The Ford-Pfenning Quantum Inequalities(QI) Analysis applied to the Natario Warp Drive Spacetime.

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October 15, 2012

Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the warp drive violates all the known energy conditions because the stress energy momentum tensor(the right side of the Einstein Field Equations) for the Einstein tensor G_{00} is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the Quantum Field Theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself.The major drawback concerning negative energies for the warp drive are the so-called Quantum Inequalities(QI) that restricts the time we can observe the negative energy density.This time is known as the sampling time.Ford and Pfenning computed the QI for the Alcubierre warp drive and concluded that the negative energy in the Alcubierre warp drive can only exists for a sampling time of approximately 10^{-10} seconds rendering the warp drive impossible for an interstellar trip for example a given star at 20 light years away with a speed of 200 times faster than light because such a trip would require months not 10^{-10} seconds. We repeated the QI analysis of Ford and Pfenning for the Natario warp drive and because the Natario warp drive have a very different distribution of negative energy when compared to its Alcubierre counterpart this affects the QI analysis.We arrived at a sampling time that can last longer than 10^{-10} seconds enough to sustain a warp bubble for the interstellar travel mentioned above.We also computed the total negative energy requirements for the Natario warp drive and we arrived at a comfortable result.This leads us to conclude that the Natario warp drive is a valid solution of the Einstein Field Equations of General Relativity physically accessible for interstellar spaceflight. We also discuss Horizons and infinite Doppler blueshifts.

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1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of large negative energy requirements enough to sustain the warp bubble.While from a classical point of view negative energy densities are forbidden the Quantum Field Theory allows the existence of such energies but the major problem affecting the Quantum Field Theory negative energies for the warp drive are the so called Quantum Inequalities(QI):The QI restricts the time we can observe a negative energy density which means to say that as large the amount of negative energy density is then the time when this amount of energy can exists to be observed becomes incredible small.This time is known as the sampling time which is inversely proportional to the magnitude of the negative energy density amount.

Ford and Pfenning computed the QI for the Alcubierre warp drive and arrived at the conclusion that the sampling time is incredible small and approximately around the order of 10^{-10} seconds[2]. This means to say that the negative energy according to them can exists for only 10^{-10} seconds making the warp drive an impracticable form of transport because for an interstellar trip for example a given star at 20 light years away with a warp drive speed of 200 times faster than light some months are required.to complete the journey but if the warp bubble can exists for only 10^{-10} seconds then the warp drive is impossible for such an interstellar trip. Ford and Pfenning also computed the negative energy density requirements for the Alcubierre warp drive and they arrived at the conclusion that in order to sustain a warp bubble able to perform interstellar travel the amount of negative energy density is of about 10 times the mass of the universe.

Again they concluded that the warp drive is impossible.

However they performed all the calculations for the Alcubierre warp drive and not the Natario one.
This means to say that while the Alcubierre warp drive is physically impossible the possibility or impossibility of Natario warp drive is still an open quest to be solved by modern science in the future.

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[1] do not violates Relativity

[2] Consider a warp bubble with a 100 meters of radius a thickness Δ of 2 meters moving at 200 times light speed.Look to eqs 19 pg 8 and eq 20 pg 9 in [3].Insert the thickness and the speed of the bubble in these equations and the sampling time of 10^{-10} seconds will appear.

Due to a different distribution of energy the Natario warp drive behaves radically different than its Alcubierre counterpart when examined under the QI analysis.The main difference is the fact that the position of the Eulerian observer[3] inside the warp bubble $rs^2 = [(x - xs)^2 + y^2 + z^2]$ cannot be removed in the Alcubierre equation for the negative energy density but can be removed in the Natario one making the behavior of the Natario warp drive when submitted to the QI integral completely different than in its Alcubierre counterpart and in consequence the integration of the QI for the Natario warp drive is much more easy to be calculated when compared to its Alcubierre counterpart leading to dramatically different results.

