Signal space and the Schwarzschild black hole

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Abstract

The geometry of the Schwarzschild black hole is compared to the geometry of the signal space from Shannon's mathematical theory of communication. One result of these considerations is that the black hole is found to leak in a way that does not introduce an information loss paradox.

1 Introduction

Shannon's mathematical theory of communication (information theory) has been successfully applied to the study of black hole thermodynamics for quite some time now, and the results can be summarized by the holographic principle [1–3]: The maximum amount of information required to describe the physics that occurs within a region – that is, the maximum average information content per microscopic state, or maximum entropy – is proportional not to the total volume of the region (as one may initially suspect), but rather to the total area of the boundary of the region. A Schwarzschild black hole embodies this limit, where the entropy is proportional to the black hole's event horizon area.

The ultimate goal of this paper is to consider some possible connections between the geometry of the black hole and the geometry of the signal space from Shannon's theory. We will begin by discussing Shannon's theory on signals. We will then discuss the entropy of the black hole and van der Pauw sheet resistance-conductance. We will finish by discussing the interior and exterior volume of the black hole. A brief review follows.

2 Noise-free and distortionless discrete binary signals

In Shannon's theory, as presented in his paper "*Communication in the Presence of Noise*" (see [4,5]), a noise-free and distortionless discrete binary signal consists of a string of n binary samples ('off' or 'on' values).

Each of the 2^n distinct signals form a distinct vector in an n dimensional flat space – the signal space. Where $x_1, x_2, ..., x_n$ are the n binary samples in an individual signal, the length

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of the signal vector in the nD signal space is

$$d = \sqrt{\sum_{m=1}^{n} x_m^2}. \tag{1}$$

In this paper we will assume that 'on' $\equiv 1$. As for the numerical value corresponding to 'off', we have to make a choice between 0 and -1:

1. If 'off' $\equiv 0$, then the signal vector lengths can be any one of $d = \sqrt{0}, \sqrt{1}, \sqrt{2}, \ldots, \sqrt{n}$. The integer factorial is

$$z! = \begin{cases} 1, & z = 0 \\ \prod_{m=1}^{z} m, & z > 0 \end{cases}. \tag{2}$$

   For large z, the integer factorial can be reasonably approximated via Stirling's continuous approximation. The distribution of the 2^n signal vector lengths is given by the binomial coefficient "n choose k"

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \tag{3}$$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n, \tag{4}$$

   where k is the number of 'on' samples per signal (or equivalently, the number of 'off' samples per signal, due to a symmetry of the distribution). For large n, the signal vector length distribution can also be reasonably approximated (indirectly) via the continuous normal distribution.

2. Otherwise, if 'off' $\equiv -1$, then the the signal vector lengths are always $d \equiv \sqrt{n}$. In this case, the tips of all of the 2^n signal vectors are constrained to a single $(n-1)$D shell in the nD signal space. Altogether, the 2^n signal vectors form a perfect binary tree in the nD signal space, where a branching in the signal tree occurs once per dimension.

In this paper we will assume that 'off' $\equiv -1$, although the alternative is clearly interesting in its own way.

The binary 'Shannon' entropy, or average information content (in bits) per distinct signal, is

$$S = \frac{-\sum_{m=1}^{2^n} p_m \ln p_m}{\ln 2}, \tag{5}$$

where p_m is the probability of occurrence for the mth distinct signal, and the probabilities add up to unity

$$\sum_{m=1}^{2^n} p_m = 1. \tag{6}$$

In this paper we will assume that the signals are all equiprobable, where $p_m \equiv 1/2^n$, and so the entropy is maximized and simplifies to

$$S = \frac{\ln 2^n}{\ln 2} = n. \tag{7}$$

In this case where the signals are all equiprobable, and thus the entropy is maximized, the average information content per signal (n bits per signal) is identical to the average data content per signal (n binary samples, or bits, per signal). If the signals had not all been equiprobable, and thus the entropy had not been maximized, then the average information content per signal would have been less than the average data content per signal.

In Shannon's theory, the value $n = d^2$ is a measure of power.

This paper will use geometrized units, where $c \equiv G \equiv \hbar \equiv k_\mathrm{b} \equiv 1$, and so measures of power are dimensionless (units of length / length). Incidentally, measures of electric potential are also dimensionless (units of length / length).

