THE GOLDBACH CONJECTURE - SOLUTIONS

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ABSTRACT

The author had published a paper on the solutions for the twin primes conjecture in an international mathematics journal in 2003 and had since then been working on the solutions for the Goldbach conjecture, which is another problem relating to the prime numbers. This paper, which comprises of 4 parts that are each self-contained, is a combination and modification of the author's 2 papers published recently in another international mathematics journal. The expected mode of solving the Goldbach conjecture appears to be the utilisation of advanced calculus or analysis, e.g., by the summation, or, integration, of the reciprocals involving directly or indirectly the primes to see whether they converge or diverge, in order to get a "feel" of the pattern of the distribution of the primes. But, such a method of solving the problem has evidently not succeeded so far. Some other approach or approaches could be more appropriate. This paper brings up a number of such approaches.

INTRODUCTION

The problem of whether there is an infinitude of cases of even numbers which are each the sum of 2 primes is an inherently difficult one to solve, as infinity (normally symbolised by: ∞) is a difficult concept and is against common sense. It is impossible to count, calculate or live to infinity, perhaps with the exception of God. Infinity is a nebulous idea and appears to be only an abstraction devoid of any actual practical meaning. How do we quantify infinity? How big is infinity? The difficulty of the problem of infinity has been compounded by Georg Cantor who proved that there are actually different sizes to infinity, an idea so bizarre to many mathematicians that he was attacked for his ideas during much of his career. The attack was so serious that he suffered mental illness and severe depression. However, after his death his ideas became widely accepted as the only consistent, accurate and powerful definition of infinity. Hilbert had honoured him by saying, "No one shall drive us from the paradise Cantor has created for us." Nevertheless, in this paper offering solutions for infinity, in this case the infinity of the even numbers which are each the sum of 2 primes, incontrovertible evidence that the peculiar characteristics of the prime numbers themselves among other things contribute to the infinite "generation" of such even numbers would be put forward.

PART 1

Theorem:- Every even number after 2 is the sum of 2 primes.

Solution:-

Christian Goldbach, tutor to the teenage Czar Peter II, had examined dozens of even numbers and noticed that he could split all of them into the sum of 2 primes. Thus, his conjecture that every even number after the number 2 is the sum of 2 prime numbers, for example:

4 = 2 + 2
6 = 3 + 3
8 = 3 + 5
10 = 3 + 7 and 5 + 5
12 = 5 + 7
14 = 3 + 11 and 7 + 7
16 = 3 + 13 and 5 + 11
18 = 5 + 13 and 7 + 11
20 = 3 + 17 and 7 + 13
50 = 19 + 31
100 = 53 + 47
21,000 = 17 + 20,983
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Computer searches completed in 2000 had verified that all even numbers up to 400 trillion (4×10^{14}), which is not a small list, are sums of 2 primes, while in 2008, a distributed computer search ran by Tomas Oliveira e Silva, a researcher at the University of Aveiro, Portugal, had further verified the Goldbach conjecture up to 12×10^{17}. But is the conjecture valid?

Proof:

By Euclid's proof, there is an infinitude of primes; that is, the list of primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, …….., continues to infinity.

Goldbach's conjecture states that every even number after the number 2 is the sum of 2 primes. How do we prove this?

First of all, we ask a "reversed" question here (as opposed to Goldbach's conjecture). We ask whether all the prime numbers in the infinite list of prime numbers would combine with each other to form a regular, continuous (*without breaks or gaps*) and infinite list of even numbers. This would lead to our proof.

Let us now take a subset of primes from the infinite set of prime numbers, say, all the primes found in the set of integers ranging from 1 to 50, that is, the following subset of prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Then, we conduct a close examination of how this subset of prime numbers "behaves", that is, how the prime numbers combine (one-to-one) with each other to form even numbers, and observe whether there is any "regularity of pattern" in the way they do so. Here, we look at how the prime numbers from 2 To 47 combine with each other to form even numbers.

We could observe the primes from 2 to 47 "generating" a regular, continuous (*without breaks or gaps*) list of even numbers from 4 to 94. This is a regular, continuous list of even numbers, the even numbers becoming evidently progressively more repetitious. For example, there are 5 discernable combinations of primes/partitions for the even number 48, which is as follows:

a) 19 + 29 = 48
b) 17 + 31 = 48
c) 11 + 37 = 48
d) 5 + 43 = 48
e) 7 + 41 = 48

and, there are 5 discernable combinations of primes/partitions for the even number 54, which is as follows:

a) 17 + 37 = 54
b) 13 + 41 = 54
c) 11 + 43 = 54
d) 7 + 47 = 54
e) 23 + 31 = 54

And many others.

There appears to be a "regularity of pattern" in the way the even numbers "pop up".

From this, we could thus conclude the following characteristic or "pattern" of the prime numbers: They would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) list of even numbers, with "overwhelming repetitions" all over the place.

Next, we select another subset of primes from the infinite set of prime numbers. We would here take the prime numbers from the set of integers 51 to 100, which is just "next to" the set of integers 1 to 50 from which we have taken our first subset of prime numbers, to be our second subset of primes. This second subset of prime numbers is as follows:

53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

As usual, we conduct a close examination of how this second subset of prime numbers "behaves," that is, how they combine (one-to-one) with each other to form even numbers, and observe whether there is "regularity of pattern" in the way they do so.

Here, as earlier, we could observe the primes from 53 to 97 "generating" a regular, continuous (*without breaks or gaps*) list of even numbers ranging from 56 to 194. This regular, continuous list of even numbers is also evidently progressively more repetitious. For example, there are 8 discernable combinations of primes/partitions for the even number 90, which is as follows:

a) 53 + 37 = 90
b) 59 + 31 = 90
c) 61 + 29 = 90
d) 67 + 23 = 90
e) 71 + 19 = 90
f) 73 + 17 = 90
g) 79 + 11 = 90
h) 83 + 7 = 90

and, there are 8 discernable combinations of primes/partitions for the even number 120, which is as follows:

a) 53 + 67 = 120
b) 59 + 61 = 120
c) 67 + 53 = 120
d) 73 + 47 = 120
e) 79 + 41 = 120
f) 83 + 37 = 120
g) 89 + 31 = 120
h) 97 + 23 = 120

And many others.

There appears to be a "regularity of pattern" in the way the even numbers "pop up" here - as a matter of fact, this "regularity of pattern" resembles that found in the earlier listing.

The list of even numbers "generated" by this second subset of prime numbers, that is, the regular, continuous list of even numbers ranging from 56 to 194, even overlaps (by a wide margin) the list of even numbers "generated" by the first subset of primes (2 to 47), that is, the regular, continuous list of even numbers ranging from 4 to 94.

From this listing also, from the "characteristics" found in all these listings, where the "regularity of pattern" of the appearance of the even numbers is evident, we could deduce the following characteristic of the prime numbers: The prime numbers would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) list of even numbers, with "overwhelming repetitions" all over the place. This could be further confirmed by studying the even numbers "generated", e.g., by the 3 subsets of primes by combining with other prime numbers, including the prime numbers before them, for the 3 consecutive sets of integers, 101 To 150, 151 To 200, and, 201 To 250, with "overwhelming repetitions" all over the place (see Item (1) in the data below, where there are also much further examples).

Lemma: The well-established self-similarity concept, which was developed by Mitchell Feigenbaum in the 1970s and which brought him fame, upon which the method of renormalization in perturbation theory is based, postulates that there is a tendency of identical mathematical structures to recur on many levels. Within a given structure, there would be smaller copies of the same structure, their sizes being determined by the scaling factor. Feigenbaum found that at the utmost tips of the fig-tree, there is some mathematical structure which remains the same when its size is changed (enlarged) by a scaling factor of 4.669, which is found to be a constant like pi (3.142); this structure is the shape of the fig-tree itself; in other words, little whorls could be found within big whorls. Renormalization has been a well-established technique in chaos theory/fractal geometry and is a mathematical trick which functions rather like a microscope, zooming in on the self-similar structure, removing any approximations, and filtering out everything else. All this shows the universality of some features of chaos. That is, some kind of order or pattern could be found in or is inherent in disorder or chaos. In other words, the elements of an infinite subset of an infinite set contain all the recursive significant properties of that set unless the process which selects the elements of the subset directly excludes a property.

To make it simpler, we re-phrase this concept as follows: The characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is sufficient for deducing the characteristic of the mountain or infinite volume of sand. Likewise, if x is a subset of y and if x is a list of prime numbers while y is another list of prime numbers, the characteristic presence of the even numbers "generated" by all the primes in x suggests (or reflects) the characteristic presence of even numbers "generated" by all the primes in y, so that if y is an infinite list of prime numbers, whence the prime numbers in it run to infinity, so do the even numbers "generated" by all the primes in it. What is described here is actually the "reflection" principle.

Therefore, by the above-mentioned principle, all the above-mentioned selected subsets of primes in the infinite set of primes would each reflect (present an image of, or, display something which has similarity with) the characteristic of this infinite set of primes; that is, all the infinite primes in the infinite set of primes, including its infinite subsets such as the selected subsets mentioned above, would combine (one-to-one) with each other to form a regular, continuous (*without breaks or gaps*) and infinite (implied by the infinitude of the primes (vide Euclid's proof) and the even numbers themselves) list of even numbers. It is evident that the higher up the infinite list of primes we go, the more "overwhelming" or dense the (one-to-one) combinations of primes (in the formation of even numbers) would become, the number of permutations of the combinations of primes tending towards infinity (with the infinity of the prime numbers), as a study with further selected subsets of primes would reveal. A study of the even numbers "generated" by all these subsets of primes would show that the higher up the subsets of primes we go the more "overwhelmingly" the even numbers "generated" would repeat themselves and overlap. This is (very) significant. Though the infinitude of the prime numbers would ensure that there would always be new even numbers being "generated", there is also the "fear" that there might be gaps, breaks or lack of continuity in the even numbers thus "generated". But, it is evident that these more and more profuse repetitions and overlaps of the even numbers thus "generated" by the primes the higher up the infinite list of prime numbers we go "ensure" that such gaps or breaks would not appear between the even numbers "generated" - they "ensure" that the even numbers thus "generated" by the primes in the infinite list of primes would be regular, continuous, *without breaks or gaps*, and, in consecutive running order. Also (very) significant is the great number of new even numbers that each of the primes in these subsets of primes helps to "generate". This "profuse generation" of "regular batches" of even numbers by the prime numbers represents a characteristic or feature of the prime numbers, a universal "pattern" or feature of the "chaotic" infinite prime numbers, or, recurrent, identical mathematical structures, which is all in accordance with the above lemma. (This "pattern of behaviour" of the prime numbers, as described in the paper, is analogous to the "self-similar mathematical pattern or structure" (which is the shape of the fig-tree itself) of the various parts of the fig-tree, that is, its trunk to bough section, bough to branch section, branch to twig section and twig to twiglet section, in Feigenbaum's famous fig-tree example, and, such self-similar mathematical pattern or structure, or, fractal characteristic, could also be found in other aspects of nature, for example, waves, turbulence or chaos, the structures of viruses and bacteria, polymers and ceramic materials, the universe and many others, even the movements of prices in financial markets, the growths of populations, the sound of music, the flow of blood through our circulatory system, the behaviour of people en masse, etc., which have all spawned a relatively new and important branch of mathematics with wide practical applications known as fractal geometry, which has been pioneered by Benoit Mandelbrot. As a matter of fact, self-similarity or fractal characteristic could be regarded as the fundamental mathematical aspect found in practically everything in nature including the numbers such as the prime numbers and the even numbers which are the subjects of our investigation here, and, this new branch of mathematics, fractal geometry, besides having a great practical impact on us also gives us a deeper vision of the universe in which we live and our place in it.) In other words, by the above lemma the infinity of the prime numbers implies the infinity of the "profuse generation" of "regular batches" of even numbers by the prime numbers, that is, the validity of the Goldbach conjecture.

