INSURING EXISTENCE OF VACUUM ENERGY TO KEEP COMPUTATIONAL "BITS" PRESENT AT START OF COSMOLOGICAL EVOLUTION, EVEN IF INITIAL SPATIAL RADIUS GOES TO ZERO, NOT PLANCK LENGTH

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Abstract

When initial radius $R_{initial} \to 0$ if Stoica actually derived Einstein equations in a formalism which remove the big bang singularity pathology, then the reason for Planck length no longer holds. . We present entanglement entropy in the early universe with a shrinking scale factor, due to Muller and Lousto , and show that there are consequences due to initial entanged $S_{Entropy} = .3 r_H^2 / a^2$ for a time dependent horizon radius r_H in cosmology, with (flat space conditions) $r_H = \eta$ for conformal time . Even if the 3 dimensional spatial length goes to zero. This construction preserves a minimum non zero Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{initial} \to 0$. We state that the presence of computational bits is necessary for cosmological evolution to commence. We also state that entanglement entropy not disappearing even if 4 dimensional length disappears is similar to the problem of a higher dimensional surface area at the tangential sheet intersecting a point of space-time, so that the presence of non zero entanglement entropy, embedded on an infinitesmally thick space-time thickness is the same as embedding the known 4 dimensional universe into a higher dimensional structure, as hypothesized by Steinhardt and Turok and Durrer.

Keywords: *Fjortoft theorem, thermodynamic potential, matter creation, vacuum energy non pathological singularity affecting Einstein equations,planck length.Braneworlds.*

1. Introduction

This article is to investigate what happens physically if there is a non pathological singularity in terms of Einsteins equations at the start of space-time. This eliminates the necessity of having then put in the Planck length since then ther would be no reason to have a minimum non zero length. The reasons for such a proposal come from [1] by Stoica who may have removed the reason for the development of Planck's length as a minimum safety net to remove what appears to be unadvoidable pathologies at the start of applying the Einstein equations at a space-time singularity, and are commented upon in this article. $\rho \sim H^2 / G \Leftrightarrow H \approx a^{-1}$ in particular is remarked upon. This is a counter part to *Fjortoft* theorem in **Appendix I** below. The idea is that entanglement entropy will help generate bits, due to the presence of a vacuum energy, as derived at the end of the article, and the presence of a vacuum energy non zero value, is necessary for comsological evolution. Before we get to that creation of what is a necessary creation of vacuum energy conditions we refer to constructions leading to extremely pathological problems which [1] could lead to minus the presence of initial non zero vacuum energy. [2] also adds more elaboration on this.

Note a change in entropy formula given by Lee [3] about the inter relationship between energy, entropy and temperature as given by

$$m \cdot c^2 = \Delta E = T_U \cdot \Delta S = \frac{\hbar \cdot a}{2\pi \cdot c \cdot k_B} \cdot \Delta S \tag{1}$$

Lee's formula is crucial for what we will bring up in the latter part of this document. Namely that changes in initial energy could effectively vanish if [1] is right, i.e. Stoica removing the non pathological nature of a big bang singularity. That is, unless entanglement entropy is used.

If the mass m, i.e. for gravitons is set by acceleration (of the net universe) and a change in enthropy $\Delta S \sim 10^{38}$ between the electroweak regime and the final entropy value of, if $a \cong \frac{c^2}{\Delta x}$ for acceleration is used, so then we obtain

$$S_{Today} \sim 10^{88} \tag{2}$$

Then we are really forced to look at (1) as a paring between gravitons (today) and gravitinos (electro weak) in the sense of preservation of information.

Having said this note by extention $\rho \sim H^2 / G \Leftrightarrow H \approx a^{-1}$. As ρ changes due to $\rho \sim H^2 / G$ and $R_{initial} \sim \frac{1}{\#} \ell_{Ng} < l_{Planck}$, t hen a is also altered i.e. goes to zero..

What will determine the answer to this question is if $\Delta E_{initial}$ goes to zero if $R_{initial} \to 0$ which happens if there is no minimum distance mandated to avoid the pathology of singularity behavior at the heart of the Einstein equations. In doing this, we avoid using the energy $E \to 0^+$ situation, i.e. of vanishing initial space-time energy, and instead refer to a nonzero energy, with

$\Delta E_{initial}$ instead vanishing. In particular, the Entanglement entropy concept as presented by Muller and Lousto [4] is presented toward the end of this manuscript as a partial resolution of some of the pathologies brought up in this article before the entanglement entropy section. No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t, rescaled,does not go to zero. This preserves a minimum non zero Λ vacuum energy, and in doing so keep non zero amounts of initial bits, for computational bits cosmological evolution even if $R_{initial} \to 0$

Before doing that, we review Ng [5] and his quantum foam hypothesis to give conceptual underpinnings as to why we later even review the implications of entanglement.entropy. I.e. the concept of bits and computations is brought up because of applying energy uncertainty, as given by [5] and the Margolis theorem appears to indicate that the universe could not possibly evolve if [1] is applied, in a 4 dimensional closed universe. This bottle neck as indicated by Ng's [5] formalism is even more striking in the author's end of article proof of the necessity of using entanglement entropy in lieu of the conclusion involving entanglement entropy, which can be non zero, even if $R_{initial} \to 0$ provided therre is a minimum non zero time length.

