IS A NON ZERO "ZEROTH" ORDER GRAVITON MASS DERIVABLE IN BRANE THEORY SETTINGS AND KALUZA KLEIN GRAVITON MODELS? HOW DOES A LOWEST ORDER KK GRAVITON NON ZERO MASS AFFECT THE BLUE SPECTRUM FOR GRAVITONS?

Andrew Beckwith
Chongqing University Department of Physics;
e-mail: abeckwith@uh.edu
Chongqing, PRC, 400044

Abstract

The lowest order mass for a KK graviton, as a non zero product of two branes interacting via a situation similar to Steinhardt's ekpyrotic universe is obtained, as to an alternative to the present dogma specifying that gravitons must be massless. The relative positions as to the branes gives a dynamical picture as to how lowest order KK gravitons could be affected by contraction and then subsequent expansion. Initially we have bulk gravitons as a vacuum state. The massless condition is just one solutino to a Stern Liuouville operator equation we discuss, which with a non zero lowest order mass for a KK graviton permits modeling of gravitons via a dynamical Casmir effect which we generalize using Durrer's 2007 work. In particular the blue spectrum for (massless gravitons), is revisited, with consequences for observational astrophysics.

Keywords: Gravitons, blue spectrum, KK theory, Casmir effect

1 Introduction

We make use of work done by Ruser and Durrer [1] which is essentially a re do of the Steinhardt model of the *ekpyrotic* universe [2] [3]. With two branes. One of which is viewed to be stationary and the other is moving toward and away from the stationary brane.

The construction used, largely based upon the Ruser and Durrer [1] article makes used of a set of differential equations based on the Sturm Liouville method which in the case of the zeroth order mass being zero have in usual parlance a zero value to lowest order KK graviton mass [1].We will turn this idea on its head by having a non zero graviton mass, zeroth order in the KK construction as to show how graviton mass, lowest order is affected by a Casmir plate treatment of graviton dynamics.

2 Setting up a Casmir effect for zeroth order 'massive' KK gravitons.

What we will do is to examine via figure 1 from [1] below the dynamics of the two branes with one stationary and the other moving, which influence a close form solution of the zeroth order graviton mass problem.

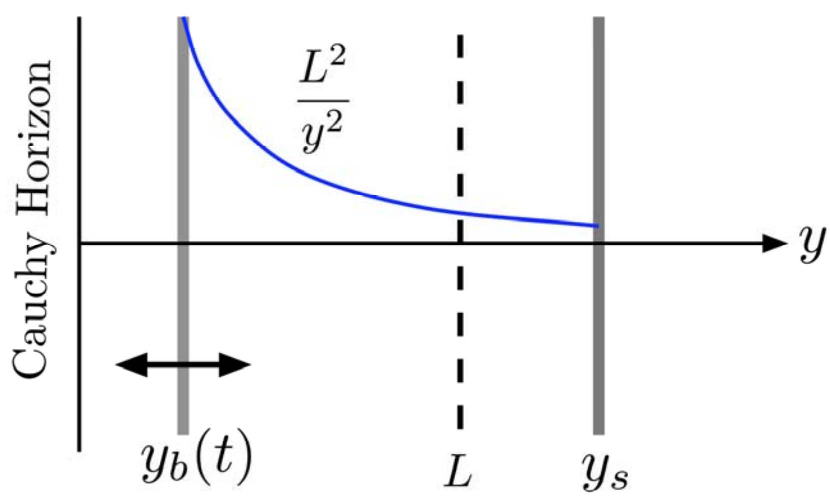

Figure 1, from [1]

Using [1] what we find is that there are two branes on the AdS_5 space-time so that with one moving and one stationary, we can look at figure 1 which is part of the geometry used in the spatial decomposition of the differential operator acting upon the $h_{\bullet}$ Fourier modes of the h_{ij} operator [1] . As given by [1], we have that

$$\left[\partial_t^2 + k^2 - \partial_y^2 + \frac{3}{y}\cdot\partial_y\right] h_{\bullet} = 0 \tag{1}$$

Spatially, (1) can be, in its configuration as having

$$\left[\partial_y^2 + \frac{3}{y}\cdot\partial_y\right]\Phi_\alpha(t,y) = m_\alpha^2(t)\,\Phi_\alpha(t,y) \qquad (2)'$$