Removing the position of the Eulerian observer in the Natario warp drive negative energy expression:

The negative energy density according to Alcubierre is given by(see eq 8 pg 6 in [3])[4]:

$$\langle T^{\mu\nu} u_\mu u_\nu \rangle = \langle T^{00} \rangle = \frac{1}{8\pi} G^{00} = -\frac{1}{8\pi} \frac{v_s^2(t)[y^2 + z^2]}{4r_s^2(t)} \left(\frac{df(r_s)}{dr_s} \right)^2, \tag{1}$$

Look when $[y^2 + z^2] = 0$ the negative energy density is zero and there is nothing to integrate.So although the motion of the Eulerian observer is along the x-axis only[5] $[y^2 + z^2] \neq 0$ implying in a non-null y or z and the factor $rs^2 = [(x - xs)^2 + y^2 + z^2]$ cannot be removed.This complicates the QI integral.

The negative energy density according to Natario is given by(see pg 5 in [2])[6]:

$$\rho = T_{\mu\nu} u^\mu u^\nu = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \cos^2\theta + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 \sin^2\theta \right] \tag{2}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu} u^\mu u^\nu = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 (\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 (\frac{y}{rs})^2 \right] \tag{3}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane :

$$\rho = T_{\mu\nu} u^\mu u^\nu = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \right] \tag{4}$$

The factor $rs^2 = [(x - xs)^2 + y^2 + z^2]$ can now be removed.This makes the integration method of the QI integral for the equatorial plane of the Natario warp drive so easy that can be integrated by sample trigonometric substitution.

[3]Note this fact:The motion is in the equatorial plane of the bubble and the x-axis is the axis where the warp drive motion occurs.

[4]$f(r_s)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c = G = 1$

[5]The equatorial plane of the warp bubble

[6]$n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

In this work we follow the line of reason of Ford and Pfenning and we reproduce their analysis of the QI for the Natario warp drive spacetime and we arrive at the conclusion that depending on the form of the Natario shape function the sampling time is higher than in its Alcubierre counterpart,much higher than 10^{-10} seconds and can last enough to sustain a warp bubble with a speed 200 times faster than light able to transport a ship in a trip to a star at 20 light years away.

We also compute the negative energy density requirements for the Natario warp drive and we find that the amount encountered by us is low and affordable making the Natario warp drive a viable form of interstellar travel

We terminate with a discussion of the problems of the Horizons that affects both Alcubierre and Natario warp drive spacetimes and infinite Doppler Blueshifts that affects the Alcubierre warp drive but not its Natario counterpart using the literature already available.

We adopted the International System of Units where $G = 6,67 \times 10^{-11} \frac{Newton \times meters^2}{kilograms^2}$ and $c = 3 \times 10^8$ $\frac{meters}{seconds}$ and not the Geometrized System of units in which $c = G = 1$.

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness $\Delta = 2$ meters moving with a speed 200 times faster than light implying in a $vs = 2 \times 10^2 \times 3 \times 10^8 = 6 \times 10^{10}$ and a $vs^2 = 3,6 \times 10^{21}$

Our warped region starts at 99 meters from the center of the bubble ($xs = 0$) and ends at 101 meters according to the limits of eq 4 pg 3 in [3].We call it the Natario warped region.

Our warp factor is defined as a dimensionless parameter with the value $WF = 2 \times 10^2$

This work is a companion work to our [4] and we advise the readers of this work to read [4] first in order to get acquaintance with our line of reason specially when we mention the warp factor.

Our value of π is $\pi = 3,1415926536$

2 Quantum Inequalities(QI) in the Natario Warp Drive Spacetime

We already know that the negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{5}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression(see eqs 21 and 23 pg 6 in [4]):

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{6}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{7}$$

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane :

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{8}$$

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

Ford and Pfenning in order to simplify the calculations of the QI integral created the piecewise shape function for the Alcubierre warp drive given by eqs 4 pg 3 in [3] and eq 5.4 pg 73 in [5].From these expressions we can define the Natario piecewise shape function as follows:

$$n_{p.c.}(r_s) = \begin{cases} 0 & r_s < R - \frac{\Delta}{2} \\ -\frac{1}{2\Delta}(r_s - R - \frac{\Delta}{2}) & R - \frac{\Delta}{2} < r_s < R + \frac{\Delta}{2} \\ \frac{1}{2} & r_s > R + \frac{\Delta}{2} \end{cases} \tag{9}$$

From above we can see that when $rs = 99,5$ meters for example the Natario piecewise shape function possesses a value of $0,375$ well within the range of $0 < n(rs) < \frac{1}{2}$ making it a valid shape function according to Natario requirements.