2. **Review of Ng, [5] with comments.**
   First of all, Ng refers to the Margolus-Levitin theorem with the rate of operations $< E/\hbar \Rightarrow \#operations < E/\hbar \times time = \frac{Mc^2}{\hbar} \cdot \frac{l}{c}$. Ng wishes to avoid black-hole formation $\Rightarrow M \leq \frac{lc^2}{G}$. This last step is not important to our view point, but we refer to it to keep fidelity to what Ng brought up in his presentation. Later on, Ng refers to the $\#operations \leq \left(R_H / l_P\right)^2 \sim 10^{123}$ with R_H the Hubble radius. Next Ng refers to the $\#bits \propto \left[\#operations\right]^{3/4}$. Each bit energy is $1/R_H$ with $R_H \sim l_P \cdot 10^{123/2}$

   The key point as seen by Ng [4] and the author is in

$$\#bits \sim \left[\frac{E}{\hbar} \cdot \frac{l}{c}\right]^{3/4} \approx \left[\frac{Mc^2}{\hbar} \cdot \frac{l}{c}\right]^{3/4} \tag{3}$$

   Assuming that the initial energy E of the universe is not set equal to zero, which the author views as impossible, the above equation says that the number of available bits goes down dramatically if one sets $R_{initial} \sim \frac{1}{\#} \ell_{Ng} < l_{Planck}$? Also Ng writes entropy S as proportional to a particle count via N.

$$S \sim N \cong \left[R_H / l_P\right]^2 \tag{4}$$

   We rescale R_H to be

$$R_H\big|_{rescale} \sim \frac{l_{Ng}}{\#}\cdot 10^{123/2} \tag{5}$$

The upshot is that the entropy, in terms of the number of available particles drops dramatically if $\#$ becomes larger.

So, as $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$ grows smaller, as $\#$ becomes larger

a. The initial entropy drops
b. The nunber of bits initially available also drops.

The limiting case of (4) and (5) in a closed universe, with no higher dimensional embedding is that both would almost vanish, i.e. appear to go to zero if $\#$ becomes very much larger. The quest4ion we have to ask is would the number of bits in computational evolution actually vanish?

3. Does it make sense to talk of vacuum energy if $R_{initial} \neq 0$ is changed to $R_{initial} \to 0$? Only answerable straightforwardly if an embedding superstructure is assigned. Otherwise difficult. Unless one is using entanglement entropy which is non zero even if $R_{initial} \to 0$

We summarize what may be the high lights of this inquiry leading to the present paper as follows.

3a. One could have the situation if $R_{initial} \to 0$ of an infinite point mass, if there is an initial nonzero energy in the case of four dimensions and no higher dimensional embedding even if [1] goes through verbatim. The author sees this as unlikely. The infinite point mass construction is verbatim if one assumes a closed universe, with no embedding superstructure and no entanglement entropy. Note this appears to nullify the parallel brane world construction used by Durrer [17].The author, in lieu of the manuscript sees no reason as to what would perturb this infinite point structure, so as to be able to enter in a big bang era. In such a situation, one would not have vacuum energy unless entanglement entropy were used. That is unless one has a non zero entanglement entropy [4] present even if $R_{initial} \to 0$. See [6] for a smilar argument.

3b. The most problematic scenario. $R_{initial} \to 0$ and no initial cosmological energy. I.e. this in a 4 dimensional closed universe. Then there would be no vacuum energy at all.initially. A literal completely empty initial state, which is not held to be viable by Volovik [7].

3c. If additional dimensions are involved in beginning cosmology, than just 4 dimensions will lead to physics which may give credence to other senarios. One scenario being the authors speculation as to initial degrees of freedom reaching up to 1000, and the nature of a phase transition from essentially very low degrees of freedom, to over 1000 as speculated by the author in 2010 [8].

3d. What the author would be particularly interested in knowing would be if actual semiclassical reasoning could be used to get to an initial prequantum cosmological state. This would be akin to using [9], but even more to the point, using [10] and [11] , with both these last references relevant to forming Planck's constant from electromagnetic wave equations. The author points to the enormous Electromagnetic fields in the electroweak era as perhaps being part of the background necessary for such a semiclassical derivation, plus a possible Octonionic space-time regime, as before inflation flattens space-time, as forming a boundary condition for such constructions to occur [12]

The relevant template for examinging such questions is given in the following table 1 as printed below.

3e. The meaning of Octonionic geometry prior to the introduction of quantum physics presupposes a form of embedding geometry and in many ways is similar to Penrose's cyclic conformal cosmology speculation.:

3f. It is striking how a semiclassical argument can be used to construct Table 1 below. In particular, we look at how Planck's constant is derived, as in the electroweak regime of space-time, for a total derivative [10],[11]

$$E_y = \frac{\partial A_y}{\partial t} = \omega \cdot A_y' \left(\omega \cdot (t - x) \right) \tag{6}$$

Similarly [10],[11]

$$B_z = -\frac{\partial A_y}{\partial x} = \omega \cdot A_y' \left(\omega \cdot (t - x) \right) \tag{7}$$

The A field so given would be part of the Maxwell's equations given by [9] as, when $[\ \ ]$ represents a D'Albertain operator, that in a vacuum, one would have for an A field [10], [11]

$$[\ \ ] A = 0 \tag{8}$$

And for a scalar field ϕ

$$[\ \ ] \phi = 0 \tag{9}$$

Following this line of thought we then would have an energy density given by, if ε_0 is the early universe permeability [10]

$$\eta = \frac{\varepsilon_0}{2} \cdot \left(E_y^2 + B_z^2 \right) = \omega^2 \cdot \varepsilon_0 \cdot A_y'^2 \left(\omega \cdot (t - x) \right) \tag{10}$$

