# The Maxwell Equations, the Lorentz Field and the Electromagnetic Nanofield with Regard to the Question of Relativity

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**Abstract**

We discuss the Electromagnetic Theory in some main respects and specifically with relation to the question of Relativity. Let us consider the Maxwell equations for the representation of the electromagnetic field and the Lorentz force for the description of the motion of a particle in a field of magnetic induction. The Lorentz force is also useful for describing the behavior of the electromagnetic field in presence of cut flux, that is the physical situation which happens with respect to interactive moving reference systems. We examine at last physics of the electromagnetic nanofield which is essential for the definition of the behaviour of single energy quanta that compose the energy radiation. Let us have to accept that physics and science have an insurmountable limit and that they can give an answer to all possible questions except the first and the last. Those who want to give an answer to these two questions have to leave physics and science with their method and to go into the the order of philosophy or of religion.

## 1. Introduction

Electromagnetism is the discipline that studies all the electric and magnetic phenomena and it got the highest point of synthesis in Maxwell's equations. Those equations nevertheless disregard the Lorentz force that really is indispensable for describing a few electromagnetic events in which the law of electromagnetic induction due to cut flux intervenes. This consideration can be related for example to the motion whether of a single particle or of a particle beam in a field of magnetic induction, but it is also useful in order to describe the relativistic behavior of Maxwell's equations with respect to Einsteinian inertial reference systems that are open and therefore interactive.
Electromagnetism is characterized by the propagation of both the electromagnetic field and electromagnetic waves. The light and energy radiations with greater frequencies than the light ( X rays, $\gamma$ rays, $\delta$ rays ) aren't really electromagnetic waves but rather beams composed of energy quanta.
From the physical viewpoint then the single quantum is an electromagnetic nanowave generated by an electromagnetic nanofield that can be described by a group of mathematical relations derived from Maxwell's equations.

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This paper was sent to the Physics Colloquium in Portland, Maine, 2012 August

## 2. The Maxwell equations

The historical group of Maxwell' s equations is defined in a vacuum by the following four equations, derived from the scientific works by J. C. Maxwell, “A Dynamical Theory of the Electromagnetic Field” (1865), “A Treatise on Electricity and Magnetism” (1873), and from the scientific work by M. Faraday “On the Physical Nature of Force Lines” (1852):

$$\text{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_o} \qquad (1)$$

$$\text{div}\,\mathbf{B} = 0 \qquad (2)$$

$$\text{rot}\,\mathbf{E} = -\frac{\delta\,\mathbf{B}}{\delta\,t} \qquad (3)$$

$$\text{rot}\,\mathbf{B} = \mu_o\,\mathbf{J} + \frac{1}{c^2}\frac{\delta\,\mathbf{E}}{\delta\,t} \qquad (4)$$

Almost all electric and magnetic phenomena can be described by the previous equations and it is surely a big advantage to have a synthesis of electromagnetism through a limited group of equations. This group of equations nevertheless raised soon a problem because it seemed it didn't respect the Principle of Relativity. In fact in the equation (4) the speed of light appears: in the first place it means electromagnetic waves (e.w.) travel at the same speed of light, secondly it seemed this equation didn' t respect the Galileo transformations for inertial reference systems. In fact it was supposed that the light and e.w. propagated with the c speed (≈300000 km/s) only with respect to a medium (called ether) which was the absolute reference system and with respect to the other inertial reference systems its speed was obtained by the theorem of vector composition of speeds.
The negative result of Michelson-Morley' s experiment convinced at first Lorentz to assign strange properties to the ether and to replace Galileo' s transformations with different transformations of space-time and later Einstein to prove and to accept the same Lorentz transformations even though he claimed the concept of ether wasn' t necessary. In Special Relativity Einstein then proved the length contraction and the time dilation, and replaced the theorem of vector composition of speeds with the theorem of addition of speeds through which he proved the speed of both light and electromagnetic waves was always the same with respect to all the inertial reference systems. It seemed to be the surmounting with regard to the contradiction of the fourth equation. As a matter of fact we proved[1][2][3] this isn' t true and new contradictions[2][3] are caused by both the Principle of Constancy of the Speed of Light and Lorentz' s Transformations that are the fundamentals of Special Relativity.