However in order to cope with the factor $\frac{c^2}{G}vs^2$ to low the negative energy density requirements we must have a piecewise shape function with very low values and also very low values in its derivatives. In our companion work pg 9 in [4] we introduced the warp factor as a dimensionless parameter that can ameliorate the negative energy density requirements.[7].A Natario piecewise shape function that can satisfy these requirements encompassing warp factors can be given by:

[7]in order to get a clear comprehension of the role performed by the warp factor read from pg 8 to pg 13 in [4]

$$n_{p.c.}(r_s) = \begin{cases} 0 & r_s < R - \frac{\Delta}{2} \\ -\frac{1}{2\Delta^{WF}}(r_s - R - \frac{\Delta}{2}) & R - \frac{\Delta}{2} < r_s < R + \frac{\Delta}{2} \\ \frac{1}{2} & r_s > R + \frac{\Delta}{2} \end{cases} \tag{10}$$

From above we can see that $\frac{1}{2\times\Delta^{WF}} = 3,11 \times 10^{-61}$ and when $rs = 99,5$ meters this Natario shape function possesses a value of $4,67 \times 10^{-61}$.This value is also in the range of $0 < n(rs) < \frac{1}{2}$ making it a valid Natario shape function.

The square of the first derivative becomes:

$$n'_{p.c.}(r_s)^2 = \frac{1}{4\Delta^{2WF}} \tag{11}$$

Note that the above expression gives a fixed value of $9,68 \times 10^{-122}$ along the entire Natario warped region. Inserting this in the Natario negative energy density we get:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \tag{12}$$

The factor $\frac{c^2}{G} = 1,35 \times 10^{27}$ and the factor $\frac{vs^2}{8\pi} = 1,43 \times 10^{20}$.Their product is $1,93 \times 10^{47}$.The factor $\frac{3}{4\Delta^{2WF}} = 2,90 \times 10^{-121}$.The final product for ρ the Natario negative energy density is $-5,61 \times 10^{-74}$.

Now a reader can understand why the warp factor is so important.The mass or the Earth is $5,9722 \times 10^{24}$ in kilograms[8].In tons this would mean $5,9722 \times 10^{21}$ tons.Our $vs^2 = 3,61 \times 10^{21}$ meaning to say that for 200 times light speed this factor vs^2 approaches in magnitude the value of the entire mass of the Earth in tons.

Without the warp factor we would never be able to low the energy density requirements rendering the warp drive as unphysical and impossible to be achieved.The first Natario piecewise shape function presented here would make the warp drive impossible but by choosing a different shape function(the second Natario piecewise shape function)we can alter the whole scenario.

According to Ford and Pfenning the QI restricts the time when we can observe a negative energy density.This time is known as the sampling time.As large the amount of negative energy density is then the sampling time becomes incredible small.(see pg 7 in [3]).The inverse is also true:as smaller the negative energy density is then the sampling time becomes higher.Fortunately for ourselves the negative energy density in the Natario warp drive using our Natario piecewise shape function is $\rho = -5,61 \times 10^{-74}$.A very low value for a negative energy density and this will allow ourselves to observe this negative energy density with a large sampling time enough to sustain a warp bubble for an interstellar travel to a star at 20 light years with a warp drive speed 200 times faster then light.

The Ford-Pfenning QI infinite integral is given by the following expression:(see eq 9 pg 6 in [3] and eq 5.9 pg 75 in [5]).