Note that even though the value of $\hbar$ is set to unity, it is still a dimensionful constant (units of length^2) and so it will be explicitly written out in this paper in order to make the dimension (or lack thereof) of each equation clear.

Also note that the signal space is not quite the same as the familiar state space from quantum field theory / quantum gravity: Although there is a one-to-one mapping between the 2^n discrete binary signals and the 2^n states (both are effectively defined by the same data – the samples – and as such are effectively the same thing), each state would instead be represented by a distinct *unit length* vector in a flat space of 2^n dimensions (the state vectors, altogether, would form the set of 2^n orthonormal basis vectors for the state space). The signal space is also not quite the same as the phase space, nor is it quite the same as the configuration space, *etc*. The signal space is *power* space.

3 Noisy and distorted discrete binary signals and the effective dimension of the signal space

For any n, there exist signals other than the 2^n *root* noise-free and distortionless discrete binary signals that were discussed in the previous section.

Consider a continuous sinusoidal toy representation of the root signals (see [6, 7] for a complex version of such), where the signal frequency (band limit) is $\nu = 1$ (and thus the Nysquist sampling rate is $2\nu = 2$).

In the presence of phase distortion, the individual sample amplitudes can range in value from -1 through 1 (they are no longer necessarily integer values, like in the case of the root signals). As such, the tips of the signal vectors are no longer constrained to end upon an $(n-1)$D shell of 'radius' $d = \sqrt{n}$, but can now end within an nD ball of 'radius' $d = \sqrt{n}$.

In the presence of noise, for instance, randomly-generated amplitudes that can range in value from -1 through 1 are added to the signal's (possibly already phase distorted) sample amplitudes. Considering the special case where all of the signal amplitudes have a value of 1, and all of the noise amplitudes have a value of 1, is it found that the tips of the signal vectors can now end within an nD ball of 'radius' $d = \sqrt{2^2 + 2^2 + 2^2 + \ldots + 2^2} = 2\sqrt{n}$. Generally, the noise power need not scale like the signal power. For instance, the noise can scale like 1, instead of like n.

Altogether, the noisy and distorted versions of the root signals are *aliases* of the root signals, and we can effectively perform a dimensional reduction of the signal space (from nD down to $(n-1)$D) by simply ignoring these *alias signals* (or at least those alias signals which do not end upon the $(n-1)$D shell). This dimensional reduction is indicative of a kind of gauge redundancy, insomuch that the transformation of a root signal into an alias signal can be ideally represented as a single 'rotation plus scale' operation in the nD signal space (a gauge transformation).

As mentioned in the previous section, the length-squared of a signal vector d^2 is a measure of power, and so the amplitude-squared of a signal's sample x_m^2 is also a measure of power. For instance, the amplitude-squared of any sample in a *root* signal corresponds to a measure of 1 (the Planck power), and so the length-squared of any *root* signal corresponds to a measure of n. As such, samples of amplitude-squared $x^2 < 1$ are sub-Planckian samples, samples of $x_m^2 = 1$ are Planckian samples, and samples of $x_m^2 > 1$ are trans-Planckian samples. Similarly, the (off-shell) signals of length-squared $d^2 < n$ are sub-Planckian signals, (on-shell) signals of $d^2 = n$ are Planckian signals, and (off-shell) signals of $d^2 > n$ are trans-Planckian signals.

Some discussion of gauge redundancy and gravity can be found in [8–14].

Some discussion of the signal space (albeit *sans* discussion of dimensional reduction and black holes) can be found in [15].

4 Leaky black holes and van der Pauw sheet resistance-conductance

As discussed in a previous section, a signal and a state are effectively defined by the same data, and so we consider them to be effectively the same thing. As such, even though we will exclusively use the word 'signal' in this section for the sake of consistency, we could have just as readily used the word 'state' in its place. In this section, we will consider a possible connection between the geometry of the signal space and the geometry of the black hole's event horizon radius.

