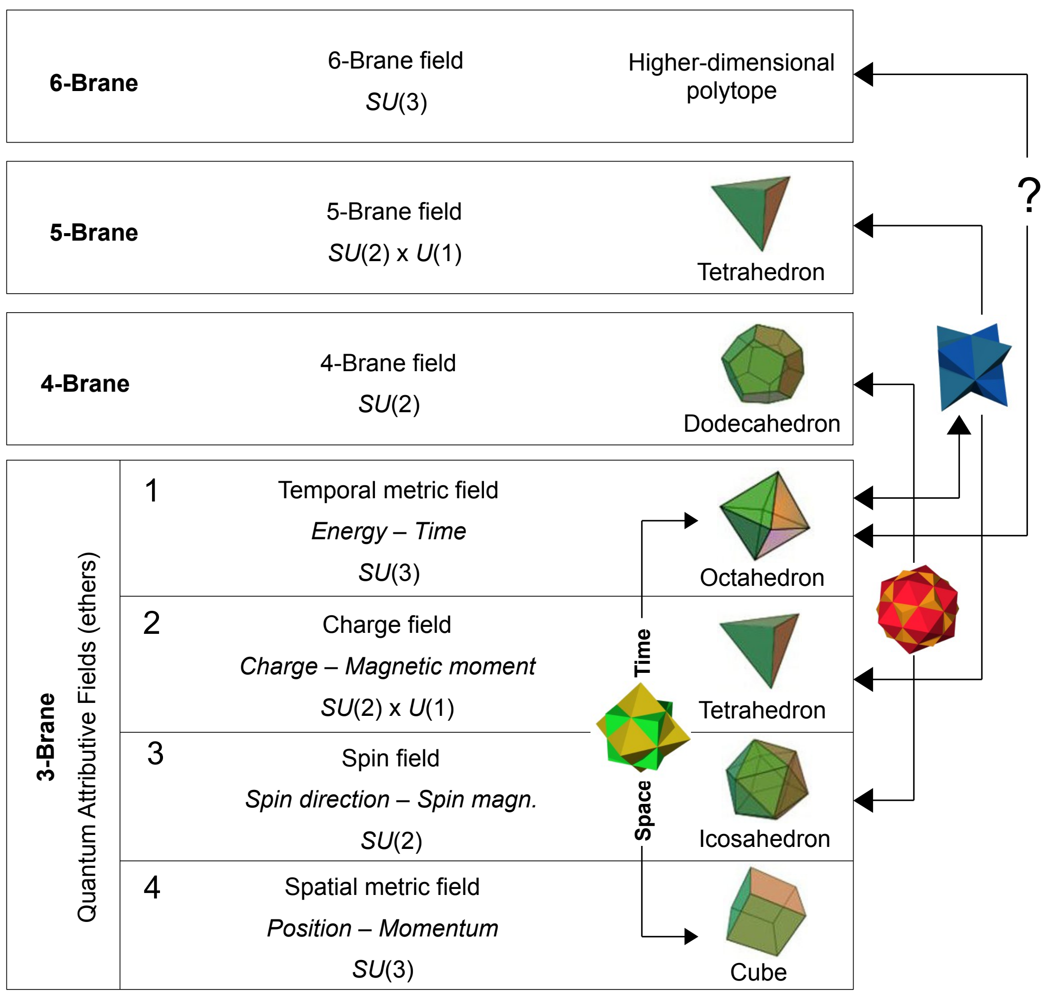

Figure 28: Dualities and symmetries among the fundamental fields

Part 6

A Context for String Theory

String theory has been criticized for not being a theory at all, but simply a mathematical framework with no connection to the "real world". Yet string theorists labor on, entranced by the storied serendipity of the string formalism, convinced that such an elegant mathematical framework must relate in some way to Nature. Given that mathematics has preceded a scientific context on numerous occasions throughout history, we may do well to take heed. Historically, mathematicians often get there first.

As string theory matures, more physicists (not necessarily string theorists) are weighing in with their considered insights regarding a possible context for string theory. Following is the abstract from a recent paper by Nobel laureate Gerard 't Hooft, titled *On the Foundations of Superstring Theory*: [1]

> Superstring theory is an extension of conventional quantum field theory that allows for stringlike and branelike material objects besides pointlike particles. The basic foundations on which the theory is built are amazingly shaky, and, equally amazingly, it seems to be this lack of solid foundations to which the theory owes its strength. We emphasize that such a situation is legitimate only in the development phases of a new doctrine. Eventually, a more solidly founded structure must be sought.
>
> Although it is advertised as a "candidate theory of quantum gravity", we claim that string theory may not be exactly that. Rather, just like quantum field theory itself, it is a general framework for a class of theories. Its major flaw could be that it still embraces a Copenhagen view on the relation between quantum mechanics and reality, while any "theory of everything", that is, a theory of the entire cosmos, should do better than that.

