Cosmic acceleration in a model of interacting massive gravitons

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Abstract: We propose an alternative understanding of gravity resulting from the extension of N.Wu's gauge theory of gravity with massive gravitons, which are minimally coupled to general relativity. Based on this, we derive the equations of state for massive gravitons. We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe. We calculate the critical points of the massive graviton dark energy and background perfect fluid. These calculations may have crucial implications for the massive gravitons and dark energy theories. They could, therefore, have important repercussions for current cosmological problems.

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1 Introduction

Observations of type Ia supernovae and of large-scale structure (LSS), in combination with measurements of the characteristic angular size of fluctuations in the cosmic microwave background (CMB) [1–10] provide evidence that the expansion of the Universe is accelerating. This acceleration is attributed to the "dark energy", a hypothetical energy with negative pressure [1–6]. Evidence for the presence of a dark energy is also provided by an independent, albeit more tentative, investigation of the integrated Sachs-Wolfe (ISW) [11].

The dark energy may result from Einstein's cosmological constant (which is of a phenomenally small value); from evolving scalar fields [12]; and from a weakening of gravity in our 3 + 1 dimensions by leaking into the higher dimensions, as required in string theories [13]. These explanations may have crucial implications for both massive gravitons and dark energy. This has stimulated further efforts to confirm the initial results on dark energy, test possible sources of error and extend our empirical knowledge of this newly discovered component of the Universe.

Although the most discussions of magnetic fields in the universe use a scalar field (because of its simplicity), the vector field also has applications in modern cosmology. In [14, 15], for instance, the vector field is considered as the source which drives the inflation; in [16, 17] the Lorentz-violated vector field and its effects to universe are studied; the quantum fluctuations of vector fields produced at the first stage of reheating after inflation is also studied in [18]; the cosmology of massive vector fields with SO(3) global symmetry is investigated in [19], For overviews of the literature on magnetic fields in the universe see [20, 21,22] and references therein.

Building on this earlier work, N. Wu proposed a mechanism which introduces massive gravitons without violating the local gauge symmetry of the Lagrangian ([23], [24], [25], and [26]). The third-order gravitational gauge field $C_{3\mu}^{a}$ is massive if mass is very large. Such a field has no contribution to the long-range gravitational force. Long-range gravitational force results exclusively from the contribution of the fouth-order gravitational gauge field $C_{4\mu}^{a}$ and obeys inverse square law.

If the mass term of gravitational gauge field is extremely small, however, the third-order gravitational gauge field $C_{3\mu}^{a}$ will contribute to the middle range gravitational force within an approximate range $L \approx \frac{hc}{m}$ (where h is the Plank constant and c is the speed of light). For graviton mass $2 \times 10^{-7} eV$ the gravitational force range will be about one meter.

In the traditional gauge treatment of gravity the Lorentz group is localized. The gravitational field is, thus, not represented by gauge potential, but by the metric field $g_{\mu\nu}$.

Here, we propose an alternative understanding of gravity, resulting from the extension of N. Wu's gauge theory of gravity with massive gravitons that are minimally coupled to general relativity. Based on this, we derive the equations of state for massive gravitons. We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe and calculate the critical points of the massive graviton dark energy and background perfect.

These calculations may have crucial implications for the theory of both massive gravitons and dark energy. They could, therefore, have important repercussions for current cosmological problems.

2 Gauge theory of gravity with massive graviton

Taking Wu's gauge model as our starting point [23-32], we introduce two gravitational gauge fields $(C^a_\mu, C^a_{2\mu})$ simultaneously. Since $(C^a_\mu, C^a_{2\mu})$ are vectors in Lie algebra ([8], [9], [25]), they can be expanded as

$$C_\mu = C^a_\mu \hat{P}_a \quad , \; C_{2\mu} = C^a_{2\mu} \hat{P}_a \, . \tag{1}$$

These correspond with two gauge covariant derivatives

$$D_\mu = \partial_\mu - igC_\mu(x) \quad , \; D_{2\mu} = \partial_\mu + iagC_{2\mu}(x) \tag{2}$$

and two different field strengths, given by

$$\begin{aligned} F_{\mu\nu} &= \frac{1}{-ig}[D_\mu, D_\nu] \\ F_{2\mu\nu} &= \frac{1}{iag}[D_{2\mu}, D_{2\nu}] \end{aligned} \tag{3}$$

