Micro-Particle Model and Black Hole

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Abstract

Modified Reissner-Nordstrom solution to Einstein's field equation for static charged spherical black hole is used in microscopic scale by redefine the potential. In addition, circular orbits and horizons obtained. The obtainable model characterized the boundaries of micro-Particle scale. Casimir force is generalized and computed without reference to zero point energies.

1 introduction

Schwarzschild solution described static uncharged black hole. Schwarzschild metric has singularity when a gravitational collapse takes the surface until it reaches Schwarzschild radius, $r_s = 2GM$.on the other hand, Reissner-Nordstrom solution is a solution to the Einstein's field equation for static charged spherical black hole [1].

Various authors, from early work of Bekenstein [2] to more recent works of 't Hooft [3] and Susskind, Thorlacius and Uglum [4] have suggested that the black hole is a normal quantum system with discrete energy levels.

The theory of differential spaces is used in gluing two Reissner-Nordstrom space-times [5].

Classical and quantum arguments supporting the existence of a maximal acceleration (MA)[6]. MA also appears in the context of Weyl space [7] and of a geometrical analogue of Vigier's stochastic theory [8]. Its existence would rid black hole entropy of ultraviolet divergencies [3],[9],[10], and circumvent inconsistencies associated with the application of the point-like concept to relativistic quantum particles [11].Some authors regard Am as a universal constant fixed by Planck's mass[12],[13], but a direct application of Heisenberg's uncertainty relations [14],[15] as well as the geometrical interpretation of the quantum commutation relations given by Caianiello, suggested that Am be fixed by the rest mass of the particle itself according to $A_m = \frac{2mc^3}{\hbar}$.

2 Micro-Particle Model

In this work we obtained scale transformation (which agree with a maximal acceleration) to apply Reissner-Nordstrom metric in microscopic scale.

Reissner-Nordstrom metric in spherical coordinate symmetric is given by:

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\emptyset^2) \qquad (1)$$

From uncertainty principle $\Delta E \Delta t \approx \hbar$, the maximum range of the force of a particle would be approximately $c\Delta t > \hbar/_{2mc}$.

Instate of the $R = 2\, {GM}/{c^2}$ we substitute $2GM \rightarrow \frac{\hbar c}{m}$ in (1).then, Rewrite Reissner-Nordstrom metric (restore c^2), the metric becomes:

$$ds^2 = -\left(1 - \frac{\hbar}{mcr} + \frac{\hbar Q_m^2}{4\pi\epsilon_0 c^3 m r^2}\right) dt^2 + \left(1 - \frac{\hbar}{mcr} + \frac{\hbar Q_m^2}{4\pi\epsilon_0 c^3 m r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\emptyset^2)$$

We redefined $Q_m^2 = \rho Q$ as the charge density ($\rho = {Q}/{M}$) multiplying by the total charge of the black hole (Q),we add $\frac{1}{4\pi\epsilon_0}$ to the electric term. we can rewrite the metric as:

$$ds^2 = -\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) dt^2 + \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta d\emptyset^2) \qquad (2)$$

Where: $r_s = \frac{\hbar}{m_s c}$, $r_Q^2 = \frac{\hbar Q_m^2}{4\pi\epsilon_0 m_Q c^3}$,m_s the total mass of black hole, m_Q the total mass of charges carriers.

The scale transformation can be obtained as:

$$2GM \rightarrow \frac{\hbar c}{m} \qquad (3)$$

3 Scale's boundaries

Quantum gravity is defined as the regime where the Planck‘s quantities, Planck‘s length $l_P = \sqrt{\frac{2\hbar G}{c^3}}$ and Planck mass $m_P = \sqrt{\frac{\hbar c}{2G}}$, became relevant. On the other hand, general relativity implies a maximal mass of black hole (maximal mass that can be contained in limited system r),is: $M = \frac{c^2 r}{2G}$,which occurred at radius (Schwarzschild‘s radius) $R_s = \frac{2GM}{c^2}$ .Also quantum effect entails mass fluctuation of order of magnitude , $\delta M(l) = \frac{\hbar}{cl}$. The lower boundary of mass is ruled by quantum effect, where $\delta M(r) = \frac{\hbar}{cl}$ .the upper bound is general relativity with $M = \frac{c^2 r}{2G}$ .Now we can define the threshold mass, as the mass which satisfies $\frac{\hbar}{cl} = \frac{c^2 r}{2G}$ (the uniqueness of this equation, is the appearance of both $\hbar$ and G in one equation, what is forbidden anywhere), the solution depends on the length, $= r = l_P$, then, the threshold mass is Planck‘s mass. consequence, the approach sign in the transformation (2), becomes exactly an equal sign with Planck‘s mass $\frac{\hbar c}{m_P} = 2Gm_P$, this what motivated Arbab[16], to build on, his characteristic Planck‘s constant, $\hbar_c$,which depends on the size of the system.

