Using Higher Dimensions to Unify Dark Matter and Dark Energy, if Massive Gravitons are Stable

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Abstract. This paper raises the following questions. Can there be a stable (massive) graviton? If so, does this massive graviton, as modeled by Kaluza-Klein dark matter, with a modification of slight four-dimensional space mass, contribute to dark energy, at least in terms of re-acceleration? The answer appears to be affirmative, if one assumes that the square of a frequency for graviton mass is real-valued and greater than zero. The author finds evidence that re-acceleration of the universe one billion years ago in a higher-dimensional setting can be justified in terms of a modification of standard Kaluza-Klein dark matter models, if one considers how an information exchange between present and prior universes occurs, which would mandate more than four-dimensional space time, according to the author.

Keywords: Kaluza-Klein dark matter, dark energy, re-acceleration parameter, massive gravitons
PACS: 04.50.Cd, 14.70.Kv, 14.80.Rt, 95.35.+d, 95.36.+x, 98.80.-k

OVERVIEW

This article first asks for criteria for massive graviton stability, then applies stable massive (four-dimensional) gravitons in terms of a Kaluza-Klein (KK) dark matter (DM) model, with small four-dimensional graviton mass, to obtain re-acceleration of the universe a billion years ago. This re-acceleration is a way to obtain dark energy (DE), at least in terms of a macro-effect in cosmological structure. To look at the problem of massive graviton stability, the author first applies a modification [1, 2] of the KK representation of DM according to Maartens [3], with a small mass in four dimensions. Then, using Visser's treatment of a stress energy tensor [4], he obtains the square of frequency of a massive graviton, which is both positive and real-valued. The paper concludes with a summary of Yurov's double inflation hypothesis [5] which contributes to understanding the results in Fig. 1, which show re-acceleration a billion years ago, due to the existence of a stable massive graviton. This would permit a better understanding of Smoot's values [6] for the initial information content of the universe as specified in his Paris Observatory talk in 2007.

IDENTIFICATION OF GRAVITON STABILITY REQUIREMENTS FROM GRAVITON FREQUENCY

If the graviton is stable, with small mass, then its macro-effects show up in the deceleration parameter behavior, indicating re-acceleration a billion years ago. Specifically for non-zero graviton mass, where we write $h \equiv \eta^{uv} h_{uv} = trace \cdot (huv)$ and $T = trace \cdot (T^{uv})$, Maggiore [7] delineated:

$$-3m_{graviton}^2 h = \frac{\kappa}{2} \cdot T \tag{1}$$

The present work uses the 1998 analysis by Visser [4], of non-zero graviton mass for both T and h. Using Eq. 1, with particle count n^f as a way to present initial gravitational wave (GW) relic inflation density and using Maggiore's definition [7] leads to the following:

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{f}{1kHz}\right)^4 \tag{2}$$

Here, n^f is the frequency-based numerical count of gravitons per unit phase space.

The reasons for applying the values of Visser [4] for T and h in Eq. (1) are as follows. To start with, a modification [1, 2] of work by Maartens [8] writes as:

$$m_n(graviton) = \frac{n}{L} + 10^{-65} grams \tag{3}$$

On the face of it, assignment of a mass of about 10^{-65} grams for a four-dimensional graviton, allowing for $m_0(graviton - 4D) \sim 10^{-65}$ grams [1, 2], violates all known quantum mechanics, and is to be avoided. Numerous authors, including Maggiore [7], have demonstrated how adding a term to the Fiertz Lagrangian for gravitons, and assuming massive gravitons, leads to results that appear to violate field theory.

Visser's Treatment of a Stress Energy Tensor for Massive Gravitons

Visser [4] stated a stress energy treatment of gravitons along the lines of

$$T_{uv}|_{m\neq 0} = \left[\left(\frac{\hbar}{l_P^2 \lambda_g^2}\right) \cdot \left(\frac{GM}{r}\right) \cdot \exp\left(\frac{r}{\lambda_g}\right) + \left(\frac{GM}{r}\right)^2\right] \times \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

Furthermore, his version of $g_{uv} = \eta_{uv} + h_{uv}$ can be written as setting

$$h_{uv} \equiv 2\frac{GM}{r} \cdot \left[\exp\left(\frac{-m_g r}{\hbar}\right)\right] \cdot \left(2 \cdot V_\mu V_\nu + \eta_{uv}\right) \tag{5}$$