The Lagrangian of the system is given by

$$\begin{aligned} \Im_0 = &-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^b_{\mu\nu} F^c_{\rho\sigma} - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^b_{2\mu\nu} F^c_{2\rho\sigma} \\ &- \frac{m^2}{2(1+a^2)}\eta^{\mu\nu} g_{bc} (C^b_\mu + aC^b_{2\mu})(C^c_\nu + aC^c_{2\nu}) \end{aligned} \tag{4}$$

where m is the constant mass parameter. The action of the system is given by

$$S = \int d^4x J(C) \Im_0 \tag{5}$$

where

$$J(C) = \sqrt{-\det(g_{ab})}, \tag{6}$$

and

$$g_{ab} = \eta_{\mu\nu} (G^{-1})^\mu_a (G^{-1})^\nu_b . \tag{7}$$

For the definition of G-matrix and its inverse G^{-1}–matrix see [23].

Equation (4) gives the mass term in gravitational gauge fields. To obtain the eigenstates of mass matrix the following rotation is needed

$$C_{3\mu} = \cos\theta C_{\mu} + \sin\theta C_{2\mu}$$
$$C_{4\mu} = -\sin\theta C_{\mu} + \cos\theta C_{2\mu} \tag{8}$$

where the angle θ is given by

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}, \quad \sin\theta = \frac{a}{\sqrt{1+a^2}} \tag{9}$$

After transformation (8), the Lagrangian of the system is given by

$$\Im_0 = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^{b}_{30\mu\nu} F^{c}_{30\rho\sigma} - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^{b}_{40\mu\nu} F^{c}_{40\rho\sigma}$$
$$-\frac{m^2}{2}\eta^{\mu\nu} g_{bc} C^{b}_{3\mu} C^{c}_{3\nu} + \Im_I \tag{10}$$

whereas $F^{a}_{30\mu\nu}$ $F^{a}_{40\mu\nu}$ are given by:

$$F^{a}_{30\mu\nu} = \partial_{\mu} C^{a}_{3\nu} - \partial_{\nu} C^{a}_{3\mu}$$
$$F^{a}_{40\mu\nu} = \partial_{\mu} C^{a}_{4\nu} - \partial_{\nu} C^{a}_{4\mu} \tag{11}$$

From the above follows that the gauge field, $C^{a}_{3\mu}$ has mass m_c, whereas the gravitational gauge field, $C^{a}_{4\mu}$ is massless.

Transformation (8) is pure algebraic operations which do not affect the gauge symmetry of the Lagrangian [23]. They can, therefore, be regarded as redefinitions of gauge fields. The local gauge symmetry of the Lagrangian is still strictly preserved after field transformations. In other words, the symmetry of the Lagrangian before transformations is absolutely the same with the symmetry of the Lagrangian after transformations. We do not introduce any kind of symmetry breaking in this paper.

3. Interacting massive gravitons and dark energy with general relativity

Our aim is to study the dynamics of such a massive graviton in the presence of gravity. Here, those massive gravitons, $C^{a}_{3\mu}$ are minimally coupled to general relativity:

$$S = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \left[\sum_{a=1}^{3}\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{ac} F^{b}_{30\mu\nu} F^{c}_{30\rho\sigma} + V(C^{a2})\right] + \Im_m\right\}, \tag{12}$$

where g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, $F^{a}_{30\mu\nu} = \partial_{\mu} C^{a}_{3\nu} - \partial_{\nu} C^{a}_{3\mu}$, and $\Im_m$ is the Lagrangian density of the matter field.

The term $V(C^{a2})=\frac{m_c^2}{2}\eta^{\mu\nu}g_{bc}C^b_{3\mu}C^c_{3\nu}$, is a self-interaction. In this, m_c is the mass of the graviton. The self-interaction term is introduced into the theory without violating the strict local gravitational gauge symmetry.