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle T_{\mu\nu}u^\mu u^\nu\rangle}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4}\ , \tag{13}$$

[8]From Wikipedia:The free Encyclopedia

Inserting the Natario negative energy density in the QI integral we get:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \tag{14}$$

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle\rho\rangle}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4}\ , \tag{15}$$

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{\langle -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right]\rangle}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4}\ , \tag{16}$$

$$\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}-\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right]\frac{1}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4}\ , \tag{17}$$

Removing the constant terms from the integral we get:

$$-\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right]\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4}\ , \tag{18}$$

Solving the -1 terms we get:

$$\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right]\frac{\tau_0}{\pi}\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \leq \frac{3}{32\pi^2\tau_0^4}\ , \tag{19}$$

Solving for the sampling time τ_0 and π we get:

$$\frac{c^2}{G}\frac{v_s^2}{8}\left[\frac{3}{4\Delta^{2WF}}\right]\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \leq \frac{3}{32\tau_0^5}\ , \tag{20}$$

Solving for 3 we get:

$$\frac{c^2}{G}\frac{v_s^2}{8}\left[\frac{1}{4\Delta^{2WF}}\right]\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \leq \frac{1}{32\tau_0^5}\ , \tag{21}$$

Solving for 8 we get:

$$\frac{c^2}{G}v_s^2\left[\frac{1}{4\Delta^{2WF}}\right]\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \leq \frac{1}{4\tau_0^5}\ , \tag{22}$$

Solving for 4 we get:

$$\frac{c^2}{G}v_s^2\left[\frac{1}{\Delta^{2WF}}\right]\int_{-\infty}^{\infty}\frac{1}{\tau^2+\tau_0^2}d\tau \leq \frac{1}{\tau_0^5}\ , \tag{23}$$

Above is the final expression for the QI integral in the Natario warp drive spacetime in the equatorial plane in a reference frame where $xs = 0$ the center of the warp bubble.Note that this integral will fall in a very familiar form of integrals that can be integrated by trigonometric substitution.This is the reason why it was so important to ourselves to remove the Eulerian term in the QI integral allowing ourselves to arrive at this sample form.

$$\int\frac{1}{\tau^2+\tau_0^2}d\tau = \frac{1}{\tau_0}\arctan\left(\frac{\tau}{\tau_0}\right)+C \tag{24}$$

$$\int \frac{1}{\tau^2 + \tau_0^2} d\tau \tag{25}$$

The integral above is the remaining term in the QI integral for the Natario warp drive that needs to be integrated.Applying the following elemental trigonometric substitution from Integral Calculus we get[9]:

$$\tau = \tau_0 tan(\theta) \tag{26}$$

$$d\tau = \tau_0 sec(\theta)^2 d\theta \tag{27}$$

$$\theta = \arctan(\frac{\tau}{\tau_0}) \tag{28}$$

$$\int \frac{1}{\tau^2 + \tau_0^2} d\tau = \int \frac{\tau_0 sec(\theta)^2 d\theta}{\tau_0^2 tan(\theta)^2 + \tau_0^2} = \int \frac{\tau_0 sec(\theta)^2 d\theta}{\tau_0^2 (1 + tan(\theta)^2)} = \int \frac{\tau_0 sec(\theta)^2 d\theta}{\tau_0^2 sec(\theta)^2} = \int \frac{d\theta}{\tau_0} \tag{29}$$

Using the following elemental trigonometric identity

$$sec(\theta)^2 = 1 + tan(\theta)^2 \tag{30}$$

Arriving at the final result

$$\int \frac{d\theta}{\tau_0} = \frac{\theta}{\tau_0} + C = \frac{1}{\tau_0} \arctan(\frac{\tau}{\tau_0}) + C \tag{31}$$

Solving the infinite limits of integration :

$$\int_{-\infty}^{\infty} \frac{1}{\tau^2 + \tau_0^2} d\tau = |_{-\infty}^{+\infty} \frac{1}{\tau_0} \arctan(\frac{\tau}{\tau_0}) + C \ , \tag{32}$$

$$\arctan(+\infty) = \frac{\pi}{2} \tag{33}$$

$$\arctan(-\infty) = -\frac{\pi}{2} \tag{34}$$

$$\int_{-\infty}^{\infty} \frac{1}{\tau^2 + \tau_0^2} d\tau = \frac{1}{\tau_0}(\frac{\pi}{2} - -\frac{\pi}{2}) = \frac{1}{\tau_0}(+\frac{2\pi}{2}) = \frac{1}{\tau_0}(\pi) = \frac{\pi}{\tau_0} \ , \tag{35}$$

