WORKING WITH THE EARLY UNIVERSE VERSION OF THE WHEELER DE WITT EQUATION, AND FINDING THE INITIAL CONDITIONS ARE UNDEFINABLE.

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This document is due to reviewing an article by Maydanyuk and Olkhovsky, of a Nova Science conpendium as of " The big bang, theory assumptions and Problems", as of 2012, which uses the Wheeler De Witt equation as an evolution equation assuming a closed universe. Having the value of k, not as the closed universe, but nearly zero of a nearly flat universe, which leads to serious problems of interpretation of what initial conditions are. These problems of interpretations of initial conditions tie in with difficulties in using QM as an initial driver of inflation. And argue in favor of using a different proceedure as far as forming a wave function of the universe initially.

A Introduction

What we are looking at, in Maydanyuk and Olkhovsky [1] , is a way to define the initial Wheeler De Witt equation, not as what they did, for a closed universe, but to get to the actual nearly flat space Euclidian universe conditions which suggest that QM will not work well as to initial conditions, and that a different procedure than what was done for closed unverse conditions [1] needs to be considered for the start of cosmological evolution. Note that the difficulty in initial conditions has startling similarities as to the problem with gravitions having mass as noted by by Maggiorie [2] which specifically delineated for non zero graviton mass, where $h \equiv \eta^{uv} h_{uv} = Trace \cdot (h_{uv})$ and $T = Trace \cdot \left(T^{uv}\right)$ that

$$- 3m_{graviton}^2 h = \frac{\kappa}{2} \cdot T \tag{1}$$

As noted by Maggiore, one gets into serious analytical difficulties from the beginning, with (1) and the reader is invited to look at his massive Graviton section [2] which delineates some of the problems. In a similar manner, the closed universe analysis done in [1] encounters serious problems in initial conditions if we used flat space in the onset which sheds light upon the vulnerabilities of QM in forming appropriate initial conditions, which we will comment upon and offer a solution for.

B. Looking at the way to form a Wheeler De Witt equation via a nearly flat space model

The author is quite aware of work discussed with him in conferences, noticiably Rencontres De Moriond, in the experimental gravity conference, which alledges that from the initial conditions that inflation mandated almost completely flat space. For the sake of argument in this work, we will work with flat space, and will commence a derivation which shows serious issues with the Wheeler De Witt analysis of Quantum space time offered in [1] which works passably well in a closed universe condition.

To do this, we will reproduce, using instead of k=1 (closed universe), $k \cong \varepsilon^{+} \sim 0^{+}$, and use that to reproduce the Wheeler De Witt argument and wave functions in [1], designating what we think are serious initial condition problems inherient in the $k \cong \varepsilon^{+} \sim 0^{+}$ nearly flat space conditions, so as to look at first the mini super space Langrangian, which is written in [1] as

$$L(a,\dot{a}) = \frac{3a}{8\pi G} \cdot \left(-\dot{a}^{2} + [k \cong \varepsilon^{+} \sim 0^{+}] - \frac{8\pi G}{3} a^{2} \rho(a) \right) \tag{2}$$

A Chapylgin gas equation of state was used, in working with Eq. 2 using $0 < \alpha < 1$ so that

$$p_{Chapyglin} = -A / \rho^{\alpha}_{Chapyglin} \tag{3}$$

And, in conditions which specify $A = \rho_{\Lambda}$ and $B = \rho_{Dust}$

$$\rho_{Chapyglin}(a) = (A + B / a^{3\cdot(1+\alpha)}) \xrightarrow[\alpha \to 0]{} \rho_{Dust} + \rho_{\Lambda} \tag{4}$$

and a general density equation we will write up as

$$\rho(a) = \left(\rho_{\Lambda} + \rho_{Dust} / a^{3\cdot(1+\alpha)} \right)^{(1/1+\alpha)} + \rho_{Radiation} \big/ a^{4} \tag{5}$$

The end result as given is that [3] one has a S.E. with a wavefunction $\phi(a)$

$$\left\{ -\frac{\partial^{2}}{\partial a^{2}} + V(a) \right\} \phi(a) = E_{Radiation} \phi(a) \tag{6}$$

With

$$V(a) = \left(\frac{3}{4\pi G} \right)^{2} \cdot \left(k \cong \varepsilon^{+} \sim 0^{+} \right) \cdot a^{2} - \frac{3}{2\pi G} \cdot a^{4} \cdot \left(\rho_{\Lambda} + \rho_{Dust} / a^{3\cdot(1+\alpha)} \right)^{(1/3+\alpha)} \tag{7}$$

The difficulty in the change of variables comes next and is attributed to $k \cong \varepsilon^{+}$. Set $8\pi G = M^{-2} = 1$, and then the Eq. (7) becomes, instead, if $E_{radiation} = 12\rho_{radiation}$

$$V(a) = 36\cdot\left(k \cong \varepsilon^{+}\right)\cdot a^{2} - 12\cdot a^{4}\cdot\left(\rho_{\Lambda} + \rho_{Dust} / a^{3\cdot(1+\alpha)}\right)^{(1/3+\alpha)} \quad (8)$$

This potential is almost identitcal to what was done in [1] but the term $k \cong \varepsilon^{+}$ is what creates initial conditions which simply do not work out and are to be commented upon directly. If one does an expansion of Eq. (8) as given above by $q = a - \bar{a}$ then by [1]

$$V_{Chapyglin}(q) = V_0 - V_1 q \quad (9)$$

$$V_0 = V_{Chapyglin}(a = \bar{a});$$
$$V_1 = 72\cdot\left[k = \varepsilon^{+}\right]\cdot a + 12\cdot\left\{-4\Lambda - \rho_{Dust} \big/ a^{3\cdot(1+\alpha)}\right\}\cdot\left(\Lambda + \rho_{Dust} / a^{3\cdot(1+\alpha)}\right)^{-(\alpha/1+\alpha)} \quad (10)$$

