# Interband Alias-Free Subband Adaptive Filtering with Critical Sampling


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**Abstract** – Adaptive Filtering is an important concept in the field of signal processing and has numerous applications in fields such as speech processing and communications. An Adaptive filter is a filter that self-adjusts its transfer function according to an optimizing algorithm. Because of the complexity of the optimizing algorithms, most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal. An adaptive filter is often employed in an environment of unknown Statistics for various purposes such as system identification, inverse modeling for channel equalization, adaptive prediction, and interference canceling. Knowing nothing about the environment, the filter is initially set to an arbitrary condition and updated in a step-by-step manner toward an optimum filter setting. For updating, the least mean-square algorithm is often used for its simplicity and robust performance. However, the LMS algorithm exhibits slow convergence when used with an ill-conditioned input such as speech and requires a high computational cost, especially when the system to be identified has a long impulse response. To overcome the limitations of a conventional full band adaptive filtering, various sub band adaptive filtering (SAF) structures have been proposed. Properly designed, an SAF will converge faster at a lower computational cost than a full band structure. However, its design should consider the following two facts: the inter band aliasing introduced by the down sampling process degrades its performance, and the filter bank in the SAF introduces additional computational overhead and system delay. In this project, a critically sampled SAF structure that is almost Alias-free is proposed to reap all the benefits of using an SAF. Since the proposed SAF is performed using subbands that is almost alias-free, there is little inter band aliasing error at the output. In each sub band, the inter band aliasing is obtained using a bandwidth-increased linear-phase FIR analysis filter, whose pass band has almost-unit magnitude response in the subband interval, and is then subtracted from the sub band signal. This aliasing cancellation procedure, however, causes the spectral dips of the sub band signals. These spectral dips can be reduced by using a simple FIR filter. Simulations show that the proposed structure converges faster than both an equivalent full band structure at lower computational complexity and recently proposed SAF structures for a colored input. The analysis is done using MATLAB, a language of technical computing, widely used in Research, Engineering and scientific computations.



**Keywords** – Adaptive Filtering, Interband Aliasing, LMS Algorithm, Multirate Signal Processing, Subband Adaptive Filtering


## 1. Introduction

A critically sampled subband adaptive filtering (SAF) structure that is almost alias-free is proposed to reap all the benefits of using an SAF [1]. Since the proposed SAF is performed using subbands that are almost alias-free, there is little interband aliasing error at the output. The interband aliasing is removed from the subband signal by isolating the aliasing using a bandwidth-increased analysis filter. Computer simulations show that the proposed structure converges faster than both an equivalent full band structure at lower computational complexity and recently proposed SAF structures for a colored input. In the previous we have considered a variety of different problems including signal modeling, Wiener filtering, and spectrum estimation. In each case we made an important assumption that the signals that were being analyzed were stationary, since the signals that arise in almost every application will be non-stationary, the approaches and techniques that we have been considering thus far would not be appropriate. In order to motivate the approach that we consider the above case, we go for an adaptive filter approach.

Signal processing has become indispensable in communications and control algorithms, geo- physical and medical analysis, and entertainment and home appliances. This is mainly due to its immense growth in the last two and half decades and availability of fast and cost effective digital signal processors in the last few years. The field of signal processing was, initially, dominated by Fourier Transform (FT), as Fast Fourier Transform algorithm was available for its efficient implementation, and provides alternate representation of signals in the frequency domain, when the signals are stationary. In power spectral estimation problem, data length limits the frequency resolution of the techniques based on FT. Parametric methods employ system representation of the signals to enhance the frequency resolution. Signal models are increasingly being used in data compression, transmission and storage. Both FT and parametric methods are valid only for stationary signals in the strict sense. However, real world signals, like speech, provide information only because of their non-stationary nature and it has to be preserved while processing. In speech coding, the parameters of the under- lying model have to be time varying to capture the non stationary features of the speech signal. In applications like noise cancellation and equalization, the unknown physical system, which is possibly time varying or non-linear or both, has to be identified. Linear models are preferred to represent the physical systems because of convenience and mathematical tractability. Usually, in all these applications, input and output signals are used along with an appropriately chosen cost function, to find optimal parameters. Because of inbuilt time variations of the physical system, estimation of model parameters only once will not sufficient. Time dependent parameters, due to non stationary and time variations, can be estimated by block processing, over short segments of data.

The high computational complexity and excessive processing delays may prohibit block processing methods in real time applications. Sequential estimation of the parameters, are most suitable to the real time applications and are being done by adaptive algorithms. An adaptive filter is a computational device that attempts to model the relationship between two signals in real time in an iterative manner. Adaptive filters are often realized either as a set of program instructions running on an arithmetical processing device such as a microprocessor or DSP chip, or as a set of logic operations implemented in a field-programmable gate array (FPGA) or in a semi custom or custom VLSI integrated circuit. However, ignoring any errors introduced by numerical precision effects in these implementations, the fundamental operation of an adaptive filter can be characterized independently of the specific physical realization that it takes. For this reason, we shall focus on the mathematical forms of adaptive filters as opposed to their specific realizations in software or hardware.

### *1.1 Definition of Adaptive filter*

An adaptive filter is defined by four aspects

- The signals being processed by the filter.
- The structure that defines how the output signal of the filter is computed from its input Signal.
- The parameters within this structure that can be iteratively changed to alter the filters Input- output relationship.
- The adaptive algorithm that describes how the parameters are adjusted from one time to the next by choosing a particular adaptive filter structure, one specifies the number and type of parameters that can be adjusted [1]. The adaptive algorithm used to update the parameter values of the system can take on a myriad of forms and is often derived as a form of optimization procedure that minimizes an error criterion that is useful for the task at hand.