We use the Einstein summation convention throughout, where a sum is implied only over indices in opposite positions. The indices that label the different vectors are raised and lowered with the flat "metric" δ_{ab}. Varying action (12) with respect to the metric, we obtain the Einstein equations $G_{\mu\nu}=8\pi T_{\mu\nu}$. In these, the energy momentum tensor of the triad is given by

$$^{(C)}T_{\mu\nu}=\sum_a\left\{C^a_{3\mu\rho}C^{a\rho}_{3\nu}+2\frac{dV}{dC^{2a}}C^a_{3\mu}C^a_{3\nu}-\left[\frac{1}{4}C^a_{\rho\sigma}C^{a\rho\sigma}+V(C^{2a})\right]\right\} \tag{13}$$

This energy momentum tensor is the sum of the three different energy momentum tensors of the decoupled $^{(C)}T_{\mu\nu}=\sum_a {}^{(a)}T_{\mu\nu}$, neither of which is of perfect-fluid form.

By varying the action with respect to the massive gravitons $C^a_{3\mu}$, one obtains their equations of motion,

$$\partial_\mu\left(\sqrt{-g}C^{a30\mu\nu}\right)=2\sqrt{-g}\,\frac{dV}{dC^a}C^{a3\nu} \tag{14}$$

The four-divergence of the last equation yields a constrain equation for the massive graviton. As a consequence, each massive graviton has three dynamical degrees of freedom, as it should. We shall study the dynamics of these massive gravitons in a flat, homogeneous and isotropic FRW universe with metric

$$ds^2=-dt^2+a^2(t)dx^2 \tag{15}$$

An ansatz for the massive gravitons that turns out to be compatible with the symmetries (homogeneity and isotropy) of this metric is

$$C^a_{3\mu}=\delta^b_\mu C(t)^{(2)}\alpha \tag{16}$$

Hence, the three vectors point to three mutually orthogonal spatial directions and share the same time-dependent length, $C^{2a}=C^a_{3\mu}C^{a3\mu}=C(t)$. Substituting ansatz (16) and the metric (15) into the vector equations of motion (14) we find

$$\ddot{C}+3H\dot{C}+\left(H^2+\frac{\ddot{a}}{a}\right)C+\frac{dV}{dC}=0 \tag{17}$$

where a dot stands for a derivative with respect to cosmic time t. Note that the 0-component of equation (14) forces C^a_{30} to vanish, as in ansatz (16). Substituting metric (15) into the Einstein equations we obtain

$$H^2 = \frac{8\pi G}{3}\rho, \tag{18}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{19}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble "constant". The energy density of the universe is $\rho = -T_0^0$ and its pressure is defined by $p\delta_j^i = T_j^i$, where i and j run over the spatial space-time components. Note that the energy momentum tensor has to be compatible with the symmetries of the metric. For the FRW metric (15), should be $G_i^0 = 0$, so that $T_i^0 = 0$ should also vanish. It can be easily verified that this is indeed the case for ansatz (16). The requirement of isotropy is non-trivial for a single vector, since its energy momentum tensor is

$$^{(a)}T_j^i = \left(\frac{1}{2}(\dot{C} + HC)^2 - V\right)\delta_j^i + \left(2\frac{dV}{dC^2}C - (\dot{C} + HC)^2\right)\delta_j^i\delta_j^a \tag{20}$$

Although this energy momentum tensor is diagonal, its value along the j = a direction is different from the one along the directions perpendicular to it. Nevertheless, the total energy momentum tensor $^{(C)}T_{\mu\nu} = \sum_a {}^{(a)}T_{\mu\nu}$ has isotropic stresses. The corresponding energy density ρ_C and (isotropic) pressure p_C are given by

$$\rho_C = \frac{3}{2}(\dot{C} + HC)^2 + 3V(C^2), \tag{21}$$

$$p_C = \frac{1}{2}(\dot{C} + HC)^2 - 3V(C^2) + 2\frac{dV}{dC^2}C^2 \tag{22}$$

Note that the equation of motion (17) can be also derived from the condition

$$\dot{\rho}_C + 3H(\rho_C + p_C) = 0. \tag{23}$$

The most remarkable feature of the massive graviton dark energy is that its Equations of State (EoS) $w_C = p_C / \rho_C$ can be smaller than −1, while possessing a conventional positive kinetic term. This is due to the additional term in proportion to dV / dC^2 in p_C. While the energy density ρ_C is positive, one can find that the condition for $w_C < -1$ is

$$\frac{dV}{dC^2}C^2 < -\left(\dot{C} + HC\right)^2. \tag{24}$$

Thus, it is necessary that $dV / dC^2 < 0$.