We integrate (10) over a specified E and M boundary, so that, then we can write the following condition namely [10],[11].

$$\iiint \eta d(t - x) dydz = \omega \varepsilon_o \iiint A_y'^2 \left(\omega \cdot (t - x) \right) d(t - x) dydz \tag{11}$$

(11) would be integrated over the boundary regime from the transition from the Octonionic regime of space time, to the non Octonionic regime, assuming an abrupt transition occurs, and we can write, the volume integral as representing [10],[11]

$$E_{gravitational-energy} = \hbar \cdot \omega \tag{12}$$

Then by applying [10], [11] we get $\hbar$ formed by semiclassical reasons In semi classical reasoning similar to [9]

$$\hbar(t) \xrightarrow[Apply-Machs-Relations]{} \hbar \text{ (Constant value)} \tag{13}$$

The question we can ask, is that can we have a prequantum regime commencing for (11) and (12) for $\hbar$ if $R_{initial} \sim \frac{1}{\#} \ell_{Ng} < l_{Planck}$? And a closed 4 dimensional universe? If so, then what is the necessary geometrial regime of space-time so that the integration performed in (11) can commence properly? Also, what can we say about the formation of (12) above, as a number, $\#$ gets larger and larger, effectively leading to. Also,with an Octonionic geometry regime which is a pre quantum state.[12]

TABLE 1

.Time Interval	Dynamical consequences	Does QM/WdW apply?
Just before Electroweak era	Form $\hbar$ from early E & M fields, and use Maxwell's Equations with necessary to implement boundary conditions created from change from Octonionic geometry to flat space	NO
Electro-Weak Era	$\hbar$ kept constant due to Machian relations	YES
Post Electro-Weak Era to today	$\hbar$ kept constant due to Machian relations	YES Wave function of Universe

In so many words, the formation period for $\hbar$ is our pre-quantum regime. This table 1 could even hold if $R_{initial} \to 0$ but that the 4 dimensional space-time exhibiting such behavior is embedded in a higher dimensional template. That due to $R_{initial} \to 0$ not removing entanglement entropy as is discussed near the end of this article.

4. ***If*** $R_{initial} \to 0$ **then if there is an isolated, closed universe, there is a disaster unless one uses entanglement entropy.**

One does not have initial entropy, and the number of bits initially disappears. That is if one is not using entanglement entropy, as will be examined at the end of this article.

Abandoning the idea of a completely empty universe, this unperturbed point of matter-energy appears to be a recipede for a static point with no perturbation, as may be the end result of applying Fjortoft theorem [13] to the thermodynamic potential as given in [14], i.e. the non definitive anwer for fufillment of criteria of instability by applying Fjortoft's theorem [13] to the potential [14] leading to no instability as given by the potential given in [14] may lead to a point of space-time with no change, i.e. a singular point with 'infinite' mass which does not change at all. This issue will be reviewed in [15] a different procedure, i.e. a so called nonsingular universe construction. To get there we will first of all review an issue leading up to implimentation of [15].

5. **Can an alternative to a minimum length be put in? Consider the example of Planck time as the minimal component, not Planck length.**

From J. Dickau, [16] the following was given to the author, as a counter point to $R_{initial} \to 0$ *leading to a disaster*.

"If we examine the Mandelbrot Set along the Real axis, it informs us about behaviors that also pertain in the Quaternion and Octonic case-because the real axis is invariant over the number types. If numbers larger than .25 are squared and summed recursively (i.e. –z = z^2 +c) the result will blow up, but numbers below this threshold never get to infinity, no matter how many times they are iterated. But once space-like dimensions are added-i.e. an imaginary compoent- the equation blows up exponentially, faser than when iterated"

Dickau concludes:

"Anyhow there may be a minimum (space-time length) involved but it is probably in the time direction".

This is a counter pose to the idea of minimum length, looking at a beginning situation with a crucial parameter $R_{initial}$ even if the initial time step is "put in by hand". First of all, look at [4], if E is M, due to setting c = 1, then

$$\Delta E_{initial} \approx 4\pi\rho\left(R_{initial}\right)^2 \Delta R_{initial} \qquad (14)$$

Everything depends upon the parameter $R_{initial}$ which can go to zero. We have to look at what (14) tells us, even if we have an initial time step for which time is initially indeterminate, as given by a redoing of Mitra's g_{00} formula [6] which we put in to establish the indeterminacy of the initial time step if quantum processes hold.

$$\left(g_{00} = \exp\left[\frac{-2}{1+\left(\rho(t)/p(t)\right)} \right] \right) \xrightarrow[\rho+p=0]{} 0 \tag{15}$$

What Dickau is promoting is, that the Mandelbrot set, if applicable to early universe geometry, that what the author wrote, with $R_{initial} \sim \frac{1}{\#} \ell_{Ng} < l_{Planck} \xrightarrow[\# \neq \infty]{} small - value$ potentially going to zero, is less important than a minimum time length. The instability issue is reviewed in **Appendix II**. for those who are interested in the author's views as to lack proof of instability. It uses [14] which the author views as THE reference as far as thermodynamic potentials and the early universe.

6. Muller and Lousto Early universe entanglement entropy, and its implications. Solving the spatial length issue, provided a minimum time step is preserved in the cosmos, in line with Dickau's suggestion.