The exerted force between two micro-Particles, according to the model is given by:

$$F_{12} = \frac{\hbar c}{2m_1 r^2} m_2 \qquad (c)$$

This force, in Planck's length and mass) is $\sim 10^{43} N$. Furthermore, the micro-Particles force in (c) is the Casimir attractive force between two plates, $F_{casimir} = \frac{\pi^2 \hbar c}{240 L^4} A$, where L is the distance between the two plates, A is plate area. When $A = L^2$, (square plate). The Casimir force construes to micro-Particles force, $F_{casimir} = \alpha \frac{\hbar c}{L^2}$. Where $\alpha = \frac{\pi^2}{240}$ in Casimir's calculation. In Micro-Particle model, $= \frac{m_2}{2m_1}$.

The potential energy is

$$E_g = \frac{\hbar c}{2 m_1 r} m_2 \qquad (d)$$

The potential energy,when $m_1 = m_2 \equiv m$ and $r = \frac{\hbar}{mc}$, Compton length, is $E_g = \frac{1}{2} mc^2$

The region of applying, $\hbar$ constant, and, G constant, is shown in bellow diagram and in figure 1.

$$\left\{\frac{\hbar c}{m}\right\} \qquad m < m_P < M \qquad 2GM$$

Obviously, the scale identification is the mass. Accordingly, the transformation in (3), translates GR equations (macro scale) to micro scale, no more constants need to be added. universal gravitational constant G and Planck's constant $\hbar$ are both scale constant.

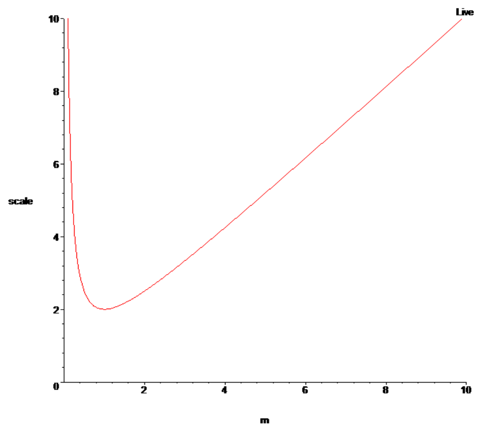

Fig 1: shows the scale responds to mass, it manifested into two regions, $m < m_P < M$. The turn point occurs at Planck mass, where the scale length is equal to Planck's length.

4 Micro-Black hole orbits

There is true singularity in (2) at $m = 0$, mass singularity add additional constraint to this model, which leads to redefine the micro-Particle scale as the scale of mass in the range , $0 < m_0 < m_p$, where m_0 is the rest mass of the micro-Particle. In addition, there are two singularities (horizons) at:

$$r_{\pm} = \tfrac{1}{2} r_s \left(1 \pm \sqrt{1 - 4\frac{r_Q^2}{r_s^2}} \right) \tag{4}$$

The geodesic equation and constraints for the line element metric (2) are:

$$\ddot{t} + \frac{-r_s + 2{r_Q}^2/r}{r^2 - r_s r + {r_Q}^2}\dot{r}\dot{t} = 0$$

$$\ddot{\theta} + \frac{2}{r}\dot{r}\dot{\theta} - \sin\theta\cos\theta\,\dot{\emptyset}^2 = 0$$

$$\ddot{\emptyset} + \frac{2}{r}\dot{r}\dot{\emptyset} + 2\cot\theta\,\dot{\theta}\dot{\emptyset} = 0$$

$$E = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)\dot{t}$$

$$L = r^2\dot{\emptyset}$$

For massless particles, E and L are the energy and angular momentum. For massive particles, they are energy and angular momentum per unit mass of the particle.

$$-\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)\dot{t}^2 + \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1}\dot{r}^2 + r^2(d\theta^2 + sin^2\theta d\emptyset^2) = -\varepsilon$$

We assume that $\theta = \frac{\pi}{2}$ and $\dot{\theta} = 0$

$$\tfrac{1}{2}\dot{t}^2 = \tfrac{1}{2}E^2 - V(r)$$

The effective potential is

$$V_{eff} = \frac{1}{2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)\left[\frac{L^2}{r^2} + \varepsilon\right]$$

$$V_{eff} = \frac{\varepsilon}{2} - \frac{\varepsilon}{2}\frac{r_s}{r} + \frac{\varepsilon}{2}\frac{r_Q^2}{r^2} + \frac{L^2}{2r^2} - \frac{r_s L^2}{2r^3} + \frac{r_Q^2 L^2}{2r^4} \tag{5}$$

For massless particle, $\varepsilon = 0$

At minimum or maximum potential, circular orbit (r=constant) is possible.

