DOES THE QUANTUM SUM RULE HOLD AT THE BIG BANG AND WHAT ABOUT QUANTUM MEASURES AT COSMIC SINGULARITIES?

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Abstract

In Dice 2010 Sumati Surya brought up a weaker Quantum sum rule as a biproduct of a quantum invariant measure space. Our question is, does it make sense to have disjoint sets to give us quantum conditions for a measure at the origin of the big bang? We argue that the answer is no, which has implications as to quantum measures and causal set structure. The entire supposition as to the incompatibility of quantum measures at a singularity means that our assumptions of Quantum gravity have to be revisited as to 4 dimensional singularities, and that some embedding of 4 dimensional space time, inititally in line with work brought up by Dowker, et al, 2007 may suggest a solution to initial measures giving credence to t'Hoofts embedding of QM within a deterministic semi classical structure. A Weyl quantization wave function (4 dimensions) as embedded in a 5 dimensional extended version of the Wheeler De Witt equation with an added pseudo time component added serves as an analogy as to how to get about the problem mentioned in the introduction of this text.

A Introduction

First of all, we are working with the formalism introduced by Surya [1] and submit that it breaks down spectularly at a singularity. We introduce the formalism by appealing to the concept of spatial diffeomorphism [2] as a necessary condition for linking the physics of what happens at a singularity to outside of the singularity of inflation generated space time geometry. Trivially, a diffeomorphism involves an infinitely differentiable, one-to-one mapping of the model to itself. In contrast, there is a breakdown of differentiability at the start of the big bang, based on non-loop-quantum-gravity theories. We submit that the difficulties in terms of consistency of Eq. (1) of this document. In terms of initial causal structural breakdown -- which we claim leads to Eq. (1) being re written as an inequality -- one has to come up with a different way to embed quantum measures within a superstructure, as noted in the conclusions of this paper. Spatial diffiomorphisms as stated in [2] do not work unless there is a lattice structure, effectively doing away with a singularity. If the lattice structure is not used, differentiability breaks down and one does not have one-to- one mapping of the physics of the big bang singularity onto the rest of the inflationary process. We submit that this breakdown would then make Eq. (1) and then later Eq. (19) not definable.

As to the measure set structure, the readers are referred to [3] to get the foundations of the measure theory structure understood. The rest of this text is an adoptation of what was done in [1], with the author's re interpretation of what the significance is of quantum measures as stated in [1], in the vicinity of a singularity. The author's main point is that there is a break down of measurable structure, starting with definitions given in [1] and [2] where the concept of disjoint sets becomes meaningless in a point of space.

The text takes this spectacular break down of dis joint sets and asks what can be done to replace the sigularity, where the break down occurs.

In the causal set approach, the probabilities are held to be Markovian [1], label-independent and adhere to Bell's inequality. The author of [1] refers to a sequential growth called a classical transition percolation model. Then reference [1] extends the classical transition percolation model to complex models involving quantum measures in the definition of a (quantum) complex percolation model. Reference [1] defines the amplitude of transition as follows. For a quantum measure space defined as triple as given by (Ω, A, μ_V), with μ_V a yet to be defined vector measure, A is an event algebra or set of propositions about the system, and Ω is the sample space of histories or space-time configurations.

Let $p \ \varepsilon \ C$ be amplitude of transition, instead of a probability; and set $\psi\left(C^n\right)$ as the amplitude for a transition from an empty set to n element of a causal set C^n, and with $Cyl\left(C^n\right)$ cylinder set as a subset of Ω containing labeled past finite causal sets whose first n elements form the causal subset C^n. Note that the cylinder sets form event algebra A with measure given by form the sub-causal set C^n. Here, ψ is a complex measure on A, so then ψ is a vector measure [1]. This is the primary point of breakdown that occurs in the case of a space time singularity. Away from the singularity we will be working with the physics of

$$D\left(Cyl\left(C^n\right), Cyl\left(C'^n\right)\right) = \psi^*\left(C^n\right)\psi\left(C'^n\right) \tag{1}$$