What we will do, instead of looking at a Sturm Liouville operator, as was done in [ ] is instead to look at an inner product treatment of the zeroth order mass as can be accessed in a KK decomposition of a graviton, and to consider though using

$$\left[\partial_y^2 + \frac{3}{y}\cdot\partial_y\right]\Phi_0(t,y) = m_0^2(t)\,\Phi_0(t,y) \tag{3}$$

Standard treatment of the problem represented in (3) is to use the RHS of (3) as set equal to zero. That allows for the "solution" to (3), namely $\Phi_0(t,y) = \Phi_0 =$ constant with respect to space. Our substition is given below:

An ansatz can be placed into the (3) results above, with, say,

$$\Phi_0(t,y) = \Phi_0\left(\widehat{k},y\right) = A\cos\left(\widehat{ky}\right) \tag{4}$$

Our next approximation is to keep the product $\widehat{ky}$ real valued and do a power series expansion of (4) above. Also, we keep the following normalization intact from [1]

$$\int_{y1}^{y2}\frac{dy}{y^3}\cdot\Phi_\alpha(y)\,\Phi_\beta(y) = \delta_{\alpha,\beta} \tag{5}$$

The right hand side is a Kroniker delta, and so it is equal to zero often. So we look at, then if we take an "inner product" procedure as to (4) above we have then the zeroth order mass for a graviton as written up as

$$m_0^2(t) = \widehat{k}^2 \cdot \left(1 - \left[\frac{ -\frac{2}{y^2}\Big|_{y1}^{y2} - 8\widehat{k}^2 \ln y\Big|_{y1}^{y2} + 16\widehat{k}^4 y^2\Big|_{y1}^{y2} - \ldots }{ -\frac{2}{y^2}\Big|_{y1}^{y2} - \widehat{k}^2 \ln y\Big|_{y1}^{y2} + \widehat{k}^4 y^2\Big|_{y1}^{y2} - \ldots } \right] \right) \tag{6}$$

The time dependence as to the above zeroth value comes from looking at if $y1 = y_b$, and $y2 = y_S$ are such with having ,by figure 1 above, $y1 = y_b$ moving $y2 = y_S$ not able to move, so that (6) definitely has a time dependence. The term $\widehat{k}$ is a term which can be fixed by requirements as to the initial conditions in (5) are met, and equal to 1 when $\alpha = \beta$ and = 0. The end result is that the (6) is the zeroth order mass term which is not equal to 0. We submit that the entire procedure behind (6) is similar in part to how QM is not essential in setting up initial conditions in the matter of Planck's constant [4].

3 Lessons from Gryzinski, as far as semiclassical derivation of a usually assumed quantum derivation of Inelastic Scattering in Atomic Hydrogen and its implications as to (3) and (6) as given in [4].

We will review the derivation of what is normally assumed to be a quantum result, with the startling implications that a cross section formula, normally quantum, does not need usual Hilbert space construction (usually Hilbert space means quantum mechanics). We will briefly review the Gryzinski result [5], [6] which came from something other than Hilbert space construction and then make our comparison with the likelihood of doing the same thing with respect to forming the zeroth order value of a graviton mass, as not equal to zero, by (3) above without mandating the existence of Hilbert spaces in the electroweak era. Gryzinski [5], [6] starts off with what is called an excitation cross section given by

$$Q(U_n) = \frac{\sigma_0}{U_n^2} g_j \left(\frac{E_2}{U_n}; \frac{E_1}{U_n} \right) \tag{7}$$

Where

$$g_j \left(\frac{E_2}{U_n}; \frac{E_1}{U_n} \right) = \left(\frac{E_2}{E_1 + E_2} \right)^{3/2} \cdot \Phi \tag{8}$$

And

$$\Phi \equiv \frac{2}{3} \cdot \frac{E_1}{E_2} + \frac{U_n}{E_2} \cdot \left(1 - \frac{E_1}{E_2}\right) - \left(\frac{U_n}{E_2}\right) \quad if \quad U_n + E_1 \le E_2 \tag{9}$$

And

$$\Phi \equiv \left[\frac{2}{3} \cdot \frac{E_1}{E_2} + \frac{U_n}{E_2} \cdot \left(1 - \frac{E_1}{E_2}\right) - \left(\frac{U_n}{E_2}\right)\right] \cdot \sqrt{\ } \quad if \quad U_n + E_1 \ge E_2 \tag{10}$$

With

$$\sqrt{\ } = \sqrt{\left(1 + \frac{U_n}{E_1}\right) \cdot \left(1 - \frac{U_n}{E_2}\right)} \tag{11}$$