Clearly, every physicist would like to know what string theory is. If indeed the string formalism in some way reflects Nature, then any "theory of everything" should provide a solidly founded context for string theory. Early in this paper, the concepts of extra dimensions and branes were introduced from string theory, interpreted and applied as follows:

- There are a total of ten imaginary dimensions in our universe (which includes time, understood as spatial motion), or nine dimensions when our real 3-space is counted as three.
- Our physical universe is a 3-brane, interpenetrated by six branes of increasing dimension (the highest being a 9-brane), all sharing the same higher-dimensional space (bulk).

Beyond these crucial concepts the current framework may appear to have little in common with string theory. Most obviously, string theory models elementary particles as miniscule open or closed strings, while the framework models particles as composite excitations of discrete attributive fields. But let us put that detail aside for a moment and take a closer look at the structure of string/M-theory.

6.1 S-Dualities and the Platonic Solids

As 't Hooft points out, string theory is not just one theory but "a class of theories", in a similar sense that quantum field theory constitutes a class of theories. While quantum field theories are a dime a dozen, however, there appear to be just five consistent string theories, as follows:

- Type I
- Type IIA
- Type IIB
- Heterotic-O
- Heterotic-E

The existence of five consistent theories was considered an embarrassment for string theorists until the five theories were shown to be aspects of a more general theory, M-theory, encompassing various relationships among the five theories known as *dualities*. The dualities that concern us here are known as *strong-weak* dualities, or S-dualities, referring to *coupling strength,* which sets the strength of interactions. A big problem in string theory is that the coupling constant is an unknown parameter. While calculations assuming weak coupling are relatively straightforward, calculations with strong coupling break down in all but a few highly symmetric cases due to the limitations of the perturbative mathematical methods inherited from quantum field theory. The S-dualities provide a workaround, since calculations in one theory with weak coupling can describe the same physics as in a dual theory with strong coupling. String theorists are thus able to pass back and forth between dual theories to perform calculations that otherwise would not be possible [2]. The S-dualities are as follows:

- Type IIB is dual to Itself
- Type I is dual to Heterotic-O
- Type IIA is dual to Heterotic-E (each in ten spatial dimensions)

The first two dualities are in nine spatial dimensions (plus one of time), and each maps a theory with weak coupling to the dual theory with strong coupling. For instance, the Type IIB theory with coupling g will yield physics identical to the same theory with coupling $1/g$. Similarly, Type I with coupling g will yield the same physics as Heterotic-O with coupling $1/g$, and vice versa.

The Type IIA/Heterotic-E duality is more subtle. In each theory, coupling g is mapped to a tenth spatial dimension of size g. The tenth spatial dimension is interpreted as the spatial extension of a string to yield a membrane, forming the basis of M-theory in eleven (10+1) dimensions.

We are in sufficiently rarefied territory that the subtlest of clues can prove crucial, and this basic picture of the string theory S-dualities offers important guidance. First, let us note that there are five string theories and five Platonic solids. This means little until we recognize that the pattern of S-dualities correlates with the geometric dualities among the five Platonic solids, as follows:

- Tetrahedron is dual to Itself
- Octahedron is dual to Cube
- Icosahedron is dual to Dodecahedron

Consider that we have noted a corresponding pattern of dualities between the five string theories and *pure geometry* in three real dimensions – nothing less. Is this pure coincidence?

6.2 String Fields

Let us assume that the correspondence holds. The implications are far-reaching. In a nutshell, it implies that each of the five string theories corresponds to a particular quantum attributive field or, even more profoundly, to a field *outside* our world, in the 4-brane or 5-brane. The proposal is as follows:

- String theory is the mathematical theory of Nature's fundamental fields.

What, then, are elementary particles? Are they excitations of tiny open or closed strings? Or, are they composite excitations of up to four quantum attributive fields? Thus far we know just a few details about the seven fields. We know that each is characterized (in some sense) by a regular polytope. We require that the four attributive fields manifest harmonics reflecting the respective quantum observables. We have surmised that each of the fields has a discrete structure constructed of some "material" preceding physical matter, while enjoying complex degrees of freedom by way of geometric dualities with higher-dimensional fields.

On the foregoing basis it is not difficult to imagine a Planck-scale mesh or grid, formed from one-dimensional filaments of "pre-material" (string stuff), extended throughout our real 3-space. According to this model, the fundamental entities of string theory turn out to be directly related to the fundamental fields – both are strings. And, whether tiny filamentary loops or snippets, or filamentary fields somehow characterized by regular polyhedra, we could expect the harmonics of these two varieties of strings to be related. Might the primary entities of string theory actually be filamentary fields rather than isolated open or closed strings? Could it be that string theorists have actually been studying filamentary fields all along, only approached in the wrong context? In support of this conjecture, we set out to map M-theory and the five string theories unambiguously to the current framework.