Therefore we want to consider now the question within the Theory of Reference Frames[2][3][4] . If the equations (1), (2), (3) and (4) are valid with respect to the reference system supposed at rest, then as per the Principle of Relativity, which claims physical laws are independent of the considered inertial reference system, with respect to any other inertial reference system the same equations become

$$\text{div}\,\mathbf{E'} = \frac{\rho'}{\varepsilon_o} \qquad (5)$$

$$\text{div}\,\mathbf{B'} = 0 \qquad (6)$$

$$\text{rot}\,\mathbf{E'} = -\frac{\delta\,\mathbf{B'}}{\delta\,t'} \qquad (7)$$

$$\text{rot}\,\mathbf{B'} = \mu_o\,\mathbf{J'} + \frac{1}{c'^2}\frac{\delta\,\mathbf{E'}}{\delta\,t'} \qquad (8)$$

where t‘=t is the inertial time and c‘=c±v is the relativistic speed of light.
In order to prove the complete equivalence of equations (5), (6), (7), (8) with equations (1), (2), (3), (4) we distinguished[2][3] between Galilean reference systems and Einsteinian reference systems. Galilean reference systems are closed, isolated and do not interact among them and with the universe. In that case, being always within the Theory of Reference Frames ρ‘=ρ, **E'=E, B'=B,** for the equivalence of the two groups of equations [(1),(2),(3),(4)] and [(5),(6),(7),(8)], it must be

$$\mathbf{J'} = \mathbf{J} + \varepsilon_o \frac{\delta\mathbf{E}}{\delta t}\left(1 - \left(\frac{c}{c \pm v}\right)^2\right) \qquad (9)$$

The term

$$\varepsilon_o \frac{\delta\mathbf{E}}{\delta t}\left(1 - \left(\frac{c}{c \pm v}\right)^2\right) \qquad (10)$$

represents the “relativistic current density“ and for v<<c it matchs[2]

$$\pm\, 2\,\varepsilon_o \frac{v}{c}\frac{\delta\mathbf{E}}{\delta t} \qquad (11)$$

Einsteinian reference systems are open, not isolated and interactive for which in that case the Lorentz force $\mathbf{F_L}=q\mathbf{u}\Lambda\mathbf{B}$ causes in the moving reference system an induced electric field for cut flux $\mathbf{E_L}=\mathbf{u}\Lambda\mathbf{B}$ (Lorentz’s field, Λ is the vector product) that is added to the electric field **E** for which

$$\mathbf{E_t} = \mathbf{E} + \mathbf{u}\Lambda\mathbf{B} \qquad (12)$$

### 3. The Lorentz force

The existence of the Lorentz field suggests to make some changes in Maxwell' s equations. We can observe that in the classic group of equations the second equation (div**B**=0) is indeterminate, and therefore little revealing, because it is always true for every value of **B** for which we can think to replace it just with the Lorentz field and so to obtain the following new group of Maxwell' s equations

$$\mathrm{div}\,\mathbf{E} = \frac{\rho}{\varepsilon_o} \tag{13}$$

$$\mathrm{rot}\,\mathbf{B} = \mu_o \mathbf{J} + \frac{1}{c^2}\frac{\delta \mathbf{E_t}}{\delta t} \tag{14}$$

$$\mathrm{rot}\,\mathbf{E_t} = -\frac{\delta \mathbf{B}}{\delta t} \tag{15}$$

$$\mathbf{E_t} = \mathbf{E} + \mathbf{u} \wedge \mathbf{B} \tag{16}$$

In the equation (16) the Lorentz field is considered a physical event and it represents the force which works on moving electric charges with speed **u** inside a field of magnetic induction **B**. The equation of continuity of electric charge

$$\mathrm{div}\,\mathbf{J} = -\frac{\delta \rho}{\delta t} \tag{17}$$

isn' t independent and can be derived from equations (13) and (14) because the equation (13) is valid also for not static electric fields when the $\rho$ electric charge density isn' t constant with respect to time. Therefore the group of equations (13), (14), (15) and (16) is a complete group in order to describe electromagnetism.
The Lorentz field can be considered also the cause of electromagnetic induction due to cut flux with respect to the S' moving reference system which has **v** speed with respect to the resting reference system S (fig.1).

