The acceleration of in the 2-demension inertial system

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ABSTRACT

In the special relativity theory, the acceleration a about the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration a' about the accelerated matter that has not the initial velocity v_0 in 2-Dimension inertial coordinate system $S'(t',x')$ are same.

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I.Introduction

Use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}), t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

In the special relativity, the formula about 2-Dimension inertial coordinate system $S(t,x)$ and $S'(t',x')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0}\ ,\ V=\frac{dx}{dt}, u=\frac{dx'}{dt'}\ ,\ dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}},\ dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}),\ a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (2)$$

The velocity V has the initial velocity v_0 and the velocity u is the velocity by the pure acceleration a'.

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (3)$$

In this time , if the pure acceleration a' of the velocity u is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})\ ,\ u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (4)$$

Eq(3) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$= a'+v_0 \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \frac{a'}{c^2} = a'(1+\frac{v_0}{c^2}\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$= a'(1+\frac{v_0}{c^2}u) \qquad (5)$$

Therefore, the acceleration a about the accelerated matter that has the initial velocity v_0 in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration a' about the accelerated matter that has not the initial velocity v_0 in 2-Dimension inertial coordinate system $S'(t',x')$ are same. In this time, if the acceleration a' is the constant acceleration a_0, the inertial acceleration in 2-Dimension inertial coordinate system $S(t,x)$ and in 2-Dimension inertial coordinate system $S'(t',x')$ is the constant acceleration a_0.

$$a_0 = a' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) = a = \frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \quad (6)$$

II. Additional chapter-I

Therefore,

$$V = \frac{dx}{dt} = \frac{a_0 t + C}{\sqrt{1+\frac{1}{c^2}(a_0 t + C)^2}}, \quad u = \frac{dx'}{dt'} = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}, x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)$$

$$= \frac{\gamma a_0(t'+\frac{v_0}{c^2}x') + C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x') + C)^2}}$$, C is the constant number

$$= \frac{\gamma a_0(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C)^2}}$$

$$= \frac{\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C}{\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C)^2}}$$

$$= \frac{u+v_0}{1+\frac{u}{c^2}v_0} = \frac{\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}+v_0}{1+\frac{v_0}{c^2}\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}} = \frac{a_0 t' + v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}+\frac{v_0}{c^2}a_0 t'} \quad (7)$$

In this time,

$$\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2})^2} = \gamma(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}+\frac{v_0}{c^2}a_0 t') \quad (8)$$

Therefore,

$$C = \gamma v_0 \qquad (9)$$

Hence,

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2} - \sqrt{1+\frac{1}{c^2}(\gamma v_0)^2})$$

$$= \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2} - \gamma) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}), V = \frac{a_0 t+\gamma v_0}{\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}}$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1) \ , \ u = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}},$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (10)$$

And

$$d\tau = \sqrt{1-V^2/c^2}\,dt = \frac{dt}{\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}}, \ d\tau = \sqrt{1-u^2/c^2}\,dt' = \frac{dt'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}$$

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}) - \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}) - \tau_0$$

$$\tau + \tau_0 = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}), \qquad \tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c})$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma \frac{v_0}{c}), \ \ \gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (11)$$

Therefore,

$$t + \gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau + \frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c}) + \cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (12)$$

In this time,

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c}) \rightarrow \ \sinh(\frac{a_0\tau}{c}) = \frac{a_0 t'}{c},$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1) \ \ \rightarrow \ \cosh(\frac{a_0\tau}{c}) = \sqrt{1+\frac{{a_0}^2 t'^2}{c^2}} = 1 + \frac{a_0}{c^2}x'$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) \rightarrow \sinh(\frac{a_0\tau_0}{c}) = \frac{\gamma v_0}{c}, \ \ \cosh(\frac{a_0\tau_0}{c}) = \sqrt{1+\frac{\gamma^2{v_0}^2}{c^2}} = \gamma,$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (13)$$

Therefore, Eq(12) is

$$t + \gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau + \frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c}) + \cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$= \frac{c}{a_0}[\gamma\sinh(\frac{a_0\tau}{c}) + \cosh(\frac{a_0\tau}{c})\frac{\gamma v_0}{c}]$$

$$= \frac{c}{a_0}[\frac{a_0 t'}{c}\cdot\gamma + (1+\frac{a_0}{c^2}x')\cdot\frac{\gamma v_0}{c}] = \gamma(t' + \frac{v_0}{c^2}x') + \gamma\frac{v_0}{a_0}, \gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (14)$$

Therefore, Eq(10) is

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t + \gamma v_0)^2} - \gamma), \quad x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x')+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1))+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(\gamma a_0t'+\gamma v_0\sqrt{1+\frac{1}{c^2}(a_0t')^2})^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{(\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma a_0\frac{v_0}{c^2}t')^2}-\gamma)$$

$$=\gamma\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1)+\gamma v_0t'=\gamma(x'+v_0t')\ \ ,\ \gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (15)$$

or by Eq(13),Eq(14)

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\sinh^2(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)}-\gamma)=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\cosh(\frac{a_0}{c}\tau_0)+\sinh(\frac{a_0}{c}\tau)\sinh(\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\gamma+\sinh(\frac{a_0}{c}\tau)\frac{\gamma v_0}{c}-\gamma)$$

$$=\frac{c^2}{a_0}\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma v_0t'-\frac{c^2}{a_0}\gamma,\quad \gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (16)$$

V. Conclusion

Hence, Eq(1) is in the 2-Dimension inertial coordinate system $S'(t',x')$

$$x'=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)$$

$$t' = \frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \qquad (17)$$

Therefore, in the 2-Dimension inertial coordinate system $S(t,x)$

$$t = \gamma(t' + \frac{v_0}{c^2}x') = \gamma(\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{a_0}(\cosh(\frac{a_0}{c}\tau) - 1))$$

$$x = \gamma(x' + v_0 t') = \gamma(\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c}) - 1) + \frac{v_0 c}{a_0}\sinh(\frac{a_0\tau}{c})), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \qquad (18)$$

$$dt = \gamma(\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\sinh(\frac{a_0}{c}\tau))d\tau,$$

$$dx = \gamma(c\sinh(\frac{a_0}{c}\tau) + v_0\cosh(\frac{a_0}{c}\tau))d\tau,$$

$$V = \frac{dx}{dt} = (c\tanh(\frac{a_0}{c}\tau) + v_0)/(1 + \frac{v_0}{c}\tanh(\frac{a_0}{c}\tau)), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} \qquad (19)$$

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