6.3 Type IIB and the Tetrahedron

Among the two sets of dualities, the Type IIB theory and the tetrahedron single themselves out by being self-dual, implying that the Type IIB theory is in some sense related to the tetrahedron, and hence to the charge field and the 5-brane field. On the theoretical basis that *weak* coupling produces physics containing elementary particles, Type IIB with weak coupling is ascribed to the charge field in our 3-brane, while the same theory with *strong* coupling is ascribed to the 5-brane field [3].

This model deepens our appreciation of Kaluza's 5-dimensional Einstein-Maxwell theory, or rather, how Kaluza's theory makes contact with our physical world. As previously described, Kaluza's theory yields both Einstein's field equations and Maxwell's equations of electromagnetism in three real dimensions (plus time) in the 5-brane. Note that Maxwell's equations, so derived, are the *very same* equations as those underpinning electromagnetism in our 3-brane. This we explain on the basis of the 5-brane field and the charge field each being characterized by the tetrahedron. Being superimposed throughout real 3-space, it is suggested that these two tetrahedral fields somehow establish an inextricable connection across the boundaries of their respective branes. Analogously, from the perspective of string theory it is proposed that the same Type IIB string theory describes each of these two fields while promising to further illuminate the duality between the charge field and the 5-brane.

Because electromagnetism is an energetic phenomenon, we can assume that this same mechanism connects our 3-brane not just to natural law in the 5-brane (to Maxwell's equations), but also to the primordial real energy generated by natural processes in the 5-brane (see Figure 20), hence powering the charge field and all electromagnetic phenomena throughout the physical universe.

6.4 Spin Strings

Besides being a dual pair, the Type I and Heterotic-O string theories single themselves out by both coming under the symmetry group $SO(32)$, the special orthogonal group of 32 variables. Just as the $SO(3)$ symmetry group describes rotations in three-dimensional space, the $SO(32)$ group governs rotation (or spin) in a space of 32 dimensions. One would assume that such a high degree of rotational symmetry would provide the appropriate machinery for the manifestation of quantum spin phenomena in the 3-brane along with related phenomena in the 4-brane. It follows that the Type I and Heterotic-O theories are ascribed to the spin field and its dual, the 4-brane field.

According to the formalism of string theory, in the *heterotic* theories right-moving and left-moving string excitations differ, suggesting a notion of *directionality* consistent with the measurement of *spin direction*. It follows that the Heterotic-O theory is ascribed to the spin field in our 3-brane, while the Type I theory is ascribed to the 4-brane field.

6.5 Metric Strings

We are left with the Type IIA and Heterotic-E theories to describe the energy-time and position-momentum fields, being the metric fields measuring time and space in our 3-brane. Once again we are guided by the directionality of the heterotic theory. In our physical world, space looks the same in every direction, while time always flows in just one direction and energy is always positive, suggesting that the Heterotic-E theory be ascribed to the time metric field and the Type IIA theory to the space metric field.