The event horizon radius and event horizon area of a Schwarzschild black hole [16–18], is

$$R_{\mathrm{s}} = 2E_{\mathrm{bh}}, \tag{8}$$
$$A = 4\pi R_{\mathrm{s}}^2. \tag{9}$$

The binary 'Bekenstein-Hawking' (also known as 'von Neumann') entropy, or average information content per root signal, of a Schwarzschild black hole is

$$S = n = \frac{A}{4\hbar \ln 2} = \frac{\pi R_{\mathrm{s}}^2}{\hbar \ln 2} = \frac{4\pi E_{\mathrm{bh}}^2}{\hbar \ln 2}. \tag{10}$$

The equation $S = n$ means that each of the black hole's 2^n equiprobable root signals requires an average of n bits of information content to be described, as was exactly the case in Eq. (7). The proportionality $S \propto A$ means that the average information content per root signal is proportional to the bounding area (event horizon area) of the black hole, as per the holographic principle. Taking the holographic principle quite literally, the (data/information of the) 2^n root signals of the black hole are generated by a shell of $S = n$ qubits (spin-like degrees of freedom) at radius R_{s}.

Where

$$\alpha = \frac{\pi}{\ln 2}, \tag{11}$$

the root signal shell 'radius' $d = \sqrt{n}$ is related to the event horizon radius by

$$d = R_\mathrm{s}\sqrt{\alpha/\hbar}. \tag{12}$$

Likewise, a root signal's power is related to the black hole energy by a dimensionless integer constant (units of length^3 / length^3)

$$\frac{\hbar(x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2)}{\alpha E_\mathrm{bh}^2} = 4. \tag{13}$$

In the presence of noise, it is inevitable that some of the alias signals will be trans-Planckian (where $d^2 > n$). If the relationship $r = d/\sqrt{\alpha/\hbar}$ holds, then these trans-Planckian alias signals exist in spacetime at a radius of $r > R_\mathrm{s}$, in which case it would be possible for these trans-Planckian alias signals to escape to infinite distance. Clearly these leaked signals must be represented by some form of energy, such as Hawking radiation of temperature (at infinite distance)

$$T = \frac{\hbar}{8\pi E_\mathrm{bh}}. \tag{14}$$

The photons constituting the Hawking radiation at infinite distance should not possess any kind of overtly trans-Planckian properties when sampled by a detector in spacetime. In terms of Shannon's theory, this is to say that the trans-Planckian nature of the leaked alias signals should be suppressed by a mapping from the signal space to the *message space* that corresponds to the sampled photon property [19].

Given that noise is the mechanism by which the alias signals are leaked in the first place, and that the black hole's temperature decreases as a root signal's power increases, it seems that the noise power does not scale at all. This is to say that $E \propto \hbar/T \propto \sqrt{n}$, and so $T \propto \hbar/\sqrt{n}$. In effect, the black hole's temperature is a kind of indirect measure of '*noise*-to-*signal*' which indicates that the noise is constant like $\hbar$ (like a quantum of noise), and thus independent of the root signal shell 'radius' $d = \sqrt{n}$.

Since we have no reason to suspect *a priori* that the entanglement of any of the signals is broken when the trans-Planckian alias signals are leaked [20], we shall assume that the entropy of the entire system (the black hole proper plus the radiation) does not reduce as the black hole evaporates. This semi-classical interpretation is not quite the same as the standard semi-classical interpretation of black hole thermodynamics, in which case the black hole is found to leak energy, but there is still some remaining ambiguity as to what happens to the entropy (and the signals that generate it) – does it stay behind in the black hole proper; does it leak out; does it vanish? Through consideration of the signal space representation of the black hole, we instead find that the black hole evaporates over time in a way which does not produce an entropy (information) loss paradox, because we know from the very beginning that the signals are leaked and we have no reason to suspect *a priori* that the entanglement of any of the signals is broken in the process. This possibility is remarkable largely because it is based on a classical theory of communication that pre-dates the work on black hole thermodynamics. Altogether, the black hole becomes a trans-Planckian alias of itself as it evaporates due to *the presence of noise*.

What may or may not be immediately obvious about the entropy and temperature equations is the following set of circumstances, altogether:

1. The event horizon is a 2D sheet.

2. The entropy is dimensionless (units of length^2 / length^2), which is dimensionally equivalent to resistance-conductance (both also have units of length^2 / length^2).

3. The factor $\alpha = \pi / \ln 2$ is van der Pauw's constant from his theory of sheet resistance-conductance [21], and so the entropy is similar to resistance-conductance in terms of how to calculate the numerical magnitude.