### *1.2 Algorithms Used*

Most of the algorithms are designed with squared error based cost function, achieving best tradeoff among various performance criteria. Computationally attractive Least Mean Square (LMS), and fast converging Recursive Least Squares (RLS) adaptive algorithms are most popular representative algorithms commonly used, in practice. A unified view of various existing algorithms has been provided. In applications like acoustic echo cancellation, where the number of coefficients to be adapted is very high, LMS algorithm implementation is computationally expensive. The Eigen spread of the input speech signal is very high, and the convergence rate is also poor. Other fast algorithms which provide better convergence rates are too costly to implement. The least-mean-square (LMS) algorithm is similar to the method of steepest-descent in that it adapts the weights by iteratively approaching the MSE minimum. Wiener and Hoff invented this technique in 1960 for use in training neural networks. The key is that instead of calculating the gradient at every time step, the LMS algorithm uses a rough approximation to the gradient [2].

The limitations of a conventional full band adaptive filtering (converge slowly at a high computational cost) overcome by using various subband adaptive filtering (SAF) structures. Properly designed, an SAF will converge faster at a lower computational cost than a full band structure.

However, its design should consider the following two facts: the inter band aliasing introduced by the down sampling process it degrades its performance, and the filter bank in the SAF introduces additional computational overhead and system delay. In this project, to fully exploit the benefits of using an SAF, an almost alias-free SAF structure with critical sampling is proposed. The inter band aliasing is removed from the sub band signal by isolating the aliasing using a bandwidth-increased analysis filter.

## 2. Problem Definition

The LMS algorithm exhibits slow convergence when used with an ill-conditioned input such as speech and requires a high computational cost, especially when the system to be identified has a long impulse response. The limitations of a conventional full band adaptive filtering (converge slowly at a high computational cost) overcome by using various subband adaptive filtering (SAF) structures. Properly designed, an SAF will converge faster at a lower computational cost than a full band structure.

However, its design should consider the following two facts:

- The interband aliasing introduced by the down sampling process it degrades its performance, and
- The filter bank in the SAF introduces additional computational overhead and system delay.

## 3. Proposed System Methodology

### *3.1 Scope of the Proposed System.*

An adaptive filter is often employed in an environment of unknown statistics for various purposes such as system identification, inverse modeling for channel equalization, adaptive prediction, and interference canceling. Knowing nothing about the environment, the filter is initially set to an arbitrary condition and updated in a step-by-step manner toward an optimum filter setting.

For updating, the least mean square (LMS) algorithm is often used for its simplicity and robust performance. However, the LMS algorithm exhibits slow convergence when used with an ill-conditioned input such as speech and requires a high computational cost, especially when the system to be identified has a long impulse response. One promising method that improves the performance and reduces the computational cost is sub band adaptive filtering (SAF), in which the input is decomposed into a number of sub band signals, and the adaptive filtering is performed on each sub band. It has the potential for a faster convergence and a lower computational complexity than a full band structure.

However, a sub band structure suffers from two deficiencies. First, the interband aliasing that is introduced by the down sampling process required in reducing the data rate is unavoidable and degrades the performance. Second, the filter bank introduces additional computation and system delay. For these reasons, various SAF structures were proposed. In this project, a critically sampled SAF structure that is almost alias-free is proposed to reap all the benefits of using an SAF.

Since the proposed SAF is performed using sub bands that is almost alias-free, there is little inter band aliasing error at the output. In each sub band, the inter band aliasing is obtained using a bandwidth-increased linear-phase FIR analysis filter, whose pass band has almost-unit magnitude response in the sub band interval, and is then subtracted from the sub band signal. This aliasing cancellation procedure, however, causes the spectral dips of the sub band signals. These spectral dips can be reduced by using a simple FIR filter.

*3.2 Adaptive Filter Problem*

An adaptive filter is a computational device that attempts to model the relationship between two signals in real time in an iterative manner. Adaptive filters are often realized either as a set of program instructions running on an arithmetical processing device such as a microprocessor or DSP chips. We shall focus on the mathematical forms of adaptive filters. An Adaptive filter is a filter that self-adjusts its transfer function according to an optimizing algorithm. Because of the complexity of the optimizing algorithms, most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal.

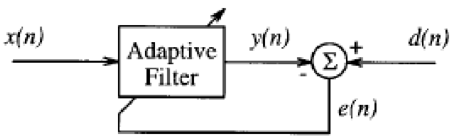


Figure1. The general adaptive filtering problem

Figure 1 shows a block diagram in which a sample from a digital input signal x(n) is fed into a device, called an adaptive filter*,* that computes a corresponding output signal sample y(n) at time n.For the moment, the structure of the adaptive filter is not important, except for the fact that it contains adjustable parameters whose values affect how y(n*)* is computed. The output signal is compared to a second signal d(n), called the desired response signal*,* by subtracting the two samples at time n. This difference signal, given by

$$e(n)=d(n) - y(n) \quad (1)$$

is known as the error signal*.* The error signal is fed into a procedure which alters or adapts the parameters of the filter from time n to time (n+1) in a well-defined manner. This process of adaptation is represented by the oblique arrow that pierces the adaptive filter block in the figure. As the time index *n* is incremented, it is hoped that the output of the adaptive filter becomes a better and better match to the desired response signal through this adaptation process, such that the magnitude of e(n) decreases over time. In the adaptive filtering task, adaptation refers to the method by which the parameters of the system are changed from time index n to time index (n+1). The number and types of parameters within this system depend on the computational structure chosen for the system. We now discuss different filter structures that have been proven useful for adaptive filtering tasks. We could define a general input-output relationship for the adaptive filter as

$$y(n)=f(W(n), y(n-1),y(n-2)..,y(n-N), x(n), x(n-1), .., x(n-M+1)) \quad (2)$$

$$y(n) = \sum_{i=0}^{L-1} w_i(n)x(n-i) = \mathbf{W}^T(n)\mathbf{X}(n), \quad (3)$$

$$\mathbf{X}(n) = [x(n)\, x(n-1) \cdots x(n-L+1)]^T \quad (4)$$

It denotes the input signal vector and T denotes vector transpose

*3.3 LMS Algorithm*

The cost function (error function) J(n) chosen for the steepest descent algorithm determines the coefficient solution obtained by the adaptive filter. If the MSE cost function is chosen, the resulting algorithm depends on the statistics of x(n) and d(n) because of the expectation operation that defines this cost function. Since we typically only have measurements of d(n) and of x(n) available to us, we substitute an alternative cost function that depends only on these measurements. One such cost function is the least-squares cost function given by