Here, we take a close look at the following data:-

(1) <u>No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps "Generated" (By The Additions/Combinations Of 2 Primes), For Integers 1 To 1,250 (*See Appendix 1 For Example Of Computation Method*)</u>

(a) Set Of Integers, 1 To 50, With 14 Primes Within It = Not Applicable
(aa) Percentage Increase In Repetition = Not Applicable

(b) Set Of Integers, 51 To 100, With 10 Primes Within It = **20** Repeated Even Nos.
(bb) Percentage Increase In Repetition = Not Applicable

(c) Set Of Integers, 101 To 150, With 10 Primes Within It = 46 Repeated Even Nos.
(cc) Percentage Increase In Repetition = (46 - 20) ÷ 20 x 100% = **130%**

(d) Set Of Integers, 151 To 200, With 11 Primes Within It = 73 Repeated Even Nos.
(dd) Percentage Increase In Repetition = (73 - 46) ÷ 46 x 100% = 58.7%

(e) Set Of Integers, 201 To 250, With 7 Primes Within It = 93 Repeated Even Nos.
(ee) Percentage Increase In Repetition = (93 - 73) ÷ 73 x 100% = 27.4%

(f) Set Of Integers, 251 To 300, With 9 Primes Within It = 115 Repeated Even Nos.
(ff) Percentage Increase In Repetition = (115 - 93) ÷ 93 x 100% = 23.66%

(g) Set Of Integers, 301 To 350, With 8 Primes Within It = 139 Repeated Even Nos.
(gg) Percentage Increase In Repetition = (139 - 115) ÷ 115 x 100% = 20.87%

(h) Set Of Integers, 351 To 400, With 8 Primes Within It = 172 Repeated Even Nos.
(hh) Percentage Increase In Repetition = (172 - 139) ÷ 139 x 100% = 23.74%

(i) Set Of Integers, 401 To 450, With 9 Primes Within It = 196 Repeated Even Nos.
(ii) Percentage Increase In Repetition = (196 - 172) ÷ 172 x 100% = 13.95%

(j) Set Of Integers, 451 To 500, With 8 Primes Within It = 220 Repeated Even Nos.
(jj) Percentage Increase In Repetition = (220 - 196) ÷ 196 x 100% = 12.24%

(k) Set Of Integers, 501 To 550, With 6 Primes Within It = 247 Repeated Even Nos.
(kk) Percentage Increase In Repetition = (247 - 220) ÷ 220 x 100% = 12.27%

(l) Set Of Integers, 551 To 600, With 8 Primes Within It = 268 Repeated Even Nos.
(ll) Percentage Increase In Repetition = (268 - 247) ÷ 247 x 100% = 8.5%

(m) Set Of Integers, 601 To 650, With 9 Primes Within It = 298 Repeated Even Nos.
(mm) Percentage Increase In Repetition = (298 - 268) ÷ 268 x 100% = 11.19%

(n) Set Of Integers, 651 To 700, With 7 Primes Within It = 320 Repeated Even Nos.
(nn) Percentage Increase In Repetition = (320 - 298) ÷ 298 x 100% = 7.38%

(o) Set Of Integers, 701 To 750, With 7 Primes Within It = 340 Repeated Even Nos.
(oo) Percentage Increase In Repetition = (340 - 320) ÷ 320 x 100% = 6.25%

(p) Set Of Integers, 751 To 800, With 7 Primes Within It = 367 Repeated Even Nos.
(pp) Percentage Increase In Repetition = (367 - 340) ÷ 340 x 100% = 7.94%

(q) Set Of Integers, 801 To 850, With 7 Primes Within It = 392 Repeated Even Nos.
(qq) Percentage Increase In Repetition = (392 - 367) ÷ 367 x 100% = 6.81%

(r) Set Of Integers, 851 To 900, With 8 Primes Within It = 412 Repeated Even Nos.
(rr) Percentage Increase In Repetition = (412 - 392) ÷ 392 x 100% = 5.1%

(s) Set Of Integers, 901 To 950, With 7 Primes Within It = 433 Repeated Even Nos.
(ss) Percentage Increase In Repetition = (433 - 412) ÷ 412 x 100% = 5.1%

(t) Set Of Integers, 951 To 1,000, With 7 Primes Within It = 470 Repeated Even Nos.
(tt) Percentage Increase In Repetition = (470 - 433) ÷ 433 x 100% = 8.55%

(u) Set Of Integers, 1,001 To 1,050, With 8 Primes Within It = 492 Repeated Even Nos.
(uu) Percentage Increase In Repetition = (492 - 470) ÷ 470 x 100% = 4.68%

(v) Set Of Integers, 1,051 To 1,100, With 8 Primes Within It = 523 Repeated Even Nos.
(vv) Percentage Increase In Repetition = (523 - 492) ÷ 492 x 100% = 6.3%

(w) Set Of Integers, 1,101 To 1,150, With 5 Primes Within It = 545 Repeated Even Nos.
(ww) Percentage Increase In Repetition = (545 - 523) ÷ 523 x 100% = 4.21%

(x) Set Of Integers, 1,151 To 1,200, With 7 Primes Within It = 553 Repeated Even Nos.
(xx) Percentage Increase In Repetition = (553 - 545) ÷ 545 x 100% = **1.47%**

(y) Set Of Integers, 1,201 To 1,250, With 8 Primes Within It = **592** Repeated Even Nos.
(yy) Percentage Increase In Repetition = (592 - 553) ÷ 553 x 100% = **7.05%**

It could be seen that on the whole the No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps "Generated" (By The Additions/Combinations Of 2 Primes) increases progressively from 20 in (b) to 592 in (y), while it could be seen that the Percentage Increase In Repetition on the whole thins out from 130% in (cc) to 7.05% in (yy), with the lowest percentage increase of 1.47% found in (xx). This statistical trend or feature is not surprising and represents (very) significant evidence that lends support to the validity of the Goldbach conjecture - the infinitude of both the primes and the even numbers implies that the above overlaps increase progressively to infinity.

(2) Density Of New Even Numbers "Generated" (*See Appendix 1 For Example Of Computation Method*)

(a) Set Of Integers, 51 To 100, With 10 Primes Within It = 5 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 10.)

(b) Set Of Integers, 101 To 150, With 10 Primes Within It = 5.2 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 10.)

(c) Set Of Integers, 151 To 200, With 11 Primes Within It = **4.55 New Even Nos. Per Prime No.** (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 11.)

(d) Set Of Integers, 201 To 250, With 7 Primes Within It = 6 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 42. No. Of Primes = 7.)

(e) Set Of Integers, 251 To 300, With 9 Primes Within It = 5.78 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 9.)

(f) Set Of Integers, 301 To 350, With 8 Primes Within It = 7 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 56. No. Of Primes = 8.)

(g) Set Of Integers, 351 To 400, With 8 Primes Within It = 6 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 48. No. Of Primes = 8.)

(h) Set Of Integers, 401 To 450, With 9 Primes Within It = 5.78 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 9.)

(i) Set Of Integers, 451 To 500, With 8 Primes Within It = 6.25 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 8.)

(j) Set Of Integers, 501 To 550, With 6 Primes Within It = 8 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 48. No. Of Primes = 6.)

(k) Set Of Integers, 551 To 600, With 8 Primes Within It = 6.5 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 8.)

(l) Set Of Integers, 601 To 650, With 9 Primes Within It = 5.33 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 48. No. Of Primes = 9.)

(m) Set Of Integers, 651 To 700, With 7 Primes Within It = 6.29 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 44. No. Of Primes = 7.)

(n) Set Of Integers, 701 To 750, With 7 Primes Within It = 7.43 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 7.)

(o) Set Of Integers, 751 To 800, With 7 Primes Within It = 7.71 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 54. No. Of Primes = 7.)

(p) Set Of Integers, 801 To 850, With 7 Primes Within It = 6 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 42. No. Of Primes = 7.)

(q) Set Of Integers, 851 To 900, With 8 Primes Within It = 6 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 48. No. Of Primes = 8.)

(r) Set Of Integers, 901 To 950, With 7 Primes Within It = 8.57 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 60. No. Of Primes = 7.)

(s) Set Of Integers, 951 To 1,000, With 7 Primes Within It = 7.14 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 7.)

(t) Set Of Integers, 1,001 To 1,050, With 8 Primes Within It = 6.5 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 52. No. Of Primes = 8.)

(u) Set Of Integers, 1,051 To 1,100, With 8 Primes Within It = 6 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 48. No. Of Primes = 8.)

(v) Set Of Integers, 1,101 To 1,150, With 5 Primes Within It = 6.4 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 32. No. Of Primes = 5.)

(w) Set Of Integers, 1,151 To 1,200, With 7 Primes Within It = **9.14 New Even Nos. Per Prime No.** (No. Of New Even Nos. "Generated" = 64. No. Of Primes = 7.)

(x) Set Of Integers, 1,201 To 1,250, With 8 Primes Within It = 7 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 56. No. Of Primes = 8.)

Average Density For The Above 24 Items ((a) To (x)) = 155.54 ÷ 24 = 6.48 New Even Nos. Per Prime No.

Maximum Density = 9.14 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 64. No. Of Primes = 7.)

Minimum Density = 4.55 New Even Nos. Per Prime No. (No. Of New Even Nos. "Generated" = 50. No. Of Primes = 11.)

Such a "profuse generation" of "regular batches" of even numbers by the prime numbers is (very) significant and represents a characteristic or feature of the prime numbers, a universal "pattern" or feature of the "chaotic" infinite prime numbers (or, recurrent, identical mathematical structures), which is excellently in accordance with the above lemma. This lends further support to the validity of the Goldbach conjecture, which, as stated above, is implied by both the infinitude of the primes and the even numbers.

There is indeed further incontrovertible proof which is obtainable by analysing a number of even numbers; e.g., we could split a group of 240 even consecutive numbers, from 4 to 482, into 8 equal batches (30 even numbers per batch) and analyse the batches, which buttresses the above evidence that the infinite quantity of primes would "generate" a regular, continuous (*without breaks or gaps*) and infinite list of even numbers. The density of distribution or prime additions/combinations per even number evidently become greater and greater the higher up the infinite list of the even numbers we go - this increase in density evidently represents a definite pattern in the "behaviour" of the prime numbers. This pattern is (very) significant and is discernable in the following example:-

(1) 1 st. Batch Of 30 Even Numbers (4 To 62) (*See Appendix 2 For Example Of Computation Method*)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = **5**
b) Minimum No. Of Prime Additions/Combinations Per Even Number = **1**
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **2.77** Prime Additions/Combinations Per Even Number (see Appendix for computation method)

(2) 2 nd. Batch Of 30 Even Numbers (64 To 122)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 14
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 2
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **6.1** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (6.1 - 2.77) ÷ 2.77 x 100% = 120.22%

(3) 3 rd. Batch Of 30 Even Numbers (124 To 182)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 16
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 4
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **9.07** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (9.07 - 6.1) ÷ 6.1 x 100% = 48.69%

(4) 4 th. Batch Of 30 Even Numbers (184 To 242)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 22
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 5
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **10.53** Prime Additions/Combinations Per Even Number

d) Percentage Increase In Density Of Distribution = (10.53 - 9.07) ÷ 9.07 x 100% = 16.1%

(5) 5 th. Batch Of 30 Even Numbers (244 To 302)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 21
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 7
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **12.37** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (12.37 - 10.53) ÷ 10.53 x 100% = 17.47%

(6) 6 th. Batch Of 30 Even Numbers (304 To 362)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 27
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 7
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **13.77** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (13.77 - 12.37) ÷ 12.37 x 100% = 11.32%

(7) 7 th. Batch Of 30 Even Numbers (364 To 422)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = 30
b) Minimum No. Of Prime Additions/Combinations Per Even Number = 7
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **15.23** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (15.23 - 13.77) ÷ 13.77 x 100% = 10.6%

(8) 8 th. Batch Of 30 Even Numbers (424 To 482)

a) Maximum No. Of Prime Additions/Combinations Per Even Number = **30**
b) Minimum No. Of Prime Additions/Combinations Per Even Number = **9**
c) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = **16.93** Prime Additions/Combinations Per Even Number
d) Percentage Increase In Density Of Distribution = (16.93 - 15.23) ÷ 15.23 x 100% = 11.16%

The Density Of Distribution is expected to increase to infinity, though the Percentage Increase In Density Of Distribution is expected to thin out towards infinity - it could be seen above to increase from 2.77 prime additions/combinations per even number for batch of even numbers, 4 to 62, all the way up to 16.93 prime additions/combinations per even number for batch of even numbers, 424 to 482. This is nevertheless (very) significant evidence that lends support to the validity of the Goldbach conjecture. Also, the Maximum No. Of Prime Additions/Combinations Per Even Number and the Minimum No. Of Prime Additions/Combinations Per Even Number could be seen to range from 5 and 1 respectively for batch of even numbers, 4 to 62, to 30 and 9 respectively for batch of even numbers, 424 to 482. This trend of “upward increase” of the (maximum and minimum) numbers of prime additions/combinations for each even number implies that at some points toward infinity the numbers of prime additions/combinations for each even number could be thousands, millions, billions, trillions, and more, if only we have the computing power to compute/check such prime additions/combinations. This is (very) significant too and is also evidence that lends support to the validity of the Goldbach conjecture. By the infinitude of the primes and even numbers and the above lemma, these “patterns”, as described here, would be there all the way to infinity, which would be in accordance with the Goldbach conjecture.