We look at [4]

$S_{Entropy} = .3 r_H^2 / a^2$ for a time dependent horizon radius r_H in cosmology (16)

Equation (16) above was shown by the author to be fully equivalent to

$$S_{Entropy} = .3 r_H^2 / a^2 \sim \frac{.3}{a^2} \exp\left[-t \cdot \sqrt{\frac{\Lambda}{3}} \right] \tag{17}$$

i.e.

$$\left[-t \sqrt{\frac{\Lambda}{3}} \right] \sim \ln_e \left(\frac{a^2}{.3} \cdot S_{entropy} \right) \tag{18}$$

So, then one has

$$\Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{.3} \cdot S_{entropy} \right) \right]^2 \tag{19}$$

No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t, re scaled does not go to zero. *This preserves a minimum non zero* Λ *vacuum energy, and in doing so keep the non zero initial bits, for computational bits contributions to evolving space time behavior even if* $R_{initial} \to 0$

7. **Reviewing a suggestion as to how to quantify the shrinkage of the scale factor and its connections with entanglement entropy.**

We are given by [15] if there is a non singular universe, a template as to how to evaluate scale factor a against time scaled over Planck time, with the following results.

$$\ln_e a + \frac{a^6}{6} + \frac{2 \cdot a^3}{3} = \sqrt{\frac{8\pi}{3}} \cdot \frac{t}{t_{Planck}} \tag{20}$$

Two time and scale factor values in tandem particularly stand out. Namely,

$$a \sim \frac{a_{scale}}{\left[a_{Planck} \sim 10^{-25}\right]} \equiv 1.344 \Leftrightarrow t \propto t_{Planck} \sim 5.4 \times 10^{-44} \sec \tag{21}$$

Also

$$a \sim \frac{a_{scale}}{\left[a_{Planck} \sim 10^{-25}\right]} \equiv .7414 \Leftrightarrow t \propto 0^{+} \tag{22}$$

The main thing we can take from this, is to look at the inter-relationship of how to pin down an actual initial Hubble "constant" expansion parameter, where we look at:

$$1.813 = \exp(H_{Planck} \cdot t_{Planck}) \Leftrightarrow H_{Planck} = \frac{\ln_e (1.813)}{t_{Planck}} \tag{23}$$

Recall that $\Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{.3} \cdot S_{entropy} \right) \right]^2$, which is predicated upon , if the time is close to Planck time the initial maximal density of

$$\rho_{Planck} \sim 5.2 \times 10^{96} kg / m^3 \tag{24}$$

And length given by

$$Length(Planck) = l_{Planck} \sim 1.6 \times 10^{-35} meters \tag{25}$$

So (24) is implying that the amount of matter in a region of space $(l_{Planck})^3$ is initially about

$$\rho_{initial} \sim 2 \times 10^{-10} Kg \sim 2 \times 10^{-7} grams \tag{26}$$

Using 1 GeV/c^2 = 1.783×10^{-27} kg means that (26) above is

$$\rho_{initial} \sim 2 \times 10^{-10} Kg \sim 2 \times 10^{-7} grams \sim 10^{+17} GeV \tag{26a}$$

Then if

$$10^{+17} GeV \sim \Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{.3} \cdot S_{entropy} \right) \right]^2 \tag{27}$$

It will lead to

$$10^{+17} GeV \times 10^{-70} \sec^2 \sim \left[\ln_e \left(\frac{a^2}{.3} \cdot S_{entropy} \right) \right] \tag{28}$$

Then , to first order, one is looking at Initial entropy to get a non zero but definite vacuum energy as leading to an entanglement entropy of about (just before the electro weak regime)

$$S_{entropy} \sim 1/a^2 \sim 10^{20} - 10^{40} \tag{29}$$

8. Reviewing the geometry for embedding (29) above.

. In line with Stoica [1] shrinking the minimum length and referring to both (29) and (27), the idea is to use a surface area treatment as to getting the initial entropy values as given in (29). To do so, the author looks at the following diagram:

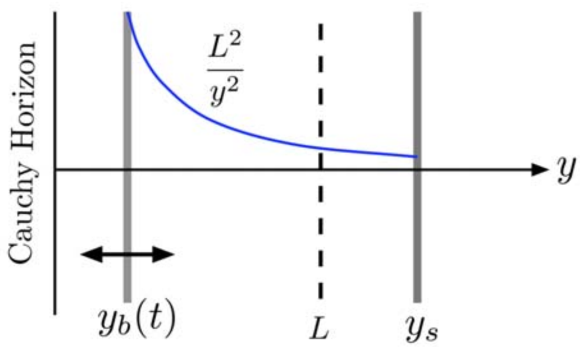

Figure 1, from [17]

The two branes given at y_b and y_s refer to the two Brane world states, especially in line with [20], [21]. The first one, namely y_b is the brane where our physical universe lives in, and is embedded in. If one uses this construction, with higher dimensions than just 4 dimensions, then it is possible to have a single point in 4 dimensional space as a starting point to a tangential sheet which is part of an embedding in more than 4 dimensions. Along the lines of having a 4 dimensional cusp with its valley (lowest) point in a more than 4 dimensional tangential surface. The second brane is about 10^{-30} centimeters away from the brane our physical world lives in, and moves closer to our own brane in the future, leading to a slapping of the two branes together about a trillion years ahead in our future [20],[21]. The geometry we are referring to with regards to embedding is in the first brane y_b . [17] uses this geometry to have graviton production which the author has used to model Dark Energy [17a]

9. Conclusion. Making computational bits, via (19).

As stated by Ng. the idea would be to have to give imputs into (3) i.e.