$$J_{LS}(n) = \sum_{k=0}^{n} \alpha(k)(d(k) - \mathbf{W}^T(n)\mathbf{X}(k))^2 \quad (5)$$

where α(*n)* is a suitable weighting sequence for the terms within the summation. This cost function, is complicated by the fact that it requires numerous computations to calculate its value as well as its derivatives with respect to each $w_i(n)$, although efficient recursive methods for its minimization can be developed. Alternatively, we can propose the simplified cost function $J_{LMS}(n)$ given by

$$J_{LMS}(n) = \frac{1}{2}e^2(n) \quad (6)$$

This cost function can be thought of as an instantaneous estimate of the MSE cost function, as $J_{LMS}(n) = E\{J_{LMS}(n)\}$. Although it might not appear to be useful, the resulting algorithm obtained when $J_{LMS}(n)$ is used for J(n) is extremely useful for practical applications. Taking derivatives of $J_{LMS}(n)$ with respect to the elements of W(n) and substituting the result , we obtain the LMS adaptive algorithm given by

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu(n)e(n)\mathbf{X}(n) \quad (7)$$

It also requires only multiplications and additions to implement. In fact, the number and type of operations needed for the LMS algorithm is nearly the same as that of the FIR filter structure with fixed coefficient values, which is one of the reasons for the algorithm's popularity.

The average behavior of the LMS algorithm is quite similar to that of the steepest descent algorithm that depends explicitly on the statistics of the input and desired response signals. The iterative nature of the LMS coefficient updates is a form of time-averaging that smooth's the errors in the instantaneous gradient calculations to obtain a more reasonable estimate of the true gradient. The task of the LMS algorithm is to find a set of filter coefficients **c** that minimizes the expected value of the quadratic error signal, i.e., to achieve the least mean squared error.The squared error and its expected value is the dependence of all variables on time n

$$e^2 = \left(d - c^H x\right)^2 = d^2 - 2d\, c^H x + c^H x x^H c,$$

$$\begin{aligned} E(e^2) &= E(d^2) - E(2d\, c^H x) + E(c^H x x^H c) \\ &= E(d^2) - c^H 2 E(dx) + c^H E(x x^H) c \end{aligned} \quad (8)$$

*3.4 Summary of the LMS algorithm*

*1) Filter operation:*

$$y[n] = c^H[n]x[n] \quad (9)$$

*2) Error calculation:*

$$e[n] = d[n] - y[n], \quad (10)$$

*where d[n] is the desired output*

*3) Coefficient adaptation:*

$$c[n + 1] = c[n] + \mu\, e^*[n]\, x[n] \quad (11)$$

## 4. Subband Adaptive Filtering

To overcome the limitations of a conventional full band adaptive filtering, various sub band adaptive filtering (SAF), structures have been proposed. Properly designed, an SAF will converge faster at a lower computational cost than a full band structure.

Subband adaptive filtering was thought as an effective method to reduce the complexity of the adaptive algorithms. Subband approach was intuitively found suitable to introduce parallelism in realizing filtering and adaptation with minimum overhead. Significant reduction in complexity was envisaged by using decimated versions of sub band signals [4]. Insightful considerations predicted sub band signals to have more flat spectrum than the full band signals from which they were derived. LMS type adaptive algorithms were known to converge faster when the input signal autocorrelation matrix has less Eigen value spread.

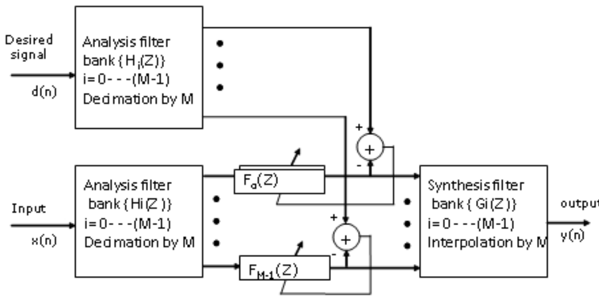


Figure2. Block diagram of subband adaptive filtering algorithm

Early efforts to reduce the processing requirement of an adaptive algorithm in acoustic echo cancellation yield encouraging results. The complexity of the filtering and the adaptation was reduced by a factor of M, when an M channel filter bank is used to derive the sub band signals and to recombine them. The initial convergence rate in every band was reported to be more in comparison with the full band convergence rate. But very soon, some problems associated with the sub band approach came to the fore. The maladjustment in the steady state performance used to be high. There was a delay due to the insertion of the perfect reconstruction filter bank in the signal path [5].

It was found that the non ideal analysis filter bank created aliasing and the aliased signals were responsible for the increased maladjustment. Many attempts were initialized simultaneously to overcome these difficulties. The sub band adaptive filtering literature can broadly be divided into algorithms reducing complexity, improving convergence rate and delay less algorithms. The full band performance with normalized LMS adaptation is taken as reference for convergence improvement and complexity comparisons. Whenever, the maladjustment is reduced, either additional adaptive filters are introduced or more number of taps/adaptive filter is used at an increased sampling rate, reducing the margin of complexity. Complexity reduction is normally found to go along with higher maladjustments or decreased convergence rate or both. Algorithms that enhance the convergence rate are available, but at a cost comparable to that of the full band LMS algorithm. Delays less SAF algorithms generally do not reduce the filtering complexity but, the adaptation will be done in the sub band domain. Algorithms which combine two or more of these features are always sought and is an active area of research. A few algorithms achieve superior performance in terms of convergence rate and complexity requirements. But, till now no single structure which could provide both complexity reduction and convergence improvement over the full band solution using LMS algorithm, without introducing processing delay, has been proposed. The requirements of an ideal subband adaptive filtering algorithm have been summarized, in the following, as the statement of the subband adaptive filtering problem.