Back to the Natario warp drive QI integral and inserting the result above:

$$\frac{c^2}{G} vs^2 \frac{1}{\Delta^{2WF}} \int_{-\infty}^{\infty} \frac{1}{\tau^2 + \tau_0^2} d\tau \leq \frac{1}{\tau_0^5} \ , \tag{36}$$

We get:

$$\frac{c^2}{G} vs^2 \frac{1}{\Delta^{2WF}} \frac{\pi}{\tau_0} \leq \frac{1}{\tau_0^5} \ , \tag{37}$$

[9]The integration method given here explicitly is destined to potential readers of E-print versions of this work with interest in the Natario warp drive or in generic warp drives but without acquaintance with or needing to review Integral Calculus in order to avoid the consultation of a textbook.Specially readers of Internet forums where the warp drive is the main theme of discussion

$$\frac{c^2}{G}vs^2\frac{1}{\Delta^{2WF}}\frac{\pi}{\tau_0} \leq \frac{1}{\tau_0^5} \ , \tag{38}$$

Solving for the sampling time τ_0 we get:

$$\frac{c^2}{G}vs^2\frac{1}{\Delta^{2WF}}\pi \leq \frac{1}{\tau_0^4} \ , \tag{39}$$

Rearranging the terms we get:

$$\tau_0^4\frac{c^2}{G}vs^2\frac{1}{\Delta^{2WF}}\pi \leq 1 \ , \tag{40}$$

The factor $\frac{c^2}{G}vs^2 = 4,86 \times 10^{48}$ and the factor $\frac{1}{\Delta^{2WF}}\pi = 1,22 \times 10^{-120}$.Their product is $5,91 \times 10^{-72}$ Hence we should expect for:

$$\tau_0^4 \times 5,91 \times 10^{-72} \leq 1 \ , \tag{41}$$

Giving the $4th$ power of the sampling time as

$$\tau_0^4 \leq 1,69 \times 10^{71} \ , \tag{42}$$

Finally we arrive at the sampling time τ_0 which means to say the time when according to the Ford-Pfenning QI we can observe a negative energy density for a Natario warp drive bubble that have a radius of 100 meters a thickness Δ of 2 meters and moves with a warp drive speed 200 times faster than light with a negative energy density of $\rho = -5,61 \times 10^{-74}$ and needs to be stable for a couple of months able to complete an interstellar travel to a star at 20 light years away.

This maximum sampling time in seconds τ_0 is:

$$\tau_0 \leq 6,41 \times 10^{17} \ , \tag{43}$$

This agrees with Ford and Pfenning:A small negative energy density can be observed during a large sampling time.A year of 365 days possesses $3,15 \times 10^7$ seconds.In years this would mean:

$$\tau_0 \leq 2,03 \times 10^{10} \ , \tag{44}$$

20 billion years.So the maximum time we can observe this Natario warp bubble is 20 billion years.More than enough for our interstellar travel.

3 Total Energy needed to maintain the Natario Warp Drive Spacetime

We must now calculate the total energy required to maintain our Natario warp bubble with 100 meters radius thickness Δ of 2 meters dimensionless warp factor WF of 200 moving with 200 times light speed.From the negative energy density equation:

$$\rho = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] == -5,61 \times 10^{-74} \tag{45}$$

We must integrate this negative energy density ρ over the volume to get the total energy in the Natario warped region as follows:

$$E = \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} \rho dV = \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] dV \tag{46}$$

Since all the terms in ρ are constants these can be placed outside the integral giving:

$$E = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} dV \tag{47}$$

$$\tag{48}$$

But remember that we are working in the equatorial plane of the Natario warp bubble using cartezian coordinates and $rs^2 = (x - xs)^2$ with $xs = 0$ being the center of the bubble and also the center of our coordinates frame.Then $rs = x$ and the integral above reduces to:

$$E = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} drs = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] |_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} rs + C \tag{49}$$

$$E = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] |_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} rs + C = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \left[\left[R + \frac{\Delta}{2}\right] - \left[R - \frac{\Delta}{2}\right]\right] \tag{50}$$

$$E = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \left[\left[R + \frac{\Delta}{2}\right] - \left[R - \frac{\Delta}{2}\right]\right] = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \Delta \tag{51}$$