This leaves us with a choice to make as to whether we shall interpret the entropy to be a measure of sheet resistance or a measure of sheet conductance. Given that an increase in conductance is generally related to a decrease in temperature, it seems more likely that the entropy is a measure of sheet conductance.

Some discussion on black hole superconductivity and temperature can be found in [22,23]. Some discussion of the thermodynamics of gravity can be found in [24–30]. Some discussion of fuzzy black holes can be found in [31]. Some further discussion of the relation between Shannon's theory and noise-based uncertainty can be found in [32,33], which were the direct inspiration for considering the possible relation between Shannon's theory, van der Pauw sheet resistance-conductance, and black holes. Some further discussion of the entanglement of the signals can be found in [34–37].

5 Finite volume and volume derivative of the black hole interior

In this section, we will consider a possible connection between the geometry of the signal space and the geometry of the black hole interior and exterior.

In Shannon's theory, it is often noted that as the dimension n of the signal space increases, the majority of the volume of the space becomes more and more distributed toward the $(n-1)$D shell at 'radius' $d = \sqrt{n}$. This can be stated in terms of the volume ratio

$$\frac{\mathrm{Vol}(n,d)}{\mathrm{Vol}(n,\frac{d}{2})} = 2^n. \tag{15}$$

For instance: where $n = 2$, the 'volume' ratio for a 2D disk of radius $r = d$ and a 2D disk of radius $r = d/2$ is $2^2 = 4$; where $n = 3$ the ratio is $2^3 = 8$; where $n = 4$ the ratio is $2^4 = 16$; *etc.* This is ultimately because the derivative of the volume with respect to radius r is proportional to r^{n-1}.

The integer Gamma function is

$$\Gamma(z) = (z-1)!. \tag{16}$$

The half-integer Gamma function is

$$\Gamma_{1/2}(z+1/2) = \sqrt{\pi}\frac{(2z)!}{z!4^z}. \tag{17}$$

Again, for large z, the integer factorial can be reasonably approximated via Stirling's continuous approximation.

The volume of a zD ball of radius r is proportional to r^z

$$\mathrm{Vol}(z,r) = \begin{cases} \frac{\sqrt{\pi}^z r^z}{\Gamma(z/2+1)}, & z \bmod 2 = 0 \\ \frac{\sqrt{\pi}^z r^z}{\Gamma_{1/2}([z+1]/2)}, & z \bmod 2 = 1 \end{cases}, \tag{18}$$

and the derivative of the volume with respect to r is proportional to r^{z-1}

$$\frac{\mathrm{dVol}(z,r)}{\mathrm{d}r} = \mathrm{Vol}(z,r)\frac{z}{r}. \tag{19}$$

Consider the possibility that the volume (and/or the derivative of the volume) of the black hole interior is also related to the signal space. In the paper [38], an effectively time-independent total interior volume of $(4/3)\pi R_\mathrm{s}^3$ is calculated (also see [39,40]), which is equal to $\mathrm{Vol}(3, R_\mathrm{s})$. If we wish to use this calculation as a starting point, then the signal space would affect only the derivative of the volume. For instance, where $z = n$ and $r \leq R_\mathrm{s}$

$$\mathrm{Vol}_\mathrm{bh}(r) = \mathrm{Vol}(3, R_\mathrm{s})\frac{\mathrm{Vol}(n,r)}{\mathrm{Vol}(n, R_\mathrm{s})}. \tag{20}$$

In effect, we have cast the volume derivative of nD space into the role of the volume derivative of 3D space, thus possibly curving the 3D space.

For instance, where $n = 3$, the volume derivative would be the same as flat 3D space – the volume ratio from Eq. (15) holds as $\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s})/\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s}/2) = 2^3$ – and so the 3D space would not become curved.

Where $n = 1$ and $n = 2$, the volume would contract near the centre of the black hole – the volume ratio would be $\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s})/\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s}/2) < 2^3$ – and so the 3D space would become positively curved. The contraction of space near the black hole centre would accelerate anything within the black hole toward the centre, somewhat like how the contraction of space in the black hole exterior region near the event horizon causes anything outside of the black hole to accelerate toward the event horizon. We take this to mean that for $n = 1$ and $n = 2$ that all of the black hole's matter would reside at the centre of the black hole, but at least both the volume and the volume derivative are always finite – no central singularity arises.