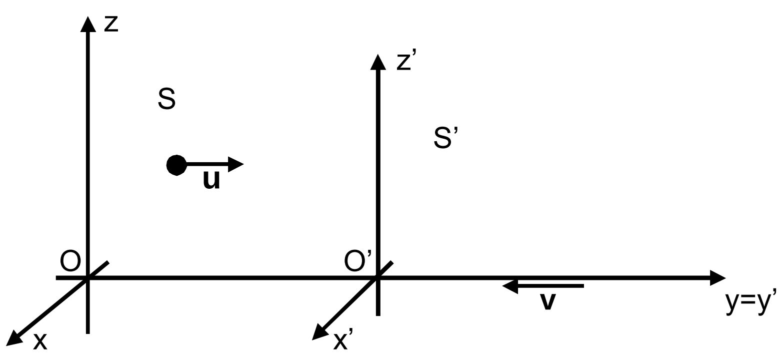


Fig.1 Representation of inertial reference systems. S is at rest and S' is moving with **v** speed.

As per the Principle of Relativity, with respect to the S' inertial reference system, the Maxwell equations are independent of the motion of the reference system and therefore the Maxwell equations become

$$\text{div}\,\mathbf{E'} = \frac{\rho'}{\varepsilon_o} \qquad (18)$$

$$\text{rot}\,\mathbf{B'} = \mu_o\,\mathbf{J'} + \frac{1}{c'^2}\frac{\delta\,\mathbf{E'}}{\delta\,t'} \qquad (19)$$

$$\text{rot}\,\mathbf{E'} = -\frac{\delta\,\mathbf{B'}}{\delta\,t'} \qquad (20)$$

$$\mathbf{E'} = \mathbf{E} + (\mathbf{u}+\mathbf{v})\Lambda\mathbf{B} \qquad (21)$$

Considering the physical event due only to the cut flux and supposing therefore that **u**=0 the equation (21) becomes

$$\mathbf{E'} = \mathbf{E} + \mathbf{v}\Lambda\mathbf{B} \qquad (22)$$

If the electromagnetic wave propagates along the y axis with respect to the S resting reference frame, in the Theory of Reference Frames it is

$$t' = t \qquad \text{and} \qquad c' = c + v \qquad (23)$$

From (22) we deduce

$$\text{div}\,\mathbf{E'} = \text{div}\,\mathbf{E} + \text{div}\,\mathbf{v}\Lambda\mathbf{B} \qquad (24)$$

Assuming that $\mathbf{v}\Lambda\mathbf{B}=\mathbf{E_L}$ (Lorentz' s electric field) we have

$$\text{div}\,\mathbf{E_L} = \frac{\rho_L}{\varepsilon_o} \qquad (25)$$

where $\rho_L$ is the Lorentz electric charge density.
From (24) we have

$$\text{div}\,\mathbf{E'} = \text{div}\,\mathbf{E} + \text{div}\,\mathbf{E_L}$$

$$\rho' = \rho + \rho_L \qquad (26)$$

From (20) and (22)

$$\text{rot } \mathbf{E'} = \text{rot } \mathbf{E} + \text{rot } \mathbf{v}\Lambda\mathbf{B} = -\frac{\delta \mathbf{B'}}{\delta t}$$

$$-\frac{\delta \mathbf{B}}{\delta t} + \text{rot } \mathbf{v}\Lambda\mathbf{B} = -\frac{\delta \mathbf{B'}}{\delta t}$$

$$\mathbf{B'} = \mathbf{B} - \int_0^t \text{rot } \mathbf{v}\Lambda\mathbf{B}\, \delta t \qquad (27)$$

