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Identification of Corneal Aberrations by using Computer Techniques

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Abstract. The objective was to study the relative contributions of optical aberrations of the cornea and determine the irregularities across its surface area. Corneal topographic imaging data is used and corneal aberrations are computed by using corneal height maps. Mathematical modeling of cornea surface is developed by using Zernike polynomials and they are compared with the patient corneas. Simulation techniques are utilized to determine the amount of corrections with respect to an ideal cornea in computer environment.

Keywords: cornea, corneal aberration, Zernike polynomial, optic, simulation of the cornea.

1 Introduction

The cornea is a transparent, clear and protective layer at the front part of the eye. It plays an important role in human vision. When light enters the eye, cornea's task is to focus the light on the retina at the back of the internal surface of the eye. It also shapes the incident light beam inside the eye to have an ideal refraction on the retina [1].

If the cornea surface has an irregular shape or discontinuous contour lines due to some eye disease or injury, the incident light is prevented from focusing properly on the retina and its beam shape is disturbed. These disfunctions in return affect a good vision with the eye. See Fig. 1.

(a) (b)

Fig. 1. a) Perfect cornea b) Diseased cornea

Purpose built spectacles or contact lenses prescribed by the doctors assist the light to be focused on the retina of an eye which has refraction problem due to focusing and light beam shape [2]. Similar corrections can also be achieved by performing cornea reshaping in refractive surgery. A series of corrective measures are evolved over the years which aim to surgically reshape cornea and to give the people with spectacles or contact lenses a chance of clearer sight and healthier life. These surgical operations can be analyzed in two groups: Pre-surgery and post-surgery. During the pre-surgery, a detailed eye examination is carried out in order to determine status of the eye disorder. Thickness of cornea is measured and a topographical image map of the cornea is extracted. See Fig. 2.

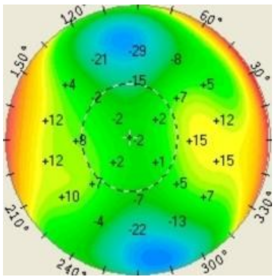

Fig. 2. Topographic image map of the cornea

Construction of the topographical map consists of two dimensional shape of the cornea surface. This map is generated by utilizing emitted light circles of different radius and the perceived reflected light with a computer-assisted device [3]. Detailed knowledge of corneal topography or corneal surface geometry is an important factor in planning of refractive surgery and post-surgery examination. This is because the surface topography determines most of the optical properties for a perfect eye vision.

In this study, the primary purpose is to develop a technique to determine the corneal surface deformation. Secondly, the cornea shape deformation will be corrected by using 3D modeling in computer environment. Zernike polynomials play an important role to quantize and model the cornea surface topography [4]. The corneal surface, as well as, the optical aberrations caused by the optical refraction at this surface, is often represented in terms of Zernike polynomials [5]. They are a sequence of polynomials that are orthogonal on the unit disk [6]. Corneal shape disorders can be corrected in two stages:

1. Calculation of the equivalent Zernike polynomials from the topographical map and construction of a computer model of the cornea surface,
2. Simulating the corneal surface correction by using the mathematical model created in stage (1).

2 Zernike Polynomials

In the study, Zernike polynomials are utilized to represent the shape of the cornea to be corrected. In mathematics, Zernike polynomials are a sequence of polynomials which are orthogonal on a unit circle [6][7]. They are defined by using polar coordinates (ρ, θ). Cartesian coordinates are not used in identification of Zernike polynomials. See Fig. 3.

