The twin transformations of the light wave's the frequency and the wavelength in the 2-Dimension inertial system

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ABSTRACT

In the special relativity theory, in the 2-Dimension inertial coordinate system, study the twin transformations of the light wave's the frequency υ and the wavelength λ.

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I.Introduction

Treat the twin transformations of the light wave's the frequency υ and the wavelength λ.

The total energy E', the momentum p' in 2-Dimension inertial coordinate system $S'(t',x')$ and the total energy E, the momentum p in 2-Dimension inertial coordinate system $S(t,x)$ is

$$t=\gamma(t'+\frac{v_0}{c^2}x'),\ x=\gamma(x'+v_0t')\ (1),\quad \frac{dx}{dt}=c,\ \frac{dx'}{dt'}=c,\ \gamma=\frac{1}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}\qquad(2)$$

$$E=\frac{E'+v_0p'}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}},\ ,\ p=\frac{p'+\frac{v_0}{c^2}E'}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}\qquad(3)\quad E'=\frac{E-v_0p}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}},\ p'=\frac{p-\frac{v_0}{c^2}E}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}\ (4)$$

In this time, the frequency υ and the wavelength λ about the light wave is

$$E=mc^2=h\upsilon\ ,\quad E'=m'c^2=h\upsilon',\qquad(5)$$

$$\lambda=\frac{h}{p},\ \ \lambda'=\frac{h}{p'},\ \ p=mc\ ,\ \ p'=m'c,$$

$$\frac{1}{\lambda}=\frac{p}{h}\ ,\ \frac{1}{\lambda'}=\frac{p'}{h}\qquad(6)$$

$$\omega=2\pi\upsilon,\omega'=2\pi\upsilon',\ \ k=\frac{2\pi}{\lambda},k'=\frac{2\pi}{\lambda'}\qquad(7)$$

$$c=\frac{dx}{dt}=\frac{d\omega}{dk},c=\frac{dx'}{dt'}=\frac{d\omega'}{dk'}\quad(8)$$

$$\upsilon\lambda=\frac{E}{h}\cdot\frac{h}{p}=c\ ,\ \upsilon'\lambda'=\frac{E'}{h}\cdot\frac{h}{p'}=c\qquad(9)$$

II.Additional chapter

Therefore, if Eq(3) is inserted by Eq(5),Eq(6)

$$E=h\upsilon=\frac{h\upsilon'+v_0\frac{h}{\lambda'}}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}=\frac{h\upsilon'+hv_0(\frac{1}{c}\upsilon')}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}=\frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}h\upsilon'\ (10),$$ In this time, $\frac{1}{\lambda'}=\upsilon'\frac{1}{c}$

$$p=\frac{h}{\lambda}=\frac{p'+\frac{v_0}{c^2}E'}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}=\frac{\frac{h}{\lambda'}+\frac{v_0}{c^2}h\upsilon'}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}=\frac{\frac{h}{\lambda'}+\frac{v_0}{c^2}h(\frac{c}{\lambda'})}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}=\frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{v_0^{\ 2}}{c^2}}}\frac{h}{\lambda'}\ (11)$$

In this time, $\upsilon'=\frac{c}{\lambda'}$

$$\frac{h}{\lambda}=\frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\frac{h}{\lambda'}\rightarrow\lambda=\frac{\sqrt{1-\frac{{v_0}^2}{c^2}}}{1+\frac{v_0}{c}}\lambda' \quad (12)$$

Therefore, in Eq(10),in Eq(11),in Eq(12), the frequency υ 's and the wavelength λ 's transformation is

$$\upsilon=\frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\upsilon'=\frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\upsilon' \;,\; \lambda=\frac{\sqrt{1-\frac{{v_0}^2}{c^2}}}{1+\frac{v_0}{c}}\lambda'=\frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\lambda' \quad (13)$$

If Eq(4) is inserted by Eq(5),Eq(6)

$$E'=h\upsilon'=\frac{h\upsilon-v_0\frac{h}{\lambda}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}=\frac{h\upsilon-hv_0(\frac{1}{c}\upsilon)}{\sqrt{1-\frac{{v_0}^2}{c^2}}}=\frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}h\upsilon \quad (14),$$

In this time, $\frac{1}{\lambda}=\upsilon\frac{1}{c}$

$$\upsilon'=\frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\upsilon \quad (15)$$

$$p'=\frac{h}{\lambda'}=\frac{p-\frac{v_0}{c^2}E}{\sqrt{1-\frac{{v_0}^2}{c^2}}}=\frac{\frac{h}{\lambda}-\frac{v_0}{c^2}h\upsilon}{\sqrt{1-\frac{{v_0}^2}{c^2}}}=\frac{\frac{h}{\lambda}-\frac{v_0}{c^2}h(\frac{c}{\lambda})}{\sqrt{1-\frac{{v_0}^2}{c^2}}}=\frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\frac{h}{\lambda} \quad (16)$$

In this time, $\upsilon=\frac{c}{\lambda}$

$$\frac{1}{\lambda'}=\frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{{v_0}^2}{c^2}}}\frac{1}{\lambda} \quad (17) \quad \rightarrow \quad \lambda'=\frac{\sqrt{1-\frac{{v_0}^2}{c^2}}}{1-\frac{v_0}{c}}\lambda \quad (18)$$