The write up of (7) to (11) has $\sigma_0 = 6.53 \times 10^{-14} cm^2 eV^2$, and U_n being energy of level n, and E_1 being the energy of the bound electron , and E_2 being the energy of the incident electron. We refer the reader to access [5] as to what the value of the Born approximation used as a comparison with (11) above. The result was that the Gryzinski's approximation gives scattering cross sections lower than those of the Born approximation although the shape of the curves for cross sectional values are almost the same, with the difference between the Gryzinski approximation and the Born approximation in value closed in magnitude, with principal quantum numbers increased The net effect though is that having a Hilbert space, I.e. assuming that the presence of a Hilbert space implies the Quantum condition, is not always necessary for a typical quantum result. Now, how does that argument as to Hilbert spaces not being necessary for presumed quantum results relate to how to obtain (3)?

4 **In particular the blue spectrum for (massless gravitons), no longer holds, if gravitons have a slight mass with consequences for observational astrophysics. If (6) holds, the spectrum for light mass gravitons has a different character.**

We refer to (3) and (6) as giving a non zero value of the zeroth order mass of a graviton in KK theory, and then try to re focus upon the more traditional 4 space definition of GW expansion in order to come up with normal modes. To do this, look at the mode equation in 4 space and its analogy to higher dimensions [1]. In 4 space, the mode equation reads as

$$\ddot{\chi}_k + \left[k^2 - \frac{\ddot{a}}{a} \right] \chi_k \sim \ddot{\chi}_k + \left[k^2 - m_0^2 \right] \chi_k = 0 \qquad (12)$$

Usually $m_0 = 0$, but if it is not equal to zero, then the (12) equation has a more subtle meaning. Consider from Ruser and Durrer [1] what (12) is turned into, in a more general setting. It gets exotic, namely

$$\ddot{q}_{\alpha,k,\bullet} + \left[k^2 - m_\alpha^2 \right] q_{\alpha,k,\bullet} + \sum_\beta \left[M_{\beta,\alpha} - M_{\alpha,\beta} \right] \cdot \dot{q}_{\beta,k,\bullet} + \sum_\beta \left[\dot{M}_{\alpha,\beta} - N_{\alpha,\beta} \right] \cdot q_{\beta,k,\bullet} = 0 \qquad (13)$$

The obvious connection between the two (12) and (13) is that one will have if $\alpha = 0$, then one observes

$$\sum_\beta \left[M_{\beta,\alpha=0} - M_{\alpha=0,\beta} \right] \cdot \dot{q}_{\beta,k,\bullet} + \sum_\beta \left[\dot{M}_{\alpha=0,\beta} - N_{\alpha=0,\beta} \right] \cdot q_{\beta,k,\bullet} = 0 \qquad (14)$$

So, does one have, then, that we can ask if the coefficients in (14) are going to be zero? i.e. can we say that

$$\left[M_{\beta,\alpha=0} - M_{\alpha=0,\beta} \right] = \left[\dot{M}_{\alpha=0,\beta} - N_{\alpha=0,\beta} \right] = 0 \qquad (15)$$

For them to become zero, then we should note by Ruser and Durrer [1] , that by [7]

$M_{\beta,\alpha=0}, M_{\beta,\alpha\neq 0}, M_{\alpha=0,\beta}, M_{\alpha\neq 0,\beta}, N_{\alpha=0,\beta}, N_{\alpha\neq 0,\beta}$ have been already derived in detail in [7] . Furthermore matrix M is defined by brane motion and

$$N = M^T M \qquad (16)$$

The claim we have is that **if (6) holds, then (15), and (16) does not hold.** We claim that if (15) does not hold, one is observing conditions for which the blue spectrum for massless gravitons cannot be true, if the initially massless zeroth order KK gravitons becomes massive. In order to understand this though, we should look at what an expert had to say about massive gravitons, h_{ij} and the formation of $h_\bullet$

5 Conclusion, a necessary Review of Physics of linkage between $h_{\bullet}, h_{ij}$ and massive Gravitons

First of all, review the details of a massive graviton imprint upon h_{ij}, and then we will review the linkage between that and certain limits upon $h_{\bullet}$

