

Twisted Radio Waves and Twisted Thermodynamics

(September 28, 29, October 1, 2012)

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Abstract. We point out that the assumption that more than two spatially orthogonal far-field wave modes (the two polarization modes) can leave an antenna and propagate in free space violates the Second Law of Thermodynamics and is thus incorrect.

Recently it has been claimed in scientific literature and popular news media—and purportedly demonstrated by experiment—that it is possible to generate radio waves with more spatially orthogonal *far-field* modes than the usual two polarization modes [1-4]. Such radio waves would have angular momenta (so called orbital angular momenta) different from the angular momentum of circularly polarized waves in a similar way as the orthogonal (l) wave modes of electrons belonging to the same main quantum number (n) exist at the same frequency. Communication utilizing such modes would expand the available frequency band by a factor given by the number N of such spatially orthogonal modes. Experimental demonstration for $N = 2$, which is equal to the number of modes of a standard circularly polarized wave, has been carried out and published [2]. If the number of spatially orthogonal modes can be more than two, or even infinite as claimed in news media [1], this would revolutionize wireless communication because the information channel capacity of the radio waves scales linearly with N in the case of fixed bandwidth and signal-to-noise ratio.

However, the most important question before the idea of a vastly increased capacity for radio communication can be realized in practical applications is whether such far-field modes can exist in free space. If the polarization is circular—a common situation in wireless technology—one has $N = 2$ and the waves in the two polarization modes are phase-shifted by 90° , which leads to non-zero angular momentum. Thus the situation with $N = 2$ is obvious because of the existence of the two polarization modes and the circularly polarized waves.

It should first be noted that an assumption of more than the two far-field polarization modes is counter-intuitive. In the atom, the existence of waves with different angular momenta at the same energy originates from the potential and the ensuing localized nature of the waves. A charge revolving in a Coulomb potential field will have an infinite number of different classical physical paths with the same energy, and Bohr-Sommerfeld quantization will select a finite number of states that are allowed within quantum theory. But, in stark contrast, no such state components exist for free electron waves. In the light of this intuitive argument, the existence of spatially orthogonal modes for electromagnetic waves is fine for photons propagating under spatially confined conditions such as in wave guides and optical fibers [5,6], or in the immediate surroundings of a black hole [7]; however it is difficult to imagine them in the free space. Thus the ultimate question for possible applications in wireless communication [1,2] is this: Can far-field waves with $N > 2$ exist and be radiated by an antenna in free space? We reiterate that the existing experimental radio wave demonstrations [2] hold only for $N = 2$.

Rather than analyzing the theoretical treatments for errors, we use another approach and prove that the, hypothetical, existence of more than two independent far-field harmonic modes would violate the Second Law of Thermodynamics, which states the impossibility of constructing a perpetual motion machine of the second kind.

According to Planck's Law [8], each of the spatially orthogonal modes of a black-body (with unity emissivity) radiates with a spectral intensity

$$I(f) = \frac{4\pi hf^3}{c^2} \frac{1}{e^{hf/kT} - 1} \quad , \quad (1)$$

where f is frequency, $h = 6.626 * 10^{-34}$ Js is Planck's constant, $k = 1.381 * 10^{-23}$ JK⁻¹ is Boltzmann's constant and T is absolute temperature. This means that a unit surface area of the black-body emits the power

$$P(f, \Delta f) = I(f)\Delta f = \frac{4\pi hf^3}{c^2} \frac{\Delta f}{e^{hf/kT} - 1} \quad (2)$$

within an infinitesimally small frequency band Δf around f and for each of the spatially orthogonal modes. The total radiated power from a unit area is

$$NP(f, \Delta f) = NI(f)\Delta f = N \frac{4\pi hf^3}{c^2} \frac{\Delta f}{e^{hf/kT} - 1} \quad , \quad (3)$$

where N is the number of spatially orthogonal modes. In Planck's work [8] and related radiation measurements one has $N = 2$ because the two spatially orthogonal modes are the two polarization modes. Thus one then retrieves the standard Planck formula, *i.e.*,

$$P(f, \Delta f) = \frac{8\pi hf^3}{c^2} \frac{\Delta f}{e^{hf/kT} - 1} \quad . \quad (4)$$

Inspired by Nyquist's treatment of Johnson noise [9], we now devise the following gedanken experiment: A large box is located in a thermal reservoir. For the sake of simplicity we assume that its walls are ideally black. Furthermore a thermally isolated resistor and a "twisted-wave" antenna with N spatially orthogonal modes are located within the box, and the resistor is connected to the electrodes of the antenna. We start from thermal equilibrium, *i.e.*, a uniform temperature within the box, including the walls, the inherent thermal radiation, the antenna, the resistor, and the thermal isolation.

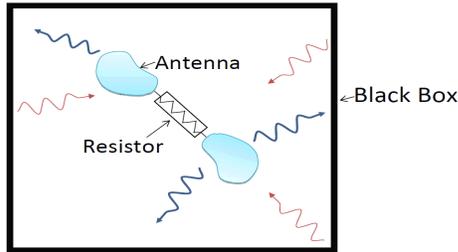


Figure 1. Outline of the gedanken experiment

For $N = 2$ we have the classical Nyquist/Planck case, and the energy supplied by the resistor and radiated by the antenna gets absorbed in the walls. However the antenna will absorb the same average amount of energy from the radiation field—originating from the walls—and transform this energy in into a voltage, so the resistor will dissipate the same amount of energy. The reason for this is Boltzmann’s Principle of Detailed Balance [10], which guarantees that each of the two polarization modes of the antenna maintains a separate dynamical equilibrium with the corresponding polarization mode of the black-body radiation of the walls. Thus the Second Law of Thermodynamics is not violated, and the temperature stays homogeneous in the system.

In the case of $N > 2$, the antenna radiates more energy than it can absorb, because its radiation would have $N - 2$ extra orthogonal modes which do not match the radiation originating from the walls. Thus the resistor will cool down, which implies that a temperature inhomogeneity is induced in the system and hence violates of the Second Law of Thermodynamics.

There are only two ways to avoid violation of the Second Law of Thermodynamics with the mentioned set-up: *Firstly*, we suppose that the black-body radiation from the walls has at least as many (*i.e.*, $N > 2$) spatially orthogonal modes as the antenna. This assumption would preserve the Second Law of Thermodynamics but violate the theory and experiments on the intensity of black-body radiation. And it deserves mentioning that these experiments are part of the very foundation of quantum physics and thereby of one of the most thoroughly verified experimental effects in physics. For example, $N = 20$ would result in a ten-fold increase of the rate of radiation cooling of bodies and would impact not only on everyday experience and thermal engineering but also on cosmology. Thus we are compelled to discard this assumption. *Secondly*, the only remaining way to save the Second Law of Thermodynamics is to suppose that N cannot be greater than two.

In conclusion, we have shown that an assumption that more than two spatially orthogonal wave modes—the actual polarization modes—can leave the near-field range of an antenna and propagate in free space violates the Second Law of Thermodynamics, and thus this assumption cannot be correct. If some experiment appears to support an assumption that violates a Law of Physics, then *either* the experiment and/or its interpretation is incorrect *or* that particular Law of Physics is invalid for the situation in case. We see no reason to doubt the validity of the Second Law of Thermodynamics!

Acknowledgements

Discussions with Bo Thidé are appreciated. We are grateful for discussions and constructive comments to Claes-Göran Granqvist, Carl-Gustaf Ribbing, Derek Abbott, Kai Chang and Greg Huff.

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