$$\#bits \sim \left[\frac{E}{\hbar}\cdot\frac{l}{c}\right]^{3/4} \approx \left[\frac{Mc^2}{\hbar}\cdot\frac{l}{c}\right]^{3/4} \tag{3}$$

Here in this case, even if the spatial contribution, due to [1] goes to zero, the idea would be to have the time length non zero so as to have a space-time version of l non zero. . This would also be in tandem with calling E, in (3) as proportional to $\Lambda \approx \frac{3}{t^2}\cdot\left[\ln_e\left(\frac{a^2}{.3}\cdot S_{entropy}\right)\right]^2$, where if the time is Planck time, in minimum value, and $S_{entropy} \sim 1/a^2 \sim 10^{20} - 10^{40}$ in value, one would have before the electro-weak an input into E, which would require an entropy (entanglement)

What remains to be seen is , if there is a geometric sheet in more than 4 dimensions, allowing for non zero time, as argued for $\Lambda \approx \frac{3}{t^2}\cdot\left[\ln_e\left(\frac{a^2}{.3}\cdot S_{entropy}\right)\right]^2$, even if the spatial componet goes to zero, according to [1]. We suggest an update as to what was written by Seth Lloyd [22] with

$$I = S_{total}/k_B \ln 2 = [\#operations]^{3/4} = \left[\rho\cdot c^5\cdot t^4/\hbar\right]^{3/4} \tag{30}$$

when [23]

$$\rho \equiv T^{00} \sim \Lambda_{vacuum-energy} \tag{31}$$

While doing this, a good thing to do, would be to keep in mind the four dimensional version of vacuum energy as given by Park, [24] namely

$$\Lambda_{4-\dim} \approx c_2 \cdot T^{\beta} \tag{32}$$

As well as the transition given by a combination of [24], with [25], Barvinskey et. al.

$$\Lambda_{4-\dim} \propto c_2 \cdot T \xrightarrow[graviton-production]{} 360\cdot m_P^2 << c_2 \cdot \left[T \approx 10^{32} K\right] \tag{33}$$

Quantifying the above, and giving it experimental proof, via detector technology may allow us to investigate an old suggestion by the author as to four dimension and five dimensional vacuum energy which was given for small time values $t \approx \delta^1 \cdot t_P, 0 < \delta^1 \leq 1$ and for temperatures sharply lower than $T \approx 10^{12}\, Kelvin$, Beckwith [23] (2007), where for a positive integer n

$$\frac{\Lambda_{4-\dim}}{|\Lambda_{5-\dim}|} - 1 \approx \frac{1}{n} \tag{34}$$

In particular, the author is interested in investigating if the following is true, i.e.

Look at an argument provided by Thanu Padmanabhan [26], leading to the observed cosmological constant value suggested by Park[23] . Assume that $l_P \sim 10^{-33}cm \xrightarrow[Quantum-Gravity-threshold]{} \tilde{N}^{\alpha} \cdot l_P$, but that when we make this substitution that $1 \leq \tilde{N}^{\alpha} \leq 10^2$ [27]

$$\begin{aligned} \rho_{VAC} &\sim \frac{\Lambda_{observed}}{8\pi G} \sim \sqrt{\rho_{UV} \cdot \rho_{IR}} \\ &\sim \sqrt{l_{Planck}^{-4} \cdot l_H^{-4}} \sim l_{Planck}^{-2} \cdot H_{observed}^2 \end{aligned} \tag{35}$$

i.e. looking at if

$$\Delta\rho \approx \text{a dark energy density} \sim H_{observed}^2 / G \tag{36}$$

Now to make it more interesting:

We can replace $\Lambda_{observed}, H_{observed}^2$ by $\Lambda_{initial}, H_{initial}^2$. In addition we may look at inputs from the initial value of the Hubble parameter to get the necessary e folding needed for inflation, according to

$$\begin{aligned} &E - foldings = H_{initial} \cdot \left(t_{End\ of\ \inf} - t_{beginning\ \ of\ \inf} \right) \equiv N \geq 100 \\ &\Rightarrow H_{initial} \geq 10^{39} - 10^{43} \end{aligned} \tag{37}$$

Leading to

$$a(End - of - \inf) / a(Beginning - of - \inf) \equiv \exp(N) \tag{38}$$

If we set $\Lambda_{initial} \sim c_1 \cdot \left[T \sim 10^{32}\, Kelvin \right]$ implying a very large initial cosmological constant value, we get in line with what Park suggested for times much less than the Planck interval of time at the instant of nucleation of a vacuum state

$$\Lambda_{initial} \sim \left[10^{156}\right] \cdot 8\pi G \approx huge \quad number \tag{39}$$

.**Question**. Do we always have this value of (39)? At the onset of Inflation? When we are not that far away from a volume of space characterized by l_P^3, or at most 100 or so times larger ? Contemporary big bang theories imply this. I.e. a very high level of thermal energy. We need to ask if this is something which could be transferred from a prior universe , i.e. could there be a pop up nucleation effect , i.e. emergent space time? This question is what should be investigated throughly. **Appendix III** and **Appendix IV** give suggestions which the author has thought of which may contribute to, if anything, models of how instantons from a prior universe may be transmitted to our present universe, i.e. **Appendix V** which is based in part on what Wesson formulated as to five dimensional universe constructions, and instantons [29]. The very interesting topic of vacuum fluctuations in such space-time has also been reviewed briefly in **Appendix VI, and Appendix VII**

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Appendix I. Fjortoft theorem:

A **necessary condition for instability** is that if z_* is a point in spacetime for which $\frac{d^2U}{dz^2}=0$ for any given potential U , then there must be some value z_0 in the range $z_1 < z_0 < z_2$ such that

$$\left.\frac{d^2U}{dz^2}\right|_{z_0} \cdot \left[U(z_0)-U(z_*)\right] < 0 \quad (1)$$

For the proof, see [11] and also consider that the main discussion is to find instability in a physical system which will be described by a given potential U . Next, we will construct in the boundary of the EW era, a way to come up with an optimal description for U

Appendix II. **Constructing an appropriate potential for using Fjortoft theorem in cosmology for the early universe cannot be done. We show why**

To do this, we will look at Padamanabhan [14] and his construction of (in Dice 2010) of thermodynamic potentials he used to have another construction of the Einstein GR equations. To start, Padamanabhan [14] wrote

If P^{ab}_{cd} is a so called Lovelock entropy tensor, and T_{ab} a stress energy tensor

$$\begin{aligned} &U(\eta^a) = -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b + T_{ab}\eta^a\eta^b + \lambda(x)\, g_{ab}\eta^a\eta^b \\ &= U_{gravity}(\eta^a) + U_{matter}(\eta^a) + \lambda(x)\, g_{ab}\eta^a\eta^b \\ &\Leftrightarrow U_{matter}(\eta^a) = T_{ab}\eta^a\eta^b ; U_{gravity}(\eta^a) = -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b \end{aligned} \quad (1)$$

We now will look at

$$\begin{aligned} &U_{matter}(\eta^a) = T_{ab}\eta^a\eta^b \qquad ; \\ &U_{gravity}(\eta^a) = -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b \end{aligned} \quad (2)$$

So happens that in terms of looking at the partial derivative of the top (1) equation, we are looking at

$$\frac{\partial^2 U}{\partial\left(\eta^a\right)^2} = T_{aa} + \lambda(x)\, g_{aa} \quad (3)$$

Thus, we then will be looking at if there is a specified η^a_* for which the following holds.

$$\left[\begin{array}{l}\left[\frac{\partial^2 U}{\partial\left(\eta^a\right)^2}=T_{aa}+\lambda(x)\,g_{aa}\right]_{\eta_0^a} * \\ \left[\begin{array}{l}-4\cdot P_{ab}^{cd}\left(\nabla_c\eta_0^a\nabla_d\eta_0^b-\nabla_c\eta_*^a\nabla_d\eta_*^b\right)+ \\ T_{ab}\cdot\left[\eta_0^a\eta_0^b-\eta_*^a\eta_*^b\right]+\lambda(x)\,g_{ab}\cdot\left[\eta_0^a\eta_0^b-\eta_*^a\eta_*^b\right]\end{array}\right]\end{array}\right. < 0 \qquad (4)$$

What this is saying is that there is no unique point, using this η_*^a for which (4) holds. Therefore, we say there is no official point of **instability of** η_*^a due to (3). The Lagrangian structure of what can be built up by the potentials given in (3) with respect to η_*^a mean that we cannot expect an inflection point with respect to a 2nd derivative of a potential system. Such an inflection point designating a speed up of acceleration due to DE exists a billion years ago [19]. Also note that the reason for the failure for (4) to be congruent to Fjoroft's theorem is due to

$$\left[\frac{\partial^2 U}{\partial\left(\eta^a\right)^2}=T_{aa}+\lambda(x)\,g_{aa}\right]\neq 0,\ for\,\forall\,\eta_*^a choices \qquad (5)$$

Appendix III, Details as to forming Crowell's time dependent Wheeler De Witt equation, and its links to Worm holes

This will be to show some things about the worm hole we assert the instanton traverses en route to our present universe. From Crowell [28]

$$-\frac{1}{\eta r}\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{\eta r^2}\cdot\frac{\partial\Psi}{\partial r}+rR^{(3)}\Psi=\left(r\eta\phi-r\ddot{\phi}\right)\cdot\Psi \qquad (1)$$

This has when we do it $\phi\approx\cos(\omega\cdot t)$, and frequently $R^{(3)}\approx$ constant, so then we can consider

$$\phi\cong\int_0^\infty d\omega\left[a(\omega)\cdot e^{ik_\varpi x^\mu}-a^+(\omega)\cdot e^{-ik_\varpi x^\mu}\right] \qquad (2)$$

In order to do this, we can write out the following with regards to the solutions to Eqn (1) put up above.

$$\begin{aligned}C_1&=\eta^2\cdot\left(4\cdot\sqrt{\pi}\cdot\frac{t}{2\omega^5}\cdot J_1(\omega\cdot r)+\frac{4}{\omega^5}\cdot\sin(\omega\cdot r)+(\omega\cdot r)\cdot\cos(\omega\cdot r)\right)\\&+\frac{15}{\omega^5}\cos(\omega\cdot r)-\frac{6}{\omega^5}Si(\omega\cdot r)\end{aligned} \qquad (3)$$

And

$$C_2=\frac{3}{2\cdot\omega^4}\cdot\left(1-\cos(\omega\cdot r)\right)-4e^{-\omega\cdot r}+\frac{6}{\omega^4}\cdot Ci(\omega\cdot r) \qquad (4)$$

This is where $Si(\omega \cdot r)$ and $Ci(\omega \cdot r)$ refer to integrals of the form $\int_{-\infty}^{x} \frac{\sin(x')}{x'} dx'$ and $\int_{-\infty}^{x} \frac{\cos(x')}{x'} dx'$.