And the final energy is:

$$E = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{4\Delta^{2WF}}\right] \Delta == -5,61 \times 10^{-74} \Delta = -1,12 \times 10^{-73} Joules \tag{52}$$

Dividing by c^2 we get the amount of negative mass:

$$M = -1,25 \times 10^{-90} Kilograms \tag{53}$$

By choosing the correct Natario piecewise shape function given below

$$n_{p.c.}(r_s) = \begin{cases} 0 & r_s < R - \frac{\Delta}{2} \\ -\frac{1}{2\Delta^{WF}}(r_s - R - \frac{\Delta}{2}) & R - \frac{\Delta}{2} < r_s < R + \frac{\Delta}{2} \\ \frac{1}{2} & r_s > R + \frac{\Delta}{2} \end{cases} \quad (54)$$

Which gives the values of $\frac{1}{2\times\Delta^{WF}} = 3,11\times10^{-61}$ and for $rs = 99,5$ meters gives the value of $4,67\times10^{-61}$ well within the range of $0 < n(rs) < \frac{1}{2}$ according to the Natario requirements for the Natario warp bubble (pg 5 in [2]) making it a valid Natario shape function we were able to overcome the Ford-Pfenning QI sampling time of 10^{-10} seconds for the Alcubierre warp drive and also the total energy of 10 times the mass of the Universe found by Ford and Pfenning also for the Alcubierre warp drive.

For the Natario warp drive we found a sampling time of 20 billion years and a total exotic mass of $-1,25 \times 10^{-90} Kilograms$.The rest-mass of the electron is $9,11 \times 10^{-31}$ Kilograms.So we are moving a Natario warp bubble with 100 meters of radius large enough to contain a spaceship the size of the Space Shuttle at 200 times light speed with a total negative mass being 10^{60} times lighter than the mass of the electron.

Many readers will find our proposal physically unrealistic however we did choose a warp factor WF of 200 to demonstrate that all the restrictions pointed towards the Alcubierre warp drive can mathematically be solved for the Natario warp drive because the Natario energy density differs radically from its Alcubierre counterpart and the Eulerian term were removed from the Natario QI integral.

Of course by adjusting the warp factor WF we can get more physically reasonable results that can be achieved or obtained making the Natario warp drive a valid proposal for interstellar travel.

4 Horizon and Infinite Doppler Blueshifts in both Alcubierre and Natario Warp Drive Spacetimes

According to pg 6 in [2] warp drives suffers from the pathology of the Horizons and according to pg 8 in [2] warp drive suffer from the pathology of the infinite Doppler Blueshifts that happens when a photon sent by an Eulerian observer to the front of the warp bubble reaches the Horizon.This would render the warp drive impossible to be physically feasible.

The Horizon occurs in both spacetimes.This means to say that the Eulerian observer cannot signal the front of the warp bubble wether in Alcubierre or Natario warp drive because the photon sent to signal will stop in the Horizon..The solution for the Horizon problem must be postponed until the arrival of a Quantum Gravity theory that encompasses both General Relativity and Non-Local Quantum Entanglements of Quantum Mechanics-

The infinite Doppler Blueshift happens in the Alcubierre warp drive but not in the Natario one.This means to say that Alcubierre warp drive is physically impossible to be achieved but the Natario warp drive is perfectly physically possible to be achieved.

Consider again the negative energy density distribution in the Alcubierre warp drive spacetime:

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{55}$$

And considering again the negative energy density in the Natario warp drive spacetime:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{56}$$

In pg 6 in [2] a warp drive with a x-axis only is considered.In this case for the Alcubierre warp drive $[y^2+z^2]=0$ and the negative energy density is zero but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{57}$$

The Alcubierre shape function $f(rs)$ is defined as being 1 inside the warp bubble and 0 outside the warp bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region according to eq 7 pg 4 in [1] or top of pg 4 in [2].