This is done for a cylinder set [1], where γ is a given path, and γ^t as a truncated path, with $cyl\left(\gamma^t\right)$ a subset of Ω and $\mu\left(cyl\left(\gamma^t\right)\right) = P\left(\gamma^t\right)$, with $P\left(\gamma^t\right)$ the probability of a truncated path, with a given initial $\left(x_i, t_i\right)$ to final $\left(x_f, t_f\right)$ spatial and times. Note that the μ measure would be for $\mu : A \to R^+$ obeying the weaker Quantum sum rule [4]

$$\mu(\alpha \cup \beta \cup \gamma) = \mu(\alpha \cup \beta) + \mu(\alpha \cup \gamma) + \mu(\beta \cup \gamma) - \mu(\alpha) - \mu(\beta) - \mu(\gamma) \tag{2}$$

This probability would be a quantum probability which would *not* obey the classical rule of Kolmogrov [1]

$$P\left(\gamma_1 \cup \gamma_2\right) = P\left(\gamma_1\right) + P\left(\gamma_2\right) \tag{3}$$

The actual probability used would have to take into account quantum interference. That is due to Eq. (1a) and Kolmogrov probability no longer applying, leading to [1]

$$cyl\left(\gamma^{t}\right)\equiv\left\{\gamma\in\Omega\,|\,\gamma\left(t'\right)=\gamma^{t}\left(t'\right)\ for\quad all\quad 0\le t'\le t\right\} \tag{4}$$

Here, $D:\mathrm{A}\times\mathrm{A}\rightarrow C$ is a decoherence functional [1], which is (i) Hermitian, (ii) finitely biadditive, and (iii) strongly additive [5], i.e., the eignvalues of D constructed as a matrix over the histories $\{\alpha_i\}$ are non-negative.

A quantum mesurement is then defined via

$$\mu\left(\alpha\right)=D(\alpha,\alpha)\ge 0 \tag{5}$$

A quantum vector measurment is defined via

$$\mu_V\left(\alpha\right):=\left[\chi_\alpha\right]\in H \tag{6}$$

Where

$$\chi_\alpha\left(\beta\right)=\begin{cases}1\\0\end{cases},\qquad \chi_\alpha\left(\beta\right)=\ 1\text{ if }\quad\beta=\alpha\,,\ \chi_\alpha\left(\beta\right)=0\text{ if }\quad\beta\ne\alpha \tag{7}$$

Also V is the vector space over A with an inner product given by

$$\left\langle u,v\right\rangle_V\equiv\sum_{\alpha\in A}\sum_{\beta\in A}u^{*}\left(\alpha\right)v\left(\beta\right)\cdot D\left(\alpha,\beta\right) \tag{8}$$

with a Hilbert space H constructed by taking a sequence of Cauchy sequences $\{u_i\}$ sharing an equivalence relationship

$$\{u_i\}\sim\{v_i\}\text{ if }\lim_{i\to\infty}\left\|u_i-v_i\right\|_V=0 \tag{9}$$

So then as given in [1], the following happens,

$$\left[\{u_i\}\right]+\left[\{v_i\}\right]\equiv\left[\{u_i+v_i\}\right] \tag{10}$$

$$\left[\{\lambda u_i\}\right]\equiv\lambda\left[\{u_i\}\right] \tag{11}$$

$$\left\langle\left[\{u_i\}\right],\left[\{v_i\}\right]\right\rangle\equiv\lim_{i\to\infty}\left\langle u_i,v_i\right\rangle_V \tag{12}$$

This is for all $\left[\{u_i\}\right],\left[\{v_i\}\right] \in H$ and $\lambda \in C$ so then the quantum measure is defined for $\mu_V : A \rightarrow H$ so the inner product on H is

$$\left\langle \mu_V(\alpha), \mu_V(\beta) \right\rangle = D(\alpha,\beta) \tag{13}$$

The claim associated with Eq. (1) above is that since ψ is a complex measure of A , Eq. (1) corresponds to an unconditional convergence of the vector measure over all partitions. Secondly according to the Caratheodary-Hahn theorem there is unconditional convergence for classical stochastic growth, but this is not necessarily always true for a quantum growth process.

