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Capacity Analysis of Adaptive Transmission Techniques over TWDP Fading Channel

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Abstract: Expressions for the single-user capacity have been presented for different power and rate adaptive transmission techniques over Two Wave Diffuse Power [TWDP] fading channels. Different power and rate adaptation techniques available in the literature have been considered in these analyses. A study of the effect of fading parameters on the channel capacity of different techniques has been presented. Also the results have been verified against the known special case results.

Keywords: Channel capacity, TWDP fading channel, Power and rate adaptation techniques.

1. Introduction

In mobile radio channels the propagation of transmitted signal is characterized by various effects including fading and shadowing. In different physical scenario different distributions exist that describe the statistics of the mobile radio channels. For example Rayleigh, Rice, Nakagami-*m*, Weibull and Hoyt are the well known distributions for short term signal fluctuations. However, in some specific occasion, none of the above distributions is able to characterize the narrowband fading measurement. Even the Nakagami-*m* distribution does not seem to yield a good fitting to experimental data [1].

A new fading model has been introduced in [2] the two-wave with diffuse power (TWDP) which provide more flexibility to the fading model. TWDP fading model consist of two specular multipath components in the presence of diffusely propagating waves. This fading model can better represent the real-world frequency-selective fading data obtained from wireless sensor networks [3]. TWDP fading can also represent Rayleigh, Rician, one-wave fading model as special cases [2]. The TWDP fading is observed in a variety of propagation scenarios and may occur for typical narrow-band receiver operation, directional antennas and wide-band signals increase the likelihood of TWDP small-scale fading [2]. Although in practice TWDP fading model can better represent real-world fading scenario and useful to understand performance of wireless system, but so far only a few works have been published [5]–[8] in this fading model. In [5], bit error rate performance expressions for an uncoded binary phase-shift keying (BPSK) system is derived using alternate expression of Gaussian Q function [4] and for maximal ratio combining (MRC) system, performance of BPSK is presented in [7]. Average bit error rate (ABER) performance of Gray coded QAM signaling in TWDP environment is presented in [6] using the cumulative distribution function of TWDP fading and that for MRC system is derived in [8].

Capacity analysis of over fading channels can provide useful information for design and implementation of wireless communication systems and to improve spectrum efficiency and service quality. Works have been published on the channel capacity over various fading channels [9]- [17] and provide useful analytical support to the system design engineers. But, for TWDP fading capacity analysis is not available in the literature. This generates a motive to analysis the capacity for different adaptive transmission techniques over TWDP fading channels.

The rest of this paper is organized as follows. In Section 2, the channel has been elaborated. Different capacity formulas available in literature are discussed in Section 3. In Section 4, capacity of adaptive systems have been obtained over TWDP fading channels and in Section 5, we present numerical results. Finally, the paper is concluded in Section 6.

2. Channels

The channel has been assumed to be slow, frequency nonselective, with TWDP fading statistics. The complex low pass equivalent of the received signal over one symbol duration *Ts* can be expressed as,

$$r'(t) = re^{j\varphi}s(t) + n(t) \tag{1}$$

Where, $s(t)$ is the transmitted signal with energy E_b and $n(t)$ is the complex Gaussian noise having zero mean and two sided power spectral density $2N_0$. Random variable (RV) φ represents the phase and r is the TWDP distributed fading envelope having approximate PDF given by [2]

$$f_r(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2} - K\right)\sum_{i=1}^{L} a_i D\left(\frac{r}{\sigma}; K; \Delta\cos\frac{\pi(i-1)}{2L-1}\right) \tag{2}$$