The one-to-one additions/combinations of the primes in the formation of even numbers do evidently become more and more “overwhelming” or profuse the higher up the infinite list of even numbers/prime numbers we go, thereby assuring an infinite, regular and consecutive supply of even numbers, as is evident from the example just above. This, together with the above-described evidently more and more profuse repetitions and overlaps of the even numbers “generated” by the primes the higher up the infinite list of prime numbers we go (refer to No. Of Old/Repeated (Also Appeared Earlier) Even Numbers/Overlaps “Generated” (By The Additions/Combinations Of 2 Primes), For Integers 1 To 1,250 above), go to show that the Goldbach conjecture becomes, evidently, even stronger and stronger the higher up the infinite list of prime numbers/even numbers we go. Here, we have in fact approached the problem from 2 different, but somewhat related, angles - by a statistical analysis of the “behaviour” of the primes in the formation of even numbers, and, a statistical analysis of the even numbers “generated” as a result. The statistical data thus obtained are indeed found to greatly support the Goldbach conjecture, evidently the more so the higher up the infinite list of prime numbers/even numbers we go, and, by the infinitude of the primes and even numbers and the above lemma there would be an infinitude of such statistical data thus obtained. Hence, by virtue of these imposing statistical trends, plus the statistical trend that a prolific number of new even numbers are always being "generated" (refer to Density Of New Even Numbers "Generated" above), as well as the infinitude of the prime numbers and the even numbers, together with the above lemma, we affirm the validity of the Goldbach conjecture.

Reversing the "reversed way", we hereby affirm that every even number after the number 2 in the infinite list of even numbers is a combination or sum of 2 primes. In fact, the prime numbers are the building-blocks or "atoms" of all the even numbers - and more - the prime numbers are the building-blocks of all the integers or whole numbers: every even number (with the exception of 2) is the sum of 2 prime numbers, and, every odd number is either a prime number, or, a composite of prime numbers (that is, the odd number has prime factors). It is truly the peculiar characteristics of the prime numbers themselves (as described above, whose distribution could in fact be predicted by the prime number theorem which had been proven, implying some pattern or fractal nature in the prime numbers as per the above lemma), which could be regarded as a self-similar or fractal feature as such, that are responsible for the Goldbach conjecture being true. By induction the Goldbach conjecture has been proven true - the above constitutes proof of the Goldbach conjecture (which ought to be known as the Goldbach Theorem instead).

This proof could be extended here. It has been mentioned above that the Goldbach conjecture had been tested and found to be correct for every even number up to 12×10^{17} by computer searches completed in 2008. Thus, by the above lemma, and, the infinitude of the primes and even numbers, this long list of consecutive even numbers up to 12×10^{17} reflects (indicates or implies) the fact that all the infinite even numbers above 12×10^{17} would each be the sum of 2 primes. (This is in accordance with the "reflection" principle stated above, wherein it is mentioned that the characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is enough for deducing the characteristic of the mountain or infinite volume of sand.)

We therefore declare that the Goldbach conjecture is true - every even number after the number 2 is indeed the sum of 2 primes.

APPENDIX 1

(20) Set Of Integers, 1,201 To 1,250, With 8 Primes Within It
- (a) Primes: 1,201; 1,213; 1,217; 1,223; 1,229; 1,231; 1,237 and 1,249
- (b) No. Of Primes: 8
- (c) No. Of Even Numbers "Generated" (Including Repetitions) By The 8 Primes = 648 (1,204 [1,201 + 3] To 2,498 [1,249 + 1,249])
- (d) No. Of New Even Numbers "Generated" = 56 (2,388 To 2,498)
- (e) No. Of Old/Repeated (Also Appeared In (19) Above, With Some Also Having Appeared In (18), (17), (16), (15), (14), (13), (12), (11), (10), (9) And (8) Above) Even Numbers "Generated" (I.e., Repetitions/Overlaps) = 592 (1,204 To 2,386)
- (f) Density Of New Even Numbers "Generated" = (d) ÷ 8 Primes = 56 ÷ 8 = 7 New Even Numbers Per Prime Number

APPENDIX 2

(8) 8 th. Batch Of 30 Even Numbers (424 To 482)
- (a) 424: No. Of Above-mentioned Prime Additions/Combinations = 12
- (b) 426: No. Of Above-mentioned Prime Additions/Combinations = 21
- (c) 428: No. Of Above-mentioned Prime Additions/Combinations = **9**
- (d) 430: No. Of Above-mentioned Prime Additions/Combinations = 14
- (e) 432: No. Of Above-mentioned Prime Additions/Combinations = 19
- (f) 434: No. Of Above-mentioned Prime Additions/Combinations = 14
- (g) 436: No. Of Above-mentioned Prime Additions/Combinations = 11
- (h) 438: No. Of Above-mentioned Prime Additions/Combinations = 22
- (i) 440: No. Of Above-mentioned Prime Additions/Combinations = 15
- (j) 442: No. Of Above-mentioned Prime Additions/Combinations = 13
- (k) 444: No. Of Above-mentioned Prime Additions/Combinations = 22
- (l) 446: No. Of Above-mentioned Prime Additions/Combinations = 12
- (m) 448: No. Of Above-mentioned Prime Additions/Combinations = 13
- (n) 450: No. Of Above-mentioned Prime Additions/Combinations = 29

(o) 452: No. Of Above-mentioned Prime Additions/Combinations = 14
(p) 454: No. Of Above-mentioned Prime Additions/Combinations = 12
(q) 456: No. Of Above-mentioned Prime Additions/Combinations = 26
(r) 458: No. Of Above-mentioned Prime Additions/Combinations = **9**
(s) 460: No. Of Above-mentioned Prime Additions/Combinations = 17
(t) 462: No. Of Above-mentioned Prime Additions/Combinations = **30**
(u) 464: No. Of Above-mentioned Prime Additions/Combinations = 13
(v) 466: No. Of Above-mentioned Prime Additions/Combinations = 14
(w) 468: No. Of Above-mentioned Prime Additions/Combinations = 26
(x) 470: No. Of Above-mentioned Prime Additions/Combinations = 16
(y) 472: No. Of Above-mentioned Prime Additions/Combinations = 14
(z) 474: No. Of Above-mentioned Prime Additions/Combinations = 24
(aa) 476: No. Of Above-mentioned Prime Additions/Combinations = 14
(bb) 478: No. Of Above-mentioned Prime Additions/Combinations = 12
(cc) 480: No. Of Above-mentioned Prime Additions/Combinations = **30**
(dd) 482: No. Of Above-mentioned Prime Additions/Combinations = 11
(i) Maximum No. Of Prime Additions/Combinations = 30
(ii) Minimum No. Of Prime Additions/Combinations = 9
(iii) Total No. Of Prime Additions/Combinations For (a) To (dd) = 508
(iv) Total No. Of Even Numbers = 30
(v) Density Of Distribution = Average Prime Additions/Combinations Per Even Number = (iii) ÷ (iv) = 508 ÷ 30 = 16.93 Prime Additions/Combinations Per Even Number

PART 2

Theorem:- Every even number after 2 is the sum of 2 primes.
Proof 1:-
Lemma: By Euclid's proof the primes are infinite.

The prime number theorem, which had been proven, states that the limit of the quotient of the 2 functions $\pi(n)$ and $n/\log n$ as n approaches infinity is 1, which is expressed by the formula:

$$\lim_{n\to\infty} \pi(n)/(n/\log n) = 1 , \quad \text{where } \pi(n) \text{ is approximately equal to } (n/\log n)$$

The function $\pi(n)$ represents the number of primes less than or equal to the number n. This function measures the distribution of the prime numbers. With it, we compute the ratio $n/\pi(n)$ which says what fraction of the numbers up to a given point are primes. (It is actually the reciprocal of this fraction.) The following is the result of a computation:-

n	***π(n)***	***n/ π(n)***
10	4 (a)	2.5
100	25 (b)	4.0
1,000	168 (c)	6.0
10,000	1,229 (d)	8.1
100,000	9,592 (e)	10.4
1,000,000	78,498 (f)	12.7
10,000,000	664,579 (g)	15.0
100,000,000	5,761,455 (h)	17.4
1,000,000,000	50,847,534 (i)	19.7
10,000,000,000	455,052,512 (j)	22.0

It is noticeable that as one moves from 1 power of 10 to the next, the ratio $n/\pi(n)$ increases by about 2.3, e.g., 22.0 - 19.7 = 2.3. As $\log_e 10 = 2.30258 \ldots$, we may thus regard $\pi(n)$ as approximately equal to $n/\log n$.

We have the following partitions with the primes described in the "$\pi(n)$" column above:-

1) With (a) above, we have the following "prime + prime = even number" combinations:

a) prime a + prime a: 4 x 4 "prime + prime" combinations
b) prime a + prime b: 4 x 25 "prime + prime" combinations

c) prime a + prime c: 4 x 168 “prime + prime” combinations
d) prime a + prime d: 4 x 1,229 “prime + prime” combinations
e) prime a + prime e: 4 x 9,592 “prime + prime” combinations
f) prime a + prime f: 4 x 78,498 “prime + prime” combinations
g) prime a + prime g: 4 x 664,579 “prime + prime” combinations
h) prime a + prime h: 4 x 5,761,455 “prime + prime” combinations
i) prime a + prime i: 4 x 50,847,534 “prime + prime” combinations
j) prime a + prime j: 4 x 455,052,512 “prime + prime” combinations

For example, for (j) above, a prime described in (a) in the “$\pi(n)$” column above plus a prime described in (j) in the “$\pi(n)$” column above give an even number, and there are 4 x 455,052,512 such “prime + prime = even number” combinations.