The main point of the formalism for Eq. (13) is of bi-additivity of D leading to the finite addivity of μ_V

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right) = \sum_{i=1}^{n}\mu_V(\alpha_i) \tag{14}$$

B**. Looking at Arguments against Eq. (1) in the vicinity/ origin of the big bang singularity**

The precondition for a quantum measure μ_V for a quantum measurement is given by Eq. (14) [1] for *n* disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$. This Eq. (14) is a math pre condition for μ_V being a vector measure over A . Eq (14) right at the point of the big bang cannot insure the existence of *n* disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$. Therefore at the loci of the big bang one would instead get, due to non-definable disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$, a situation definable as, at best.

$$\mu_V\left(\bigcup_{i=1}^{n}\alpha_i\right) \neq \sum_{i=1}^{n}\mu_V(\alpha_i) \tag{15}$$

Not being able to have a guarantee of having *n* disjoint sets $\alpha_i \;\; \varepsilon \;\; \mathrm{A}$ because of singular conditions at the big bang will bring into question whether Eq. (1) can hold and the overall research endeavor of analyzing the existence of quantum measures μ_V. I.e., the triple (Ω, A, μ_V) for quantum measures μ_V cannot be guaranteed to exist. More importantly, the statement that there exists $\psi(C^n)$ from an empty set to a nth element causal set cannot be adhered to, and Eq. (1) cannot exist, since there would be no causal set structure at the loci of the big bang.

C. Making sense out of QM and also wave-particle duality.

So what can be inferred? If discontinuous set structures do not exist at the onset of the big bang in effectively measure set zero space, with non existent length then what is left ? We get into all sorts of difficulties. Our assumption is that a breakdown of a quantum measure would probably be congruent with the breakdown of use of QM, at the onset of the big bang. The Eq. (16) is a simple quantum argument: how QM, i.e., the wave-particle duality structure, falls falls apart,. Assume that we have ultra-light gravitons, with a tiny rest mass. Then a simple quantum argument gives [6]

$$\begin{aligned} & m_{graviton}\Big|_{RELATIVISTIC} < 4.4\times 10^{-22} h^{-1} eV/c^2 \\ & \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 2.8\times 10^{-8} meters \end{aligned} \tag{16}$$

I.e., the smaller the right hand side of Eq. (16) gets, the heavier the rest graviton mass is, which would raise problems for at ultra-short wave lengths. The obvious generalization of Eq. (16) would be for a mass M

$$\begin{aligned} & M\ \Big|_{RELATIVISTIC} < \#\, given\quad h^{-1} eV/c^2 \\ & \Leftrightarrow \lambda \ \equiv \frac{\hbar}{M\ \cdot c} < \#\, given\ \ 10^{\beta}\, meters \end{aligned} \tag{17}$$

One could then get, as in Eq. 16 and Eq, 17 , a situation in which

$$m_{graviton}, M \to \infty \Leftrightarrow \lambda \to 0^{+} \tag{18}$$

With a point source, i.e., an infinitely small wavelength, the effective mass would go to huge, non-physical values. Since Eq.(16) and Eq. (17) are based on quantum structure, the shorter the wavelength, the less physical the problem becomes, as shown in Eq. (18) , until we get to the absurdity of an infinitely massive gravition or an infinitely massive particle for an infinitely short wavelength. I.e., not only would there be as we go to a point structure, no disjoint causal structure, our very physics as we understand QM insight would become non-tendable. This would lead to a problem with the causal set discretization procedure, for analysis next.