We deduce from (27) div **B'** = div **B** = 0.
From (19)

$$\mu_o \mathbf{J'} = \text{rot } \mathbf{B'} - \frac{1}{c'^2} \frac{\delta \mathbf{E'}}{\delta t'}$$

$$\mu_o \mathbf{J'} = \text{rot } \mathbf{B} - \text{rot rot} \int_0^t \mathbf{v}\Lambda\mathbf{B}\, \delta t - \frac{1}{(c+v)^2} \left( \frac{\delta \mathbf{E}}{\delta t} + \frac{\delta}{\delta t} \mathbf{v}\Lambda\mathbf{B} \right)$$

$$\mu_o \mathbf{J'} = \mu_o \mathbf{J} + \frac{1}{c^2}\frac{\delta \mathbf{E}}{\delta t} - \frac{1}{(c+v)^2}\frac{\delta \mathbf{E}}{\delta t} - \frac{1}{(c+v)^2}\frac{\delta}{\delta t}\mathbf{v}\Lambda\mathbf{B} - \text{rot rot}\int_0^t \mathbf{v}\Lambda\mathbf{B}\, \delta t$$

$$\mathbf{J}' = \mathbf{J} + \varepsilon_o \frac{\delta \mathbf{E}}{\delta t} \left( 1 - \frac{1}{\left(1 + \frac{v}{c}\right)^2} \right) - \frac{\varepsilon_o}{\left(1 + \frac{v}{c}\right)^2} \frac{\delta}{\delta t} \mathbf{v}\Lambda\mathbf{B} - \frac{1}{\mu_o} \text{rot rot} \int_0^t \mathbf{v}\Lambda\mathbf{B}\, \delta t \qquad (28)$$

For v<<c we have

$$\mathbf{J}' = \mathbf{J} + 2\, \varepsilon_o \frac{v}{c} \frac{\delta \mathbf{E}}{\delta t} + \varepsilon_o \left( 1 + 2\frac{v}{c} \right) \frac{\delta}{\delta t} \mathbf{v}\Lambda\mathbf{B} - \frac{1}{\mu_o} \text{rot rot} \int_0^t \mathbf{v}\Lambda\mathbf{B}\, \delta t \qquad (29)$$

The (28) gives the mathematical expression about **J'** in order that Maxwell' s equations respect the Principle of Relativity and are invariant with respect to inertial reference systems. The (29) gives the same mathematical expression[2] about **J**' for v<<c. In the Einsteinian reference systems, in addition to the relativistic current density, the " induced current density " must be considered and it depends on the Lorentz electric field **v**Λ**B**. The (22), (23), (26), (27) and (28) represent the equations of transformations for Einsteinian inertial reference systems.

## 4. Equations of the electromagnetic nanofield

Maxwell' s equations describe electromagnetic phenomena which are characterized by a propagation of continuous electromagnetic waves. Generally it is supposed that also light propagates by electromagnetic waves but we proved[5] light and the other radiations with higher frequencies (X , $\gamma$, $\delta$ rays) are discrete beams made up of energy quanta. In the propagation of both light and radiations there isn' t propagation of a macroscopic electromagnetic field but only motion of energy quanta. We have also proved nevertheless that the single photon (for the light) and in the general the single energy quantum (for other radiations) are electromagnetic nanowaves produced by an electromagnetic nanofield that can be described by Maxwell' s equivalent equations[1] (photon equations).
The difference between waves and nanowaves is defined only by the wavelength: electromagnetic waves include long, medium, short waves and microwaves. Nanowaves include infrared rays, light photons, ultraviolet rays, X rays, $\gamma$ rays and $\delta$ rays. Microwaves utilize the wavelength band (100mm – 0,1mm), infrared radiation utilizes the wavelength band (1mm – 0,8$\mu$m). In the little band (1mm – 0,1mm) there is the full transition from electromagnetic waves to nanowaves, from microwaves to infrared radiation, from the continuous macroscopic structure of the electromagnetic energy to the discontinuous microscopic structure of radiations.
Electromagnetic nanowaves are energy quanta which travel with the velocity of light. Equations of the electromagnetic nanofield connected with the single energy quantum can be derived from Maxwell' s equations considering that the single energy quantum generally is produced by an accelerated electron[6] which for example starts from the O origin at time t=0 and moves with $a_z$ constant acceleration and velocity $u_z=a_zt$ along the z axis (fig.2).
We can write Maxwell' s (14) and (16) equations like this (let us make use of small letters for the electromagnetic nanofield)