Where $n \geq 4$ the volume would instead contract near the event horizon – the volume ratio would be $\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s})/\mathrm{Vol}_\mathrm{bh}(R_\mathrm{s}/2) > 2^3$ – and so the 3D space would become negatively curved. The contraction of space near the event horizon would accelerate anything within the black hole toward the event horizon. We take this to mean that for $n \geq 4$ that all of the black hole's matter would reside at the black hole's event horizon.

Similarly, we could consider the possibility that the 2^nD *state* space affects the derivative of the volume. For instance, where $z = 2^n$ and $r \leq R_\mathrm{s}$

$$\mathrm{Vol}_\mathrm{bh}(r) = \mathrm{Vol}(3, R_\mathrm{s})\frac{\mathrm{Vol}(2^n, r/R_\mathrm{s})}{\mathrm{Vol}(2^n, 1)}, \tag{21}$$

which would simply serve to increase the degree of the contraction of space near the event horizon much more rapidly than the case where $z = n$.

If we do not wish to use the calculation from [38] as a starting point, then the signal space would affect not only the volume derivative, but also the total interior volume. For instance, where $z = n$ and $r \leq R_\mathrm{s}$, the volume could be

$$\mathrm{Vol}_\mathrm{bh}(r) = \mathrm{Vol}(n, r/R_\mathrm{s}), \tag{22}$$

in which case the total interior volume of the black hole all but vanishes as n increases toward infinity. In effect the black hole would be all but hollow, in somewhat similar fashion to what is presented in the paper [41], but with the exception that the exterior and interior solutions are not (in terms of the present formulation) connected in a continuous way (in terms of both total interior volume and volume derivative).

Some other possibilities for the volume are

$$\begin{aligned}
\mathrm{Vol}_\mathrm{bh}(r) &= \mathrm{Vol}(n, r), && (23)\\
\mathrm{Vol}_\mathrm{bh}(r) &= \mathrm{Vol}(n, d[r/R_\mathrm{s}]), && (24)\\
\mathrm{Vol}_\mathrm{bh}(r) &= \mathrm{Vol}(2^n, r/R_\mathrm{s}), && (25)\\
\mathrm{Vol}_\mathrm{bh}(r) &= \mathrm{Vol}(2^n, r), && (26)\\
\mathrm{Vol}_\mathrm{bh}(r) &= \mathrm{Vol}(2^n, \sqrt{2^n}[r/R_\mathrm{s}]). && (27)
\end{aligned}$$

In any case under consideration, both the total interior volume and the volume derivative are always finite – no singularities ever arise. In most cases (where $z \geq 4$), it is found that the black hole's matter would reside not at the centre of the black hole, but rather at the event horizon. Altogether, considering the possibility that the geometry of the nD signal (or 2^nD state) space is related to the geometry of spacetime (as far as the total interior volume and the volume derivative are concerned) leads to a set of possible black hole interior solutions that (in most cases) enforce the holographic principle via black hole interior 'antigravitation', while never suffering from the singularities that afflict standard general relativity.

Considering the possibility that a black hole also contributes to the *exterior volume derivative*, it seems likely that the vast majority of the Universe's matter currently exists in the form of black holes of $n = 3$, $E = \frac{1}{2}\sqrt{(3\ln 2)/\pi}$ since the vast majority of space is flat and $n = 3$ is the only measure for n that is accompanied by the appropriate volume derivative. One interesting conclusion that can be drawn from the possibility that the black hole also contributes to the *exterior volume* is that if the volume is all but vanishing, like with the solutions described by Eqs. (22) and (25), and if all of the Universe's matter were to be consolidated into a single black hole, then the Universe itself would be effectively constrained to exist upon a 2D shell – a dimensional reduction of the Universe would occur because both the interior and exterior volume of the black hole would be essentially zero. As such, in the case of these solutions with all but vanishing exterior volume, it seems likely that the vast majority of the Universe's matter currently exists in the form of black holes of very small n, since the volume of the Universe is known to be gigantic, and only a huge number of small black holes would be able to generate such a vast amount of volume. This is the essentially the same conclusion that was drawn at the beginning of this paragraph when we considered only the exterior volume derivative, and so regardless of which solution we choose to consider, it seems likely that the vast majority of the Universe currently exists in the form of black holes of very small n, since this is the only configuration that consistently leads to a spatially flat Universe with non-vanishing volume, like the one that we apparently live in.

