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The Mass of the Electron-Neutrino Expressed by Known Physical Constants

László I. Orbán

Institutul Agronomic Cluj
Cluj-Napoca, Romania

Abstract

Many trials attempted to understand the neutrinos ever since Pauli theoretically concluded its existence from the conservation of energy calculations. The present paper demonstrates that commencing from two appropriately chosen measurement systems, the mass of the electron-neutrino can be calculated from the mass of the electron and the fine-structure constant. The mass of the neutrino can be determined by the theoretically derived expression $m_k = \alpha^3 m_e$ (m_k is the mass of the neutrino, m_e is the mass of electron, α is the fine-structure constant).
Keywords: Electron-Neutrino, Neutrino Mass

1 Introduction

Wolfgang Pauli proposed the existence of neutrinos in a letter written on December 4th, 1930 in relation to the decay of β, in the interest of maintaining consistency with the law of conservation of energy [1]. However, 26 years passed by before Fred Reines and Clyde Cowan described their experimental observation of the electron-neutrino in 1956 [2]. Later, another group detected the existence of the μ neutrino and the τ neutrino [3]. Early on, the mass of the neutrino was considered to be zero, similar to the resting mass of the photon. A non-zero neutrino mass value was first observed through the investigation of solar-neutrinos [4]. It turns out that solar energy production produces a great amount of neutrinos, which according to Bethe's calculations reach Earth without interruptions [5]. Consequently, detectors were built for the purpose of observing solar-neutrinos, such as the Italian Gallex, the Caucasian SAGE and the Japanese Super-Kamiokande, which measured less than expected amounts of neutrinos. The missing neutrinos may be explained by Pontecorvo's [6] suggestions on neutrino-oscillations. Neutrino-oscillation proposes that neutrinos may transform into one another. This transformation may only take place if at least one of the neutrinos has a non-zero mass. The calculated energy difference is very small, approximates 0.01 eV.

Today, we know that the mass of the neutrino may deviate from zero, which directed empirical investigations to approximate the mass of the neutrino by studying β-decay and inverse-β-decay. In Mainz, the mass of the neutrino was approximated to be 2.5 eV, whereas CUORICINO's measurements gave 0.16 - 0.84 eV [7], and Klapdor et al's, values were estimated at 0.1 - 0.9 eV [8]. University College London researchers offered the latest measurements in June 2010, where they reported 0.28 eV based on studying 700,000 galaxies [9].

2 Universal Measurement Systems

Planck has introduced a measurement system in 1899 in which the speed of light (c), the gravitational constant (G) and the quantum of action (h) were considered as basic units. Distance, time and mass units in this measurement system produced:

$$\left(G\frac{h}{c^3}\right)^{\frac{1}{2}} \approx 4.05 \cdot 10^{-35} m; \quad \left(G\frac{h}{c^5}\right)^{\frac{1}{2}} \approx 1.35 \cdot 10^{-43} s; \quad \left(h\frac{c}{G}\right)^{\frac{1}{2}} \approx 5.46 \cdot 10^{-8} Kg$$

These values cannot be interpreted at the atomic or subatomic levels. Aside from G, c and h constants used in Planck's system of units, we are aware of many other universal constants. For example, elementary charge (e), mass of the electron (m_e), the classical radius of the electron (r_e), the photon's angular momentum ($\hbar$), the mass of the proton (m_p), the radius of the observable universe (R_u), and others. Different kinds of universal measurement systems can be observed based on these constants. There are universal measurement systems in which the values of certain basic and derived units correspond in the two systems, but the chosen quantities are not always identical. For example, the unit of force is the same in Planck's (G, h, c) and in the (G, e, c) systems: $\frac{c^4}{G} \approx 1.21 \cdot 10^{44} N$. ($G$, e, c) is a universal measurement system where values of G, e and c are unit length. The following discussion will make use of the observation that the units of acceleration are identical in both universal measurement systems of $(e, m_e, \hbar)$, and (m_k, c, $\hbar$) if the quantity of neutrino's mass (m_k) is defined to be 0.2 $eV \approx$ 0.354 x 10^{-36}Kg.