One can add velocity reduction with regards to speed propagation of gravitons [5]:

$$v_g = c \cdot \sqrt{1 - \frac{m_g^2 \cdot c^4}{\hbar^2 \omega_g^2}} \tag{6}$$

Then one can insert all this into Eq. (1) to obtain a real value for the square of frequency greater than zero, i.e.,

$$\hbar^2 \omega^2 \cong m_g^2 c^4 \cdot \left[1 \Big/ \left(1 - \breve{A} \right) \right] > 0 \tag{7}$$

$$\breve{A} = \left\{ 1 - \frac{1}{6 m_g c^2} \left(\frac{\hbar^2}{l_P^2 \lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar} \right] + \left(\frac{MG}{r} \right) \cdot \exp\left(\frac{m_g r}{\hbar} \right) \right) \right\}^2 \tag{8}$$

According to Kim [9], if the square of the frequency of a graviton, with mass, is greater than zero and real-valued, it is likely that the graviton is stable (with regards to perturbations). The article by Kim [9] relates to gravitons in brane/string theory, but it is likely that the same dynamic holds for semi-classical representations of a graviton with mass. The conditions for a positive value of the square of frequency in Eq. (7) are the same as the conditions for which the following inequality holds, in terms of parameter space values for the variables in Eq. (9) below. Equation (9) allows for stable giant gravitons.

$$0 < \frac{1}{6 m_g c^2} \left(\frac{\hbar^2}{l_P^2 \lambda_g^2} \cdot \exp\left[-\frac{r}{\lambda_g} + \frac{m_g \cdot r}{\hbar} \right] + \left(\frac{MG}{r} \right) \cdot \exp\left(\frac{m_g r}{\hbar} \right) \right) < 1 \tag{9}$$

OBTAINING RE-ACCELERATION OF THE UNIVERSE A BILLION YEARS AGO WITH STABLE GIANT GRAVITONS

Beckwith [1, 2] used a version of the Friedman equations as inputs into the deceleration parameter using this equation by Maartens [3]

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \tag{10}$$

Maartens [3] gave a second Friedman equation:

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2} \right] \tag{11}$$

Also, if we are in the regime for which $\rho \cong -P$ for red shift values z between zero to 1.0-1.5 with exact equality, $\rho = -P$ for z between zero and 0.5. Because of Eq. (12), the net effect will be to obtain and use $a = \lfloor a_0 = 1 \rfloor / (1 + z)$. As given by Beckwith [1, 2],

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot \left(1 + \rho/2\lambda\right)} \approx -1 + \frac{2}{2 + \delta(z)} \tag{12}$$

Equation (12) assumes $\Lambda = 0 = K$, and the net effect is to obtain a substitute for DE, by presenting how gravitons with a small mass are done with $\Lambda \neq 0$, even if curvature $K = 0$.

Re-acceleration of Universe at $z \sim 0.423$ due to giant gravitons?

In a revision of the work by Alves et al. [1, 2], Beckwith [1, 2] used a higher-dimensional model of the brane world and the KK graviton towers of Maartens [3]. For a non-zero graviton, Beckwith [1, 2] applied the density ρ of the brane world in the Friedman equation as used by Alves et al. [10]

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar = 1)^2}\right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2}\right) \tag{13}$$

Equation (3) thus creates a joint DM and DE model, with all of Eq. (12) being for KK gravitons and DM, and grams being a four-dimensional DE. Equation (13) is part of a KK graviton presentation of DM/DE dynamics. Beckwith [1] found that acceleration of the universe increased at $z \sim 0.423$, a billion years ago, as shown in Fig. 1.

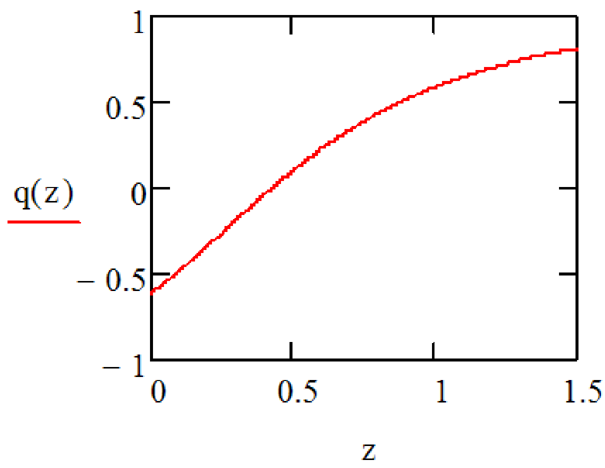

FIGURE 1. Re-acceleration of the universe according to Beckwith [1, 2]; note that $q < 0$ if $z < 0.423$.