Some discussion of the black hole volume can be found in [42]. Some discussion of a model of quantum gravity that makes use of higher order derivatives can be found in [43].

6 Review

In this paper we considered some possible connections between the geometry of the signal space and the geometry of the black hole.

First, we discussed noise-free and distortionless discrete binary signals (root signals), and we noted that these root signals are constrained to end upon a single $(n-1)$D shell in the nD signal space. We also discussed the aliasing of the root signals via noise and distortion, and we noted how aliasing increases the effective dimension of the signal space from $(n-1)$D to nD.

Next, we discussed the possibility (via the van der Pauw constant) that the radius of the $(n-1)$D shell in the signal space is related to the event horizon radius in spacetime. We also discussed how the black hole leaks trans-Planckian alias signals (due to *the presence of noise*) in a way that does not introduce an entropy (information) loss paradox. We noted how the entropy of the black hole is dimensionally and numerically similar to the van der Pauw sheet resistance-conductance, and that it is possible to interpret the entropy as a measure of sheet conductance.

Finally, we discussed the possibility that the black hole interior volume and volume derivative is related to the dimension of the signal space. The primary result was a set of possible black hole interior solutions that do not suffer from singularities. We also discussed the possibility that the black hole exterior volume and volume derivative is also related to the dimension of the signal space. The primary result was the conclusion that the vast majority of the Universe's matter likely exists in the form of microscopic black holes, which is the only configuration that consistently produces a spatially flat Universe with non-vanishing volume.

Although the model presented here may be a semi-classical toy, it is nevertheless interesting to note how the toy immediately highlights the fact that dimensional reduction is an integral part of Shannon's theory. We feel that this aspect of Shannon's theory is not mentioned nearly enough in the literature on black hole thermodynamics, even though both information theory and dimensional reduction clearly already play a critical role in the study of black holes. Perhaps there are actual physical models of the black hole based on this little-advertised coincidence. Clearly, we should seriously consider the possibility that Shannon (and his colleagues von Neumann and Pierce) effectively predicted that black hole evaporation and the holographic principle would become some of the most important concepts in the physical theories of the 21st century – we should seriously consider the possibility that Shannon was *the* progenitor of quantum gravity.

This paper is dedicated to Bob. Thank you to many people for various discussions on the FQXi website (Hi Steve).

References

[1] 't Hooft G. *Dimensional Reduction in Quantum Gravity* (1993) arXiv:gr-qc/9310026

[2] Susskind L. *The World as a Hologram* (1994) arXiv:hep-th/9409089

[3] Bousso R. *The holographic principle* (2002) arXiv:hep-th/0203101

[4] Shannon CE. *Communication in the Presence of Noise* (1949) PROCEEDINGS OF THE IRE, Vol. 37, No. 1

[5] Pierce JR. *An Introduction to Information Theory: Symbols, Signals and Noise* 2E (1980) Mineola: Dover Publications

[6] Fowles GR. *Introduction to Modern Optics, 2E* (1989) New York: Dover Publications – Section 1.4, The Complex Wave Function

[7] Fleisch D. *A Student's Guide to Maxwell's Equations* (2008) Cambridge: Cambridge University Press

[8] Maldacena JM. *The Large N Limit of Superconformal Field Theories and Supergravity* (1998) arXiv:hep-th/9711200

[9] Vasiliev MA. *Holography, Unfolding and Higher-Spin Theory* (2012) arXiv:1203.5554 [hep-th]

[10] Lasenby A, Doran C, Gull S. *Gravity, Gauge Theories and Geometric Algebra* (2004) arXiv:gr-qc/0405033

[11] Horowitz GT, Polchinski J. *Gauge/gravity duality* (2006) arXiv:gr-qc/0602037

[12] Polchinski J. *Introduction to Gauge/Gravity Duality* (2010) arXiv:1010.6134 [hep-th]

[13] Motl L. *Holography: Vasiliev's higher-spin theories and O(N) models* (2010) `http://motls.blogspot.ca/2010/02/holography-vasilievs-higher-spin.html`

[14] Gibbs P. *String Theory returns to symmetry* (2012) `http://blog.vixra.org/2012/07/31/string-theory-returns-to-symmetry/`