2) With (b) above, we have the following “prime + prime = even number” combinations:

a) prime b + prime a: 25 x 4 “prime + prime” combinations
b) prime b + prime b: 25 x 25 “prime + prime” combinations
c) prime b + prime c: 25 x 168 “prime + prime” combinations
d) prime b + prime d: 25 x 1,229 “prime + prime” combinations
e) prime b + prime e: 25 x 9,592 “prime + prime” combinations
f) prime b + prime f: 25 x 78,498 “prime + prime” combinations
g) prime b + prime g: 25 x 664,579 “prime + prime” combinations
h) prime b + prime h: 25 x 5,761,455 “prime + prime” combinations
i) prime b + prime i: 25 x 50,847,534 “prime + prime” combinations
j) prime b + prime j: 25 x 455,052,512 “prime + prime” combinations

3) With (c) above, we have the following “prime + prime = even number” combinations:

a) prime c + prime a: 168 x 4 “prime + prime” combinations
b) prime c + prime b: 168 x 25 “prime + prime” combinations
c) prime c + prime c: 168 x 168 “prime + prime” combinations
d) prime c + prime d: 168 x 1,229 “prime + prime” combinations
e) prime c + prime e: 168 x 9,592 “prime + prime” combinations
f) prime c + prime f: 168 x 78,498 “prime + prime” combinations
g) prime c + prime g: 168 x 664,579 “prime + prime” combinations
h) prime c + prime h: 168 x 5,761,455 “prime + prime” combinations
i) prime c + prime i: 168 x 50,847,534 “prime + prime” combinations
j) prime c + prime j: 168 x 455,052,512 “prime + prime” combinations

4) With (d) above, we have the following “prime + prime = even number” combinations:

a) prime d + prime a: 1,229 x 4 “prime + prime” combinations
b) prime d + prime b: 1,229 x 25 “prime + prime” combinations
c) prime d + prime c: 1,229 x 168 “prime + prime” combinations
d) prime d + prime d: 1,229 x 1,229 “prime + prime” combinations
e) prime d + prime e: 1,229 x 9,592 “prime + prime” combinations
f) prime d + prime f: 1,229 x 78,498 “prime + prime” combinations
g) prime d + prime g: 1,229 x 664,579 “prime + prime” combinations
h) prime d + prime h: 1,229 x 5,761,455 “prime + prime” combinations
i) prime d + prime i: 1,229 x 50,847,534 “prime + prime” combinations
j) prime d + prime j: 1,229 x 455,052,512 “prime + prime” combinations

5) With (e) above, we have the following “prime + prime = even number” combinations:

a) prime e + prime a: 9,592 x 4 “prime + prime” combinations
b) prime e + prime b: 9,592 x 25 “prime + prime” combinations
c) prime e + prime c: 9,592 x 168 “prime + prime” combinations
d) prime e + prime d: 9,592 x 1,229 “prime + prime” combinations
e) prime e + prime e: 9,592 x 9,592 “prime + prime” combinations
f) prime e + prime f: 9,592 x 78,498 “prime + prime” combinations
g) prime e + prime g: 9,592 x 664,579 “prime + prime” combinations
h) prime e + prime h: 9,592 x 5,761,455 “prime + prime” combinations

i) prime e + prime i: 9,592 x 50,847,534 "prime + prime" combinations
j) prime e + prime j: 9,592 x 455,052,512 "prime + prime" combinations

6) With (f) above, we have the following "prime + prime = even number" combinations:

a) prime f + prime a: 78,498 x 4 "prime + prime" combinations
b) prime f + prime b: 78,498 x 25 "prime + prime" combinations
c) prime f + prime c: 78,498 x 168 "prime + prime" combinations
d) prime f + prime d: 78,498 x 1,229 "prime + prime" combinations
e) prime f + prime e: 78,498 x 9,592 "prime + prime" combinations
f) prime f + prime f: 78,498 x 78,498 "prime + prime" combinations
g) prime f + prime g: 78,498 x 664,579 "prime + prime" combinations
h) prime f + prime h: 78,498 x 5,761,455 "prime + prime" combinations
i) prime f + prime i: 78,498 x 50,847,534 "prime + prime" combinations
j) prime f + prime j: 78,498 x 455,052,512 "prime + prime" combinations

7) With (g) above, we have the following "prime + prime = even number" combinations:

a) prime g + prime a: 664,579 x 4 "prime + prime" combinations
b) prime g + prime b: 664,579 x 25 "prime + prime" combinations
c) prime g + prime c: 664,579 x 168 "prime + prime" combinations
d) prime g + prime d: 664,579 x 1,229 "prime + prime" combinations
e) prime g + prime e: 664,579 x 9,592 "prime + prime" combinations
f) prime g + prime f: 664,579 x 78,498 "prime + prime" combinations
g) prime g + prime g: 664,579 x 664,579 "prime + prime" combinations
h) prime g + prime h: 664,579 x 5,761,455 "prime + prime" combinations
i) prime g + prime i: 664,579 x 50,847,534 "prime + prime" combinations
j) prime g + prime j: 664,579 x 455,052,512 "prime + prime" combinations

8) With (h) above, we have the following "prime + prime = even number" combinations:

a) prime h + prime a: 5,761,455 x 4 "prime + prime" combinations
b) prime h + prime b: 5,761,455 x 25 "prime + prime" combinations
c) prime h + prime c: 5,761,455 x 168 "prime + prime" combinations
d) prime h + prime d: 5,761,455 x 1,229 "prime + prime" combinations
e) prime h + prime e: 5,761,455 x 9,592 "prime + prime" combinations
f) prime h + prime f: 5,761,455 x 78,498 "prime + prime" combinations
g) prime h + prime g: 5,761,455 x 664,579 "prime + prime" combinations
h) prime h + prime h: 5,761,455 x 5,761,455 "prime + prime" combinations
i) prime h + prime i: 5,761,455 x 50,847,534 "prime + prime" combinations
j) prime h + prime j: 5,761,455 x 455,052,512 "prime + prime" combinations

9) With (i) above, we have the following "prime + prime = even number" combinations:

a) prime i + prime a: 50,847,534 x 4 "prime + prime" combinations
b) prime i + prime b: 50,847,534 x 25 "prime + prime" combinations
c) prime i + prime c: 50,847,534 x 168 "prime + prime" combinations
d) prime i + prime d: 50,847,534 x 1,229 "prime + prime" combinations
e) prime i + prime e: 50,847,534 x 9,592 "prime + prime" combinations
f) prime i + prime f: 50,847,534 x 78,498 "prime + prime" combinations
g) prime i + prime g: 50,847,534 x 664,579 "prime + prime" combinations
h) prime i + prime h: 50,847,534 x 5,761,455 "prime + prime" combinations
i) prime i + prime i: 50,847,534 x 50,847,534 "prime + prime" combinations
j) prime i + prime j: 50,847,534 x 455,052,512 "prime + prime" combinations

10) With (j) above, we have the following "prime + prime = even number" combinations:

a) prime j + prime a: 455,052,512 x 4 "prime + prime" combinations
b) prime j + prime b: 455,052,512 x 25 "prime + prime" combinations
c) prime j + prime c: 455,052,512 x 168 "prime + prime" combinations
d) prime j + prime d: 455,052,512 x 1,229 "prime + prime" combinations
e) prime j + prime e: 455,052,512 x 9,592 "prime + prime" combinations

f) prime j + prime f: 455,052,512 x 78,498 “prime + prime” combinations
g) prime j + prime g: 455,052,512 x 664,579 “prime + prime” combinations
h) prime j + prime h: 455,052,512 x 5,761,455 “prime + prime” combinations
i) prime j + prime i: 455,052,512 x 50,847,534 “prime + prime” combinations
j) prime j + prime j: 455,052,512 x 455,052,512 “prime + prime” combinations

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The above partitions/“prime + prime = even number” combinations are evidently progressively more “overwhelming”, dense, and repetitive. That is, the Goldbach conjecture becomes evidently progressively stronger and stronger towards infinity. It is not surprising that computer searches completed in 2000 had verified that all even numbers up to 400 trillion (4×10^{14}), which is not a small list, are sums of 2 primes, while in 2008, a distributed computer search ran by Tomas Oliveira e Silva, a researcher at the University of Aveiro, Portugal, had further verified the Goldbach conjecture up to 12×10^{17}, which is a long, impressive list.

The infinitude of the primes, as per the above lemma, together with the infinitude of the even numbers, however imply that the above partitions/“prime + prime = even number” combinations would become increasingly more “overwhelming”, dense, and repetitive towards infinity (the Goldbach conjecture becoming evidently stronger and stronger the higher up the infinite list of prime numbers/even numbers we go), hence “ensuring” the continuity (without any breaks or gaps) of the even numbers generated, and would be so all the way to infinity, thus proving that every even number after 2 is the sum of 2 primes.

Proof 2:-
Lemma: According to the principle of complete induction in set theory, if a set of natural numbers contains 1 and, for each n, it contains $n + 1$ whenever it contains all numbers less than $n + 1$, then it must contain every natural number, e.g., complete induction proves that every natural number is a product of primes.

By the above lemma, every even number after 2 in the infinite set of the integers is the sum of 2 primes; as per the distributed computer search completed in 2008 at the University of Aveiro, Portugal, stated above, 12×10^{17} in the infinite set of the integers is the largest even number found to be the sum of 2 primes while all the consecutive even numbers before it, from 4 to $(12 \times 10^{17}) - 2$, which is not a small list of numbers (it is in fact a long, impressive list, obtainable only with the help of modern computer technology), are also found to be sums of 2 primes - the principle of complete induction implies that all even numbers after 12×10^{17} in the infinite set of the integers must also be sums of 2 primes, i.e., it implies that every even number after 2 in the infinite set of the integers must be the sum of 2 primes - in other words, the Goldbach conjecture must be true.

Proof 3:-
Lemma: By Euclid’s proof the primes are infinite.

We make use of the proof by “reductio ad absurdum” here. For this indirect proof, we assume that the Goldbach conjecture is false. (Before we proceed further, we should again note that a long, impressive list of consecutive even numbers, from 4 to 12×10^{17}, had already been verified to be sums of 2 primes, and, these partitions/“prime + prime = even number” combinations would become increasingly more “overwhelming”, dense, and repetitive towards infinity (the Goldbach conjecture becoming evidently stronger and stronger the higher up the infinite list of prime numbers/even numbers we go), as is described above. The moot question now is, of course, whether after 12×10^{17} there would be an even number in the infinite list of even numbers which is the last, or, largest, even number which is the sum of 2 primes - this largest even number, if it exists (thereby proving the falsehood of the Goldbach conjecture), must (of necessity) be the sum of 2 primes which are each the largest existing prime. Before we continue, this point should be clearly held in mind.) This assumption implies that there is a limit to the even numbers which are sums of 2 primes and that there is a largest even number (e) which is, and must necessarily be, the sum of 2 primes that are each the largest existing prime ($e = x + x$, this largest even number, e, representing the ultimate limit of the even numbers which are sums of 2 primes, the 2 primes which add up to give e being of necessity each the largest existing prime (x)). This is of course a contradiction of the above lemma, which would imply that the lemma is false. But the lemma cannot be false - it is in fact a theorem (which had been proven by Euclid); there cannot be a largest existing prime (x) - the primes are infinite. This means that our assumption that the Goldbach conjecture is false is untenable and that the Goldbach conjecture must be true, i.e., every even number after 2 must be the sum of 2 primes. As a matter of fact, the above lemma implies that there would be an infinite number of double primes which sum up to an even number.

By both induction and contradiction the Goldbach conjecture is hence proved.

PART 3

Theorem:- Every even number after 2 is the sum of 2 primes.
Proof 1:-
Every even number after 2 is the sum of 2 odd numbers. Every odd number is either a prime which is odd or a composite - product of primes which are odd; notably, every prime with the exception of 2 is an odd number. Every even number after 2 is also a composite, but, a composite with at least 1 even prime factor, namely, 2, while the rest of its prime factors are odd, i.e., it is an even composite.

Therefore, every even number after 2 is the sum of 2 primes which are odd and/or the sum of 1 prime which is odd and 1 odd composite whose prime factors are odd and/or the sum of 2 odd composites whose prime factors are odd, besides being an even composite with at least 1 even prime factor, namely, 2, while the rest of its prime factors are odd.

Lemma:
By Euclid's proof, the primes are infinite; this implies that there would be an infinitude of sums of 2 primes as per the Goldbach conjecture. The even numbers, which are sums of 2 primes as per the conjecture, are also infinite. Thus, there are an infinite number of even numbers which are sums of 2 primes, both the even numbers and sums of 2 primes being infinite.

Corollary:
The odd numbers, which are either prime, every prime with the exception of 2 being an odd number, or composite (have prime factors which are odd), are infinite; this implies that there would be an infinite number of sums of 2 odd numbers, each of which is equal to an even number. Hence, as there is an infinitude of even numbers which are sums of 2 primes, as per the above lemma, and as all primes with the exception of 2 are odd numbers, there are an infinite number of even numbers which are sums of 2 odd numbers that are prime, all the even numbers, sums of 2 odd numbers and primes being infinite; i.e., every even number after 2 is also the sum of 2 odd numbers that are prime.

We thereby see the close interlink or relationship between the primes, even numbers and odd numbers, which are all infinite, which is significant.