Where, $$\mathrm{D}(\mathrm{x};\mathrm{K},\propto) = \frac{1}{2}\exp(\propto \mathrm{K})\mathrm{I}_0\left(\mathrm{x}\sqrt{2\mathrm{K}(1-\propto)}\right) + \frac{1}{2}\exp(-\propto \mathrm{K})\mathrm{I}_0\left(\mathrm{x}\sqrt{2\mathrm{K}(1+\propto)}\right)$$

and $\alpha = \Delta\cos\times\pi(\mathrm{i}-1)/2L-1$, $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order, K is the ratio of total specular power to diffused power, Δ indicates the relative strength of the two specular component, L is the order of the approximate TWDP PDF and $L \geq (1/2)K\Delta$ should be used so that (2) does not deviate significantly from the exact PDF. PDF of -envelop and -SNR $f_\gamma(\gamma)$ can be related as [4],

$$f_\gamma(\gamma) = \frac{E_b}{N_0} f_{r^2}\left(r^2\right) \tag{3}$$

2.1 PDF of Signal-to-Noise Ratio

From (2), by performing square transformation of random variable, $f_{r^2}\left(r^2\right)$ can be expressed as,

$$f_{r^2}(r^2) = \frac{1}{2\sigma^2} e^{-K-\frac{r}{2\sigma^2}} \sum_{i=1}^{L} a_i D\left(\frac{\sqrt{r}}{\sigma}; K; \alpha\right) \tag{4}$$

Using [2, 42] and [4] the factor $\frac{E_b}{N_0}$ can be expressed as $\frac{E_b}{N_0} = \frac{\bar{\gamma}}{2\sigma^2(K+1)}$ where, $\bar{\gamma}$ is the average SNR. From (4), PDF of SNR can be obtain by performing transformation of RV (corresponding to multiplying by a factor $\frac{E_b}{N_0}$) as,

$$f_\gamma(\gamma) = \frac{(K+1)}{2\bar{\gamma}} e^{(-K)} \sum_{i=1}^{L} a_i \left[I_0\left(\sqrt{\frac{(4K(K+1)(1-\alpha_i)\gamma)}{\bar{\gamma}}}\right) e^{\alpha_i K} e^{\frac{-(K+1)}{\bar{\gamma}}\gamma} + I_0\left(\sqrt{\frac{(4K(K+1)(1+\alpha_i)\gamma)}{\bar{\gamma}}}\right) e^{-\alpha_i K} e^{\frac{-(K+1)}{\bar{\gamma}}\gamma} \right] \tag{5}$$

3. Capacity Formulas

Channel capacity has been analyzed for various situations in literature of which power and rate adaptive techniques are frequently used. Analytical expressions for capacities based on different transmission techniques have been presented in [9] and [17]. In the analysis we use these formulas to obtain expressions of adaptive transmission technique over TWDP fading channels. For convenience, we reproduce these formulas below.

3.1 Optimal Power and Rate Adaptation at the Transmitter

For a system with a constraint on the average transmitting power, using optimal power and rate adaptation (OPRA) technique at the transmitter the channel capacity (bits/s) is given by [9],

$$C_{OPRA} = B \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) f_\gamma(\gamma) d(\gamma) \tag{6}$$

Where, B is the channel bandwidth, $f_\gamma(\gamma)$ is the PDF of the output SNR and g_0 is the optimal cutoff SNR, below which no transmission is allowed, has to satisfy the condition,

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) f_\gamma(\gamma) d(\gamma) = 1 \tag{7}$$

3.2 Constant Transmitting Power

When the transmitting power of the system is constant and optimal rate adaptation (ORA) technique is used at the transmitter, the channel capacity (bits/s) can be given by [9],

$$C_{ora} = B\int_0^\infty \log_2(1+\gamma) f_\gamma(\gamma) d(\gamma) \tag{8}$$

3.3 Channel Inversion with Fixed Rate

When the transmitter adapts its power to maintain constant received SNR so that inversion of the channel fading effects is possible, the system is said to be adopting channel inversion with fixed rate (CIFR) technique. The channel capacity (bits/s) for this case is given by [9],

$$C_{cifr} = B\log_2\left(1+\frac{1}{R_{cifr}}\right) \tag{9}$$

Where,

$$R_{cifr} = \int_0^\infty \frac{1}{\gamma} f_\gamma(\gamma) d(\gamma)$$

3.4 Truncated Channel Inversion with Fixed Rate

This is a modified version of CIFR. When the channel goes into deep fades, to maintain constant receiver SNR a large amount of power is required at transmitter. So, to overcome this problem truncated channel inversion with fixed rate (TIFR) method is employed. In this case, the channel inversion is done when the receiver SNR is above a threshold value γ_0. The capacity formula for TIFR can be given by [17],

$$C_{tifr} = B\log_2\left(1+\frac{1}{R_{tifr}}\right)\left(1-P_{OUT}(\gamma_0)\right) \tag{10}$$

Where, $R_{tifr} = \int_{\gamma_0}^\infty \left(\frac{1}{\gamma}\right) f_\gamma(\gamma) d(\gamma)$ and $P_{OUT}(\gamma_0) = \int_0^{\gamma_0} f_\gamma(\gamma) d(\gamma)$ is the probability of outage for a threshold value γ_0.

