Van der Pauw sheet resistance and the Schwarzschild black hole

S. Halayka*†

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Abstract

The entropy of the Schwarzschild black hole is considered in terms of Shannon's mathematical theory of communication and van der Pauw's theory of sheet resistance.

1 Introduction

In Shannon's mathematical theory of communication [1, 2], a discrete binary signal consists of a string of n binary samples ('off' or 'on' values). Each of the 2^n distinct signals form a distinct vector in an n dimensional flat space (signal space). Where $x_1, x_2, ..., x_n$ are the n binary samples in an individual signal, the length of the signal vector in the nD signal space is

$$d = \sqrt{\sum_{m=1}^{n} x_m^2}. \tag{1}$$

Note that the signal space is not quite the same as the state space, where each state would instead be represented by a *unit length* vector in a space of 2^n dimensions.

This paper will default to taking 'on' $\equiv$ '1'.

If 'off' $\equiv$ '0', then the lengths of the signal vectors can be any one of $\sqrt{0}, \sqrt{1}, \sqrt{2}, ..., \sqrt{n}$. The distribution of the 2^n signal vector lengths is given by the binomial coefficient "n choose k", where k is the number of 'on' samples per signal (or equivalently, the number of 'off' samples per signal, due to a symmetry of the distribution).

Otherwise, if 'off' $\equiv$ '-1', then the lengths of the signal vectors are always $d \equiv \sqrt{n}$. In this case, the tips of all of the 2^n signal vectors are constrained to a single $(n-1)$D shell in the nD signal space. Altogether, the 2^n signal vectors form a perfect binary tree in the nD signal space, where a branching in the signal tree occurs once per dimension.

This paper will default to taking 'off' $\equiv$ '-1'.

In Shannon's theory, the value $n \equiv d^2$ is a measure of power. This paper will use geometrized units, where $c \equiv G \equiv \hbar \equiv k_{\mathrm{b}} \equiv 1$, and so measures of power are dimensionless (units of length / length). Incidentally, measures of electric potential are also dimensionless (units of length / length).

*darkessay@gmail.com

†Saskatoon, SK Canada

2 Sheet resistance

The event horizon radius of a Schwarzschild black hole [3] is

$$R_{\rm s} = 2E_{\rm bh}. \tag{2}$$

The binary entropy of a Schwarzschild black hole is

$$S = \frac{\pi R_{\rm s}^2}{\ln 2} = \frac{4\pi E_{\rm bh}^2}{\ln 2}. \tag{3}$$

The Bekenstein-Hawking temperature of a Schwarzschild black hole is

$$T = \frac{1}{8\pi E_{\rm bh}}. \tag{4}$$

For ordinary matter (a car), an increase in entropy goes along with an increase in temperature. For a Schwarzschild black hole, an increase in entropy goes along with a decrease in temperature

$$S = \frac{E_{\rm bh}}{2T\ln 2}. \tag{5}$$

Since the relationship between entropy and temperature is reversed for a Schwarzschild black hole (compared to the relationship for ordinary matter), it can be taken that the relationship between resistance and temperature is also reversed for the black hole (in which case an increase in resistance goes along with a decrease in temperature). As such, in the context of signals and electric potential, the entropy S can possibly be taken to be a dimensionless measure of van der Pauw sheet resistance (units of length^2 / length^2), given that

$$\alpha = \frac{\pi}{\ln 2} \tag{6}$$

is van der Pauw's resistance constant [4]. Presumably, this is a measure of the resistance along the event horizon, not perpendicular to the event horizon.

The signal shell "radius" $d \equiv \sqrt{n}$ is related to the event horizon radius and the sheet resistance by

$$d = R_{\rm s}\sqrt{\alpha}. \tag{7}$$

An individual signal relates to the black hole energy and the sheet resistance by a unitless integer constant

$$\frac{x_1^2 + x_2^2 + x_3^2 + ... + x_n^2}{\alpha E_{\rm bh}^2} = 4, \tag{8}$$

which may or may not be the dimension of $(3+1)$D spacetime.

Altogether, if the interpretation given here is correct, then even a black hole with zero net electric charge possesses electrical properties such as resistance.

Some discussion of thermodynamic models of gravity can be found in [5, 6, 7, 8, 9]. Some discussion of the possible relation between Shannon's theory and the measurement/uncertainty problem can be found in [10, 11], which were the direct inspiration for considering the possible relation between Shannon's theory, sheet resistance, and black holes.

References

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