*4.1 Multirate signal processing*

Multirate Signal Processing techniques are used in designing phase shifts, in interfacing digital systems with different sampling rates, in implementation of narrowband low pass filters. Filter Banks (FB) with Perfect Reconstruction (PR) property can be realized using the multirate fundamentals. These FBs are extensively used in the subband coding of speech signals [6], [7].

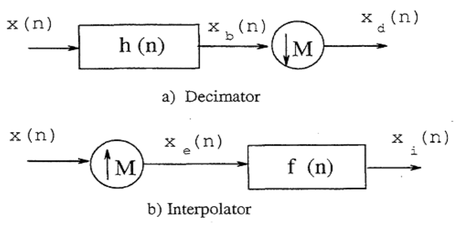


Figure3. Multirate building blocks

*1) Down sampling and Decimation*

The sampling rate of band limited signal $x_b(n)$ can be reduced by decimation operation [12]

$$\begin{aligned} x_d(n) &= (x_b(n)) \downarrow M \\ &= x_b(Mn), M \text{ being an integer} \end{aligned} \quad (9)$$

This is represented in z-domain as

$$x_d(z)=\frac{1}{M}\sum_{k=0}^{M-1}X_b\left(z^{1/M}W_M{}^{k}\right), \tag{10}$$

With $W_M = e^{-2\pi/M}$. The decimated version of a signal x(n) using band limiting filter h(n), with decimation factor M, is given by

$$x_d(n)=\sum_k x(Mn-k)h(k) \tag{11}$$

*2) Up sampling and Interpolation*

The over sampling operation, called interpolation is carried out by a cascade of an expander

$$\begin{aligned} x_e(n) &= (x(n))\uparrow M \\ &= x(n/M)\ \text{if}\ n = M_p, p = 0,1,\ldots \\ &\quad 0\ \text{otherwise} \end{aligned} \tag{12}$$

and a suitable interpolating filter. Expansion in the z-domain is represented as

$$X_e(z)=X\left(z^M\right) \tag{13}$$

The interpolation of a signal x(n), with an expansion factor is given by M and interpolating filter f(n) is given by

$$x_i(n)=\sum_k x(n-Mk)f(k) \tag{14}$$

*4.2 Digital Filter Banks*

A digital filter bank is typically a set of filters operating in parallel. It can take the form of an analysis filter bank where in the filters are driven by a single input or as a synthesis filter bank in which a set of input signals are filtered through different filters and their outputs are combined to generate a single output [12]. The output of the analysis filter bank is usually sub sampled and the input of the synthesis filters is expanded by suitable factors [7]. The output of a filter $h_k(n)$, of an AFB, sub sampled by a factor of D, is given in time domain by

$$x_{kd}(n)=\sum_l h_k(l)x(Dn-l) \tag{15}$$

and is represented in z-domain as

$$X_{kd}(z)=\frac{1}{D}\sum_{t=0}^{D-1}X\left(z^{1/D}W^{t}\right)H_k\left(z^{1/D}W^{t}\right) \tag{16}$$

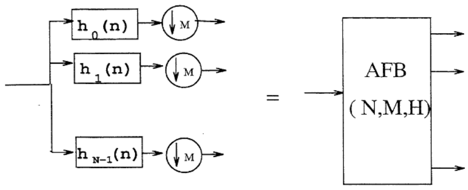


Figure4. (a) Analysis Filter Bank

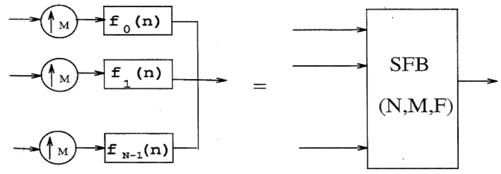


Figure4. (b) Synthesis Filter Bank

*4.3 Analysis Filter bank Design*

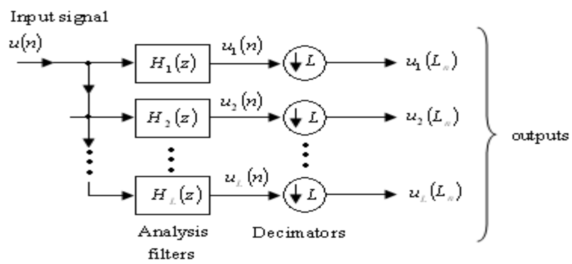


Figure5. Analysis section

The practical virtue of the analysis section of the multirate digital filter in Figure 5 is that it permits the processing of each decimated signal $u_{k.D}(n)$ in such a way that the special properties of the $k^{th}$ decimated subband signal. The signals that result from this processing are then applied to the synthesis section of the multirate digital filter for further processing.

*4.4 Synthesis Filter bank Design*

The synthesis section consists of two functional blocks of its own, as illustrated in Figure 6. The bank of expanders, which up-sample their respective inputs. The *k*th L-fold expander takes the input signal $v_k(n)$ to produce an output signal.

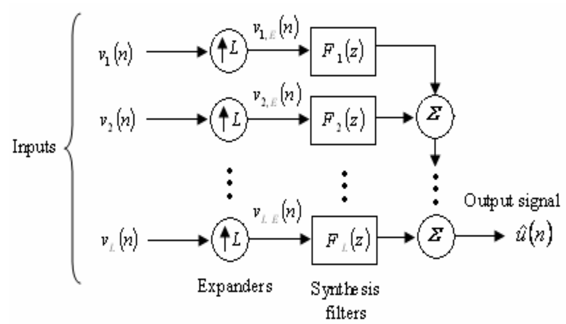


Figure6. Synthesis section

$$v_{k,E}(n)=\begin{cases} v_k(n/L) & \text{if } n \text{ is an integer multiple of } L \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

In Figure 6, the *L*-fold expanders are represented by upward arrows, followed by the expansion factor *L*. Each expander is essential to performing the process of interpolation; however, a filter is needed to convert the zero-valued samples of the expander into interpolated samples and thereby completed the interpolation. To explain this need, we recognize, from the time-frequency duality, which is an inherent property of Fourier transformation, that the spectrum

$V_{k.E}\left(e^{j\omega}\right)$ of the $k^{th}$ expander output is an $L$-fold compressed version of the spectrum $V_k\left(e^{j\omega}\right)$ of the expander input [8]. The synthesis filter bank, which consists of the parallel connection of a set of $L$ digital filters with a common output. The transfer functions of the synthesis filters are denoted by $F_1(z), F_2(z) \ldots F_L(z)$, and the resulting output of the synthesis section is denoted by $\hat{u}(n)$. The output signal $\hat{u}(n)$ differs from the input signal $u(n)$ due to

- The external processing performed on the decimated signals in the analysis section
- Aliasing errors.