As read from Hinterbichler [8],if $r = \sqrt{x_i x_i}$, and we look at a mass induced h_{ij} suppression factor put in of $\exp(-m \cdot r)$, then if

$$h_{00}(x) = \frac{2M}{3M_{Planck}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \tag{17}$$

$$h_{0i}(x) = 0 \tag{18}$$

$$\begin{aligned} h_{ij}(x) &= \left[\frac{M}{3M_{Planck}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \right] \cdot \\ &\left(\frac{1 + m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^2} \cdot \delta_{ij} - \left[\frac{3 + 3m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^4} \right] \cdot x_i \cdot x_j \right) \end{aligned} \tag{19}$$

Here, we have that these h_{ij} values are solutions to the following equation, as given by [8], [9], with D a dimensions value put in.

$$\left(\partial^2 - m^2 \right) h_{\mu\nu} = -\kappa \cdot \left[T_{uv} - \frac{1}{D-1} \cdot \left(\eta_{uv} - \frac{\partial_\mu \partial_\nu}{m^2} \right) \cdot T \right] \tag{20}$$

To understand the import of the above equations, set

$$\begin{aligned} M &= 10^{50} \cdot 10^{-27} g \equiv 10^{23} g \propto 10^{61} - 10^{62} eV \\ M_{Plank} &= 1.22 \times 10^{28} eV \end{aligned} \tag{21}$$

We should use the $m_{massive-graviton} \sim 10^{-26} eV$ value in (21) above.If the h_{ij} massive graviton values are understood, then we hope we can make sense out of the general uncertainty relationship given by [10]

$$\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}^{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{vol}{}^2} \quad (22)$$

In reviewing what was said about (22) we should keep in mind the overall Fourier decomposition linkage between $h_{\bullet}, h_{ij}$ which is written up as

$$h_{ij}\left(t, x; k\right) = \frac{1}{\left(2\pi\right)^{3/2}} \int d^3k \sum_{\bullet=+,\otimes} e^{ik\cdot x} e_{ij}^{\bullet} h_{\bullet}\left(t, y; k\right) \quad (23)$$

The bottom line is that the simple de composition with a basis in two polarization states, of $+, \otimes$ will have to be amended and adjusted , if one is looking at massive graviton states, and if we are going to have a coupling as given by (6), 4 dimensional zeroth order mass massive graviton values, and the input of information given in (17) to (21) as given by [1] Having a a simple set of polarization states as given by $+, \otimes$ will have to be replaced, mathematically by a different de composition structure, with the limit of massive gravitons approaching zero reducing to the simpler $+, \otimes$ basis states.

Acknowledgements

Thanks to Dr. Corda to suggesting that QM does not necessary be the initial starting point of gravititational physics [11].The main point of the Gryzinski derivation is that one does not need Hilbert space (usually associated with QM) to obtain what is thought to be a quantum physics result. This work is supported in part by National Nature Science Foundation of China grant No110752

References

[1] Ruth Durrer et.al., “Dynamical Casmir effect for gravitons in Bouncing Braneworlds”, 2009,
http://theory.physics.unige.ch/~durrer/papers/casimir.pdf

[2] J. Koury, P.J. Steinhardt,N. Turkok, PRL92, 031302(2004),hep-th/0307132

[3] J. Koury, P.J. Steinhardt,N. Turkok, PRL91, 161301(2003),astro-ph/0302012

[4] A. Beckwith, “Is Quantum Mechanics Involved at the Start of Cosmological Evolution? Does a Machian Relationship Between Gravitons and Gravitinos Answer This Question?”, http://vixra.org/abs/1206.0023; accepted for publication in Hadronic journal

[5]C.F. Monnin, and G.M. Prok, “Comparison of Gryzinski’s and Born’s Approximation for Inelastic scattering in Atomic Hydrogen”,

http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19650017671_1965017671.pdf
[6]M. Gryzinski, “Classical theory of Electronic and Ionic Inelastic Collisions”**Phys Rev** Vol. 115, no2, July 15, 1959, pp 374-383.
[7] C.Cartier, Ru. Durrer, M. Ruser, PRD 72, 104018(2005),hep-th/0510155
[8]K.Hinterbichler, “Theoretical Aspects of Massive Gravity”, http://arxiv.org/abs/1105.3735.pdf
[9] http://web.mit.edu/redingtn/www/netadv/Xgravitati.html
[10] T.G.Downes,G.J.Milburn,“Optimal Quantum Estimation for Gravitation”, http://xxx.lanl.gov/abs/1108.5220
[11]G. t’ Hooft, “Quantum Mechanics as a dissipative deterministic system” http://arxiv.org/PS_cache/gr-qc/pdf/9903/9903084v3.pdf