Then Eq. (6) becomes, with $\phi(q)$ a wave function of the universe for $q = a - \bar{a}$

$$\left\{-\frac{d^{2}}{dq^{2}} + \left(V_0 - E_{radiation} + V_1\cdot q\right)\right\}\phi(q) = 0 \quad (11)$$

The following change of variables is where the problem in the Planckian regime becomes acute. I.e. set

$$\xi = \frac{V_0}{\left|V_1\right|^{2/3}} - \frac{V_1}{\left|V_1\right|^{2/3}}\cdot q \quad (12)$$

Then , Eq. (11) become an Airy style differential equation with

$$\frac{d^{2}\phi(\xi)}{d\xi^{2}} + \xi\cdot\phi(\xi) = 0 \quad (13)$$

Eq. (13) above becomes undefinable, in the Planck regime of space time due to working with

$$\xi\Big|_{Planck-regime} \sim \frac{\left[E_{radiation} - V_0\right]}{\left[\varepsilon^{+} \sim 0^{+}\right]^{2/3}} \quad (14)$$

In this case, the $\varepsilon^{+} \sim 10^{-33}$ centimeters is so small, that it is next to impossible to define Eq. (14) , with a solution as given in [1] via

$$\phi(\xi) \equiv T \cdot \psi^{+}(\xi)\,;\; \psi^{+}(\xi) = \int_{0}^{\mu \max} \exp i \cdot \left[-\frac{\mu^{3}}{3} + f(\xi) \cdot \mu \right] \cdot d\mu \qquad (15)$$

If we do a power series series expansion of the function $f(\xi)$, [1] asserts that Eq. (15) becomes proportional to an airy function with $Ai(z); Bi(z)$, provided $f_0 = 0; f_1 = 1$

C. Criticism of forming wave function of Eq. (15) if an Airy function, with using Eq. (14)

We assert that in the Planck regime of space time, that Eq. (14) is in reality undefinable due to the denominator of $k \cong \varepsilon^{+} \sim 0^{+}$ at or below 10 ^ - 33 centimeters of space time. The value of this parameter is so small, in fact, that what really needs to be addressed, to make any sense out of how small Eq. (14) really is , is the following observation. Namely in looking at an evolution of a Wheeler De Witt equation of space time, that we can define a spatial evolution, via expansion of the scale factor a, as in Eq. (11) , but we have to PUT IN BY HAND, the initial TIME STEP. i.e. the exact same problem shows up in Loop quantum gravity. In the case of scale factor $a(t)$, the spatial evolution is amendable by QM, but there is no idea as to how to get about putting in ‘ by hand’ the INITIAL time step, which we presume would be a Planck time interval.

D. So how do we know that $k \cong \varepsilon^{+} \sim 0^{+}$ would even apply at a small enough spatial grid of space time?

We DO NOT know this. The evidence though appears to be that if we do have curved space time, that there would be an end to Lorentz invariance, and there would be a break down of Special relativity. Conceivably there could be for a small enough regime of space time near the Planck regime a situation for which $k \cong \varepsilon^{+} \sim 0^{+}$ **no longer holds.**

In the end, as in the heavy gravity problem, as given by Eq. (1) above, we would be looking for some sort of signal confirmation of if $k \cong \varepsilon^{+} \sim 0^{+}$ no longer holds. The only reason it would not hold would be if inflation is not true. Good luck in proving that, i.e. like it or not, inflation appears to be the strongest argument in favor of keeping $k \cong \varepsilon^{+} \sim 0^{+}$. The analysis of [1] presumes a closed universe, and for the reasons outlined above appears to go to pieces in $k \cong \varepsilon^{+} \sim 0^{+}$ in initial time and spatial integration steps.

E. Conclusion. We need to re consider the role of Quantum gravity models at the onset of inflation.

The article given in [1] makes an assumption about the value of k which is unsupported by observations. We have gone into reasons, namely linked to the fact that an evolution in scale factor $a(t)$ may be defined in terms of spatial evolution, but we are stuck in ALL Quantum gravity models as of putting in an initial time step ‘ by hand’ so to speak which raises fundamental issues of what would form an initial time step in Quantum gravity. This indeterminate nature of time , itself, at the onset of Quantum gravity models of space time may be seen as a fundamental defect killing off all initial QM influences at the start of inflation. The other way to look at the role of an undefined initial starting point for time, which we put in by ‘ hand’ is that the special nature of time itself may be if experimentally verified, via observations, the best hope we have of falsifiable measurements of t’Hoofts conjecture [4] that QM is embedded within a classical physics frame work which we have yet to fully develop. To do that would also , if the Gravition exists with initial measurements, such as given by

$$
\begin{aligned}
& m_{graviton}\Big|_{RELATIVISTIC} < 4.4 \times 10^{-22} h^{-1} eV / c^2 \\
& \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} meters
\end{aligned} \tag{16}
$$

Perhaps lead to signals from early universe GW which may confirm or falsify the role of QM in initial univese conditions.

References

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3. G.A Monerat, et al. “ Dynamics of the Early Universe and the Initial conditions for inflation in a model with radiation and a Chaplygin Gas, Physical review D76, 024017(2007), l l pages

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