It so happens that this is for forming the wave functional permitting an instanton forming , while we next should consider if or not the instanton so farmed is stable under evolution of space time leading up to inflation. We argue here that we are forming an instanton whose thermal energy is focused into a wave functional which is in the throat of the worm hole up to a thermal discontinuity barrier at the onset , and beginning of the inflationary era.

Appendix IV: The D'Albembertain operation in an equation of motion for emergent scalar fields

We begin with the D'Albertain operator as part of an equation of motion for an emergent scalar field. We refer to the Penrose potential (with an initial assumption of Euclidian flat space for computational simplicity) to account for, in a high temperature regime an emergent non zero value for the scalar field ϕ due to a zero effective mass, at high temperatures. [29]

When the mass approaches far lower values, it, a non zero scalar field re appears.

Leading to $\phi \xrightarrow[T \to 2.7^0 \, Kelvin]{} \varepsilon^+ \approx 0^+$ as a vanishingly small contribution to cosmological evolution

Let us now begin to initiate how to model the Penrose quintessence scalar field evolution equation. To begin, look at the flat space version of the evolution equation

$$\ddot{\phi} - \nabla^2 \phi + \frac{\partial V}{\partial \phi} = 0 \tag{1}$$

This is, in the Friedman – Walker metric using the following as a potential system to work with, namely:

$$\begin{aligned} V(\phi) &\sim -\left[\frac{1}{2} \cdot \left(M(T) + \frac{\Re}{6}\right)\phi^2 + \frac{\tilde{a}}{4}\phi^4\right] \equiv \\ &-\left[\frac{1}{2} \cdot \left(M(T) + \frac{\kappa}{6a^2(t)}\right)\phi^2 + \frac{\tilde{a}}{4}\phi^4\right] \end{aligned} \tag{2}$$

This is pre supposing $\kappa \equiv \pm 1,0$, that one is picking a curvature signature which is compatible with an open universe.

That means $\kappa = -1,0$ as possibilities. So we will look at the $\kappa = -1,0$ values . We begin with.

$$\begin{aligned} &\ddot{\phi} - \nabla^2 \phi + \frac{\partial V}{\partial \phi} = 0 \Rightarrow \\ &\phi^2 = \frac{1}{\tilde{a}} \cdot \left\{ c_1^2 - \left[\alpha^2 + \frac{\kappa}{6a^2(t)} + M(T)\right]\right\} \\ &\Leftrightarrow \phi \equiv e^{-\alpha \cdot r} \exp(c_1 t) \end{aligned} \tag{3}$$

We find the following as far as basic phenomenology, namely

$$\phi^2 = \frac{1}{\tilde{a}} \cdot \left\{ c_1^2 - \left[\alpha^2 + \frac{\kappa}{6a^2(t)} + \left(M(T) \approx \varepsilon^+ \right) \right] \right\} \quad (4)$$

$$\xrightarrow[M(T\sim high)\to 0]{} \phi^2 \neq 0$$

$$\phi^2 = \frac{1}{\tilde{a}} \cdot \left\{ c_1^2 - \left[\alpha^2 + \frac{\kappa}{6a^2(t)} + \left(M(T) \neq \varepsilon^+ \right) \right] \right\} \quad (5)$$

$$\xrightarrow[M(T\sim Low)\neq 0]{} \phi^2 \approx 0$$

The difference is due to the behavior of $M(T)$. We use $M(T) \sim$ axion mass $m_a(T)$ in asymptotic limits with

$$m_a(T) \cong 0.1 \cdot m_a(T=0) \cdot (\Lambda_{QCD}/T)^{3.7} \quad (6).$$

Appendix V **Interesting speculation. Does there exist a five dimensional version of an instanton in the worm hole transition regime?**

We will attempt to build the contribution as to a Reissner-Nordstrom metric embedded in a five dimensional space-time metric , and see if this satisfied . i.e. .look at (1) below This allows us to determine, using of the Risessner-Nordstrom metric as given, by Kip Thorne, Wheeler, and Misner [31], for an added cosmological ‘constant’ Λ and ‘charge’ Q. This will be shown to lead to [30]

$$\begin{aligned} M_g(r) &= \int \left[T_0^0 - (T_1^2 + 2 \cdot T_2^2) \right] \cdot \sqrt{-g_4}\, dV_3 \\ &\approx \pi \cdot c_1^2 \cdot \left[\frac{r^3}{3} - 2M \cdot \frac{r^2}{2} + Q \cdot r - \frac{\Lambda}{15} \cdot r^5 \right] + \\ & 4\pi \cdot c_1 \cdot \left[r^2 - 8 \cdot M \cdot r - \frac{\Lambda}{3} \cdot r^4 \right] \xrightarrow[r \to \delta]{} \varepsilon^+ \approx 0 \end{aligned} \quad (1)$$

To do this, we start off with the following space time line metric in five dimensions. This is a modification of Wesson’s book .[30]

$$\begin{aligned} dS_{5-\dim} &= [\exp(i\pi/2)] \cdot \left\{ \begin{array}{l} e^{2\Phi(r)} dt^2 \\ + e^{2\tilde{\Lambda}(r)} dr^2 + R^2 d\Omega^2 \end{array} \right\} \\ &+ (-1) \cdot e^{\mu} dl^2 \end{aligned} \quad (2)$$