4. Capacity over TWDP Fading Channel

It can be observed from the expressions in (6)-(10) that an analysis of the capacity of a system requires an expression for the PDF of the system output SNR i.e. $f_\gamma(\gamma)$. For a channel whose envelop follow TWDP statistics, we have obtained the PDF of SNR in equation (5). Capacity of different power and rate adaptation techniques have been derives using the same PDF expression as given below.

4.1 Optimal power and rate adaptation at the transmitter

Putting (5) into (6), expressing the modified Bessel function in infinite series [18] form and arranging the integral, the capacity for OPRA scheme can be given as,

$$C_{OPRA} = B\log_2 e\frac{(K+1)e^{-K}}{2\bar{\gamma}}\sum_{i=1}^{L}a_i\left[\sum_{P_1=0}^{\infty}\frac{1}{(P_1!)^2}\left(\frac{K^2(K+1)}{\bar{\gamma}}\right)^{P_1}e^{\alpha_i K}(1-\alpha_i)^{P_1} J_{P_1+1}\left(\frac{\gamma_0(K+1)}{\bar{\gamma}}\right)+\sum_{P_2=0}^{\infty}\frac{1}{(P_2!)^2}\left(\frac{K^2(K+1)}{\bar{\gamma}}\right)^{P_2}e^{-\alpha_i K}(1+\alpha_i)^{P_2} J_{P_2+1}\left(\frac{\gamma_0(K+1)}{\bar{\gamma}}\right)\right] \quad (11)$$

Where, $J_n(\mu)=\int_1^{\infty}t^{n-1}\ln(t)e^{-\mu t}dt$ for $\mu>0$. For integer n, $J_n(\mu)=\frac{(n-1)!}{\mu^n}\sum_{k=0}^{n-1}\frac{\Gamma(k,\mu)}{k!}$ [16], Where, $\Gamma(a,x)=\int_x^{\infty}e^{-t}t^{a-1}dt$ is upper incomplete gamma function. Putting the value of $J_n(\mu)$ and after some simplification the expression of C_{OPRA} can be given as,

$$C_{OPRA} = B\log_2 e\frac{e^{-K}}{2}\sum_{i=1}^{L}a_i\left[\sum_{P_1=0}^{\infty}\frac{K^{P_1}}{P_1!}\sum_{t_1=0}^{P_1}\frac{\Gamma\left[(t_1),\left(\frac{K+1}{\bar{\gamma}}\right)\gamma_0\right]}{t_1!}e^{\alpha_i K}(1-\alpha_i)^{P_1}+\sum_{P_2=0}^{\infty}\frac{K^{P_2}}{P_2!}\sum_{t_2=0}^{P_2}\frac{\Gamma\left[(t_2),\left(\frac{K+1}{\bar{\gamma}}\right)\gamma_0\right]}{t_2!}e^{-\alpha_i K}(1+\alpha_i)^{P_2}\right] \quad (12)$$

As stated in (6) γ_0 should satisfy (7). For $K = 0$, which corresponds to the Rayleigh fading channel, it is easy to verify that (12) can be simplified to the results in [16, (16)]. Putting (5) into (7)the condition for OPRA scheme can be simplified to,