When: $\frac{dV}{dr} = 0$. Differentiate the effective potential in (5) to obtain the circular orbits.

$$r_c = \frac{3}{4} r_s \left(1 \pm \sqrt{1 - \frac{32}{9} \frac{r_Q^2}{r_s^2}} \right) \qquad (6)$$

Equation (6) reduces to Schwarzschild, $r_c = 3GM$, when $Q = 0$.

For massive particle $\varepsilon = 1$. The effective potential is

$$V_{eff} = \frac{1}{2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) + \frac{L^2}{2r^2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)$$

.we can rewrite massive effective potential as:

$$V_{eff} = V(r) + \frac{V(r)}{2r^2}(L^2 - r^2) \qquad (7)$$

Where $V(r) = 1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}$

The critical orbits occur at:

$$\frac{d}{dr}\left[\frac{1}{2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) + \frac{L^2}{2r^2}\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)\right] = 0$$

$$r^3 - \left(2\frac{r_Q^2}{r_s} - 2\frac{L^2}{r_s}\right) r^2 + 3L^2 r - 4L^2 \frac{r_Q^2}{r_s} = 0 \qquad (8)$$

We find the real root of (8) ,

$$r_c = \sqrt[3]{-36ba + 108C + 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54abC + 81C^2 + 12Ca^3}} \Big/ 6 -$$

$$\frac{6\left(\frac{b}{3} - \frac{a^2}{9}\right)}{\sqrt[3]{-36ba + 108C + 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54abC + 81C^2 + 12Ca^3}}} + \frac{a}{3} \qquad (9a)$$

Where: $a = \left(2\frac{r_Q^2}{r_s} + 2\frac{L^2}{r_s}\right)$, $b = 3L^2$, $C = 4L^2 \frac{r_Q^2}{r_s}$.

If we neglect the last term in (8), the equation of the circular orbits becomes:

$$r_c = \left[\frac{r_Q^2}{r_s} + \frac{L^2}{r_s}\right] \pm \sqrt{\left(\frac{r_Q^2}{r_s} + \frac{L^2}{r_s}\right)^2 - 3L^2} \qquad (9b)$$

As we define L as the angular momentum per unit mass of the particle, so L can be written as $L^2 = \frac{\hbar^2 l(l+1)}{c^2 m^2}$ we restore, c, for dimension compatibility. Equation (9b) becomes:

$$r_c = \left[\frac{r_Q^2}{r_s} + \frac{\hbar^2 l(l+1)}{c^2 m^2 r_s}\right] \pm \sqrt{\left(\frac{r_Q^2}{r_s} + \frac{\hbar^2 l(l+1)}{c^2 m^2 r_s}\right)^2 - 3\frac{\hbar^2 l(l+1)}{c^2 m^2}} \qquad (10a)$$

When $Q = 0$ in (9b), we recover Schwarzschild's circular orbit

$$r_c = \frac{L^2 \pm \sqrt{L^4 - 12G^2M^2L^2}}{2GM}$$

Equation (10) gives:

$$r_c = \left[\frac{r_Q^2}{r_s} + \frac{\hbar^2 l(l+1)}{c^2m^2r_s}\right] \pm \left(\frac{r_Q^2}{r_s} + \frac{\hbar^2 l(l+1)}{c^2m^2r_s}\right)\left[1 - \frac{3\hbar^2 l(l+1){r_s}^2}{2c^2m^2\left(r_Q^2+\frac{\hbar^2 l(l+1)}{c^2m^2}\right)^2}\right] \qquad (10b)$$

There are different regimes depending on the angular momentum.

We find the unstable orbit for massive particles at the radius r_{c-}:

$$r_{c-} = \frac{3\hbar^2 l(l+1)r_s}{2c^2m^2\left(r_Q^2 + \frac{\hbar^2 l(l+1)}{c^2m^2}\right)} \qquad (11a)$$

The other stable orbit r_{c+} , is located at:

$$r_{c+} = 2\left[\frac{r_Q^2}{r_s} + \frac{\hbar^2 l(l+1)}{c^2m^2r_s}\right] - \frac{3\hbar^2 l(l+1)r_s}{2c^2m^2\left(r_Q^2 + \frac{\hbar^2 l(l+1)}{c^2m^2}\right)} \qquad (11b)$$

When $= 0$, in equation (11)

$$r_{c-} = \frac{3}{2}r_s \qquad (12a)$$

$$r_{c+} = \frac{2L^2}{r_s} - r_{c-} \qquad (12b)$$

At this limit the stable orbit becomes farther away, while the unstable one approaches, r_s, (behaves like massless).The orbit vanishes when $Q = 0$ and $l = 0$ .as we see the existence of the orbit does not depend on electrical force, rather than gravitational force.