Expanding the quadratic term in eq 8 pg 4 in [1] and solving eq 8 for a null-like interval $ds^2 = 0$ we will have the following equation for the motion of the photon sent to the front[10]:

$$\frac{dx}{dt} = vsf(rs) - 1 \tag{58}$$

Inside the Alcubierre warp bubble $f(rs) = 1$ and $vsf(rs) = vs$.Outside the warp bubble $f(rs) = 0$ and $vsf(rs) = 0$.

[10] The coordinate frame for the Alcubierre warp drive as in [1] is the remote observer outside the ship

Somewhere inside the Alcubierre warped region when $f(rs)$ starts to decrease from 1 to 0 making the term $vsf(rs)$ decreases from vs to 0 and assuming a continuous behavior then in a given point $vsf(rs) = 1$ and $\frac{dx}{dt} = 0$.The photon stops,A Horizon is established.This due to the fact that there are no negative energy density in the front of the Alcubierre warp drive in the x-axis to deflect the photon.

Now taking the components of the Natario vector defined in the top of pg 5 in [2] and inserting these components in the first equation of pg 2 in [2] and solving for the same null-like interval $ds^2 = 0$ considering only radial motion we will get the following equation for the motion of the photon sent to the front[11]:

$$\frac{dx}{dt} = 2vsn(rs) - 1 \tag{59}$$

The Natario shape function $n(rs)$ is defined as being 0 inside the warp bubble and $\frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region according to pg 5 in [2].

Inside the Natario warp bubble $n(rs) = 0$ and $2vsn(rs) = 0$.Outside the warpb bubble $n(rs) = \frac{1}{2}$ and $2vsn(rs) = vs$. Somewhere inside the Natario warped region $n(rs)$ starts to increase from 0 to $\frac{1}{2}$ making the term $2vsn(rs)$ increase from 0 to vs and assuming a continuous behavior then in a given point we would have a $2vsn(rs) = 1$ and a $\frac{dx}{dt} = 0$ The photon would stops.A Horizon would be established.

However when the photon reaches the beginning of the Natario warped region it suffers a deflection by the negative energy density in front of the Natario warp drive because this negative energy is not null.So in the case of the Natario warp drive the photon never reaches the Horizon and the Natario warp drive never suffer from the pathology of the infinite Doppler Blueshift due to a different distribution of energy density when compared to its Alcubierre counterpart.

Adapted from the negative energy in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

[11]The coordinate frame for the Natario warp drive as in [2] is the ship frame observer in the center of the warp bubble $xs = 0$

5 Conclusion

In this work we demonstrated the major differences between Alcubierre and Natario warp drive spacetimes. Due to a different distribution of negative energy density the Natario warp drive can encompasses a QI sampling time that can last long enough to allow interstellar space travel with low energy densities and can bypass the physical problem of the infinite Doppler Blueshift and can be regarded as a valid candidate for interstellar space travel. In this work we mentioned many times our interstellar trip of 20 light years.It was inspired by the first planet discovered in the habitable zone of another star:the Gliese 581 at 20 light years away.Given the huge number of planets in the habitable zones of their parent stars example:Kepler-22 at 600 light years away or Kepler-47 at 4950 light years away these planets can only be accessible to our space exploration if faster than light space travel could ever be developed.

6 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[12]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[13][14]

7 Acknowledgements

We would like to express our most profound and sincere gratitude towards Professor Doctor Marcelo Ribeiro from Univeraidade Federal do Rio de Janeiro(UFRJ) Brazil for the opportunity he allowed to ourselves to compare the original manuscripts of Alcubierre,Ford-Pfenning and Natario published in the peer-review scientific journal Classical and Quantum Gravity available only to subscribers with their equivalents available to the general public inside the E-Print server known as http://arXiv.org.We were particularly interested in the Natario work.

8 Legacy

This work is dedicated to celebrate the $10th$ anniversary of the publication of the Natario Warp Drive Spacetime in the peer-review scientific journal Classical and Quantum Gravity.The final form was received on 28 January 2002.It was finally published on 6 March 2002

[12]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[13]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[14]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

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