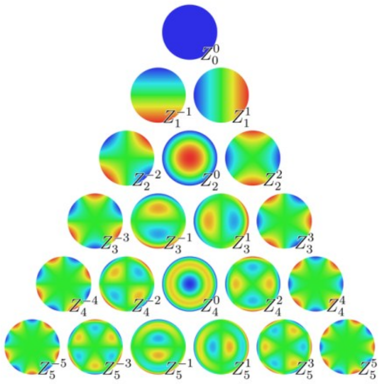

Fig. 3. Plots of Zernike values in unit disk

Zernike polynomials can be expressed as: [8,9]

$$\begin{aligned} Z_n^m(\rho,\theta) &= Z(n,\pm m) \\ &= \sqrt{(n+1)}R_n^0(\rho) \qquad \text{for } m=0\,,\ 0\le\rho\le 1\,,\ 0\le\theta\le 2\pi \\ &= \sqrt{2(n+1)}R_n^{|m|}(\rho)\cos(m\theta) \qquad \text{for } m>0\,,\ 0\le\rho\le 1\,,\ 0\le\theta\le 2\pi \\ &= -\sqrt{2(n+1)}R_n^{|m|}(\rho)\sin(m\theta) \qquad \text{for } m<0\,,\ 0\le\rho\le 1\,,\ 0\le\theta\le 2\pi \end{aligned} \tag{1}$$

where n is the highest order of the radial polynomial, m is the angular frequency of the sinusoidal component, $R_n^{|m|}(\rho)$ is the radial polynomial given by the following equation.

$$R_n^{|m|}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s(n-s)!}{s![0.5(n+|m|)-s]![0.5(n-|m|)-s]!}\rho^{n-2s} \tag{2}$$

For example, if n=3 and m=1, $R_n^{|m|}(\rho)$ is calculated as

$$\begin{aligned} &= \sum_{s=0}^{1} \frac{(-1)^s(3-s)!}{s!(2-s)!(1-s)!}\rho^{3-2s} \\ &= \frac{3!}{2!1!}\rho^3 + \frac{-1(2)!}{1!1!0!}\rho^1 \\ &= 3\rho^3 - 2\rho \end{aligned}$$

The Zernike polynomial $Z_3^1(\rho,\theta)$ is calculated by placing the calculated radial polynomial for values n=3 and m=1 obtained as

$$\begin{aligned} Z_n^m(\rho,\theta) &= Z_3^1(\rho,\theta) \\ &= \sqrt{2(n+1)}R_n^{|m|}(\rho)\cos(\theta) \\ &= \sqrt{8}(3\rho^3 - 2\rho)\cos(\theta) \end{aligned}$$

3 Corneal Aberration

Corneal aberration can be described as the difference of irregular, so called aberrated, cornea shape and an ideal cornea shape. Zernike polynomials can be used to identify the properties of a cornea shape [10][11]. Initially the shape of cornea surface is compared with the theoretical Zernike polynomial and the closest match is found. The difference between the closest matched theoretical Zernike polynomial and the shape of the corneal surface provides the numerical information about the areas of irregularities on the surface to be corrected.

4 Methodology

Corneal shape aberrations can be determined by following stages:

1. Loading patient corneal data
2. Calculating corresponding Zernike polynomial
3. Determining reference corneal data
4. Determining error margin between the reference and the patient corneal data.

Once the patient cornea data is obtained, in order to find the corresponding Zernike polynomial, all the Zernike polynomials are compared with the patient cornea data starting from the smallest one. Least squares method is used for this comparison. Least squares method (LSM) is a standard regression technique which is used to find the mathematical connection between two physical magnitudes as an equation as realistically as possible. In other words, this method is used to find a function curve which fits the measured data points as close as possible. For example;

$y_i(x)$: real values ,
$\hat{y}_i(x)$: estimated values (regression values),
q_i : difference between these values.

Therefore $Y = \sum_{i=0}^{n} q_i^2$ is the minimum equation for the best representation of the q_i distribution. q_i distribution is shown in Fig. 4.

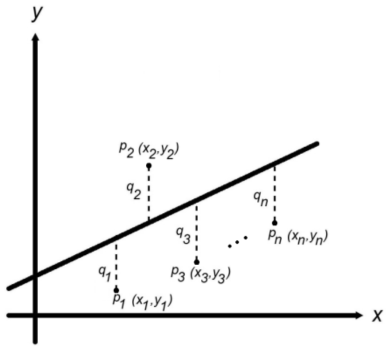

Fig. 4. minimum equation for q_i with LSM method

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There are infinite number of lines which pass between points (x_0, y_0), (x_1, y_1), (x_n, y_n). For every line, there will be distinct differences between $y_i(x)$ and $\hat{y}_i(x)$ values. Best representing line for the distribution will be the one with minimum sum of squares of the differences.