5 Conclusion

We showed that, Planck mass represents the boundary where the macroscopic world is beginning, where GR is applicable and microscopic world is vanished, where quantum mechanics is valid. Accordingly, Planck mass has a duality, micro-macro particle. Moreover, we found the equation, which gives the force between two micro particles (equation (c)), interpreted to Casimir force relation, showed that, Casimir forces can be computed without reference to zero point energies.

The potential energy of a particle is equal to $\frac{1}{2}mc^2$ in special case when, $r = \frac{\hbar}{mc}$ (Compton length).

The Modified Reissner-Nordstrom metric, showed many intrinsic features in microscopic scale, when the transformation $2GM \to \frac{\hbar c}{m}$ is used. It showed a horizon in range of nuclear radius, which does not contradict with Planck‘s length. We found the orbital radius is independent of electric charges. We

References

[1] Sean M. Caroll, Space-time and Geometry, (Addison Wesley, 2004)

[2] J. Bekenstein, Lett. Nuovo Cimento, 11 (1974) 467.

[3] G. 't Hooft, Nucl. Phys. B256 (1985) 727.

[4] L. Susskind, L. Thorlacius and J. Uglum, Phys. Rev. D48 (1993) 3743.

[5] M. Abdel-Megied and Ragab M. Gad, arXiv:gr-qc/0402077 v2 1 Apr 2004.

[6] A. Das, *J. Math. Phys.* 21 (1980) 1506;M. Gasperini, *Astrophys. Space Sci* 138 (1987) 387;M. Toller, *Nuovo Cimento* B102 (1988) 261; *Int. J. Theor. Phys.* 29(1990) 963; *Phys. Lett.* B256 (1991) 215;R. Parentani, R. Potting, *Phys. Rev. Lett.* 63 (1989) 945;P. Voracek, *Astrophys. Space Sci* 159 (1989) 181;B. Mashhoon, *Physics Letters*A143 (1990) 176; V. de Sabbata, C. Sivaram, *Astrophys. Space Sci.* 176 (1991) 145; *Spin and Torsion in gravitation*, World Scientific, Singapore, (1994); D.F. Falla, P.T. Landsberg, *Nuovo Cimento* B 106, (1991) 669;A.K. Pati, *Nuovo Cimento* B 107 (1992) 895; *Europhys. Lett.* 18 (1992)285.

[7] G. Papini and W.R. Wood, *Phys. Lett.* A170 (1992) 409; W.R. Wood and G. Papini, *Phys. Rev.* D45 (1992) 3617; *Found. Phys. Lett.* 6 (1993)409; G. Papini, *Mathematica Japonica* 41 (1995) 81.

[8] J. P. Vigier, *Found. Phys.* 21 (1991) 125.

[9] L.Susskind, J. Uglam, *Phys. Rev.* D50 (1994) 2700.

[10] M. McGuigan, *Phys. Rev.* D50 (1994) 5225.

[11] G.C. Hegerfeldt, *Phys. Rev.* 10 D (1974) 3320.

[12] H.E. Brandt, *Lett. Nuovo Cimento* 38 (1983) 522; 39 (1984) 192;*Found.Phys. Lett.* 2 (1989) 39; 4 (1991) 523; 11 (1998) 265.

[13] E.R. Caianiello, M. Gasperini, G. Scarpetta, *Class. Quantum Grav.* 8(1991) 659;M. Gasperini, in *"Advances in Theoretical Physics"* ed. E.R. Caianiello,(World Scientific, Singapore, 1991), p. 77.

[14] E.R. Caianiello, *Lett. Nuovo Cimento* 41 (1984) 370.

[15] W.R. Wood, G. Papini and Y.Q. Cai, *Il Nuovo Cimento* B104 (1989)361and (errata corrige) 727.

[16] Arbab.I.A. Quantum universe and the solution to the cosmological problems,Gen.Relativity.V.36,2465 (2004); Arbab.I.A. A cosmic Quantum mechanics, African journal of mathematical physics, Vol 2, 1 (2005).