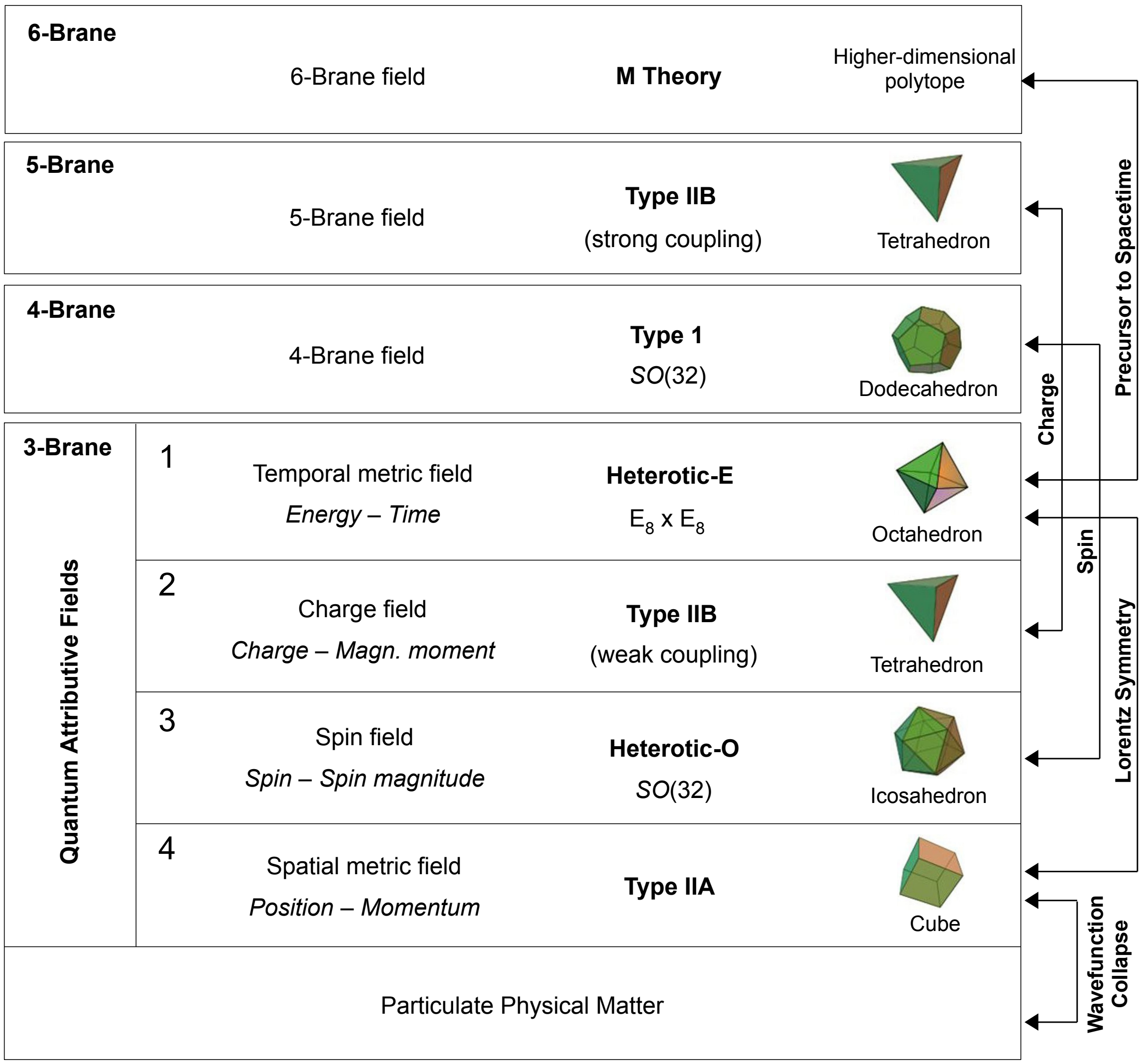

Figure 29: A cosmological context for string/M-theory

Figure 29 illustrates the resulting arrangement, unambiguously associating the five string theories with the five Platonic solids and the six real fields as previously described. Correlations emerging from this model provide an unexpected context for string theory's mysterious symmetry groups:

- The $E_8 \times E_8$ symmetry group of the Heterotic-E theory can be correlated numerically with the duality between the octahedron (characterizing the energy-time field) and the dual tetrahedron (characterizing the dual charge and 5-brane fields), whose intersection forms an octahedron, this duality constituting the required link between the energy and charge fields (see section 5.7). [4]

- The *SO*(32) symmetry group of the Heterotic-O and Type 1 theories (the spin duals) can be correlated with the total number of faces of the icosahedron (20) and dodecahedron (12).

We began this exploration of possible connections between string theory and the current framework by noting a single numerical correlation, there being five string theories and five Platonic solids. Unimpressed, we proceeded to discover that the string theory S-dualities follow the same pattern as do the Platonic duals, which perhaps piqued our interest. The above correlations, relating each of the string theory symmetry groups explicitly and unambiguously to field geometries logically fixed within the current framework, are more difficult to pass off as coincidence.

It should be mentioned that these correlations came into view only after the conceptual framework was in place (both logically and historically) and did not have to occur. The implication is that, while the symmetry groups of the standard model relate to *spatial configurations* of higher-dimensional branes, the string theory symmetry groups relate specifically to characteristic *field geometries*.

6.6 M-Theory and the 6-Brane

While the six fields in the lower three branes are each real fields extended in a real 3-manifold, space in the 6-brane constitutes a manifold of three *complex* dimensions. On this basis we have concluded that, in the 6-brane and above, there is no objective space or time. Rather, the 6-brane is the *precursor* to spacetime (see Figure 20). The 6-brane is the domain of *M-theory*, establishing the spatio-temporal substratum for spacetime and the five string theories in the lower three branes. It follows that M-theory will be formulated *outside* of time while providing a *basis* for time.

The framework suggests that the 6-brane field is characterized by a polytope extended in three complex dimensions while being dual to the octahedron. Beyond that, not much is clear. We have passed beyond the objective worlds (those with real 3-manifolds) into the dramatically different "physics" of the higher branes. M-theory is depicted in Figure 30 as the pivot or connection between processes in the higher three branes and those in the lower three branes, each triad being considered a cohesive unit. The higher triad, being so remote from the physics of our world, is deemed to lie outside the scope of this paper.