3 The (e, m_e, $\hbar$) Universal Measurement System

The advantage of (e, m_e, $\hbar$) universal measurement systems is that all basic and derived units can be expressed with universal quantities. In these systems, the electron charge (e), the electron mass (m_e) and the photon's angular momentum ($\hbar$) are basic units. Let m_u, k_u and s_u mean the length, mass and time's units in this new measurement system (herein noted as (m_u, k_u, s_u)). The relationship between the MKS system and the (m_u, k_u, s_u) measurement system would be $1m = x_1 m_u$; $1kg = x_2 k_u$ and $1s = x_3 k_u$. To determine the values of x_1, x_2 and x_3, we can use e, m_e and $\hbar$ values as units in the (m_u, k_u, s_u) measurement system. That is:

$$\begin{array}{lll} e x_2^{\frac{1}{2}} x_1^{\frac{3}{2}} x_3^{-1} = 1 & \rightarrow x_3^2 = e^2 x_2 x_1^3 & \\ m_e x_2 = 1 & \rightarrow x_2 = \frac{1}{m_e} & \rightarrow \\ \hbar x_2 x_1^2 x_3^{-1} = 1 & \rightarrow x_3 = \hbar x_2 x_1^2 & \end{array} \quad \begin{array}{l} x_1 = \frac{4\pi^2 e^2 m_e}{h^2} = \frac{\alpha^2 c^2 m_e}{e^2} \\ x_2 = \frac{1}{m_e} \\ x_3 = \frac{8\pi^3 e^4 m_e}{h^3} = \frac{\alpha^3 c^3 m_e}{e^2} \end{array}$$

Here, $\alpha = \frac{2\pi e^2}{hc}$ is the fine-structure constant.

The relationship between the MKS and the (m_u, k_u, s_u) systems is the following:

$$1m_u = \frac{1}{x_1}\text{m} = \frac{e^2}{\alpha^2 c^2 m_e}\text{m};\ 1k_u = \frac{1}{x_2}\text{kg} = m_e\text{kg};\ 1s_u = \frac{1}{x_3}\text{s} = \frac{e^2}{\alpha^3 c^3 m_e}\text{s}$$

The derived units, velocity, acceleration, force, etc., can be formed using simple algebraic operations (see Table 1). Additionally, in this universal measurement system, basic and derived units can be expressed with quantities of electron data found in the hydrogen atom's first Bohr orbit, without any quantum conditions.

Table 1. The relationship between the MKS and the $(e, m_e, \hbar)$ systems

System	Length (m)	Mass (kg)	Time (s)	Velocity ms^{-1}	Acceleration ms^{-2}	Force (N)	Energy (J)	Momentum $(kgms^{-1})$	Angular Momentum (Js)
$e, m_e, \hbar \rightarrow 1$	$\frac{e^2}{\alpha^2 c^2 m_e}$	m_e	$\frac{e^2}{\alpha^3 c^3 m_e}$	αc	$\frac{\alpha^4 c^4 m_e}{e^2}$ $\approx 9.0456 \cdot 10^{22}$	$\frac{\alpha^4 c^4 m_e^2}{e^2}$	$\alpha^2 m_e c^2$	$\alpha m_e c$	$\frac{h}{2\pi}$

$1m_u = (e^2/\alpha^3 c^3 m_e)$s is the radius of the first Bohr orbit, $1k_u = (m_e)$kg is the mass of the electron and $1s_u = (e^2/\alpha^3 c^3 m_e)$s is $1/2\pi$ times the period of the electron on the first Bohr orbit. The unit of velocity, $1m_u s_u^{-1} = (\alpha c)\text{ms}^{-1}$ is the electron's orbiting velocity on the first Bohr orbit. The unit of acceleration is $1m_u s_u^{-2} = \frac{\alpha^4 c^4 m_e}{e^2}\text{ms}^{-2} \approx 9.04558 \cdot 10^{22} ms^{-2}$. This is the acceleration of the electron in the first Bohr orbit. The unit of force is $1k_u m_u s_u^{-2} = \alpha^4 c^4 m_e^2/e^2$ N. This is the amount of force the proton attracts the electron on the first Bohr orbit. The unit of energy is $1k_u m_u^2 s_u^{-2} = \alpha^2 c^2 m_e$ J.

The energy of the electron on the first Bohr orbit is actually α^2 multiple of the electron's own energy, $m_e c^2$.

4 The $(m_k, c, \hbar)$ Universal Measurement System

We arrive at a noteworthy measurement system if we view the mass of neutrino m_k, the speed of light c and the photon's spin $\hbar$ as basic units. Let us use the following notations m_a, k_a, and s_a for length, mass and time's units (noted herein as $m_a k_a s_a$).