$$\frac{(K+1)}{2\bar{\gamma}}e^{(-K)}\sum_{i=1}^{L}a_i\left[\left(e^{\alpha_i K}\sum_{P_1=0}^{\infty}\frac{1}{(P_1!)^2}\left(K(1-\alpha_i)\right)^{P_1}\frac{1}{\frac{(K+1)\gamma_0}{\bar{\gamma}}}\Gamma\left[(P_1+1),\frac{(K+1)\gamma_0}{\bar{\gamma}}\right]+e^{-\alpha_i K}\sum_{P_2=0}^{\infty}\frac{1}{(P_2!)^2}\left(K(1+\alpha_i)\right)^{P_2}\frac{1}{\frac{(K+1)\gamma_0}{\bar{\gamma}}}\Gamma\left[(P_2+1),\frac{(K+1)\gamma_0}{\bar{\gamma}}\right]\right)-\left(e^{\alpha_i K}\sum_{P_1=0}^{\infty}\frac{1}{(P_1!)^2}\left(K(1-\alpha_i)\right)^{P_1}\Gamma\left[(P_1),\frac{(K+1)\gamma_0}{\bar{\gamma}}\right]+e^{-\alpha_i K}\sum_{P_2=0}^{\infty}\frac{1}{(P_2!)^2}\left(K(1+\alpha_i)\right)^{P_2}\Gamma\left[(P_2),\frac{(K+1)\gamma_0}{\bar{\gamma}}\right]\right)\right]-1=0 \quad (13)$$

Where, $x=\frac{(K+1)\gamma_0}{\gamma}$. Following an approach given in [16], it has been obtained that γ_0 always lies in the interval $[0, K+1]$.

4.2 Constant transmitting power

Putting (5) into (8) and writing the confluent hypergeometric function in infinite series [18], the capacity for constant transmitting power techniques can be obtained as,

$$C_{ORA} = B\log_2 e\frac{(K+1)e^{-K}}{2\bar{\gamma}}\sum_{i=1}^{L}a_i\left[\sum_{P_1=0}^{\infty}\frac{K^{P_1}}{P_1!P_1!}\left(\frac{K(K+1)}{\bar{\gamma}}\right)^{P_1}e^{\alpha_i K}(1-\alpha_i)^{P_1}I_{P_1+1}\left(\frac{(K+1)}{\bar{\gamma}}\right)\right.$$
$$\left.+\sum_{P_2=0}^{\infty}\frac{K^{P_2}}{P_2!P_2!}\left(\frac{K(K+1)}{\bar{\gamma}}\right)^{P_2}e^{-\alpha_i K}(1+\alpha_i)^{P_2}I_{P_2+1}\left(\frac{(K+1)}{\bar{\gamma}}\right)\right] \quad (14)$$

Where, $I_n(\mu)=\int_0^{\infty}t^{n-1}\ln(1+t)e^{-\mu t}dt$. For integer n, $I_n(\mu)$ can be given as,

$$I_n(\mu)=(n-1)!e^{\mu}\sum_{k=0}^{n-1}\frac{\Gamma(-n+k,\mu)}{\mu^k}$$

[16]. For Rayleigh fading case ($K = 0$) (14) can be simplified to,

$$C_{ORA} = B\log_2 e\times e^{\frac{1}{\bar{\gamma}}}\times E_1\left[\frac{1}{\bar{\gamma}}\right] \quad (15)$$

Which is matching with the result published in [16, (34)].

4.3 Channel inversion with fixed rate

The capacity for this scheme requires a solution to the integral in R_{cifr} in (9). It can be solved by putting (5) and then solving the resulting integral using [18, (3.351.2)]. The final expression after algebraic manipulation and simplification can be given as,

$$R_{cifr} = \frac{(K+1)e^{-K}}{2\bar{\gamma}}\sum_{i=1}^{L}a_i\left[\sum_{P_1=0}^{\infty}\frac{\left(K(1-\alpha_i)\right)^{P_1}(P_1-1)!}{(P_1!)^2}e^{\alpha_i k}+\sum_{P_2=0}^{\infty}\frac{\left(K(1+\alpha_i)\right)^{P_2}(P_2-1)!}{(P_2!)^2}e^{-\alpha_i k}\right] \quad (16)$$

Thus, an expression for the capacity of this scheme can be obtained by putting (16) into (9). The channel capacity of Rayleigh fading channel with total channel C_{cifr} inversion is zero [16, (Section V)]. With $K = 0$ in (16) same can be established.