The closest Zernike polynomial for the corneal data is found with the algorithm shown below using LSM.

- *For each Zernike polynomial*
- *Calculate the related Zernike polynomial value z*
- *Obtain a visual representation of z to be able to compare with eye data*
- *Calculate the distance d between z and eye data by using LSM*
- *If d is less than minimum distance*
    - *Assign d to minimum distance*
    - *Assign the index of corresponding Zernike Polynomial to the minimum distance index*

Application of the algorithm and the GUI created is presented in Fig. 5.

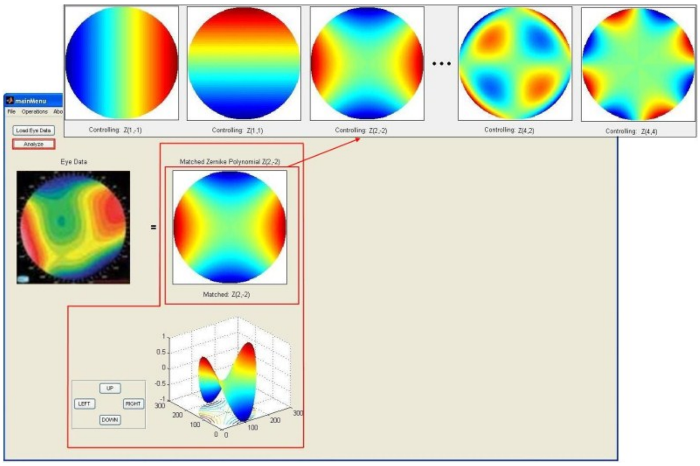

Fig. 5. GUI for finding the corresponding Zernike polynomial

Closest Zernike polynomial for the cornea data with vision impairment is acquired by using the GUI in Fig. 5. The next step would be to find the cornea surface shape after being corrected. To facilitate this, a reference cornea model is to be utilized.
Reference cornea model can be determined in two ways:

1. If we have a normal shaped cornea (perhaps the other eye of the patient), topographical map of this eye can be extracted and its parameters can be used during the correction procedures.
2. If we do not have a normal shaped cornea, radius of the eye pupil can be used to obtain the reference eye. The reference corneal equation obtained from Z_0^2 Zernike polynomial is given below.

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$$\text{Reference\_Eye} = -\frac{4\sqrt{3}Z_2^0}{(\text{pupilRadius})^2} \tag{3}$$

The difference between the reference eye and Zernike polynomial calculated from the cornea data gives the problematic areas of the eye. The matrix calculated by the simple formula laserfunction = zernikeEyeData - referenceEye is graphically shown in Fig. 6.

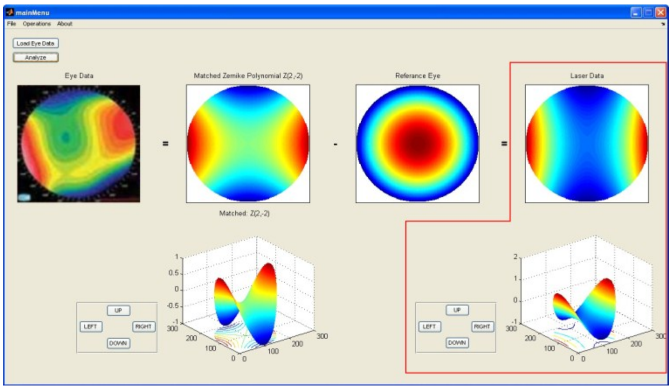

Fig. 6. GUI for representing problematic areas of the eye

4 Conclusion

Eye disorders that affect the quality of life need to be corrected. Vision defects that depend on corneal aberrations can be corrected after a series of operations based on changing the shape of cornea.