4. Dynamical system of massive graviton dark energy interacting with background perfect fluid

Let us now consider the interaction between massive gravitons and background matter. Here, the background matter is described by a perfect fluid with barotropic equation of state

$$p_m = w_m \rho_m = (\gamma - 1)\rho_m \tag{24}$$

where the barotropic index γ is a constant and satisfies the condition $0 < \gamma < 2$. In particular, $\gamma = 1$ and γ= 4/3 correspond to dust matter and radiation, respectively. We assume that massive graviton dark energy and background matter interact through an interaction term C, as follows:

$$\dot{\rho}_C + 3H(\rho_C + p_C) = -C, \tag{25}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = C, \tag{26}$$

which preserves the total energy conservation equation $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0.$

It is worth noting that the equation of motion (17) should be changed when C≠0. In this case, a new term due to C will appear in its right hand side. Following [34, 35-40, 41-50], we introduce the following dimensionless variables

$$x = \frac{k\dot{C}}{\sqrt{6}H}, y = \frac{k\sqrt{V}}{\sqrt{3}H}, z = \frac{k\sqrt{\rho_m}}{\sqrt{3}H}, u = \frac{kC}{\sqrt{6}} \tag{27}$$

With the application of equations (18) to (22), the evolution equations (25) and (26) can be rewritten as a dynamical system [33]:

$$x' = 6[(x+u)^2 + \frac{\gamma}{4}z^2 + \Theta](x+u) - 2\Theta u^{-1} - 3x - 2u - C_1, \tag{28}$$

$$y' = 6y[(x+u)^2 + \frac{\gamma}{4}z^2 + (1 + \frac{1}{3}xy^{-2}u^{-1})\Theta], \tag{29}$$

$$z' = 6z[(x+u)^2 + \frac{\gamma}{4}z^2 + \Theta - \frac{\gamma}{4}] + C_2, \tag{30}$$

$$u' = x, \tag{31}$$

where

$$\Theta = \frac{u^2}{H^2}\frac{dV}{dC^2} \tag{32}$$

and

$$C_1 = \frac{k^2 C}{18H^3}(x+u)^{-1}, \ C_2 = \frac{zC}{2H\rho_m}, \tag{33}$$

a prime denoting derivative with respect to the so-called e-folding time $N = \ln a$. Here we use

$$-\frac{\dot{H}}{H^2} = 6[(x+u)^2 + \frac{\gamma}{4} z^2 + \Theta]. \qquad (34)$$

The Friedmann constraint is

$$3[(x+u)^2 + y^2] + z^2 = 1 \text{ ([51])} \qquad (35)$$

The fractional energy densities of massive gravitons dark energy Ω_C and background matter Ω_m are given by

$$\Omega_C = \frac{\kappa^2 \rho_C}{3H^2} = 3[(x+u)^2 + y^2], \;\; \Omega_m = \frac{\kappa^2 \rho_m}{3H^2} = z^2, \qquad (36)$$

respectively. The EoS of massive graviton dark energy and the effective EoS of the whole system are

$$w_C = \frac{p_C}{\rho_C} = \frac{(x+u)^2 - 3y^2 + 4\Theta}{3[(x+u)^2 + y^2]}, \qquad (37)$$

and

$$w_{eff} = \frac{p_{tot}}{\rho_{tot}} = \Omega_C w_C + \Omega_m w_m, \qquad (38)$$

respectively.

From equation (37), it is easy to see that the condition for $w_C < -1$ is

$$(x+u)^2 + \Theta < 0, \qquad (39)$$

which is equivalent to equation (36).

Finally, it is worth noting that $y \geq 0$ and $z \geq 0$ by definition and that, in what follows, we only consider the case of expanding universe with $H > 0$. It is easy to see that equations (28)– (31) become an autonomous system when the potential $V(C^2)$ is chosen to be an inverse power-law or exponential potential and the interaction term C is chosen to be a suitable form.