D. QM, wavelengths, and problems with quantum measurement with Eq.(14) and justification for Eq. (19)

As stated by [1], one can think of causal sets as part of a partial ordering of space-time, and replace the space-time continuum with locally finite partially ordered sets [7] ,[8]. We assert that in place of Eq. (1), which uses the notion of partially ordered sets, that instead one has a break down in the applications of dis joint set measure theory mathematics in the immediate neighborhood of a singularity. So at the singularity there exists a break with the simple structure of quantum measures [1] in a way which will suggest what was brought up by t'Hooft [9] as far as fixing the problem identified below.

$$D\left(Cyl\left(C^{n}\right),Cyl\left(C'^{n}\right)\right)\neq\psi^{*}\left(C^{n}\right)\psi\left(C'^{n}\right) \tag{19}$$

The inequality as given in Eq. (19) occurs due to finite partitions no longer being relevant if we have a measure set zero (zero 'length') at a cosmic singularity . At a singularity, the finite partitions [10] as given in Eq. (20) no longer hold.

$$\pi\left(\alpha\right)=\left\{\alpha_{\rho}\right\},\alpha\in A \tag{20}$$

If there are finite partitions due to a non measure zero set (due to a cosmic singularity) , we have an equality , rather than an inequality for Eq(19) , due to Eq. (20). Note that the supremum (least upper bound) is taken over all finite partitions , with the partitions given by Eq. (20).
And then we look at whether there is sufficiently convergent behavior for μ_{V} , so that uniqueness of convergent sequenes is guaranteed by the Caratheodary-Hahn –Huvanek theorem. If so, the following supremum expression for all FINITE partitions will lead to the equality expression for vector measures.

$$\left|\mu_V\left(\alpha\right)\right| = \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{21}$$

Having a singularity removes applications of Eq. (20). The singularity will not allow us to analyze disjoint partitions. What happens if instead of Eq. (21) [1] a situation for which there is longer finite partitions, ordered sets, but the replacement for Eq. (21) is now an inequality written as:

$$\left|\mu_V\left(\alpha\right)\right| \neq \sup_{\pi(\alpha)} \sum_{\rho} \left\|\mu_V\left(\alpha_\rho\right)\right\| \tag{22}$$

Or worse, a situation where there is no finite partially ordered set, i.e., no *cuasal* set ? The inequality of Eq.(22) can occur if there is no finite disjoint sets to make a supremum over.

Suppose that Eq. (20) no longer holds. Suppose there a no partitions or ordered causal sets at a singularity, so Eq (21) no longer holds.

Eq. (1) depends upon having [1] an "*unconditional convergence of the vector measure over all partitions.*" Replace partitions with causal set structure, and one still has the same requirement of an *unconditional convergence of the vector set over all "causal set structure"* within a finite geometric regime of space-time. One does not get about the necessity of conversgence of sequences and sub sequences in a causal set structure. The convergence of sequences and sub sequences has the same rules as when causal set structure is replaced by partitions.

Surya's construction [1] of taking a least upper bound (supremum) over finite partitions does not work if there are no finite partitions at a singularity. So then, we cannot force QM, with an infinitely small "wavelength," i.e., infinitely small measure. QM geometry cannot be super imposed in space time upon a cosmic singularity, i.e. the big bang itself.

E. Conclusion? Back to a deterministic treatment of QM, as suggested by t'Hooft QM

[1] Suggests a way out of the impasse. We look at unconditional convergence over all partitions. Now if unconditional convengence is not

doable for a singular point in space time, in which we tried to have Quantum measures constructed, then we have to look at how the same singular point for the big bang is embedded via higher dimensional analogs to a non singular space time structure. Secondly, is to not insist upon the situation given in Eq. (16) and Eq. (17) as to be forced upon point space in space time geometry. I.e. looking at what was said by the author "as to " *real" complex percolation models in which one accepts that a quantum measure is not additive, as in Eq.(15), but that " the observables of the theory are identical to those of the classical transitive percolations.*