Therefore, the frequency υ 's and the wavelength λ 's transformation is

$$\upsilon' = \frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{v_0^2}{c^2}}}\upsilon = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\upsilon \ , \quad \lambda' = \frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1-\frac{v_0}{c}}\lambda = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\lambda \quad (19)$$

Therefore, by Eq(13) and Eq(19) ,the frequency υ's and the wavelength λ's transformation is in the light wave

$$\upsilon = \frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{v_0^2}{c^2}}}\upsilon' = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\upsilon' \ , \quad \upsilon' = \frac{1-\frac{v_0}{c}}{\sqrt{1-\frac{v_0^2}{c^2}}}\upsilon = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\upsilon ,$$

$$\lambda = \frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c}}\lambda' = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\lambda' \ , \quad \lambda' = \frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1-\frac{v_0}{c}}\lambda = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\lambda$$

(20)

$$\upsilon = \frac{1+\frac{v_0}{c}}{\sqrt{1-\frac{v_0^2}{c^2}}}\upsilon' = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\upsilon' \ , \lambda = \frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c}}\lambda' = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\lambda' \quad (21), \quad \upsilon\lambda = \upsilon'\lambda' = c$$

In this time, Eq(21) is different from the relativistic Doppler effect of the light. Eq(21) is the equation of the frequency υ and the wavelength λ of the observer in 2-Dimension inertial coordinate system $S(t,x)$ that observes the light phenomenon about the total energy E',the momentum p' in the 2-Dimension inertial coordinate system $S'(t',x')$.But the relativistic Doppler effect is that the light from 2-Dimension inertial coordinate system $S'(t',x')$ approach to the observer in 2-Dimension inertial coordinate system $S(t,x)$. The equation of the frequency $\upsilon_{Doppler}$ and the wavelength $\lambda_{Doppler}$ of the relativistic Doppler effect of the light is the equation of the light phenomenon about the time t in 2-Dimension inertial coordinate system $S(t,x)$.The frequency υ about the classical Doppler effect defines the inverse of the oscillation period T ($t_2 - t_1 = T$).

The equation of the relativistic Doppler effect of the light by the frequency $\upsilon_{Doppler}$ and the wavelength $\lambda_{Doppler}$ in 2-Dimension inertial coordinate system $S(t,x)$ that leaves 2-Dimension inertial coordinate system $S'(t',x')$ is

$$x = ct = (ct' + v_0 t')/\sqrt{1-\frac{v_0^2}{c^2}} = (ct' + v_0 t')/\sqrt{1-\frac{v_0^2}{c^2}} = ct'(\sqrt{1+\frac{v_0}{c}}/\sqrt{1-\frac{v_0}{c}})$$

$$t = (t' + \frac{v_0}{c^2}x')/\sqrt{1-\frac{v_0^2}{c^2}} = (t' + \frac{v_0}{c^2}ct')/\sqrt{1-\frac{v_0^2}{c^2}} = t'(\sqrt{1+\frac{v_0}{c}}/\sqrt{1-\frac{v_0}{c}}),$$

$$x = ct, x' = ct',$$

$$T = t_2 - t_1 = (\sqrt{1+\frac{v_0}{c}}/\sqrt{1-\frac{v_0}{c}})(t'_2 - t'_1) = (\sqrt{1+\frac{v_0}{c}}/\sqrt{1-\frac{v_0}{c}})T', \ T' = t'_2 - t'_1$$

$$\frac{1}{T} = \upsilon_{Doppler} = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\frac{1}{T'} = \frac{\sqrt{1-\frac{v_0}{c}}}{\sqrt{1+\frac{v_0}{c}}}\upsilon' \ , \quad \upsilon' = \frac{1}{T'}$$

$$\frac{c}{\upsilon_{Doppler}} = \lambda_{Doppler} = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\frac{c}{\upsilon'} = \frac{\sqrt{1+\frac{v_0}{c}}}{\sqrt{1-\frac{v_0}{c}}}\lambda'$$

$$\upsilon_{Doppler}\lambda_{Doppler} = \upsilon'\lambda' = c$$

T is the light's oscillation period in 2-Dimension inertial coordinate system $S(t,x)$

T' is the light's oscillation period in 2-Dimension inertial coordinate system $S'(t',x')$

(22)

III.Conclusion

In this time, in Eq(21),in Eq(22)

$$\upsilon' = \frac{E'}{h} = \frac{c}{\lambda'} = \frac{p'c}{h} = \frac{1}{T'} \quad (23)$$

But if Eq(21) compare by Eq(22),

$$\upsilon = \frac{E}{h} = \frac{c}{\lambda} = \frac{pc}{h} \neq \frac{1}{T} = \upsilon_{Doppler} = \frac{c}{\lambda_{Doppler}} \quad (24)$$

Because, the transformation of the light wave's the frequency υ and the wavelength λ is twin, Eq(24) is right.

Reference

[1]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[2]W. Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg, 1966)
[3]P.Bergman, Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V
[4]D.Halliday &R.Resnick, Fundamentals of PHYSICS(John Wiley & Sons,Inc.,1986)
[5]K.Lee,The Relativistic Doppler effect of the matter wave and the light wave, Samulli(New physics),**61,**3(2011)