Let us consider the model with exponential potential. The interaction between massive graviton dark energy and background perfect fluid, which is taken as the most familiar interaction term, an extensively considered in the literature [35-55], is expressed by $C = 3\sigma H \rho_C$, where σ dimensionless constant.

Here, we consider the massive graviton dark energy model with an exponential potential

$$V(C^2) = V_0 \exp(-\lambda \kappa^2 C^2), \qquad (40)$$

where λ is a positive dimensionless constant (required by the condition $(dV / dC^2 < 0)$. In this case,

$$\Theta = -3\lambda u^2 y^2. \tag{41}$$

One can obtain the critical points $(\bar{x},\bar{y},\bar{z},\bar{u})$ of the dynamical system expressed by equations (28)–(31) from equations (40) and (41) by imposing the conditions $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = 0.$ Note that these critical points must satisfy the Friedmann constraint (35), $\bar{x} > 0, \bar{z} > 0$ and the requirement of $\bar{x},\bar{y},\bar{z},\bar{u}$ all being real. To study the stability of these critical points, we substitute linear perturbations $x \to \bar{x} + \delta x,$ $y \to \bar{y} + \delta y,, z \to \bar{z} + \delta z,$ and $u \to \bar{u} + \delta u$ about the critical point $(\bar{x},\bar{y},\bar{z},\bar{u})$ into the dynamical system of equations (28)– (31) with Eqs. (40) and (41) and linearize these equations . Due to the Friedmann constraint (35), there are only three independent evolution equations, namely

$$\begin{aligned}\delta x' &= 6[3(1+3\lambda\bar{u}^2)(\bar{x}+\bar{u})^2 - 2\lambda\bar{u}(\bar{x}+\bar{u}) + (\lambda\bar{u} + \frac{\gamma}{4})\bar{z}^2 - \lambda\bar{u}^2 - \frac{1}{2}]\delta x \\ &+4\bar{z}[3(\bar{x}+\bar{u})(\lambda\bar{u}^2 + \frac{\gamma}{4}) - \lambda\bar{u}]\delta z + 2\{18\lambda\bar{u}(\bar{x}+\bar{u})^3 + 3(9\lambda u^2 + 3 - \lambda)(\bar{x}+\bar{u})^2 \\ &-6\lambda\bar{u}(\bar{x}+\bar{u}) + [6\lambda\bar{u}(\bar{x}+\bar{u}) + 3(\lambda\bar{u}^2 + \frac{\gamma}{4}) - \lambda](\bar{z}^2 - 1) + \frac{3}{4}\gamma - 1\}\delta u - \delta C_1,\end{aligned} \tag{42}$$

$$\begin{aligned}\delta z' &= 12\bar{z}(1+3\lambda\bar{u}^2)(\bar{x}+\bar{u})\delta x + 6[(1+3\lambda\bar{u}^2)(\bar{x}+\bar{u})^2 + (\lambda\bar{u}^2 + \frac{\gamma}{4})(3\bar{z}^2 - 1)]\delta z \\ &+12\bar{z}[3\lambda\bar{u}(\bar{x}+\bar{u})^2 + (1+3\lambda\bar{u}^2)(\bar{x}+\bar{u}) + \lambda\bar{u}(\bar{z}^2 - 1)]\delta u + \delta C_2,\end{aligned} \tag{43}$$

$$\delta u' = \delta x, \tag{44}$$

where δC_1 and δC_2 are the linear perturbations stemming from C_1 and C_2, respectively. The three eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point.

Let us now consider the case when $C = 3\sigma H \rho_C$. In this case,

$$C_1 = \frac{3}{2}\sigma[(x+u)^2 + y^2](x+u)^{-1} = \frac{\sigma}{2}(1-z^2)(x+u)^{-1}, \tag{45}$$

$$C_2 = \frac{9}{2}\sigma[(x+u)^2 + y^2]z^{-1} = \frac{3}{2}\sigma(z^{-1} - z). \tag{46}$$

The physically reasonable critical points of the dynamical system equations (28)-(31) with equations (40)-(41) are shown in Table .1,where

$$r_1 = \frac{\sigma}{-4+3\gamma}, \tag{47}$$

$$r_2 = \sqrt{-3\gamma\lambda\sigma(\gamma+\sigma) + (\gamma - \gamma\lambda + \sigma)^2}. \tag{48}$$