The relationship between the MKS and the $(m_a k_a s_a)$ system is $1m = y_1 m_a$, $1kg = y_2 k_a$ and $1s = y_3 s_a$. To determine the values of y_1, y_2 and y_3, we can use m_k, c and $\hbar$ values as units in the (m_k, c, $\hbar$) measurement system. That is:

$$m_k y_2 = 1 \qquad y_2 = \frac{1}{m_k}$$
$$c y_1 y_3^{-1} = 1 \qquad y_3 = c y_1$$
$$\hbar y_2 y_1^2 y_3^{-1} = 1 \qquad y_3 = \hbar y_2 y_1^2$$

The solution of the equation system, using the alpha constant, is the following:

$$y_1 = \frac{\alpha m_k c^2}{e^2} \qquad y_2 = \frac{1}{m_k} \qquad y_3 = \frac{\alpha m_k c^3}{e^2}$$

Substituting y_1, y_2 and y_3, we observe the following relationship:

$$1m_a = \frac{1}{y_1}\mathrm{m} = \frac{e^2}{\alpha m_k c^2}\mathrm{m};\ 1k_a = \frac{1}{y_2}\mathrm{kg} = m_k\mathrm{kg};\ 1s_a = \frac{1}{y_3}\mathrm{s} = \frac{e^2}{\alpha m_k c^3}\mathrm{s}$$

The derived units, velocity, acceleration, force, etc., can be formed using simple algebraic operations (see Table 2).

Table 2. The relationship between the MKS and the (m_k, c, $\hbar$) systems

System	Length (m)	Mass (kg)	Time (s)	Velocity ms^{-1}	Acceleration ms^{-2}	Force (N)	Energy (J)	Momentum $(kgms^{-1})$	Angular Momentum (Js)
$m_k, c, \hbar \to 1$	$\frac{e^2}{\alpha c^2 m_k}$	m_k	$\frac{e^2}{\alpha c^3 m_k}$	c	$\frac{\alpha c^4 m_k}{e^2}$	$\frac{\alpha c^4 m_k^2}{e^2}$	$m_k c^2$	$m_k c$	$\frac{h}{2\pi}$

According to Table 2, the unit of acceleration is: $1m_a s_a^{-2} = \frac{\alpha m_k c^4}{e^2} ms^{-2}$.

Note that if $m_k = 0.3540388 \cdot 10^{-36} kg$, which is comparable to CUORICINO detector's measurements, than $1m_a s_a^{-2} = 9.04559 \cdot 10^{22} ms^{-2}$ is derived. This value corresponds with the $(e, m_e, \hbar)$ system's acceleration unit's value, $1m_u s_u^{-2} = \frac{\alpha^4 c^4 m_e}{e^2} \approx 9.04559 \cdot 10^{22}\mathrm{ms}^{-2}$; therefore, we can arrive at the following relationship:

$$\frac{\alpha m_k c^4}{e^2} = \frac{\alpha^4 c^4 m_e}{e^2}$$

After reduction: $\boxed{m_k = \alpha^3 m_e}$

In words, the value of the electron-neutrino's mass is defined by the product of the fine-structure constant's third power and the mass of the electron. This is the mass of the electron-neutrino's simplest expression, derived using semi-empirical methods.

If $\alpha = \frac{1}{137.0291}$ and $m_e = 9.1093897 \cdot 10^{-31}\mathrm{kg}$, then:

$$m_k = 0.3540388 \cdot 10^{-36} kg \approx 0.198898 eV \approx 0.2 eV.$$

This is a theoretical value that empirical observations will surely confirm.

5 Conclusions

A semi-empirical examination of the measurement systems led to a prediction of the mass of the electron-neutrino that corresponds with values derived using empirical methods.

At an atomic level, the most optimal measurement system appears to be universal $(e, m_e, \hbar)$ system. Every basic and derived unit is expressed by known units, which are linked to the Universe's most common element, hydrogen. A further curious observation is that even without any quantum conditions, every basic and derived unit is linked to the hydrogen atom's first quantum orbit.

The relationship $m_k = \alpha^3 m_e$, observed for the mass of electron-neutrino, calls attention to the possibility that the exploration of universal measurement systems could yield new interrelations for both the worlds of micro-cosmos and macro-cosmos.

The basic and derived units of the $(m_k, c, \hbar)$ measurement system can probably be linked with the cosmic neutrino-medium's data. As the measurement of this data is still to be determined, the interpretation of the $(m_k, c, \hbar)$ measurement system is a task for the future.

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