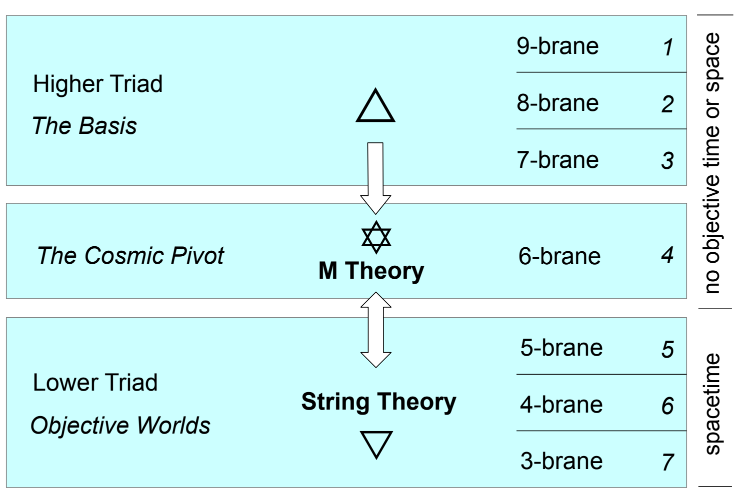

Figure 30: A context for M-theory

Whether this framework can be reflected in (or illuminated by) the mathematical formalism of string theory remains to be seen. The following points should be noted.

- On the basis that real fields cannot spread into imaginary dimensions (or into negative real dimensions, as prescribed by Kaluza), the framework does not require compactification of extra dimensions. It follows that the complex Calabi-Yau spaces of classical string theory are rendered obsolete, consigned to the dustbin of history, leaving string theory vastly simplified.
- In classical string theory a string is required to support harmonics yielding the particle's mass, charge, spin, and so on, while the current framework models particles as composite excitations of up to four attributive fields, each supporting one attribute and its conjugate. Consequently, rather than having to support the harmonics of all quantum attributes, each attributive field supports just one conjugate pair – a far simpler task.
- While the framework does not appear to require isolated open or closed strings as currently conceived by string theory, it does not necessarily exclude them. The properties of open and closed strings are called upon to explain important string physics such as the confinement of matter fields to branes, and it is not obvious how such physics might be recovered in a theory representing strings as fields.
- M-theory, incorporating Supersymmetry (SUSY), sets the number of spatial dimensions at ten, in accord with the current framework. The framework is therefore dependent on SUSY, or at least some form of SUSY, being a correct description of Nature. It is anticipated that the framework will provide the correct context for SUSY within the parameters of experimental data from the LHC.

Edward Witten offers us the following intriguing perspective on the importance of complex numbers in string theory, hinting at a more subtle relationship between string theory and the current framework: [5]

> The idea of replacing point particles by strings sounds so naive that it may be hard to believe it is truly fundamental. But in fact this naive-sounding step is probably as basic as introducing the complex numbers in mathematics. If the real and complex numbers are regarded as real vector spaces, one has $\dim_{\mathbf{R}}(\mathbf{R}) = 1$, $\dim_{\mathbf{R}}(\mathbf{C}) = 2$. The orbit of a point particle in spacetime is one-dimensional and should be regarded as a real manifold, while the orbit of a string in spacetime is two-dimensional (over the reals) and should be regarded as a complex Riemann surface. Physics without strings is roughly analogous to mathematics without complex numbers.

While Witten hints at the blind spot towards imaginary spatial dimensions endemic to canonical physics, here regarding complex numbers as real vector spaces, nonetheless the conclusion is clear that the "orbit of a string in spacetime" is complex, pointing to imaginary dimensions in Nature. Much clarity is expected to emerge when string theory is brought formally into the context of the current framework.

In the words of Gerard 't Hooft, quoted above, string theory is "an extension of conventional quantum field theory that allows for stringlike and branelike material objects besides pointlike particles". As such, it inherits the limitations of quantum field theory, which we have argued is a lower-dimensional representation of a process taking place in higher dimensions. Now we must consider this proposal in more detail.

6.7 Plato's Cave and the Holographic Principle

In Plato's Allegory of the Cave, his teacher Socrates likens our physical experience to shadows cast on the wall of a cave. In modern terms, Socrates considered our empirical experience to be a lower-dimensional representation of higher-dimensional realities occupying an eternal and transcendental realm of Ideas and Forms. According to the idealism of Socrates and his pupil Plato, the transcendent Ideas and Forms are reality, while our physical world is a mere reflection, an ephemeral shadow of the real. Aristotle, however, twenty years Plato's pupil, would have none of it. In fact, Aristotle took a view essentially juxtaposed to that of his teacher, as explained here by historian Richard Tarnas. [6]

> With Aristotle, Plato was, as it were, brought down to earth. And if, from a Platonic point of view, the luminosity of Plato's universe based on the transcendent Ideas was diminished in the process, others would point to a decisive gain in the articulate intelligibility of the world as described by Aristotle, and would indeed consider his outlook to be a necessary modification of Plato's idealism….
>
> The crux of their difference involved the precise nature of the Forms and their relation to the empirical world. Aristotle's intellectual temperament was one that took the empirical world on its own terms as fully real. He could not accept Plato's conclusion that the basis of reality existed in an entirely transcendent and immaterial realm of ideal entities. True reality, he believed, was the perceptible world of concrete objects, not an imperceptible world of eternal Ideas. The theory of Ideas seemed to him both empirically unverifiable and fraught with logical difficulties.