LINK TO INITIAL INFLATION, USING YUROV'S DOUBLE INFLATION THEORY

The following is speculative, and if confirmed through additional research would be a major step toward a cosmological linkage between initial inflation, and re-acceleration of the universe one billion years ago. Yurov's double inflation hypothesis [5] claimed that there exists one emergent complex scalar field Φ and that its evolution in both initial inflation and re-acceleration is linked. In other words, Yurov [5] stated that this scalar field would account for both the first and the second inflation potential, and in both cases, chaotic inflation of this type:

$$V = \overleftrightarrow{m}^2 \Phi^* \Phi \tag{14}$$

The mass term for early universe versions of the Friedman equation would then be, as Beckwith [1, 2] understood it:

$$\overleftrightarrow{m} \approx \sqrt{\frac{3}{8}} \cdot \left[\sqrt{\frac{3H^2}{4\pi G}} \bigg|_{time \sim 10^{-35}\,\mathrm{sec}} + \sqrt{\frac{3H^2}{4\pi G}} \bigg|_{time \sim 10^{-44}\,\mathrm{sec}} \right] \tag{15}$$

Furthermore, its bound would be specified by having

$$\left| \overleftrightarrow{m} \right| \leq \left[\frac{l^2}{4} \right] \tag{16}$$

The term l would be an artifact of five-dimensional space time, as provided in a metric given by Maartens [8] as

$$dS^2 \big|_{5-\mathrm{dim}} = \frac{l^2}{z^2} \cdot \left[\eta_{uv} dx^{\mu} dx^{\nu} + dz^2 \right] \tag{17}$$

Yurov [5] wrote the second scalar fields, contributing to the second inflation (and represented in Fig. 1), as:

$$\varphi_{0,-} = \sqrt{2/3} \cdot \overleftrightarrow{m} \cdot \left[t_{1st-EXIT} \sim 10^{-35}\,\mathrm{sec} \right] \tag{18}$$

Also,

$$\varphi_{+} = \left[\varphi_{0,+}^3 - \sqrt{3/2} \cdot \frac{3M^2 t}{\overleftrightarrow{m}} \right]^{1/3} \tag{19}$$

To make a full linkage between Yurov's formalism [5] for double inflation, Fig. 1, and initial inflationary dynamics - as referenced by obtaining $n_f \approx 10^6$ to 10^7 - would be to make the following relationships between Yurov's versions [5] of the Friedman equations and what Beckwith [1, 2] did (in terms of Fig. 1):

$$H^2 = \frac{1}{6} \cdot \left[\dot{\varphi}^2 + \overleftrightarrow{m}^2 \varphi^2 + \frac{M^2}{\varphi^2} \right] \leftrightarrow \left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) + \frac{m}{a^4} \tag{20}$$

It would also mean having:

$$\dot{H} = V - 3H \leftrightarrow \dot{H} \cong \frac{2m}{a^4} \tag{21}$$

The left-hand sides of Eq. (20) and Eq. (21) are Yurov's [5], and the right-hand sides of Eq. (20) and Eq. (21) above are Beckwith's adaptation of a modification of brane theory work by Maartens [3] which was used in part to obtain the results given in Fig. 1, i.e. the behavior of massive gravitons one billion years ago to mimic DE in terms of the re-acceleration parameter. Filling in the details for Eq. (14) to Eq. (21) would, if confirmed and linked to Fig. 1, be a way to come up with a more comprehensive cosmological picture of the linkage of geometry and space-time evolution than what exists today. Making the relationships work between the Friedman equation choices of Yurov [5] and Fig. 1 involves making the following assumption which may be falsified by experiment:

$$\frac{3H^2}{4\pi G} >> V(t)\bigg|_{time \sim 10^{-44}\,\mathrm{sec}} \tag{22}$$

In other words, the potential energy, V, of the initial inflation is overshadowed by the contributions of the Friedman equation, H, at the onset of inflation. If this Eq. (22) is false, or falsified, then the form of creating relationships between the components of Eq. (20) and Eq. (21) probably should be reconsidered.