Next, we consider the stability of these critical points. Substituting

$$\delta C_1 = -\frac{\sigma}{2}(\bar{x}+\bar{u})^{-2}(1-\bar{z}^2)\delta x - \sigma\bar{z}(\bar{x}+\bar{u})^{-1}\delta z - \frac{\sigma}{2}(\bar{x}+\bar{u})^{-2}(1-\bar{z}^2)\delta u, \tag{49}$$

$$\delta C_2 = -\frac{3}{2}\sigma(\bar{z}^{-2}+1)\delta z\,, \tag{50}$$

and the corresponding critical point $(\bar{x},\bar{y},\bar{z},\bar{u})$ into equations (42)– (44), we find that (E.I.1) is unstable if it can exist, while points (E.I.2) and (E.I.3) exist and are stable in a proper parameter-space (see [51]for more details) respectively. The late time attractors (E.I.2) and (E.I.3) both have

$$\Omega_C = \frac{\gamma}{\gamma+\sigma},\ \ \Omega_m = \frac{\sigma}{\gamma+\sigma},\ \ w_C = -1-\sigma,\ \ w_{eff} = -1, \tag{51}$$

which are scaling solutions. Note that w_C of attractors (E.I.2) and (E.I.2) are both smaller than -1, since $\sigma > 0$ is required by their corresponding Ω_m.

Table.1 Critical points for $C = 3aH\rho_C$ in the model with massive gravitons exponential potential. r_1 and r_2 are given in equations (47) and (48) respectively ([51]).

Label	Critical point $(\bar{x},\bar{y},\bar{z},\bar{u})$
E.I.1	$0, 0, (3r_1)^{1/2}, \pm(\frac{1}{3}-r_1)^{1/2}$
E.I.2	$0, \sqrt{\frac{\gamma(1+\lambda)+\sigma-r_2}{6\lambda(\gamma+\sigma)}}, \sqrt{\frac{\sigma}{\gamma+\sigma}}, \pm\sqrt{\frac{\gamma(-1+\lambda)-\sigma+r_2}{6\lambda(\gamma+\sigma)}}$
E.I.3	$0, \sqrt{\frac{\gamma(1+\lambda)+\sigma-r_2}{6\lambda(\gamma+\sigma)}}, \sqrt{\frac{\sigma}{\gamma+\sigma}}, \pm\sqrt{\frac{\gamma(-1+\lambda)-\sigma+r_2}{6\lambda(\gamma+\sigma)}}$

In the scaling attractor, the effective densities of massive graviton dark energy and background matter decrease in the same manner with the expansion of our universe, and the ratio of massive graviton dark energy and background matter becomes a constant. So, it is not strange that we are living in an $C = 3\sigma H\rho_C$ epoch when the densities of massive graviton dark energy and matter are comparable. In this sense, the (first) cosmological coincidence problem is alleviated (see [35-55] for examples).

As is explicitly shown in this work, for suitable interaction forms [for instance $C = 3\sigma H\rho_C$], regardless of the model with exponential potential, there are some attractors with $w_C < -1$ while their corresponding Ω_C and Ω_m are comparable in the interacting massive graviton dark energy model. In the case $C = 3\sigma H\rho_C$, all stable attractors have these desirable properties. So, for a fairly wide range of initial conditions with $w_C > -1$, the universe will eventually evolve to the scaling attractor(s) with $w_C < -1$. Similar to the ordinary way to alleviate the (first) cosmological coincidence problem, the second cosmological coincidence problem [35-55] is alleviated at the same time.

5 Conclusions

We propose an alternative understanding of gravity resulting from the extension of N.Wu's gauge theory of gravity with massive gravitons, which are minimally coupled to general relativity. The mass term of gravitational gauge field is introduced into the theory without violating the strict local gravitational gauge symmetry.

For these massive gravitons , we derive the equations of state that satisfy the dark energy condition.

We study the dynamics of these massive gravitons in a flat, homogeneous and isotropic FRW universe and calculate the critical points of the massive graviton dark energy and background perfect fluid.

These calculations may have crucial wider implications for the physics of massive gravitons and dark energy. They could, therefore, have important repercussions for current cosmological problems.

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