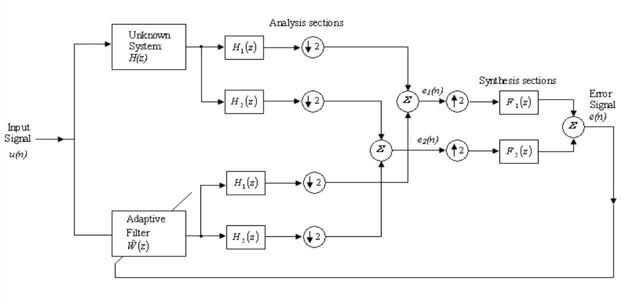


Figure7. Subband adaptive filters for two subbands

## 5. Alias-Free Subband Adaptive Filtering with Critical Sampling

### *5.1 Critical sampling*

Critical Sampling means M = K, where M = decimation factor, K = number of subbands as shown in figure 8

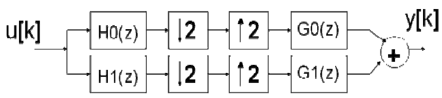


Figure8. Block diagram of critical sampling

### *5.2 SAF with Critical Sampling*

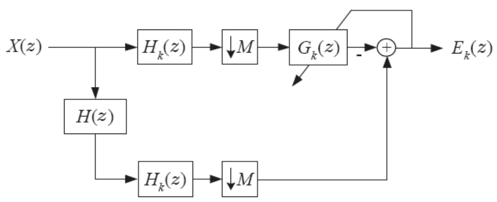


Figure9. Block diagram of the Adaptive filtering in the k th sub band

$X(z), H(z), H_k(z), G_k(z)$ and $E_k(z)$ Represent the z-transforms of input, unknown system, the kth analysis filter, the $k^{th}$ adaptive filter, and the $k^{th}$ error signal For k = 0, 1… M-1, respectively.

In SAF, signals are decomposed into a number of subband signals using an analysis filter bank, and the adaptive filtering is performed on each subband [9], [10]. The result in each sub band is combined into an output using a synthesis filter bank. For a critically sampled SAF with M sub bands, the kth sub band error $E_k(z)$, shown in Figure 9, is given as

$$E_k(z) = \frac{1}{M}\sum_{i=0}^{M-1} H_k\left(z^{1/M}W_m^i\right) H\left(z^{1/M}W_m^i\right) \times \left(z^{1/M}W_m^i\right) - \frac{1}{M} G_k(z) \times \sum_{i=0}^{M-1} H_k\left(z^{1/M}W_m^i\right) \times \left(z^{1/M}W_m^i\right) \quad (18)$$

Note: $W_m^i = e^{-j2\pi i/M}$, and the $k^{th}$ analysis filter $H_k(z)$ is a band pass filter, whose pass band is $k\pi / M \leq \omega \leq (k+1)\pi / M$. The above equation can be rewritten as

$$\begin{aligned} M.E_k(z) = {} & \left[H\left(z^{1/M}\right) - G_k(z)\right] H_k\left(z^{1/M}\right) X\left(z^{1/M}\right) + \left[H\left(z^{1/M}W_M^k\right) - G_k(z)\right] \\ & \times H_k\left(z^{1/M}W_M^k\right) X\left(z^{1/M}W_M^k\right) + \left[H\left(z^{1/M}W_M^{k+1}\right) - G_k(z)\right] \\ & \times H_k\left(z^{1/M}W_M^{k+1}\right) X\left(z^{1/M}W_M^{k+1}\right) + \xi_k(z) \end{aligned} \quad (19)$$

Where

$$\xi_k(z) = \sum_{\substack{i=1 \\ i\neq k,k+1}}^{M-1} H_k\left(z^{1/M}W_M^i\right) H\left(z^{1/M}W_M^i\right) \times X\left(z^{1/M}W_M^i\right) - G_k(z) \times \sum_{\substack{i=1 \\ i\neq k,k+1}}^{M-1} H_k\left(z^{1/M}W_M^i\right) X\left(z^{1/M}W_M^i\right) \quad (20)$$

When the filter bank is real-valued, the terms $H_k\left(z^{1/M}W_M^k\right)$ and $H_k\left(z^{1/M}W_M^{k+1}\right)$ are adjacent to $H_k\left(z^{1/M}\right)$ as shown by their magnitude responses in Figure 10, and when $H_k(z)$ is designed with high enough stop band attenuation, $\xi_k(z)$ is approximately zero

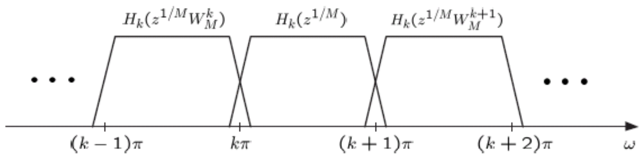


Figure10. Magnitude responses of $H_k\left(z^{1/M}\right)$ and its adjacent terms $H_k\left(z^{1/M}W_M^k\right)$ and $H_k\left(z^{1/M}W_M^{k+1}\right)$.