$$\text{rot}\,\mathbf{b} = \frac{u_z}{c^2}\frac{\delta \mathbf{e}}{\delta z} + \mu_o \mathbf{j} \qquad (30)$$

$$\mathbf{e} = \mathbf{e_o} + \mathbf{u}\Lambda\mathbf{b} \qquad (31)$$

where $\mathbf{j}$ and $\mathbf{e_o}$ ( $\mathbf{e_o}=\rho_t\mathbf{j}$ , $\rho_t$ is the electric resistivity) are the current nanodensity and the electric nanofield connected with the moving electron. The equations (30) and (31) describe the beginning of the electromagnetic nanofield. It is well-known in that case $\mathbf{u}\Lambda\mathbf{b}=0$, because the $\mathbf{u}$ and $\mathbf{b}$ vectors are perpendicular, and $\mathbf{e}=\mathbf{e_o}$.
The following equations (32) and (33) describe instead the propagation of the electromagnetic nanowave connected with the single energy quantum with respect to the S resting reference system

$$\text{rot}\,\mathbf{e} = -\frac{\delta\,\mathbf{b}}{\delta\,t} \quad (32)$$

$$\text{rot}\,\mathbf{b} = \frac{1}{c^2}\frac{\delta\mathbf{e}}{\delta\,t} + \mu_o\mathbf{j} \quad (33)$$

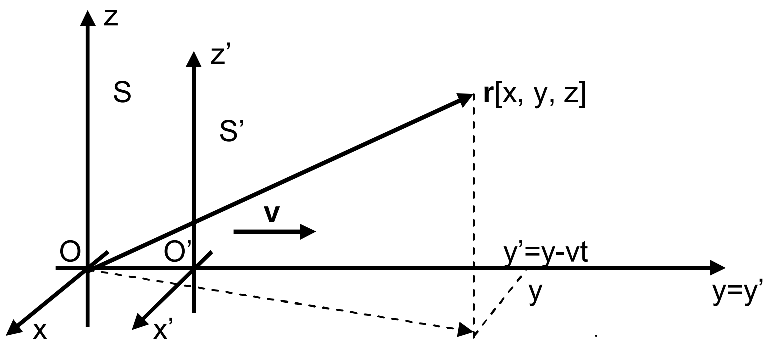


Fig.2 The accelerated electron, which generates the single energy quantum, moves with $u_z=a_zt$ speed along the z axis. At initial time t=t'=0 the two reference systems S and S' coincide. The single energy quantum moves along the y axis like the S' reference frame.

With respect to the resting reference system S we have in the general

$$\mathbf{r}(x, y, z)=x\mathbf{i}+y\mathbf{j}+z\mathbf{k} \quad (34)$$

Because for the sake of argument the electron motion happens along the z axis and supposing that the electron has a point structure, which doesn' t invalidate the correctness of the reasoning, we have x=y=0 and

$$\mathbf{r}=z\mathbf{k} \quad (35)$$

$$\mathbf{u}(u_x, u_y, u_z) = \frac{\delta\mathbf{r}}{\delta t} = \frac{\delta x}{\delta t}\mathbf{i} + \frac{\delta y}{\delta t}\mathbf{j} + \frac{\delta z}{\delta t}\mathbf{k}$$

$$\mathbf{u} = \mathbf{u_z} = \frac{dz}{dt}\,\mathbf{k} \quad (36)$$

$$a_z = \frac{du_z}{dt} \quad (37)$$

$$z = a_z\frac{t^2}{2} \quad (38)$$

where $dz/dt=u_z$ is the speed of the electron with respect to S.
With respect to the S' moving inertial reference system, the equations that describe the beginning of the electromagnetic nanofield become

$$\text{rot } \mathbf{b'} = \frac{u_{z'}}{c'^2} \frac{\delta \mathbf{e'}}{\delta z'} + \mu_o \mathbf{j'} \quad (39)$$

$$\mathbf{e'} = \mathbf{e} - \mathbf{v} \Lambda \mathbf{b} \quad (40)$$

and the equations that describe the propagation become

$$\text{rot } \mathbf{e'} = - \frac{\delta \mathbf{b'}}{\delta t'} \quad (41)$$

$$\text{rot } \mathbf{b'} = \frac{1}{c'^2} \frac{\delta \mathbf{e'}}{\delta t'} + \mu_o \mathbf{j'} \quad (42)$$