This work contains a method which defines the shape abnormality in order to remove visual disorders in computer environment.

Corneal shape is defined by using Zernike polynomials and eye disorder is calculated from the difference between this shape definition and ideal eye shape. Effects of aberrations on the cornea surface are mathematically expressed and shown in detail by using Zernike polynomials.

In this study, eye disorders are identified and corrected by computer software. The amount of disorder is quantized with respect to a healthy eye and displayed to be used in laser cutting machines. For example when the software program is run with data from an astigmatic eye, the results are shown in Fig. 7.

The corresponding Zernike polynomial is calculated as Z_2^{-2} where n is 2 and m is -2. See Fig. 8. Reference eye is calculated for a patient who has an eye pupil radius of 2.90 mm and shown in GUI. See Fig. 9. Finally, error margin is calculated and shown in GUI in both 2D and 3D.

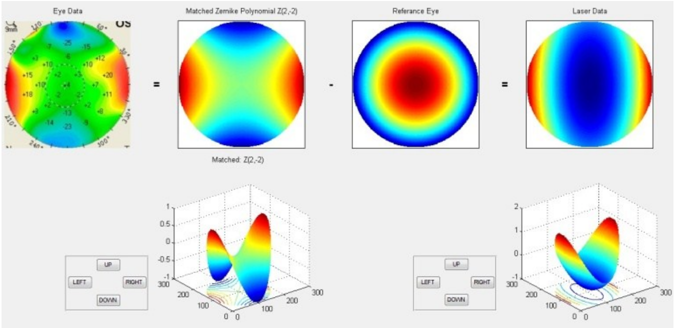

Fig. 7. GUI with an astigmatic eye

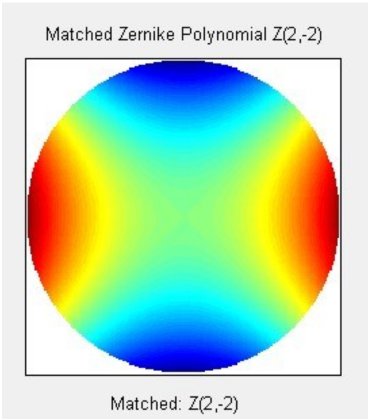

Fig. 8. Corresponding Zernike polynomial of an astigmatic eye

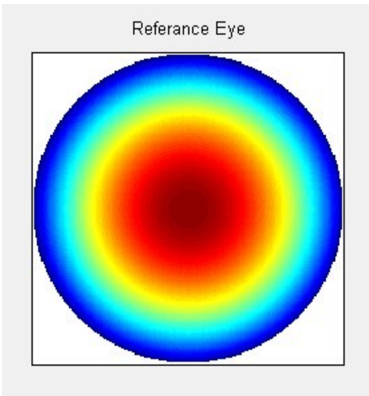

Fig. 9. Reference eye with pupil radius 2.90 mm

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The results of another example eye with different level of astigmatism with a pupil radius 3.43 mm, is presented in Fig. 10. The corresponding Zernike polynomial for the disorder is calculated as Z_2^{-2} .

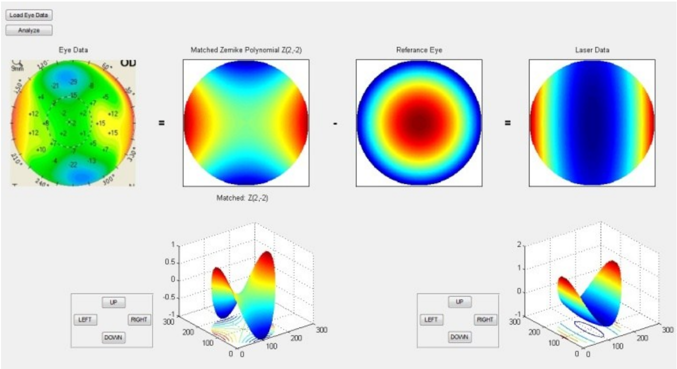

Fig. 10. GUI for an astigmatic eye with 3.43 mm pupil radius

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