The following are thus evident:

a) Every sum of 2 primes which are odd numbers is equal to an even number, as is below in consecutive order:

2 + 2 = 1 + 3 = **4**
3 + 3 = 1 + 5 = **6**
3 + 5 = 1 + 7 = **8**
5 + 5 = 3 + 7 = **10**
5 + 7 = 1 + 11 = **12**
7 + 7 = 3 + 11 = 1 + 13 = **14**
3 + 13 = 5 + 11 = **16**
7 + 11 = 5 + 13 = 1 + 17 = **18**
7 + 13 = 3 + 17 = 1 + 19 = **20**
11 + 11 = 3 + 19 = 5 + 17 = 11 + 11 = **22**
11 + 13 = 5 + 19 = 7 + 17 = 1 + 23 = **24**
13 + 13 = 3 + 23 = 7 + 19 = **26**
11 + 17 = 5 + 23 = **28**
13 + 17 = 11 + 19 = 7 + 23 = 1 + 29 = **30**
3 + 29 = 13 + 19 = 1 + 31 = **32**
17 + 17 = 3 + 31 = 5 + 29 = 11 + 23 = 17 + 17 = **34**
17 + 19 = 5 + 31 = 7 + 29 = 13 + 23 = **36**
19 + 19 = 7 + 31 = 1 + 37 = **38**
3 + 37 = 11 + 29 = 17 + 23 = **40**
19 + 23 = 5 + 37 = 11 + 31 = 13 + 29 = 1 + 41 = **42**
3 + 41 = 7 + 37 = 13 + 31 = 1 + 43 = **44**
23 + 23 = 3 + 43 = 5 + 41 = 17 + 29 = **46**
5 + 43 = 7 + 41 = 11 + 37 = 17 + 31 = 19 + 29 = 1 + 47 = **48**
3 + 47 = 7 + 43 = 13 + 37 = 19 + 31 = **50**
23 + 29 = 5 + 47 = 11 + 41 = **52**

7 + 47 = 11 + 43 = 13 + 41 = 17 + 37 = 23 + 31 = 1 + 53 = **54**
3 + 53 = 13 + 43 = 19 + 37 = **56**
29 + 29 = 5 + 53 = 11 + 47 = 17 + 41 = 29 + 29 = **58**
29 + 31 = 7 + 53 = 13 + 47 = 17 + 43 = 19 + 41 = 23 + 37 = 1 + 59 = **60**
31 + 31 = 3 + 59 = 19 + 43 = 1 + 61 = **62**
3 + 61 = 5 + 59 = 11 + 53 = 17 + 47 = 23 + 41 = **64**
5 + 61 = 7 + 59 = 13 + 53 = 19 + 47 = 23 + 43 = 29 + 37 = **66**
7 + 61 = 31 + 37 = 1 + 67 = **68**
3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41 = **70**
5 + 67 = 11 + 61 = 13 + 59 = 19 + 53 = 29 + 43 = 31 + 41 = 1 + 71 = **72**
37 + 37 = 3 + 71 = 7 + 67 = 13 + 61 = 31 + 43 = 37 + 37 = 1 + 73 = **74**
3 + 73 = 5 + 71 = 17 + 59 = 23 + 53 = 29 + 47 = **76**
37 + 41 = 5 + 73 = 7 + 71 = 11 + 67 = 31 + 47 = 37 + 41 = **78**
7 + 73 = 13 + 67 = 19 + 61 = 37 + 43 = 1 + 79 = **80**
41 + 41 = 3 + 79 = 11 + 71 = 23 + 59 = 29 + 53 = **82**
41 + 43 = 5 + 79 = 11 + 73 = 13 + 71 = 17 + 67 = 23 + 61 = 31 + 53 = 37 + 47 = 1+ 83 = **84**
43 + 43 = 3 + 83 = 7 + 79 = 13 + 73 = 19 + 67 = 43 + 43 = **86**
5 + 83 = 17 + 71 = 29 + 59 = 41 + 47 = **88**
7 + 83 = 11 + 79 = 17 + 73 = 19 + 71 = 23 + 67 = 29 + 61 = 31 + 59 = 37 + 53 = 43 + 47 = 1 + 89 = **90**
3 + 89 = 13 + 79 = 19 + 73 = 31 + 61 = 1 + 91 = **92**
47 + 47 = 5 + 89 = 11 + 83 = 23 + 71 = 41 + 53 = 47 + 47 = **94**
5 + 91 = 7 + 89 = 13 + 83 = 17 + 79 = 23 + 73 = 29 + 67 = 37 + 59 = 43 + 53 = **96**
7 + 91 = 19 + 79 = 31 + 67 = 37 + 61 = 1 + 97 = **98**
47 + 53 = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53 = **100**
5 + 97 = 11 + 91 = 13 + 89 = 19 + 83 = 23 + 79 = 29 + 73 = 31 + 71 = 41+ 61 = 43 + 59 = 1 + 101 = **102**

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b) Every sum of 1 prime which is an odd number & 1 odd composite which is the product of primes which are odd, is equal to the sum of 2 primes which are odd numbers, which are all each equal to an even number, as is below in consecutive order:

3 + 9 = 5 + 7 = 1 + 11 = **12**
5 + 9 = 3 + 11 = 7 + 7 = 1 + 13 = **14**
7 + 9 = 3 + 13 = 5 + 11 = **16**
3 + 15 = 7 + 11 = 5 + 13 = 1 + 17 **= 18**
11 + 9 = 3 + 17 = 7 + 13 = 1 + 19 = **20**
13 + 9 = 3 + 19 = 5 + 17 = 11 + 11 = **22**
3 + 21 = 11 + 13 = 5 + 19 = 7 + 17 = 1 + 23 **= 24**
17 + 9 = 3 + 23 = 7 + 19 = 13 + 13 = **26**
19 + 9 = 5 + 23 = 11 + 17 = **28**
5 + 25 = 13 + 17 = 11 + 19 = 7 + 23 = 1 + 29 = **30**
23 + 9 = 3 + 29 = 13 + 19 = 1 + 31 = **32**
7 + 27 = 17 + 17 = 3 + 31 = 5 + 29 = 11 + 23 = 17 + 17 **= 34**
3 + 33 = 17 + 19 = 5 + 31 = 7 + 29 = 13 + 23 **= 36**
29 + 9 = 7 + 31 = 19 + 19 = 1 + 37 = **38**
31 + 9 = 3 + 37 = 11 + 29 = 17 + 23 = **40**
3 + 39 = 19 + 23 = 5 + 37 = 11 + 31 = 13 + 29 = 1 + 41 **= 42**
5 + 39 = 3 + 41 = 7 + 37 = 13 + 31 = 1 + 43 **= 44**
37 + 9 = 3 + 43 = 5 + 41 = 17 + 29 = 23 + 23 = **46**
3 + 45 = 5 + 43 = 7 + 41 = 11 + 37 = 17 + 31 = 19 + 29 = 1 + 47 **= 48**
41 + 9 = 3 + 47 = 7 + 43 = 13 + 37 = 19 + 31 = **50**
43 + 9 = 5 + 47 = 11 + 41 =23 + 29 = **52**
5 + 49 = 7 + 47 = 11 + 43 = 13 + 41 = 17 + 37 = 23 + 31 = 1 + 53 **= 54**
47 + 9 = 3 + 53 = 13 + 43 = 19 + 37 = **56**
3 + 55 = 29 + 29 = 5 + 53 = 11 + 47 = 17 + 41 = 29 + 29 = **58**
5 + 55 = 29 + 31 = 7 + 53 = 13 + 47 = 17 + 43 = 19 + 41 = 23 + 37 = 1 + 59 = **60**
53 + 9 = 3 + 59 = 19 + 43 = 31 + 31 = 1 + 61 = **62**
7 + 57 = 3 + 61 = 5 + 59 = 11 + 53 = 17 + 47 = 23 + 41 **= 64**
11 + 55 = 5 + 61 = 7 + 59 = 13 + 53 = 19 + 47 = 23 + 43 = 29 + 37 **= 66**
59 + 9 = 7 + 61 = 31 + 37 = 1 + 67 = **68**

61 + 9 = 3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41 = **70**
3 + 69 = 5 + 67 = 11 + 61 = 13 + 59 = 19 + 53 = 29 + 43 = 31 + 41 = 1 + 71 = **72**
5 + 69 = 37 + 37 = 3 + 71 = 7 + 67 = 13 + 61 = 31 + 43 = 37 + 37 = 1 + 73 **= 74**
67 + 9 = 3 + 73 = 5 + 71 = 17 + 59 = 23 + 53 = 29 + 47 = **76**
3 + 75 = 37 + 41 = 5 + 73 = 7 + 71 = 11 + 67 = 31 + 47 = 37 + 41 **= 78**
71 + 9 = 7 + 73 = 13 + 67 = 19 + 61 = 37 + 43 = 1 + 79 = **80**
73 + 9 = 3 + 79 = 11 + 71 = 23 + 59 = 29 + 53 = 41 + 41 = **82**
3 + 81 = 41 + 43 = 5 + 79 = 11 + 73 = 13 + 71 = 17 + 67 = 23 + 61 = 31 + 53 = 37 + 47 = 1+ 83 = **84**
5 + 81 = 43 + 43 = 3 + 83 = 7 + 79 = 13 + 73 = 19 + 67 = 43 + 43 = **86**
79 + 9 = 5 + 83 = 17 + 71 = 29 + 59 = 41 + 47 = **88**
3 + 87 = 7 + 83 = 11 + 79 = 17 + 73 = 19 + 71 = 23 + 67 = 29 + 61 = 31 + 59 = 37 + 53 = 43 + 47 = 1 + 89 **= 90**
83 + 9 = 3 + 89 = 13 + 79 = 19 + 73 = 31 + 61 = 1 + 91 = **92**
7 + 87 = 47 + 47 = 5 + 89 = 11 + 83 = 23 + 71 = 41 + 53 = 47 + 47 **= 94**
3 + 93 = 5 + 91 = 7 + 89 = 13 + 83 = 17 + 79 = 23 + 73 = 29 + 67 = 37 + 59 = 43 + 53 **= 96**
89 + 9 = 7 + 91 = 19 + 79 = 31 + 67 = 37 + 61 = 1 + 97 = **98**
91 + 9 = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53 = **100**
3 + 99 = 5 + 97 = 11 + 91 = 13 + 89 = 19 + 83 = 23 + 79 = 29 + 73 = 31 + 71 = 41+ 61 = 43 + 59 = 1 + 101 = **102**

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c) Every sum of 2 odd composites which are products of primes which are odd, is equal to the sum of 2 primes which are odd numbers, which are all each equal to an even number, as is below in consecutive order:

9 + 9 = 5 + 13 = 7 + 11 = 1 + 17 = **18**
9 + 15 = 5 + 19 = 7 + 17 = 11 + 13 = 1 + 23 = **24**
15 + 15 = 7 + 23 = 11 + 19 = 13 + 17 = 1 + 29 = **30**
9 + 25 = 7 + 27 = 17 + 17 = 3 + 31 = 5 + 29 = 11 + 23 = 17 + 17 = **34**
15 + 21 = 5 + 31 = 7 + 29 = 13 + 23 = 17 + 19 = **36**
15 + 25 = 3 + 37 = 11 + 29 = 17 + 23 **= 40**
21 + 21 = 5 + 37 = 11 + 31 = 13 + 29 = 19 + 23 = 1 + 41 = **42**
9 + 35 = 3 + 41 = 7 + 37 = 13 + 31 = 1 + 43 **= 44**
21 + 25 = 3 + 43 = 5 + 41 = 17 + 29 = 23 + 23 = **46**
9 + 39 = 5 + 43 = 7 + 41 = 11 + 37 = 17 + 31 = 19 + 29 = 1 + 47 **= 48**
25 + 25 = 3 + 47 = 7 + 43 = 13 + 37 = 19 + 31 = **50**
25 + 27 = 5 + 47 = 11 + 41 =23 + 29 = **52**
27 + 27 = 7 + 47 = 11 + 43 = 13 + 41 = 17 + 37 = 23 + 31 = 1 + 53 = **54**
21 + 35 = 3 + 53 = 13 + 43 = 19 + 37 **= 56**
9 + 49 = 29 + 29 = 5 + 53 = 11 + 47 = 17 + 41 = 29 + 29 **= 58**
27 + 33 = 7 + 53 = 13 + 47 = 17 + 43 = 19 + 41 = 23 + 37 = 29 + 31 = 1 + 59 = **60**
27 + 35 = 31 + 31 = 3 + 59 = 19 + 43 = 1 + 61 **= 62**
9 + 55 = 3 + 61 = 5 + 59 = 11 + 53 = 17 + 47 = 23 + 41 = **64**
33 + 33 = 5 + 61 = 7 + 59 = 13 + 53 = 19 + 47 = 23 + 43 = 29 + 37 = **66**
33 + 35 = 7 + 61 = 31 + 37 = 1 + 67 = **68**
35 + 35 = 3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41 = **70**
9 + 63 = 5 + 67 = 11 + 61 = 13 + 59 = 19 + 53 = 29 + 43 = 31 + 41 = 1 + 71 **= 72**
35 + 39 = 3 + 71 = 7 + 67 = 13 + 61 = 31 + 43 = 37 + 37 = 1 + 73 = **74**
21 + 55 = 3 + 73 = 5 + 71 = 17 + 59 = 23 + 53 = 29 + 47 **= 76**
39 + 39 = 5 + 73 = 7 + 71 = 11 + 67 = 31 + 47 = 37 + 41 = **78**
15 + 65 = 7 + 73 = 13 + 67 = 19 + 61 = 37 + 43 = 1 + 79 **= 80**
25 + 57 = 41 + 41 = 3 + 79 = 11 + 71 = 23 + 59 = 29 + 53 **= 82**
39 + 45 = 5 + 79 = 11 + 73 = 13 + 71 = 17 + 67 = 23 + 61 = 31 + 53 = 37 + 47 = 41 + 43 = 1 + 83 = **84**
9 + 77 = 43 + 43 = 3 + 83 = 7 + 79 = 13 + 73 = 19 + 67 = 43 + 43 **= 86**
25 + 63 = 5 + 83 = 17 + 71 = 29 + 59 = 41 + 47 **= 88**
45 + 45 = 7 + 83 = 11 + 79 = 17 + 73 = 19 + 71 = 23 + 67 = 29 + 61 = 31 + 59 = 37 + 53 = 43 + 47 = 1 + 89 = **90**
15 + 77 = 3 + 89 = 13 + 79 = 19 + 73 = 31 + 61 = 1 + 91 **= 92**
45 + 49 = 5 + 89 = 11 + 83 = 23 + 71 = 41 + 53 = 47 + 47 = **94**
9 + 87 = 5 + 91 = 7 + 89 = 13 + 83 = 17 + 79 = 23 + 73 = 29 + 67 = 37 + 59 = 43 + 53 **= 96**

49 + 49 = 7 + 91 = 19 + 79 = 31 + 67 = 37 + 61 = 1 + 97 = **98**
49 + 51 = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53 = **100**
51 + 51 = 5 + 97 = 11 + 91 = 13 + 89 = 19 + 83 = 23 + 79 = 29 + 73 = 31 + 71 = 41 + 61 = 43 + 59 = 1 + 101 = **102**

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d) From (a), (b) & (c) above, we have the even numbers from 4 to 102 … composed as follows:

1) **4** = 2 + 2 = 1 + 3 (sum of 2 primes only)
2) **6** = 3 + 3 = 1 + 5 (sum of 2 primes only)
3) **8** = 3 + 5 = 1 + 7 (sum of 2 primes only)
4) **10** = 5 + 5 = 3 + 7 (sum of 2 primes only)
5) **12** = 5 + 7 = 1 + 11 = **3 + 9** (sum of 1 prime & 1 odd composite)
6) **14** = 3 + 11 = 7 + 7 = 1 + 13 = **5 + 9** (sum of 1 prime & 1 odd composite)
7) **16** = 3 + 13 = 5 + 11 = **7 + 9** (sum of 1 prime & 1 odd composite)
8) **18** = 5 + 13 = 7 + 11 = 1 + 17 = **3 + 15** (sum of 1 prime & 1 odd composite) = **9 + 9** (sum of 2 odd composites)
9) **20** = 3 + 17 = 7 + 13 = 1 + 19 = **11 + 9** (sum of 1 prime & 1 odd composite)
10) **22** = 3 + 19 = 5 + 17 = 11 + 11 = **13 + 9** (sum of 1 prime & 1 odd composite)
11) **24** = 5 + 19 = 7 + 17 = 11 + 13 = 1 + 23 = **3 + 21** (sum of 1 prime & 1 odd composite) = **9 + 15** (sum of 2 odd composites)
12) **26** = 3 + 23 = 7 + 19 = 13 + 13 = **17 + 9** (sum of 1 prime & 1 odd composite)
13) **28** = 5 + 23 = 11 + 17 = **19 + 9** (sum of 1 prime & 1 odd composite)
14) **30** = 7 + 23 = 11 + 19 = 13 + 17 = 1 + 29 = **5 + 25** (sum of 1 prime & 1 odd composite) **= 15 + 15** (sum of 2 odd composites)
15) **32** = 3 + 29 = 13 + 19 = 1 + 31 = **23 + 9** (sum of 1 prime & 1 odd composite)
16) **34** = 17 + 17 = 3 + 31 = 5 + 29 = 11 + 23 = 17 + 17 = **7 + 27** (sum of 1 prime & 1 odd composite) = **9 + 25** (sum of 2 odd composites)
17) **36** = 5 + 31 = 7 + 29 = 13 + 23 = 17 + 19 = **3 + 33** (sum of 1 prime & 1 odd composite) = **15 + 21** (sum of 2 odd composites)
18) **38** = 7 + 31 = 19 + 19 = 1 + 37 = **29 + 9** (sum of 1 prime & 1 odd composite)
19) **40** = 3 + 37 = 11 + 29 = 17 + 23 = **31 + 9** (sum of 1 prime & 1 odd composite) = **15 + 25** (sum of 2 odd composites)
20) **42** = 5 + 37 = 11 + 31 = 13 + 29 = 19 + 23 = 1 + 41 = **3 + 39** (sum of 1 prime & 1 odd composite) = **21 + 21** (sum of 2 odd composites)
21) **44** = 3 + 41 = 7 + 37 = 13 + 31 = 1 + 43 = **5 + 39** (sum of 1 prime & 1 odd composite) = **9 + 35** (sum of 2 odd composites)
22) **46** = 3 + 43 = 5 + 41 = 17 + 29 = 23 + 23 = **37 + 9** (sum of 1 prime & 1 odd composite) = **21 + 25** (sum of 2 odd composites)
23) **48** = 5 + 43 = 7 + 41 = 11 + 37 = 17 + 31 = 19 + 29 = 1 + 47 = **3 + 45** (sum of 1 prime & 1 odd composite) = **9 + 39** (sum of 2 odd composites)
24) **50** = 3 + 47 = 7 + 43 = 13 + 37 = 19 + 31 = **41 + 9** (sum of 1 prime & 1 odd composite) = **25 + 25** (sum of 2 odd composites)
25) **52** = 5 + 47 = 11 + 41 =23 + 29 = = **43 + 9** (sum of 1 prime & 1 odd composite) = **25 + 27** (sum of 2 odd composites)
26) **54** = 7 + 47 = 11 + 43 = 13 + 41 = 17 + 37 = 23 + 31 = 1 + 53 = **5 + 49** (sum of 1 prime & 1 odd composite) = **27 + 27** (sum of 2 odd composites)
27) **56** = 3 + 53 = 13 + 43 = 19 + 37 = **47 + 9** (sum of 1 prime & 1 odd composite) = **21 + 35** (sum of 2 odd composites)
28) **58** = 29 + 29 = 5 + 53 = 11 + 47 = 17 + 41 = 29 + 29 = **3 + 55** (sum of 1 prime & 1 odd composite) = **9 + 49** (sum of 2 odd composites)
29) **60** = 7 + 53 = 13 + 47 = 17 + 43 = 19 + 41 = 23 + 37 = 29 + 31 = 1 + 59 = **5 + 55** (sum of 1 prime & 1 odd composite) = **27 + 33** (sum of 2 odd composites)
30) **62** = 3 + 59 = 19 + 43 = 31 + 31 = 1 + 61 = **53 + 9** (sum of 1 prime & 1 odd composite) = **27 + 35** (sum of 2 odd composites)
31) **64** = 3 + 61 = 5 + 59 = 11 + 53 = 17 + 47 = 23 + 41 = **7 + 57** (sum of 1 prime & 1 odd composite) = **9 + 55** (sum of 2 odd composites)
32) **66** = 5 + 61 = 7 + 59 = 13 + 53 = 19 + 47 = 23 + 43 = 29 + 37 = **11 + 55** (sum of 1 prime & 1 odd composite) = **33 + 33** (sum of 2 odd composites)
33) **68** = 7 + 61 = 31 + 37 = 1 + 67 = **59 + 9** (sum of 1 prime & 1 odd composite) = **33 + 35** (sum of 2 odd

composites)
34) **70** = 3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41 = **61 + 9** (sum of 1 prime & 1 odd composite) = **35 + 35** (sum of 2 odd composites)
35) **72** = 5 + 67 = 11 + 61 = 13 + 59 = 19 + 53 = 29 + 43 = 31 + 41 = 1 + 71 = **3 + 69** (sum of 1 prime & 1 odd composite) = **9 + 63** (sum of 2 odd composites)
36) **74** = 3 + 71 = 7 + 67 = 13 + 61 = 31 + 43 = 37 + 37 = 1 + 73 = **5 + 69** (sum of 1 prime & 1 odd composite) = **35 + 39** (sum of 2 odd composites)
37) **76** = 3 + 73 = 5 + 71 = 17 + 59 = 23 + 53 = 29 + 47 = **67 + 9** (sum of 1 prime & 1 odd composite) = **21 + 55** (sum of 2 odd composites)
38) **78** = 5 + 73 = 7 + 71 = 11 + 67 = 31 + 47 = 37 + 41 = **3 + 75** (sum of 1 prime & 1 odd composite) = **39 + 39** (sum of 2 odd composites)
39) **80** = 7 + 73 = 13 + 67 = 19 + 61 = 37 + 43 = 1 + 79 = **71 + 9** (sum of 1 prime & 1 odd composite) = **15 + 65** (sum of 2 odd composites)
40) **82** = 3 + 79 = 11 + 71 = 23 + 59 = 29 + 53 = 41 + 41 = **73 + 9** (sum of 1 prime & 1 odd composite) = **25 + 57** (sum of 2 odd composites)
41) **84** = 5 + 79 = 11 + 73 = 13 + 71 = 17 + 67 = 23 + 61 = 31 + 53 = 37 + 47 = 41 + 43 = 1 + 83 = **3 + 81** (sum of 1 prime & 1 odd composite) = **39 + 45** (sum of 2 odd composites)
42) **86** = 43 + 43 = 3 + 83 = 7 + 79 = 13 + 73 = 19 + 67 = 43 + 43 = **5 + 81** (sum of 1 prime & 1 odd composite) = **9 + 77** (sum of 2 odd composites)
43) **88** = 5 + 83 = 17 + 71 = 29 + 59 = 41 + 47 = **79 + 9** (sum of 1 prime & 1 odd composite) = **25 + 63** (sum of 2 odd composites)
44) **90** = 7 + 83 = 11 + 79 = 17 + 73 = 19 + 71 = 23 + 67 = 29 + 61 = 31 + 59 = 37 + 53 = 43 + 47 = 1 + 89 = **3 + 87** (sum of 1 prime & 1 odd composite) = **45 + 45** (sum of 2 odd composites)
45) **92** = 3 + 89 = 13 + 79 = 19 + 73 = 31 + 61 = 1 + 91 = **83 + 9** (sum of 1 prime & 1 odd composite) = **15 + 77** (sum of 2 odd composites)
46) **94** = 5 + 89 = 11 + 83 = 23 + 71 = 41 + 53 = 47 + 47 **= 7 + 87** (sum of 1 prime & 1 odd composite) = **45 + 49** (sum of 2 odd composites)
47) **96** = 5 + 91 = 7 + 89 = 13 + 83 = 17 + 79 = 23 + 73 = 29 + 67 = 37 + 59 = 43 + 53 = **3 + 93** (sum of 1 prime & 1 odd composite) = **9 + 87** (sum of 2 odd composites)
48) **98** = 7 + 91 = 19 + 79 = 31 + 67 = 37 + 61 = 1 + 97 = **89 + 9** (sum of 1 prime & 1 odd composite) = **49 + 49** (sum of 2 odd composites)
49) **100** = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53 = **91 + 9** (sum of 1 prime & 1 odd composite) = **49 + 51** (sum of 2 odd composites)
50) **102** = 5 + 97 = 11 + 91 = 13 + 89 = 19 + 83 = 23 + 79 = 29 + 73 = 31 + 71 = 41 + 61 = 43 + 59 = 1 + 101 = **3 + 99** (sum of 1 prime & 1 odd composite) = **51 + 51** (sum of 2 odd composites)