### *5.3 Alias-Free SAF with Critical Sampling*

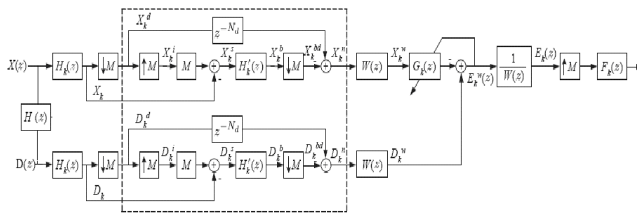

Figure11. Alias-Free SAF structure with critical sampling in the $K^{th}$ sub band.
Where W(z) = Transfer function of minimum phase filter
$H_k(z)$ = Transfer function of $K^{th}$ subband filter

The interband aliasing is a major bottleneck in using SAF, and several methods for reducing the inter band aliasing have been proposed in SAF. It can be reduced by critically sampled SAF that is almost alias-free is proposed. The interband aliasing components are caused by down sampling the signal which has passed through a non ideal analysis filter. The down sampling process is essential in almost all multirate signals processing for making the overall data rate nearly equivalent to that of the input. Figure 7.5 shows the magnitude responses $X_k(e^{j\omega})$ of the signal that has passed through the kth analysis filter, whose transition bandwidth is $\omega_\delta / M$, and its down sampled version

$$X_k^d e^{j\omega} \text{ for } k=0,1,\ldots\ M-1. \qquad (21)$$

Henceforth, only the frequency interval of $k\pi \le \omega \le (k+1)\pi$ will be considered, since the frequency responses in the other intervals are just the frequency-shifted and the frequency-flipped versions of the response. As shown in Figure 12(b), the frequency interval of $k\pi \le \omega \le (k+1)\pi$ is divided into three subintervals $\Omega_1 = \{\omega / k\pi \le \omega \le k\pi + \omega_\delta\}$, where the first and second terms coexist is given by, $\Omega_2 = \{\omega / k\pi + \omega_\delta \le \omega \le (k+1)\pi - \omega_\delta\}$, where only the first term exists, and $\Omega_3 = \{\omega / (k+1)\pi - \omega_\delta \le \omega \le (k+1)\pi\}$, where the first and third terms coexist.

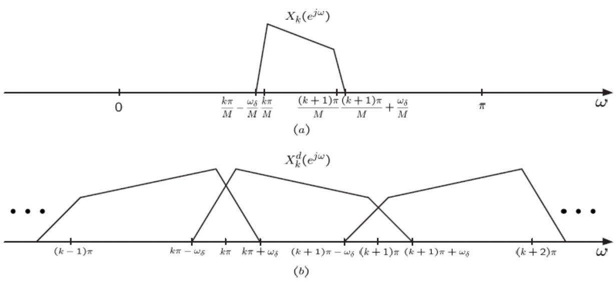

Figure12. Magnitude responses of the signals in the kth sub band. (a) Output of the kth analysis filter. (b) Its down sampled version.

As shown in Figure 12(b), the error signal $E_k(e^{j\omega})$ of the $k^{th}$ sub band is analyzed according to the subintervals as follows. The error can be approximated as

$$M \cdot E_k(e^{j\omega}) \approx [H(e^{j\omega/M}) - G_k(e^{j\omega})] \times H_k(e^{j\omega/M}) X(e^{j\omega/M}) + [H(e^{j(\omega-2\pi k)/M}) - G_k(e^{j\omega})] \times H_k(e^{j(\omega-2\pi k)/M}) X(e^{j(\omega-2\pi k)/M}) \qquad (22)$$

For $\omega \in \Omega_2$, it can be approximated as

$$M \cdot E_k(e^{j\omega}) \approx [H(e^{j\omega/M}) - G_k(e^{j\omega})] H_k(e^{j\omega/M}) X(e^{j\omega/M}) \qquad (23)$$

Finally, for $\omega \in \Omega_3$, it can be approximated as

$$M.E_k(ej\omega) \approx [H(e^{jw/M}) - G_k(e^{j\omega})] \times H_k(e^{jw/M}) \times (e^{jw/M}) + [H(e^{jw-2\pi(k+1)/M}) - G_k(e^{j\omega})] \times H_k(e^{j(w-2\pi(k+1))/M}) X(e^{j(w-2\pi(k+1))/M}) \qquad (24)$$

## 6. Results and Discussions

### 6.1 Full band Adaptive Filtering

Initialize the adaptive filter parameters: filter length m = 41, step size = 0.01, max no of iterations, the constant pi = 3.14, fsamp = 10000Hz.

Generate the input noisy sinusoidal signal having multiple frequency components (fsig1 = 500Hz, fsig2 = 2300Hz, fsig3 = 2700Hz, fsig4 = 3500Hz and the desired sinusoidal signal (fsig1 = 500Hz).

The generated input and desired signals are applied to the adaptive filters and observe the error signal is equal to the difference of desired signal and the filtered signal. For every iteration observe the Mean Square error value for different number of iterations using the MATLAB simulation.

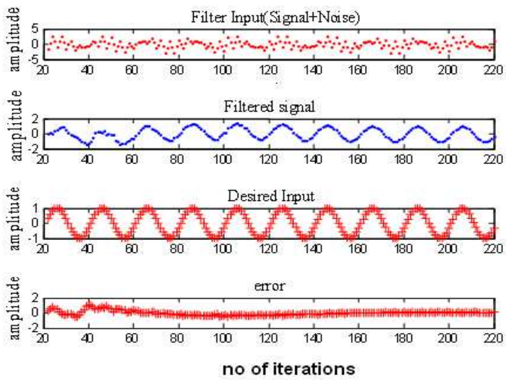


Figure13. Representation of signals

If the numbers of iterations are increased then the error value is decreased and if the input signal has longer length then the LMS algorithm converges slowly. This can be overcome by sub band adaptive filtering algorithm.

| No of iterations | Full Band Error |
|---|---|
| 100 | 0.0698 |
| 150 | 0.0387 |
| 200 | 0.0269 |
| 250 | 0.0207 |
| 300 | 0.0168 |
| 350 | 0.0141 |
| 400 | 0.0122 |
| 450 | 0.0107 |
| 500 | 0.0095 |
| 550 | 0.0086 |
| 600 | 0.0079 |
| 650 | 0.0072 |
| 700 | 0.0067 |
| 750 | 0.0062 |
| 800 | 0.0058 |
| 850 | 0.0054 |
| 900 | 0.0051 |
| 950 | 0.0048 |
| 1000 | 0.0046 |

Table1. Full band adaptive filtering algorithm

If the numbers of iterations are increased the error can be minimized (towards zero). That means the input signal closer to the desired signal. If the input signal length is increased the convergence rate will become slow. This can be improved by sub band adaptive filtering.