If we can put additional geometry necessary to avoid break down of the quantum measure at a singularity about the big bang singular point via use of classical *transitive percolations and relate that imposition of additional analytical structure to observables identical to classical transitive percolations, we are on the way to fixing the problem of the Quantum measure [10]. I.e. this may be a way to be making an analytical structure consistent* with t'Hooft's embedding of Quantum mechanics within a higher dimensional theory, as that would fix the problems of disappearance of necessary geometry permitting disjoint sets in space time with the quantum measure [9],[10] and QM as given in the limits in Eq. (16) and Eq. (17). The fundamental prloblem as stated in the beginning remains that disjoint sets, and causal structure cannot be evaluated at a point of space time.

The only way to avoid the problem of break down of the mathematics of a construction of a quantum measure is to embed the 4 dimensional representation of a point, in space time, in geometry, five or more dimensions, which no longer are a point, or singularity in space time. We contravene [11] structure as set up by Dowker' et al. if we have no disjoint sets due to no 'length' in space time physics. The problem is unsolvable in 4 dimensions, by usual cosmologies.*That is if we stick to 4 dimensional space with no higher dimensional embedding.*

The only way around the above stated problems of break down of quantum measures for 4 dimensions and a tradtional big bang singularity would be using *singular point* classical *transitive percolations and relate that to observables identical to classical transitive percolations and giving up the additivity of quantum measure. This suggested proceedure* *involving percolations suggests some sort of solution in line with t'Hoofts thoughts as to quantum structure in a deterministic embedding [9]. Further work may have similarties to reconciling structure as given by [11]. Weyl quantization in part will have to be reconciled with problems with quantum measures as outlined above. Appendix A presents material to be reconciled with giving up the additivity of Quantum measures as stated in the conclusion. And Weyl quantization can be seen to be a sub set to the Wheeler De Witt semi classical analysis on wormholes between universes in [12] Appendix B presents the sort of semiclassical embedding the quantum measure would be part of.*

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Appendix A**: Linking the thin shell approximation, Weyl quantization, and the Wheeler De Witt equation**

This is a re capitulation of what is written by S. Capoziello, [12] et al (2000) for physical review A, which is assuming a generally spherically symmetric line element. The upshot is that we obtain a dynamical evolution equation, similar in part to the Wheeler De Witt equation which can be quantified as $H\left|\Psi\right\rangle = 0$.Which in turn will lead to, with qualifications, for thin shell approximations $|x| << 1$,

$$\Psi'' + a^2 x^4 \Psi = 0 \tag{1a}$$

So that $Z_{1/6}$ is a spherical Bessel equation for which we can write

$$\Psi \equiv \sqrt{x} Z_{1/6}\left(\frac{a}{3}x^3\right) \sim x^{2/3} \tag{2a}$$

Similarly, $|x| >> 1$ leads to

$$\Psi \equiv \sqrt{x} Z_{1/6}\left(\frac{a}{3\cdot\sqrt{2}}x^3\right) \tag{3a}$$

Also, when $x \cong 1$

$$\Psi \equiv \left[\sqrt{2a^2 \cdot (x-1)}\right]^3 Z_{-3/4}\left(\frac{8}{3} \cdot a \cdot (x-1)^{3/2}\right) \tag{4a}$$

Realistically, in terms of applications, we will be considering very small x values, consistent with conditions near a singularity/ worm hole bridge between a prior to our present universe. This is for $x \equiv R / R_{equilibrium}$

Appendix B. How to obtain worm hole bridge between two universes, via the Wheeler De Witt equation: i.e. forming Crowell's time dependent Wheeler-De-Witt equation, and its links to Wormholes (higher dimensions).