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(The above is only a partial or incomplete listing of sums of 1 prime & 1 odd composite, and, sums of 2 odd composites, each of which is equal to the sum of 2 primes as well as an even number. For example, in the list of compositions for the even numbers 4 to 102 … above, in Item (48), we could also have other "combinations" such as: **98** = 7 + 91 = 19 + 79 = 31 + 67 = 37 + 61 = 1 + 97 = <u>**25 + 73** (sum of 1 prime & 1 odd composite)</u> = <u>**21 + 77** (sum of 2 odd composites), etc.</u>, in Item (49), we could also have other "combinations" such as: **100** = 3 + 97 = 11 + 89 = 17 + 83 = 29 + 71 = 41 + 59 = 47 + 53 = <u>**31 + 69** (sum of 1 prime & 1 odd composite)</u> = <u>**45 + 55** (sum of 2 odd composites), etc.</u>, and, in Item (50), we could also have other "combinations" such as: **102** = 5 + 97 = 11 + 91 = 13 + 89 = 19 + 83 = 23 + 79 = 29 + 73 = 31 + 71 = 41 + 61 = 43 + 59 = 1 + 101 = <u>**17 + 85** (sum of 1 prime & 1 odd composite)</u> = <u>**21 + 81** (sum of 2 odd composites), etc.</u>. That is, there are more "combinations" than those shown in the above listing.)

In (d) above, in the list of compositions for the 50 consecutive even numbers 4 to 102 …, the even numbers 4, 6, 8 and 10 are only formed through the summing of 2 primes and not at all through the summing of 1 prime and 1 odd composite, or, the summing of 2 odd composites, which are impossibilities here. These sums of 2 primes are present (always present) throughout the whole list of compositions, from 4 right through to 102, while this is not the case for the sums of 1 prime and 1 odd composite, and, the sums of 2 odd composites.

We reason here by the process of elimination, through analysing the information in (d) above which pertains to the compositions of the 50 consecutive even numbers 4 to 102 … taken from the infinite list of even numbers. We stated at the beginning the following about the even numbers after 2:-

<u>Firstly, every even number after 2 is:</u>
A) The sum of 2 odd numbers.

(Every odd number is either a prime which is odd or a composite - product of primes which are odd. Notably, every prime with the exception of 2 is an odd number.)

Secondly, every even number after 2 is also (the below-mentioned is the logical consequence of (A) above):
1) The sum of 2 primes which are odd.
2) And/or the sum of 1 prime which is odd and 1 odd composite whose prime factors are odd.
3) And/or the sum of 2 odd composites whose prime factors are odd.

Evidently, at least 1 of (1), (2) & (3) above has to be the "atom" or building-block of the even numbers. In (d) above, we observe the following:-

i) All the 50 consecutive even numbers 4 to 102 … in (d) above taken from the infinite list of even numbers are sums of 2 primes.
ii) It is impossible for each of the even numbers 4, 6, 8 & 10 in (d) above to be the sum of 1 prime which is odd and 1 odd composite whose prime factors are odd.
iii) It is impossible for each of the even numbers 4, 6, 8, 10, 12, 14, 16, 20, 22, 26, 28, 32 & 38 in (d) above to be the sum of 2 odd composites whose prime factors are odd.

It is evident from (i), (ii) & (iii) above that neither (2) nor (3) can be the "atom" or building-block of the even numbers since they are "incomplete". As (1) - the sum of 2 primes which are odd - is "complete", i.e., always present in the 50 consecutive even numbers 4 to 102 … in (d) above, unlike (2) & (3), it evidently is the "atom" or building-block of the even numbers. That is, every even number after 2 is evidently the sum of 2 primes which are odd. In fact, a distributed computer search completed in 2008 at the University of Aveiro, Portugal, had verified this for all even numbers up to 12×10^{17}, which is not a small list. Definitely, due respectively to (ii) & (iii) above, we cannot say that every even number after 2 is the sum of 1 prime which is odd and 1 odd composite whose prime factors are odd, or, every even number after 2 is the sum of 2 odd composites whose prime factors are odd.

By the above lemma and corollary, the infinitudes of the primes, even numbers and odd numbers indeed imply that there are an infinite number of sums of 2 primes which are odd numbers, which are each equal to an even number. As the sums of 2 primes which are odd numbers are evidently the "atoms" or building-blocks of the even numbers, it also implies that they are infinite, since the even numbers are infinite.

Hypothetically, if on the other hand just 1 of the 3 items stated above, primes, even numbers and odd numbers, were finite, the above-said sums of 2 primes which are odd numbers, each of which is equal to an even number, would be finite. The primes, even numbers and odd numbers are evidently intricately linked, with the primes playing the part of building-blocks of both the even and odd numbers through various "combinations" as is described below. However, as the primes, even numbers and odd numbers are intricately linked, the finiteness (or, infinity) of any 1 of them implies the finiteness (or, infinity) of the other 2, and vice versa. These 3 items are evidently "close comrades-in-arm" working together to give special meaning to the integers. As these 3 are all infinite, it indeed implies that there is an infinitude of even numbers which are infinitely the sums of 2 primes that are odd and infinite.

Proof 2:-
Lemma:
According to the precepts of fractal geometry and group theory, symmetry is a very important, intrinsic part of nature. There is symmetry all around us and within us. There is evident symmetry in human bodies, the structures of viruses and bacteria, polymers and ceramic materials, the permutations of numbers, the universe and many others, even the movements of prices in financial markets, the growths of populations, the sound of music, the flow of blood through our circulatory system, the behaviour of people en masse, etc.. In other words, regularity, pattern, order, uniformity or symmetry is evident everywhere.

The above-mentioned most basic, always present sums of 2 primes, each of which is equal to an even number, which are evidently the "atoms" or building-blocks of the even numbers, are characterised by the feature of symmetry (in 2008, a distributed computer search ran by Tomas Oliveira e Silva, a researcher at the University of Aveiro, Portugal, had verified that all even numbers up to 12×10^{17}, which is no small list of numbers, are sums of 2 primes, a regularity, uniformity, order, pattern, symmetry). Thus, by the above lemma, every even number after 2 is naturally or inherently the sum of 2 primes, i.e., there is an infinitude of sums of 2 primes which are each equal to an even number.

Hence, the confirmation of the following generalisation pertaining to the integers, whereby it is indeed evident that the primes play a very important role:

Let a prime = p, &, a composite = c = p x p …. .

a) Every even number after 2 = p + p = *c = *p x p …. &/V = c + p = (p x p ….) + p &/V = c + c = (p x p ….) + (p x p ….) (in *c = *p x p …. here, which is an even composite, 1 or more of the p's are 2, the only even prime, e.g., 6 = 2 x 3, 8 = 2 x 2 x 2, 10 = 2 x 5, 18 = 2 x 3 x 3, 20 = 2 x 2 x 5, 24 = 2 x 2 x 2 x 3, etc.)
b) Every odd number = p V c = p x p …. (in c = p x p …. here, which is an odd composite, like the c = p x p ….'s in (a) above, all the p's are odd, e.g., 9 = 3 x 3, 15 = 3 x 5, 21 = 3 x 7, 25 = 5 x 5, 63 = 3 x 3 x 7, 99 = 3 x 3 x 11, etc.)

It is easy to see that the Goldbach conjecture is valid, i.e., every even number after 2 is the sum of 2 primes.

PART 4

Theorem:- Every even number after 2 is the sum of 2 primes.

Solution:-
The prime numbers are evidently the atoms or building-blocks of the integers. The integers are either primes (not divisible by other integers except 1) or composites (divisible by other integers, e.g., the prime numbers), and, even (the sums of 2 primes as conjectured by Goldbach) or odd (primes, or, composites whereby they are divisible by prime factors). Therefore, to determine whether the conjecture that every even number (except the number 2) is the sum of 2 primes is true, it would be appropriate to analyse the evident atoms or building-blocks of the even numbers, viz., the prime numbers. For the solution to this conjecture we note that the primes (vide Euclid's proof) and the even numbers are infinite, which implies that this conjecture should be true.

We here analyse the "behaviour" of the first 2,400 consecutive prime numbers (divided into 12 batches of consecutive primes, each subsequent batch with an increment of 200 primes), leaving out 2 (because it is an even prime) and commencing with 3, which is the 2$^{nd.}$ consecutive prime, the latter to be the first prime in our list of 2,400 consecutive primes (3 to 21,391), as follows:-

(1) <u>200 Consecutive Primes From 3 To 1,229</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 2,458
- (b) No. of even numbers = 1,227
- (c) No. of primes = 200
- (d) Average no. of even numbers "generated" by each of these 200 consecutive primes = 1,227 ÷ 200 = **6.14**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 200 primes = 200 x 200 = 40,000
- (f) Average no. of summings of 2 primes/permutations for each of the 1,227 even numbers = 40,000 ÷ 1,227 = **32.60**

(2) <u>400 Consecutive Primes From 3 To 2,749</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 5,498
- (b) No. of even numbers = 2,747
- (c) No. of primes = 400
- (d) Average no. of even numbers "generated" by each of these 400 consecutive primes = 2,747 ÷ 400 = **6.87**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 400 primes = 400 x 400 = 160,000
- (f) Average no. of summings of 2 primes/permutations for each of the 2,747 even numbers = 160,000 ÷ 2,747 = **58.25**

(3) <u>600 Consecutive Primes From 3 To 4,421</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 8,842
- (b) No. of even numbers = 4,419
- (c) No. of primes = 600
- (d) Average no. of even numbers "generated" by each of these 600 consecutive primes = 4,419 ÷ 600 = **7.37**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 600 primes = 600 x 600 = 360,000
- (f) Average no. of summings of 2 primes/permutations for each of the 4,419 even

numbers = 360,000 ÷ 4,419 = **81.47**

(4) <u>800 Consecutive Primes From 3 To 6,143</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 12,286
- (b) No. of even numbers = 6,141
- (c) No. of primes = 800
- (d) Average no. of even numbers "generated" by each of these 800 consecutive primes = 6,141 ÷ 800 = **7.68**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 800 primes = 800 x 800 = 640,000
- (f) Average no. of summings of 2 primes/permutations for each of the 6,141 even numbers = 640,000 ÷ 6,141 = **104.22**

(5) <u>1,000 Consecutive Primes From 3 To 7,927</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 15,854
- (b) No. of even numbers = 7,925
- (c) No. of primes = 1,000
- (d) Average no. of even numbers "generated" by each of these 1,000 consecutive primes = 7,925 ÷ 1,000 = **7.93**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 1,000 primes = 1,000 x 1,000 = 1,000,000
- (f) Average no. of summings of 2 primes/permutations for each of the 7,925 even numbers = 1,000,000 ÷ 7,925 = **126.18**

(6) <u>1,200 Consecutive Primes From 3 To 9,739</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 19,478
- (b) No. of even numbers = 9,737
- (c) No. of primes = 1,200
- (d) Average no. of even numbers "generated" by each of these 1,200 consecutive primes = 9,737 ÷ 1,200 = **8.11**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 1,200 primes = 1,200 x 1,200 = 1,440,000
- (f) Average no. of summings of 2 primes/permutations for each of the 9,737 even numbers = 1,440,000 ÷ 9,737 = **147.89**