4.4 Truncated channel inversion with fixed rate

The capacity for this scheme requires a solution to the integral in R_{tifr} and $P_{out}(\gamma_0)$ in (10). Using (5),R_{tifr} can be obtained by solving the resulting integral using [18, (3.381.3)]. The final expression after simplification can be given as,

$$R_{tifr} = \frac{(K+1)e^{-K}}{2\bar{\gamma}}\sum_{i=1}^{L}a_i\left[\sum_{P_1=0}^{\infty}\frac{\left(K(1-\alpha_i)\right)^{P_1}}{(P_1!)^2}e^{\alpha_i k}\Gamma\left[P_1,\left(\frac{K+1}{\bar{\gamma}}\right)\gamma_0\right]\right.$$
$$\left.+\sum_{P_2=0}^{\infty}\frac{\left(K(1+\alpha_i)\right)^{P_2}}{(P_2!)^2}e^{-\alpha_i k}\Gamma\left[P_2,\left(\frac{K+1}{\bar{\gamma}}\right)\gamma_0\right]\right] \quad (17)$$

Outage probability, as per definition, can be given as [4],

$$P_{OUT}(\gamma_0) = \int_0^{\gamma_0} f_\gamma(\gamma) d(\gamma) \tag{18}$$

Where, γ_0 is the threshold value of the SNR. Putting $f_\gamma(\gamma)$ from (5), the involved integral can be solved by expressing the hypergeometric function in infinite series and then applying [18, (3.381.1)]. The final expression of outage probability can be given as,

$$P_{OUT} = \frac{e^{-K}}{2} \sum_{i=1}^{L} a_i \left[e^{-\alpha_i k} \sum_{p_1=0}^{\infty} \frac{(\mathrm{K}(1-\alpha_i))^{\mathrm{P}_1}}{(\mathrm{P}_1!)^2} g\left(\mathrm{P}_1+1, \left(\frac{\mathrm{K}+1}{\bar{\gamma}}\right)\gamma_0 \right) \right. \tag{19}$$
$$\left. + e^{-\alpha_i k} \sum_{P_2=0}^{\infty} \frac{(\mathrm{K}(1+\alpha_i))^{\mathrm{P}_2}}{(\mathrm{P}_2!)^2} g\left(\mathrm{P}_2+1, \left(\frac{\mathrm{K}+1}{\bar{\gamma}}\right)\gamma_0 \right) \right]$$

where $g(.,.)$ is the incomplete gamma function. Thus, a final expression for the capacity of this scheme can be obtained by putting (17) and (19) into (10). For Rayleigh fading ($K = 0$) case, C_{tifr} can be shown as,

$$C_{tifr} = B \log_2 \left(1 + \frac{\bar{\gamma}}{E_1\left[\frac{\gamma_0}{\bar{\gamma}}\right]} \right) e^{-\frac{\gamma_0}{\bar{\gamma}}} \tag{20}$$

which is matching with [16, (48)].