We claim that what is in the $\{\ \}$ brackets is just the Reissner-Nordstrom line metric in four dimensional space. The parameters in the $\{\ \}$ bracket are linked to the Reissner-Nordstrom metric via

$$e^{2\Phi(r)} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) \tag{3}$$

And

$$e^{2\tilde{\Lambda}(r)} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} \tag{4}$$

And this is assuming that $R \sim r$ as well as using $\mu \approx c_1 \cdot r$ with a maximum value topped off by a Planck's length value due to $\mu_{Maximum} \approx c_1 \cdot r_{Maximum} \sim l_P \equiv 10^{-35}\,cm$. So being the case, we get the following stress tensor values

$$T_0^0 = \left(\frac{-1}{8\pi}\right) \cdot \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2\right) \cdot \left(\frac{c_1^2}{4} + \frac{c_1}{r} + \frac{c_1}{4} \cdot \left[\frac{\frac{2M}{r^2} - \frac{2Q}{r^3} - \frac{2\Lambda r^2}{3}}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2}\right]\right) \tag{5}$$

$$T_1^1 = \left(\frac{-1}{8\pi}\right) \cdot \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2\right) \cdot \left(\frac{c_1}{r} + \frac{c_1}{4} \cdot \left[\frac{\frac{2M}{r^2} - \frac{2Q}{r^3} - \frac{2\Lambda r^2}{3}}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2}\right]\right) \tag{6}$$

$$T_2^2 = T_3^3 = \left(\frac{-1}{8\pi}\right) \cdot \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2\right) \cdot \left(\frac{c_1^2}{4} + \frac{c_1}{r} + \frac{c_1}{2} \cdot \left[\frac{\frac{2M}{r^2} - \frac{2Q}{r^3} - \frac{2\Lambda r^2}{3}}{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2}\right]\right) \tag{7}$$

Furthermore, we get the following determinant value

$$\sqrt{-g_4} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} r^2\right) \tag{8}$$

All these together lead to (1) being satisfied. Let us now see how this same geometry contributes to a worm hole bridge and a solution as to forming the instanton flux wave functional between a prior to a present universe. The Reissner-Nordstrom metric permits us to have a radiation dominated 'matter' solution whose matter 'contribution' drops off rapidly as the spatial component of geometry goes to zero. This is in tandem with radiation pressure and density falling off rapidly, as we leave the center of such a purported soliton/ instanton. This is extremely useful because it ties in with the notion of fractional branes contributing to entropy calculations. In fact it is useful to state that these two notions dove tail with each other quite closely. The only difference is that the construction above does not in itself lend to the complexity of what we would observe, which is in itself a multiple – joined net work of charge centers and of shifting geometry.

Appendix VI. Basic physics of achieving minimum precision in CMBR power spectra measurements

Begin first of all looking at

$$\frac{\Delta T}{T} \equiv \sum_{l,m} a_{lm} Y_{l,m}(\theta, \phi) \tag{1}$$

This leads to consider what to do with

$$C_l = \left\langle \left| a_{l,m} \right|^2 \right\rangle \tag{2}$$

Samtleben et al [32] consider then what the experimental variance in this power spectrum, to the tune of an achievable precision given by

$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}} \cdot \left(\frac{1}{\sqrt{f_{sky}}} + \frac{4\pi \cdot (\Delta T_{\exp})^2}{C_l} \cdot \sqrt{f_{sky}} \cdot e^{l^2 \sigma_b^2} \right) \tag{3}$$

f_{sky} is the fraction of the sky covered in the measurement , and $\Delta T_{\exp}$ is a measurement of the total experimental sensitivity of the apparatus used. Also σ_b is the width of a beam , while we have a minimum value of $l_{\min} \approx (1/\Delta\Theta)$ which is one over the fluctuation of the angular extent of the experimental survey.

I.e. contributions to C_l uncertainty from sample variance is equal to contributions to C_l uncertainty from noise. The end result is

$$4\pi \cdot f_{sky} = C_l \cdot \left(\exp\left[-l^2\sigma^2\right]\right)/(\Delta T)^2 \tag{4}$$

Appendix VII. Vacuum fluctuations which may occur : Cosmological perturbation theory and tensor fluctuations (Gravity waves)

Durrer reviews how to interpret C_l in the region where we have $2 < l < 100$, roughly in the region of the Sachs-Wolf contributions due to gravity waves. We begin first of all by looking at an initial perturbation , using a scalar field treatment of the ' Bardeen potential' Ψ This can lead us to put up, if H_i is the initial value of the Hubble expansion parameter

$$k^3|\Psi|^2 \cong \left(\frac{H_i}{M_P}\right)^2 \tag{1}$$

And

$$\left\langle|\Psi|^2\right\rangle \cdot k^3 = A^2 k^{n-1} \cdot \eta_0^{n-1} \tag{2}$$

Here we are interpreting $A =$ amplitude of metric perturbations at horizon scale, and we set $k = 1/\eta_0$, where η is the conformal time, according to $dt \equiv a d\eta =$ physical time, where we have a as the scale factor.

Then for $2 < l < 100$, and $-3 < n < 3$, and a pure power law given by

$$\left\langle\left|H(k, \eta = 1/k)\right|^2\right\rangle \cdot k^3 = A_T^2 k^{n_T} \cdot \eta_0^{-n_T} \tag{3}$$

We get for tensor fluctuation, i.e. gravity waves,, and a scale invariant spectrum with $n_T = 0$

$$C_l^{(T)} \approx \frac{A_T^2}{(l+3)\cdot(l-2)} \cdot \frac{1}{15\pi} \tag{4}$$