CONCLUSION

The end result of a massive graviton may also lead to information exchange between a prior and our present universe as has been commented upon by Beckwith [1, 2]. Note that Beckwith [1, 2] used Ng's counting algorithm [11] with regard to entropy, and non-zero mass (massive) gravitons, where

$$S \approx N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N \tag{23}$$

Furthermore, we can make an initial count of gravitons with $S \approx N \cdot \sim 10^7$ gravitons [7] with Lloyd's equation [12]:

$$I = S_{total}/k_B \ln 2 = [\#operations]^{3/4} \sim 10^7 \tag{24}$$

as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton [7]. Note that Smoot [6] gave initial values of the operations as

$$[\#operations]_{initially} \sim 10^{10} \tag{25}$$

It would be useful to determine if there were connections between the parameter space defined by Eq. (9) above, in terms of input variables, and optimal conditions as well for both Eq. (24) and Eq. (25), to be confirmed experimentally. In addition, it would be good to understand optimal space time geometric conditions for the development of KK particle physics allowing for implementation of Eq. (24) above, which assumes stable giant gravitons are possible. The number of operations, if tied into bits of information, may allow for space-time linkages of the value of the fine structure constant, as given in Eq. (26) below, between a prior and the present universe, once initial conditions of inflation may be examined experimentally, i.e., looking at inputs into the following equation [1, 2]

$$\tilde{\alpha} \equiv e^2 \big/ \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \tag{26}$$

After this is done, then the next step would be to look at inputs into the near-present time value for a Friedman equation, leading to a fuller understanding of Eq. (12) above. All this is possible if a non-brane theory version of stability of the graviton is obtained, if an extension of the frequency-based criteria as to giant graviton stability according to Kim [9] is confirmed experimentally. It would also confirm the energy flux expression for GW by Alcubierre [13], also used for the present paper. Filling in the steps to give exact representations of Eq. (14) to Eq. (21) can hopefully commence in the future.

Finally, also desirable would be to understand more on how to apply Valev's graviton wave length calculations [14], as given by

$$\begin{aligned} &m_{graviton}\big|_{RELATIVISTIC} < 4.4 \times 10^{-22} h^{-1} eV/c^2 \\ &\Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} meters \end{aligned} \tag{27}$$

Equation (28) below gives the relationship of that wavelength calculation with the gravity wave energy flux of Alcubierre [13].

$$\frac{dE}{dt} = [\lim r \to \infty] \left[\frac{r^2}{16\pi}\right] \oint \left| \int_{-\infty}^{t} \Psi_4 dt' \right|^2 \cdot d\Omega \tag{28}$$

Key to making better use of a relationship between Eq. (27) and Eq. (28) would be a better understanding of the observationally based Friedman equation by Sanders [15], as given below [16], namely:

$$H_{friedman} \equiv \left[(T_{temp})^2 \big/ c \right] \cdot \sqrt{N(T_{temp})} \cdot \sqrt{\frac{4\pi G \cdot \hat{a}}{3}} \tag{29}$$

Kolb and Turner [17] expressed the electro-weak boundary of the number of degrees of freedom as:

$$N(T_{temp})\big|_{electro-weak} \sim 10^2 \tag{30}$$

In contrast, early universe, near the Big Bang values, may have $N(T_{temp}) \sim 10^3$. Obviously, obtaining better values of the degrees of freedom, as Beckwith [16] attempted, for early-to-later universe conditions would be of crucial importance toward that goal. Note also, that if Eq. (6) had graviton speed as the speed of light, none of the graviton stability considerations would apply, and the above analysis would be moot.

ACKNOWLEDGMENTS

Dr. Lukasz Glinka is thanked for encouraging the author to think of semi-classical and Wheeler-DeWitt style issues applied to physics, and their relevance to the research areas of cosmology and gravity. Dr. Fangyu Li of Chongqing University (China) is thanked for lending his personal notes to give substance to the content of page 10 of the companion document that complements this article, plus his week-long hospitality to the author in China. Dr. Christian Corda is thanked for encouraging the author to explore semi-classical issues in GW and in GR, and for forwarding the paper by Alves et al. to the author to expand the author's scientific horizon. In addition, the author thanks the DSU 2010 staff for their hospitality and Dr. Debasish Majumdar of the Saha Institute, India, for ensuring the author addressed giant graviton stability in his write-up for DSU 2010. All these individuals affected the presentation of this paper [16] as given on June 5, 2010.

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