### *6.2 Full band Adaptive Filtering (for system identification)*

Full band adaptive filtering for system identification applications, we implement as initialize the adaptive filter parameters: filter length m = 31, step size = 0.008, max no of iterations, the constant pi = 3.14.

The input signal given to the filter is signal and the desired signal is the signal with some observation noise signal. The filter coefficients are updated by the adaptive filter using LMS adaptive algorithm. The adaptive filter array is initially set to zeros, and these are updated such that the estimated filter coefficients are almost equal to the unknown system, such a way that we select the maximum number of iterations. If we increasing the number of iterations, the mean square error is reduced (Mat lab simulations shows the estimated weights are almost equal to the actual weights).

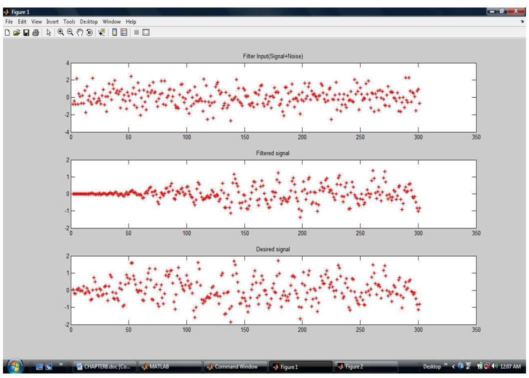


Figure14. Representation of input, desired and filtered signals

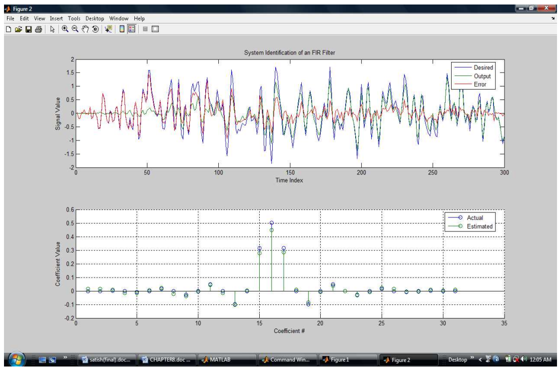


Figure15. Representation of actual and estimated filter coefficients (for 300 iterations)

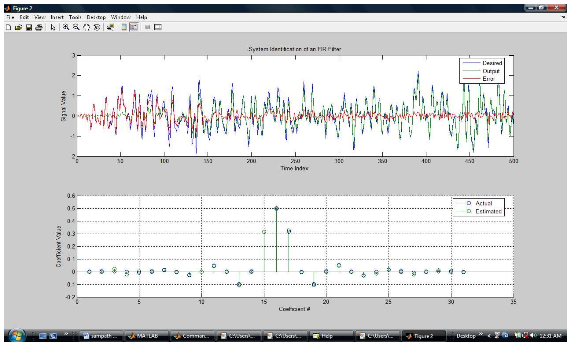


Figure16. Representation of actual and estimated filter coefficients (for 500 iterations)

### *6.3 Alias-Free Subband Adaptive Filtering (M=2)*

In full band adaptive filtering algorithm converges slowly for the sub band adaptive filtering algorithm [13], [14].

Initialize the adaptive filter parameters: filter length m = 41, step size = 0.01, max no of iterations , the constant pi = 3.14, fsamp = 10000Hz. Generate the input noisy sinusoidal signal having multiple frequency components (fsig1 = 500Hz, fsig2 = 2300Hz, fsig3 = 2700Hz, fsig4 = 3500Hz and the desired sinusoidal signal (fsig1 = 500Hz).In sub band adaptive filtering algorithm the input noisy sinusoidal signal and the desired signal are decomposed and decimated into sub bands ( M = 2) by using the analysis filter bands. Each subband input signal and desired signals are applied to the subband adaptive filters, due to the down sampling interband aliasing will occur between two adjacent subbands. Before applying to the subband adaptive filters first minimize the interband aliasing error in each subband. The interband aliasing error will obtain by using a band width increased analysis filter and this can be subtracted from the each subband signal. The resultant alias free subband input signal applied to the adaptive filters. The subband error signals are recombined by using the synthesis filter banks. Observe the total error for various no of iterations by using MATLAB simulation and calculate the Mean Square error. Each sub band adaptive filter error is combined by using the synthesis filter bank. These filter banks are designed by using windowing technique. Observe the total error signal for various no of iterations by using MATLAB simulations and also calculate the Mean Square error.

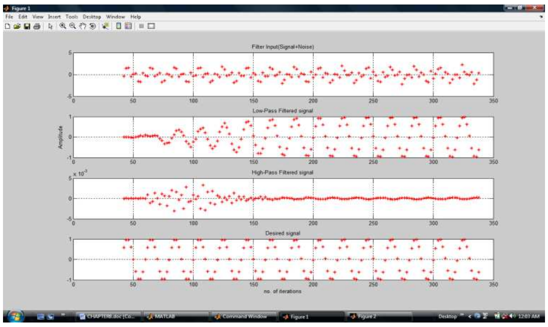


Figure17. Representation of input, desired and filtered signals

If the no of iterations are increased then the total error value is decreased and that value is less than the full band algorithm for the same no of iterations and convergence rate also is improved.