While Aristotle's position has since become the cornerstone of empirical science, Plato's Cave has recently been brought into sharp focus by a theory known as the *holographic principle*, first put forward by Gerard 't Hooft and Leonard Susskind in the early 1990s on the basis of theoretical work with black holes. Just as a two-dimensional holographic plate projects a three-dimensional image (when illuminated by laser light), 't Hooft and Susskind proposed that our physical universe is a projection of data stored on a spatial boundary of lower dimension, as Susskind explains in his book *The Cosmic Landscape*. [7]

> One of the strangest discoveries of modern physics is that the world is a kind of holographic image… Picture yourself in an enormous room bounded by walls, a ceiling, and a floor. Better yet, think of yourself as being in a large spherical space. According to the Holographic Principle, that fly just in front of your nose is really a kind of holographic image of data stored on the two-dimensional boundary of the room. In fact you and everything else in the room are images of data stored on a *quantum hologram* located on the boundary.

While the holographic principle was initially considered little more than a curiosity, attitudes changed in 1997 when string theorist Juan Maldacena published a seminal paper titled *The Large N Limit of Superconformal Field Theories and Supergravity* [8]. The so called *Maldacena conjecture*, also known as the AdS/CFT correspondence, represented a breakthrough for string theorists for it established an explicit correspondence between string theory and particular type of quantum field theory, hence bringing string theory "in from the cold". The paper quickly became the most cited in the field, while advocates of the holographic principle seized upon the result as convincing evidence for a holographic universe, as opined here by Susskind: [9]

> Maldacena, by cleverly using String Theory and Polchinski's D-branes, had discovered a completely explicit holographic description of, if not our world, a world similar enough to make a convincing case for the Holographic Principle. Slightly later Ed Witten put his stamp of approval on the Holographic Principle with a follow-up to Maldacena's paper titled "Anti De Sitter Space and Holography." Since then the Holographic Principle has matured into one of the cornerstones of modern theoretical physics.

Advocates of the multiverse have taken the holographic principle to new heights by proposing that, if indeed our universe is a hologram as the Maldacena conjecture may appear to suggest, there could be any number of such holographic universes, perhaps an infinite number. String theorist Michael Greene presents the multiverse interpretation in his book *The Hidden Reality:* [10]

> In a particular hypothetical setting, Maldacena's result *realized explicitly the holographic principle, and in doing so provided the first mathematical example of Holographic Parallel Universes.* Maldacena achieved this by considering string theory in a universe whose shape differs from ours but for the purpose at hand proves easier to analyze. In a precise mathematical sense, the shape has a boundary, an impenetrable surface that completely surrounds its interior. By zeroing in on this surface, Maldacena argued convincingly that everything taking place within the specified universe is a reflection of laws and processes acting themselves out on the boundary.

The emphasis is Greene's. Note that everything taking place *within* the universe is considered a *reflection* of the lower-dimensional boundary, corresponding to a conventional hologram whereby a three-dimensional image is projected from a two-dimensional holographic plate. Here we are not questioning Maldacena's result, but the *holographic interpretation* of the result. In a nutshell, Maldacena considered a specific physical model from two different perspectives, reasoning that because the two perspectives are describing the same physics they are *dual* formulations of the same physical law. Greene describes Maldacena's result on a more technical basis as follows: [11]

> A particular *non-gravitational, point-particle* quantum field theory in *four* spacetime dimensions (the first perspective) describes the same physics as *strings, including gravity*, moving through a particular swath of *ten* spacetime dimensions (the second perspective).

Following from the above descriptions, the first perspective is on the spatial boundary, while the second perspective reflects the enclosed space, as schematically represented in Figure 31.