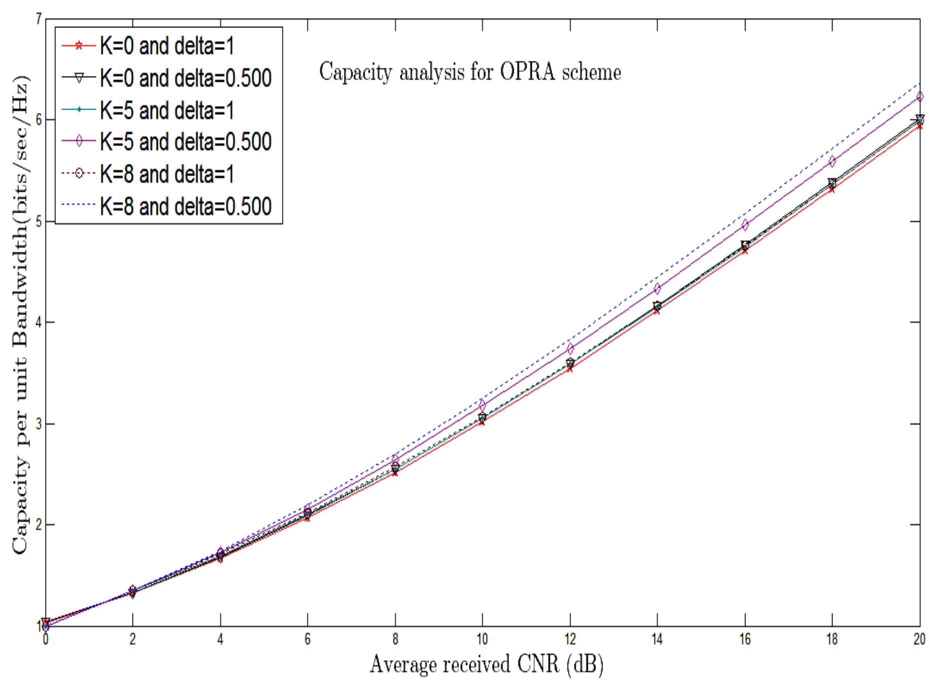

Fig 1: Capacity of OPRA Scheme

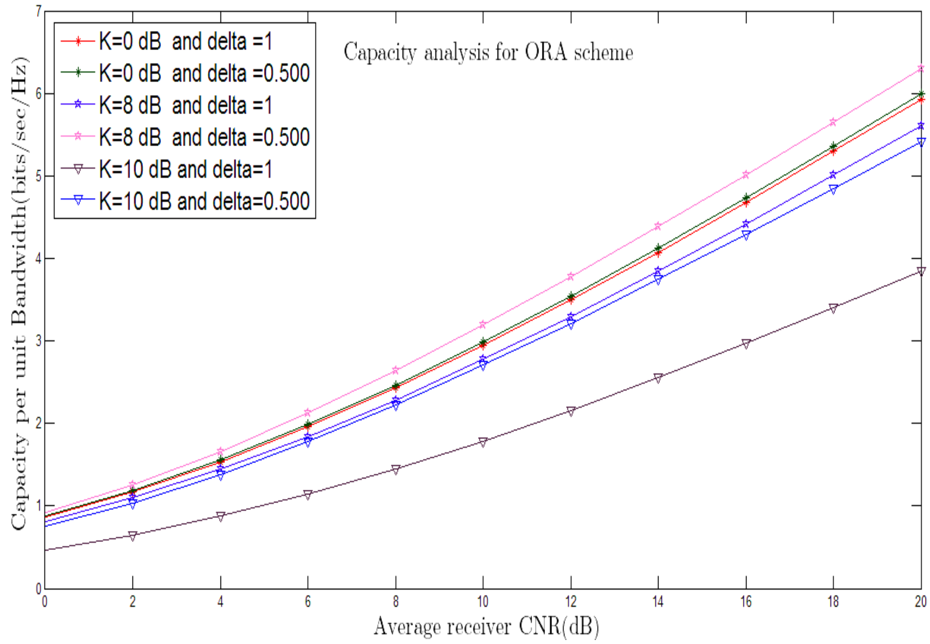

Fig 2: Capacity of ORA Scheme

6

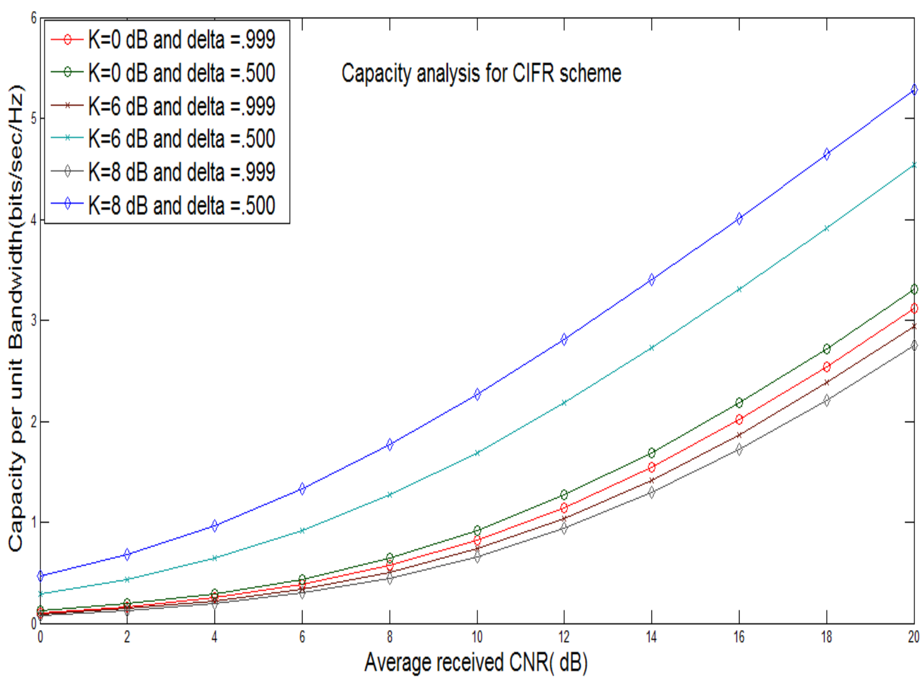

Fig 3: Capacity for CIFR Scheme

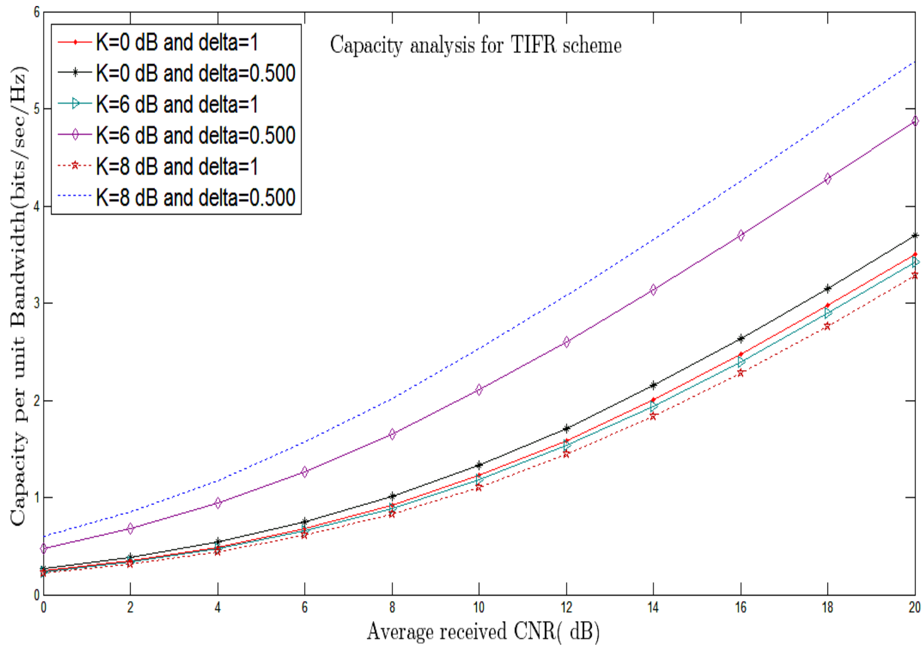

Fig 4: Capacity for TIFR Scheme

5. Numerical Results and Discussion

The obtained expressions for capacity over TWDP fading channel with different power and rate adaptation techniques have been numerically evaluated for different parameters of interest and plotted for the purpose of illustration. Capacity (per unit bandwidth) vs. average SNR ($\bar{\gamma}$) (in dB) of OPRA a scheme have been plotted in Fig.1.For OPRA scheme the value of γ_0 have been derived numerically from (13). Capacity of ORA scheme has been given in Fig. 2. Maximum capacity is observed in OPRA scheme as it is the optimal scheme. CIFR and TIFR schemes have been plotted in Figs. 3 and 4, respectively. For TIFR scheme the threshold value is considered as 1dB. Capacity of TIFR scheme has been more than CIFR scheme, as TIFR is a better scheme. From the observations it is clear that the capacity per unit bandwidth decrease with increase in the value of Δ.Here it is studied for different K values. As expected decrease in parameter Δ improve the fading condition, which is also reflected in the capacity plot.

6. Conclusion

In this paper, we analyze the capacity of a communication system over slow varying TWDP fading channels, for different known power and rate adaptation transmission techniques. Numerically evaluated results have been plotted for different parameter of interest and compared with the available special case results.

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