(7) <u>1,400 Consecutive Primes From 3 To 11,677</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 23,354
- (b) No. of even numbers = 11,675
- (c) No. of primes = 1,400
- (d) Average no. of even numbers "generated" by each of these 1,400 consecutive primes = 11,675 ÷ 1,400 = **8.34**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 1,400 primes = 1,400 x 1,400 = 1,960,000
- (f) Average no. of summings of 2 primes/permutations for each of the 11,675 even numbers = 1,960,000 ÷ 11,675 = **167.88**

(8) <u>1,600 Consecutive Primes From 3 To 13,513</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 27,026
- (b) No. of even numbers = 13,511
- (c) No. of primes = 1,600
- (d) Average no. of even numbers "generated" by each of these 1,600 consecutive primes = 13,511 ÷ 1,600 = **8.44**
- (e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 1,600 primes = 1,600 x 1,600 = 2,560,000
- (f) Average no. of summings of 2 primes/permutations for each of the 13,511 even numbers = 2,560,000 ÷ 13,511 = **189.48**

(9) <u>1,800 Consecutive Primes From 3 To 15,413</u>
- (a) Even numbers (obtained by summing of 2 primes) = 6 to 30,826
- (b) No. of even numbers = 15,411
- (c) No. of primes = 1,800
- (d) Average no. of even numbers "generated" by each of these 1,800 consecutive primes

= 15,411 ÷ 1,800 = **8.56**
(e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 1,800 primes = 1,800 x 1,800 = 3,240,000
(f) Average no. of summings of 2 primes/permutations for each of the 15,411 even numbers = 3,240,000 ÷ 15,411 = **210.24**

(10) 2,000 Consecutive Primes From 3 To 17,393
(a) Even numbers (obtained by summing of 2 primes) = 6 to 34,786
(b) No. of even numbers = 17,391
(c) No. of primes = 2,000
(d) Average no. of even numbers "generated" by each of these 2,000 consecutive primes = 17,391 ÷ 2,000 = **8.70**
(e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 2,000 primes = 2,000 x 2,000 = 4,000,000
(f) Average no. of summings of 2 primes/permutations for each of the 17,391 even numbers = 4,000,000 ÷ 17,391 = **230.00**

(11) 2,200 Consecutive Primes From 3 To 19,427
(a) Even numbers (obtained by summing of 2 primes) = 6 to 38,854
(b) No. of even numbers = 19,425
(c) No. of primes = 2,200
(d) Average no. of even numbers "generated" by each of these 2,200 consecutive primes = 19,425 ÷ 2,200 = **8.83**
(e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 2,200 primes = 2,200 x 2,200 = 4,840,000
(f) Average no. of summings of 2 primes/permutations for each of the 19,425 even numbers = 4,840,000 ÷ 19,425 = **249.16**

(12) 2,400 Consecutive Primes From 3 To 21,391
(a) Even numbers (obtained by summing of 2 primes) = 6 to 42,782
(b) No. of even numbers = 21,389
(c) No. of primes = 2,400
(d) Average no. of even numbers "generated" by each of these 2,400 consecutive primes = 21,389 ÷ 2,400 = **8.91**
(e) No. of summings of 2 primes/permutations (3 + 3, 3 + 5, 3 + 7, 3 + 11, …… etc.) for these 2,400 primes = 2,400 x 2,400 = 5,760,000
(f) Average no. of summings of 2 primes/permutations for each of the 21,389 even numbers = 5,760,000 ÷ 21,389 = **269.30**

There would evidently be more and more profuse repetitions and overlaps of the even numbers "generated" by the primes the higher up the infinite list of prime numbers we go, which is significant.

We compare all the (d)s and (f)s in (1) to (12) above, which is as follows:-

(d) Average no. of even numbers "generated" by each of the consecutive primes in (1) to (12) above, as follows according to the listings (1) to (12):

(1) **6.14**, (2) **6.87**, (3) **7.37**, (4) **7.68**, (5) **7.93**, (6) **8.11**, (7) **8.34**, (8) **8.44**, (9) **8.56**, (10) **8.70**, (11) **8.83**, (12) **8.91**

(f) Average no. of summings of 2 primes/permutations for each of the even numbers in (1) to (12) above, as follows according to the listings (1) to (12):

(1) **32.60**, (2) **58.25**, (3) **81.47**, (4) **104.22**, (5) **126.18**, (6) **147.89**, (7) **167.88**, (8) **189.48**, (9) **210.24**, (10) **230.00**, (11) **249.16**, (12) **269.30**

The following is evident from the above information:-

(A): (d) Average no. of even numbers "generated" by each of the consecutive primes in the above 12 listings increases continually all the way from the list: (1) 200 Consecutive Primes From 3 To 1,229 to the list: (12) 2,400 Consecutive Primes From 3 To 21,391, from **6.14** even numbers per prime number in List (1) to **8.91** even numbers per prime number in List (12).

(B): (f) Average no. of summings of 2 primes/permutations for each of the even numbers in the above 12 listings increases continually all the way from the list: (1) 200 Consecutive Primes From 3 To 1,229 to the list: (12) 2,400 Consecutive Primes From 3 To 21,391, from **32.60** number of summings of 2 primes/permutations per even number in List (1) to **269.30** number of summings of 2 primes/permutations per even number in List (12).

Proof 1:

Lemma: According to the principle of complete induction in set theory, if a set of natural numbers contains 1 and, for each *n*, it contains $n + 1$ whenever it contains all numbers less than $n + 1$, then it must contain every natural number, e.g., complete induction proves that every natural number is a product of primes.

By induction, we now deduce the following:

The larger the list of consecutive primes becomes, the greater would be the average number of even numbers "generated" by each of the primes in the list of consecutive primes (inferred from (A) above).

The larger the list of consecutive primes becomes, the greater would be the average number of summings of 2 primes/permutations for each of the even numbers in the infinite list of even numbers (inferred from (B) above).

Furthermore, the Goldbach conjecture had been tested and found to be correct for every even number up to 12×10^{17}, which is not a small list, by a distributed computer search carried out at the University of Aveiro, Portugal, in 2008.

As the primes and the even numbers are infinite, by the above lemma and all the above deductions and information, it could be inferred that the increases stated in (A) and (B) above, with the even numbers each being the sum of 2 primes, continue to infinity, i.e., the Goldbach conjecture becomes stronger and stronger the higher up the infinite list of prime numbers/even numbers we go - all the way to infinity.

The validity of the Goldbach conjecture is thereby confirmed - every even number after 2 is the sum of 2 primes.

Proof 2:

Next, we resort to the proof by contradiction. The above deduction would be reversed if, e.g., the following takes place (which is the reversal of the above-mentioned information):

(A): (d) Average no. of even numbers "generated" by each of the consecutive primes in the above 12 listings decreases continually all the way from the list: (1) 200 Consecutive Primes From 3 To 1,229 to the list: (12) 2,400 Consecutive Primes From 3 To 21,391, from **8.91** even numbers per prime number in List (1) to **6.14** even numbers per prime number in List (12).

(B): (f) Average no. of summings of 2 primes/permutations for each of the even numbers in the above 12 listings decreases continually all the way from the list: (1) 200 Consecutive Primes From 3 To 1,229 to the list: (12) 2,400 Consecutive Primes From 3 To 21,391, from **269.30** number of summings of 2 primes/permutations per even number in List (1) to **32.60** number of summings of 2 primes/permutations per even number in List (12).

If this reversed state happens, the implication is that there would reach a point when there are no more batches of 2 prime numbers summing together to form even numbers, in which case the Goldbach conjecture would be false. Evidently this would happen when the prime numbers are finite. As the prime numbers are infinite (as Euclid had proved long ago) this would never happen.

Since the above information indicate otherwise, and, the prime numbers are infinite, we accept the above induction/deduction and infer that the Goldbach conjecture could not be false, i.e., the Goldbach conjecture is true, and, every even number (except 2) is indeed the sum of 2 prime numbers. This concludes the proof by contradiction.

Thus, by both induction and contradiction or *reductio ad absurdum* the validity of the Goldbach conjecture is proved.

CONCLUSION

It is evident here that the Goldbach conjecture could be approached in a number of different ways; a number of methods have been adopted in this paper in proving the Goldbach conjecture.

The inductive method, which is a well-established proof, is one of the methods utilised. The following lends support to this inductive proof of the Goldbach conjecture: (a) The characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is enough for deducing the characteristic of the mountain or infinite volume of sand, to ascertain the quality of a batch of products it is only necessary to inspect some carefully selected samples from that batch of products and not everyone of the products and to carry out a population census, i.e., find out the characteristics of a population, it is only necessary to carry out a survey on some carefully selected respondents and not the whole population; in like manner, by the same principle, we just need to study a carefully selected list of even numbers, find out whether they are all sums of 2 primes and deduce by induction whether all even numbers after this list would also be sums of 2 primes - this act is rather like extrapolation. (For example, a distributed computer search completed in 2008 at the University of Aveiro, Portugal, had confirmed that every even number up to 12×10^{17}, which is no small list of numbers, is the sum of 2 primes. By the principle of induction in this case we could deduce that all the even numbers after 12×10^{17} would also be sums of 2 primes.) (b) Thus, in this way every even number after 2 could be reasonably proved to be the sum of 2 primes. In fact, induction plays an important part in a number of the proofs.

The other argument used to prove the conjecture is the indirect (reductio ad absurdum) method, which had been used by Euclid and other mathematicians after him. Logically, 1 or 2 examples of "contradiction" should be sufficient proof of infinity, for it does not make sense to have a need for an infinite number of cases of "contradiction", as our proof would then have to be infinitely and impossibly long, an absurdity. This method of proof is "proof by implication" as a result of "contradiction" - which is a "short-cut" and smart way in proving infinity, instead of "proving infinity by counting to infinity", which is ludicrous, and, impossible. Hence, 1 or 2 cases of "contradiction" should be sufficient for implying that there would be an infinitude of even numbers which are sums of 2 primes, which of course also tacitly implies that there would be an infinitude of the number of cases of such "contradiction". (Euclid evidently had this logical point in mind when he formulated the indirect (reductio ad absurdum) proof of the infinity of the primes.) This method of proof had been cleverly used by a number of mathematicians, not the least by the great German mathematician, David Hilbert. For example, Hilbert had used an indirect method (the "reductio ad absurdum" proof) to prove Gordan'1s Theorem without having to show an actual "construction", a proof which had been accepted by his peers.

There is also the involvement of concepts from set theory, group theory, geometry, etc..

One important query here, which many might not have considered: What if the list of prime numbers is not infinite? Of course, if that is the case, the Goldbach conjecture would be false. It would then have been absurd for the Goldbach conjecture to have been conceived at all. However, the list of primes is infinite (vide Euclid's proof). This gives credence to the Goldbach conjecture.

A very important related point, in fact a most important point, must be highlighted here. If the Goldbach conjecture were indeed false, there must be an ultimate (largest) even number which is (and must necessarily be) the result of the summation of 2 primes that are each the largest existing prime. It must be noted that this is actually an impossibility, as there can never be a largest existing prime - by Euclid's proof, the primes are infinite (refer to Part 2, Proof 3 above). Hence, the Goldbach conjecture cannot be false, and, by both *reduction ad absurdum* (contradiction), and, induction (wherein all even numbers up to 12×10^{17}, not a small list, had been confirmed to be sums of 2 primes), has to be true.

Another important point is that the Goldbach conjecture becomes evidently stronger and stronger the higher up the infinite list of prime numbers/even numbers we go, as has been shown above. Thus, by implication, induction, extrapolation, it could be concluded that the Goldbach conjecture is valid - that every even number after 2 is the sum of 2 primes.

So far, there has been no indication or confirmation at all that the number of even numbers after the number 2 which are each the sum of 2 primes is finite and the largest existing even number which is the sum of 2 primes has not been found and confirmed. (This would of course be the case if the Goldbach conjecture is true.) Also, no counter-example (i.e., an even number which is never the sum of 2 primes) has been found so far. On the other hand, practically everyone could intuit that the list of even numbers after the number 2 which are each the sum of 2 primes is infinite. Besides, the evidence, as shown in this paper, is overwhelmingly in support of the infinity of this list.

We have no other more logical choice but to take the stand that every even number after the number 2 is the sum of 2 prime numbers.

In conclusion, we state that the Goldbach conjecture is true - every even number after the number 2 is indeed the sum of 2 primes.

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