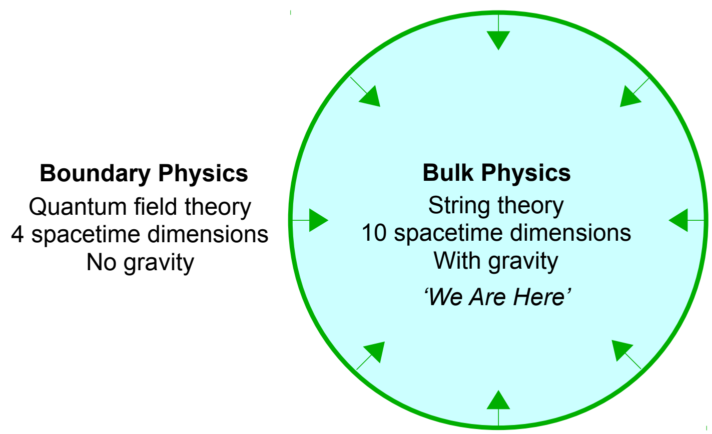

Figure 31: Holographic interpretation of AdS/CFT

Note that Maldacena's result establishes a *duality* between theories without specifying a causal direction between them. Greene points out that the two perspectives describe *the same physics*, which does not necessarily mean that the two descriptions are equivalent or contain the same information. Yet the metaphor of the hologram clearly implies that the lower-dimensional surface – being a certain quantum field theory, without gravity, in 3+1 spacetime – projects into existence a higher-dimensional representation, being string theory in 10 dimensions *with* gravity, as indicated by the arrows in Figure 28. Greene makes this interpretation clear by turning around Plato's Cave. [12]

> In Plato's parable of the cave, our senses are privy only to a flattened, diminished version of the true, more richly textured, reality. Maldacena's flattened world is very different. Far from being diminished, it tells the full story. It's a profoundly different story from the one we're used to. But his flattened world may well be the primary narrator.

The thoughtful reader may be asking whether this interpretation is reasonable. We are being asked to accept that a universe with ten dimensions and gravity can miraculously manifest out of a flattened boundary, the "primary narrator", having just four dimensions and no gravity. Greene and Susskind each go to some lengths to reassure us that the data on a holographic plate, while containing the *same information* as the resulting three-dimensional image, is completely scrambled and unrecognizable. They imply that quantum field theory does indeed contain the same information as string theory, but it is scrambled beyond recognition. Of course the reasoning is circular. Indeed, if we *assume* that our universe is a hologram, it follows that the boundary physics contains the same information as the bulk physics. But what if we *don't* make that assumption?

Figure 32 provides an alternate interpretation of the facts. From this perspective, our physical universe is the exact *opposite* of a hologram. (Our *perception* of the universe is another matter.) Rather than being the projection of a lower-dimensional boundary, our physical universe *is* the lower-dimensional boundary, projected from higher dimensions. According to this picture, we live on the outermost layer of a grand, multidimensional space. In principle, string/M-theory provides a mathematical description of the outer four branes of our seven-braned space, which, by the processes outlined in this paper, project our physical universe into existence. In Figure 32 the terms *objective* and *subjective* foreshadow the philosophical depths of the framework.

In section 4.2 it was argued that quantum field theory does not present a true picture of Nature, but is a limiting case resulting inevitably from the formulation of a higher-dimensional process in 3+1 spacetime. The promise was made to provide corroboration from an unexpected source, and finally we have arrived at that source.