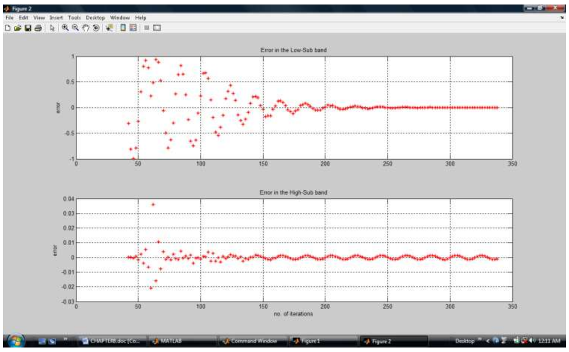


Figure18. Error in subbands

| No of iterations | Sub band Error(M=2) | Sub band Error(M=4) |
|---|---|---|
| 100 | 0.0175 | 0.0081 |
| 150 | 0.0144 | 0.0058 |
| 200 | 0.0109 | 0.0045 |
| 250 | 0.0087 | 0.0036 |
| 300 | 0.0073 | 0.0030 |
| 350 | 0.0062 | 0.0026 |
| 400 | 0.0054 | 0.0023 |
| 450 | 0.0048 | 0.0020 |
| 500 | 0.0043 | 0.0018 |
| 550 | 0.0039 | 0.0016 |
| 600 | 0.0036 | 0.0015 |
| 650 | 0.0033 | 0.0014 |
| 700 | 0.0031 | 0.0013 |
| 750 | 0.0029 | 0.0012 |
| 800 | 0.0027 | 0.0011 |
| 850 | 0.0026 | 0.0011 |
| 900 | 0.0024 | 0.0010 |
| 950 | 0.0023 | 0.0009 |
| 1000 | 0.0022 | 0.0008 |

Table2. Comparison between alias free Subbands (for 2&4)

*6.4 Alias-Free Sub band Adaptive Filtering (M=4)*

The convergence rate is improved by subband adaptive filtering algorithm for a lengthy input noisy signals compare to the full band adaptive filter algorithm with lower computational cost. For the same no of iterations the error is minimized in sub band algorithm (M=4) compared to two subband filtering and full band algorithm and the convergence rate is improved by sub band adaptive filtering.

The inter band aliasing error is occurred while down sampling the input signal. Due to this, the convergence rate improvement is very small. The interband aliasing error is reduced by alias free subband adaptive filtering with critical sampling.

For the same no of iterations the error is minimized in Alias free sub band algorithm with critical sampling (M=4) when compared to full band and sub band algorithm and the convergence rate is also improved. The interband aliasing error is minimized by using the band width increased analysis filter and also convergence rate is improved. If the number of subbands is increased then the convergence rate will be improved. In an alias free subband adaptive filtering algorithm, the convergence rate is improved compared to full band, subband adaptive filtering algorithms. For the same number of iterations the error is minimum in an alias free subband adaptive filtering algorithm compared to full band, sub band adaptive filtering algorithms.

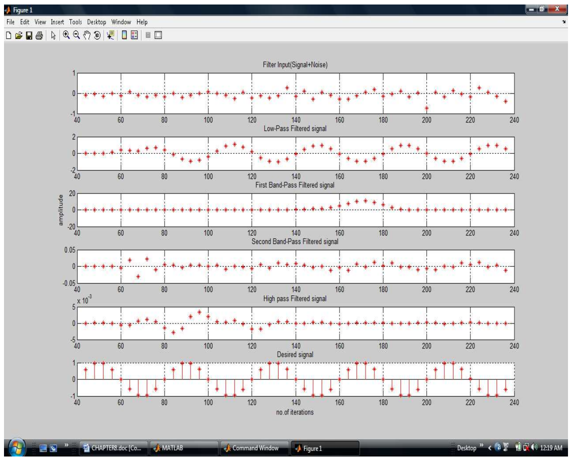


Figure19. Alias free sub band adaptive filtering with critical sampling (M=4)

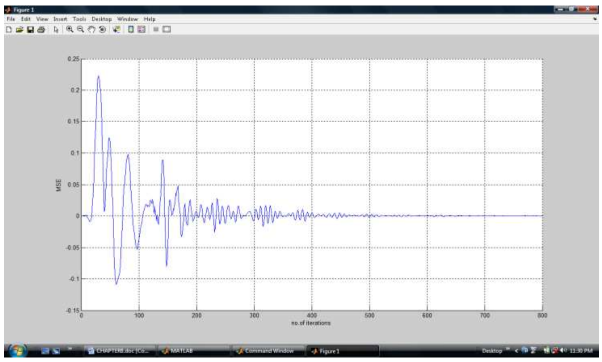

Figure20. Total error in alias free sub band adaptive filtering with critical sampling (m=4)

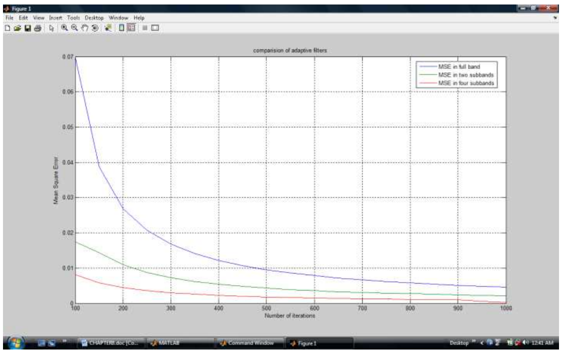


Figure21. Comparison of MSE in full band, two subbands and four Subbands

## 7. Conclusions

In this paper in order to exploit the benefits of SAF, a structure with critical sampling that is virtually alias–free is proposed. The interband aliasing is extracted in each subband using the bandwidth–increased FIR linear phase analysis filters and then subtracted from each subband signal. The use of the bandwidth increased analysis filters introduces an extra computational load. The almost alias free subband signals have the spectral flatness and then the outputs are used for adaptive filtering in each sub band.

The computational complexity of the proposed SAF algorithm is approximately reduced by M compared to that of the full band algorithm. Simulations results show the that the proposed sub band structure achieves similar convergence rate to the full band structure for white noise input and better convergence rate than both the equivalent full band at lower computational complexity and the conventional SAF structures for colored input.

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