[15] Khrennikov A. *Towards a Field Model of Prequantum Reality* (2011) Found Phys 42:725741 (2012)

[16] Schutz B. *A First Course in General Relativity* (1985) Cambridge: Cambridge University Press

[17] Bekenstein JD. *Black Holes and Entropy* (1973) Phys. Rev. D 7, 23332346

[18] Hawking SW. *Particle Creation by Black Holes* (1975) Commun. math. Phys. 43, 199-220

[19] Percacci R. *Asymptotic Safety* (2007) arXiv:0709.3851 [hep-th]

[20] Motl L. *In awe about entanglement* (2012) http://motls.blogspot.com/2012/10/in-awe-about-entanglement.html

[21] van der Pauw LJ. *A method of measuring the resistivity and Hall coefficient on lamellae of arbitrary shape.* (1958/1959) Eindhoven: Philips Technical Review, 1958/1959, Vol. 20, No. 8

[22] Chamblin A, Emparan R, Gibbons GW. *Superconducting p-branes and Extremal Black Holes* (1998) arXiv:hep-th/9806017v1

[23] Hartnoll SA, Herzog CP, Horowitz GT. *Holographic Superconductors* (2008) arXiv:0810.1563 [hep-th]

[24] Sidharth BG. *The Planck Oscillator Model* (2009) arXiv:0905.3896 [physics.gen-ph]

[25] Ghosh S. *Black Hole Entropy: From Shannon to Bekenstein* (2010) arXiv:1008.0946 [hep-th]

[26] Jacobson T. *Thermodynamics of Spacetime: The Einstein Equation of State* (1995) arXiv:gr-qc/9504004v2

[27] Jacobson T. *Introduction to Quantum Fields in Curved Spacetime and the Hawking Effect* (2003) arXiv:gr-qc/0308048

[28] Visser M. *Sakharov's induced gravity: a modern perspective* (2002) arXiv:gr-qc/0204062v1

[29] Verlinde, EP. *On the Origin of Gravity and the Laws of Newton* (2010) arXiv:1001.0785v1 [hep-th]

[30] Verlinde, EP. *CERN Colloquium – On the Origin of Gravity, Dark Energy and Matter* (2011) CERN Conference ID: 136461

[31] Mathur SD. *The fuzzball proposal for black holes: an elementary review* (2005) arXiv:hep-th/0502050

[32] McEachern RH. *Misinterpreting Reality: Confusing Mathematics for Physics* (2012) FQXi Essay Contest – Spring, 2012 – http://fqxi.org/community/forum/topic/1372

[33] Millette PA. *The Heisenberg Uncertainty Principle and the Nyquist-Shannon Sampling Theorem* (2011) arXiv:1108.3135v1 [quant-ph]

[34] Almheiri A, Marolf D, Polchinski J, Sully J. *Black Holes: Complementarity or Firewalls?* (2012) arXiv:1207.3123 [hep-th]

[35] Wootters WK. *Entanglement Sharing in Real-Vector-Space Quantum Theory* (2010) arXiv:1007.1479 [quant-ph]

[36] Hasegawa Y. *Entanglement Between Degrees of Freedom in a Single-Particle System Revealed in Neutron Interferometry* (2010) Found Phys 42:2945 (2012)

[37] Chang D, Chu C-S, Lin F-L. *Transplanckian Entanglement Entropy* (2003) arXiv:hep-th/0306055

[38] Parikh MK. *The Volume of Black Holes* (2005) arXiv:hep-th/0508108

[39] DiNunno BS, Matzner RA. *The Volume Inside a Black Hole* (2008) arXiv:0801.1734 [gr-qc]

[40] Ballik W, Lake K. *The volume of stationary black holes and the meaning of the surface gravity* (2010) arXiv:1005.1116 [gr-qc]

[41] Davidson A, Gurwich I. *Hollowgraphy Driven Holography: Black Hole with Vanishing Volume Interior* (2010) arXiv:1007.1170 [gr-qc]

[42] Motl L. *How empty is the black hole interior?* (2012) `http://motls.blogspot.ca/2012/10/how-empty-is-black-hole-interior.html`

[43] Briscese F, Rodríguez Y, González GA. *On the True Nature of Renormalizability in Horava-Lifshitz Gravity* (2012) Found Phys 42:14441451 (2012)