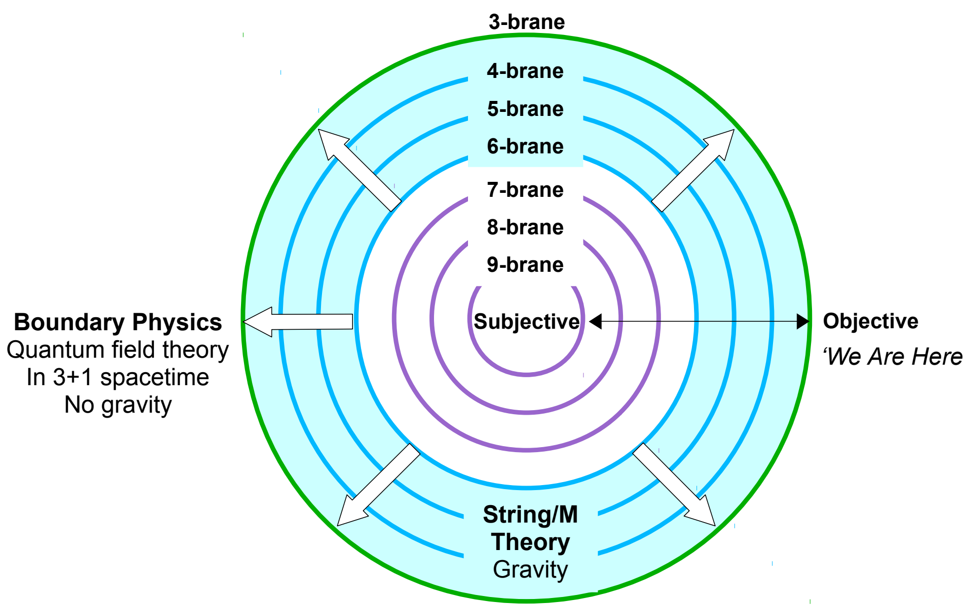

Figure 32: The holographic principle reversed

Once freed of its holographic shackles, the true power and beauty of Maldacena's result come into view. While the *holographic conjecture* is wrong, the result itself reflects a profound truth. According to this perspective, the AdS/CFT correspondence is essentially telling us that if you take string theory (in ten dimensions, with gravity) and project it into 3+1 spacetime, you end up with quantum field theory without gravity. That is, quantum field theory is simply a lower-dimensional approximation or representation of the actual objective processes taking place *inside* space, in higher dimensions. Hence, quantum field theory is no more than a mathematical duality representing the limiting case of ten-dimensional string theory projected into 3+1 spacetime. Accordingly, the quantum fields of the standard model are no more than mathematical abstractions – they don't objectively exist.

One could say that because hardware is missing (the interpenetrating branes and the four attributive fields), the missing hardware has to be emulated in software. The “quantum fields” of quantum field theory are that software. But of course the emulation is not perfect, because gravity (General Relativity) and quantum field theory are fundamentally incompatible in the context of 3+1 spacetime alone.

Maldacena’s result is telling us that physical phenomena derive from processes in higher dimensions. That is, our flattened physical world is a projection from higher dimensions. Note the accord with our previous conclusion that the quantum wavefunction, from which all physical matter and phenomena derive, evolves in higher dimensions *outside* our spacetime while exciting attributive fields *within* our spacetime. The Maldacena conjecture, far from “realizing explicitly the holographic principle”, brings us to examine the conviction that “reality” is encompassed by our 3+1 spacetime. Plato would be pleased.

Conclusion

A consistent cosmological framework has been erected from first principles, subsuming and extending essential features of quantum theory, relativity theory, and string/M-theory. The strength of the framework lies in its rich logical structure, constituting a symbiotic tapestry built upon the most primary principles of geometry, symmetry, and number. Even whilst skeletal and unadorned, the resulting framework illuminates enduring paradoxes and establishes explicit connections between previously estranged physical domains. However closely these concepts and principles reflect Nature as it is, the framework itself speaks of something beyond our limited understanding of its parts, suggesting that investigation along these lines may herald a new era of progress in theoretical physics.

Notes and References

Introduction: pages 5 – 6

1. Penrose (2004), p. 71.
2. Cramer (1986), p. 653.
3. Penrose (2004), pp. 1034–35.

Part 1: pages 7 – 13

1. Woit (2007), p 49.
2. Ibid.
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4. Ibid, p. 71.
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6. Penrose (2004), p. 200.
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8. See Penrose (2004), p. 201.

Part 2: pages 14 – 28

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2. Penrose (2004), p. 510.
3. Pusey et al. (2011).
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7. For a general introduction to Minkowski spacetime see Penrose (2004), pp. 412-15.
8. Penrose (2004), p. 769.
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10. For descriptions of the Hartle-Hawking no boundary proposal, see Hawking (1998), pp. 141-146; Penrose (2004), pp. 769-772.
11. This general information regarding branes and string theory can be found in many popular books, such as Greene (2003, 2005, 2011), Randall (2005), Gubser (2010). See also, Witten (1997, 1998).
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18. Maudlin (2011) presents a particularly clear account of Bell's theorem and the Aspect experiment.

Part 3: pages 29 – 41

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4. Overduin & Wesson (1998), p. 2.
5. Penrose (2004), p. 883.

6. See Greene (2003), pp. 308-12.
7. Greene (2003), p. 310.
8. Longo (2011).
9. Gubser (2010), p. 148.
10. Translation by Benjamin Jowett. http://classics.mit.edu/Plato/timaeus.html
11. See Maudlin (2011), pp. 180-86.

Part 4: pages 42 – 54

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2. See Penrose (1989), p. 199; Wilczek (2008), pp. 109-10.
3. Hawking et al. (2010), pp. 107-08.
4. Penrose (2004), p. 610; Penrose (1989), p. 374.
5. Wilczek (2008), p. 90.
6. Quoted in Wilczek (2008), p. 97.
7. Susskind (2006), p. 97]
8. Penrose (1989), pp. 316-17.
9. For a lucid popular presentation of these concepts I highly recommend Herbert (1985), while Bowman (2008) provides a more technical presentation accessible to those with basic math.
10. Penrose (2004), pp. 528-529.
11. Herbert (1985), p. 104.
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13. See http://en.wikipedia.org/wiki/Energy_operator
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Part 5: pages 55 – 60

1. Majid (2008), pp. 73-74.
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Part 6: pages 61 – 70

1. ’t Hooft (2012).
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3. See Witten (1997), p. 28.
4. Experts may recognize here a possible context for Garrett Lisi’s E_8 theory: http://arxiv.org/abs/0711.0770
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