This appendix will be to show some things about the wormhole we assert the instanton traverses en route to our present universe. This is the Wheeler-De-Witt equation with pseudo time component added. From Crowell [13]

$$-\frac{1}{\eta r}\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{\eta r^2} \cdot \frac{\partial \Psi}{\partial r} + rR^{(3)}\Psi = \left(r\eta\phi - r\ddot{\phi}\right) \cdot \Psi \tag{1b}$$

This has when we do it $\phi \approx \cos(\omega \cdot t)$, and frequently $R^{(3)} \approx$ constant, so then we can consider

$$\phi \cong \int_0^\infty d\omega \left[a(\omega) \cdot e^{ik_\varpi x^\mu} - a^+(\omega) \cdot e^{-ik_\varpi x^\mu}\right] \tag{2b}$$

In order to do this, we can write out the following for the solutions to Eqn. (1b) above.

$$C_1 = \eta^2 \cdot \left(4 \cdot \sqrt{\pi} \cdot \frac{t}{2\omega^5} \cdot J_1(\omega \cdot r) + \frac{4}{\omega^5} \cdot \sin(\omega \cdot r) + (\omega \cdot r) \cdot \cos(\omega \cdot r)\right)$$

$$+ \frac{15}{\omega^5}\cos(\omega \cdot r) - \frac{6}{\omega^5} Si(\omega \cdot r) \tag{3b}$$

And

$$C_2 = \frac{3}{2 \cdot \omega^4} \cdot (1 - \cos(\omega \cdot r)) - 4e^{-\omega \cdot r} + \frac{6}{\omega^4} \cdot Ci(\omega \cdot r) \tag{4b}$$

This is where $Si(\omega \cdot r)$ and $Ci(\omega \cdot r)$ refer to integrals of the form $\int_{-\infty}^{x} \frac{\sin(x')}{x'} dx'$ and $\int_{-\infty}^{x} \frac{\cos(x')}{x'} dx'$. It so happens that this is for forming the wave functional that permits an instanton to form. Next, we should consider whether or not the instanton so formed is stable under evolution of space-time leading up to inflation. To model this, we use results from Crowell [13] on quantum fluctuations in space-time, which gives a model from a pseudo time component version of the Wheeler-De-Witt equation, with use of the Reinssner-Nordstrom metric to help us obtain a solution that passes through a thin shell separating two space-times. The radius of the shell $r_0(t)$ separating the two space-times is of length l_P in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom metric'

$$dS^2 = -F(r) \cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \tag{5b}$$

This has:

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3} \cdot r^2 \xrightarrow[T \to 10^{32}\, Kelvin \sim \infty]{} - \frac{\Lambda}{3} \cdot (r = l_P)^2 \tag{6b}$$

This assumes that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park [14], leading to

$$\frac{\partial F}{\partial r} \sim -2 \cdot \frac{\Lambda}{3} \cdot (r \approx l_P) \equiv \eta(T) \cdot (r \approx l_P) \tag{7b}$$

As a wave functional solution to a Wheeler-De-Witt equation bridging two space-times. This solution is similar to that being made between these two space-times with "instantaneous" transfer of thermal heat, as given by Crowell [13]

$$\Psi(T) \propto -A \cdot \{\eta^2 \cdot C_1\} + A \cdot \eta \cdot \omega^2 \cdot C_2 \tag{8b}$$

This has $C_1 = C_1(\omega, t, r)$ as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function. This also applies to the second cyclical wave function $C_2 = C_2(\omega, t, r)$, where we have $C_1 =$ Eqn (3b) above, and $C_2 =$ Eqn. (4b) above. Eqn. (8b) is an approximate solution to the pseudo time dependent Wheeler-De-Witt equation. The advantage of Eqn. (8b) is that it represents to good first approximation of gravitational squeezing of the vacuum state. When examining this solution, we shold keep in mind that the Wheeler De Witt equation as given by Crowell [13] is a semi classical approximation, with a pseudo time component, as opposed to the time independent Wheeler De Witt equation [15] Kolb and Turner outline which is time INDEPENDENT. The situation in [13] inevitably involves higher dimensions, whereas [15] is for a 4 dimensional space time geometry.