Because

$$\mathbf{r'}(x', y', z') = x'\mathbf{i} + y'\mathbf{j} + z'\mathbf{k}$$
$$x' = 0$$
$$\mathbf{r'} = y'\mathbf{j} + z'\mathbf{k} \quad (43)$$

we have

$$\mathbf{u'}(u_{x'}, u_{y'}, u_{z'}) = \frac{\delta \mathbf{r'}}{\delta t'} = \frac{\delta x'}{\delta t'} \mathbf{i} + \frac{\delta y'}{\delta t'} \mathbf{j} + \frac{\delta z'}{\delta t'} \mathbf{k}$$

$$\mathbf{u'} = \frac{\delta y'}{\delta t'} \mathbf{j} + \frac{\delta z'}{\delta t'} \mathbf{k} \quad (44)$$

where $\delta \mathbf{r'}/\delta t' = \mathbf{u'}$ is the speed of the electron with respect to S'.
According to results obtained in the Theory of Reference Frames[2][4][7] and to results here obtained relative to the electromagnetic nanofield, the equations of transformations from S to S' in order to describe both the beginning and the propagation of the electromagnetic nanofield are

$$t' = t \quad \text{(inertial time)}$$

$$c' = c - v \quad \text{(relativistic speed of light)}$$

$$x' = x = 0 \ ; \quad y' = y - vt \ ; \quad z' = z = \frac{a_z t^2}{2} \ ;$$

$$u_{x'} = u_x = 0 \ ; \quad u_{y'} = u_y - v \ ; \quad u_{z'} = u_z = a_z t \ ;$$

$$\mathbf{e'} = \mathbf{e} - \mathbf{v}\Lambda\mathbf{b} = \mathbf{e_o} + (\mathbf{u}-\mathbf{v})\Lambda\mathbf{b}$$

$$\mathbf{b'} = \mathbf{b} + \int_0^t \text{rot}\, \mathbf{v}\Lambda\mathbf{b}\, \delta t$$

$$\mathbf{j}' = \mathbf{j} + \varepsilon_o \frac{\delta \mathbf{e}}{\delta t} \left(1 - \frac{1}{\left(1 - \frac{v}{c}\right)^2}\right) - \frac{\varepsilon_o}{\left(1 - \frac{v}{c}\right)^2} \frac{\delta}{\delta t} \mathbf{v}\Lambda\mathbf{b} + \frac{1}{\mu_o} \text{rot rot} \int_0^t \mathbf{v}\Lambda\mathbf{b}\, \delta t \qquad (45)$$

For v<<c the (44) equation becomes

$$\mathbf{j}' = \mathbf{j} - 2\, \varepsilon_o \frac{v}{c} \frac{\delta \mathbf{e}}{\delta t} + \varepsilon_o \left(1 - 2\frac{v}{c}\right) \frac{\delta\, \mathbf{v}\Lambda\mathbf{b}}{\delta t} + \frac{1}{\mu_o} \text{rot rot} \int_0^t \mathbf{v}\Lambda\mathbf{b}\, \delta t \qquad (46)$$

The theory of the electromagnetic nanofield, that we have here developed, is fine whether for atomic energy quanta (infrared rays, light photons, ultraviolet rays, X rays) or for energy quanta due to particle transformations ( $\gamma$ rays and $\delta$ rays ). Consequently this theory is valid also for neutrinos that are electromagnetic nanowaves[7][8] belonging to the gamma and delta radiation.
We have also seen[4][7] that moving electrodynamic particles have a time relativistic effect because of the variation of electrodynamic mass with the speed but this effect concerns only the average lifes of particles with respect to the resting reference frame and not the kinematic inertial time that is the same for all inertial reference frames.

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