The Hamiltonian in covariant theory of gravitation

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In the framework of covariant theory of gravitation the Euler-Lagrange equations are written and equations of motion are determined by using the Lagrange function, in the case of small test particle and in the case of continuously distributed matter. From the Lagrangian transition to the Hamiltonian was done, which is expressed through three-dimensional generalized momentum in explicit form, and also is defined by the 4-velocity, scalar potentials and strengths of gravitational and electromagnetic fields, taking into account the metric. The definition of generalized 4-velocity, and the description of its application to the principle of least action and to Hamiltonian is done. The existence of a 4-vector of the Hamiltonian is assumed and the problem of mass is investigated. To characterize the properties of mass we introduce three different masses, one of which is connected with the rest energy, another is the observed mass, and the third mass is determined without taking into account the energy of macroscopic fields. It is shown that the action function has the physical meaning of the function describing the change of such intrinsic properties as the rate of proper time and rate of rise of phase angle in periodic processes.

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Introduction

There are several approaches to describing and constructing any physical theory. In the simplest case, the content of the theory is reduced to several physical laws and principles that conform to the experimental data. By analyzing and simplifying them, the system of axioms can be found, based on which the whole theory can be derived by axiomatic method, as a logical consequence of the initial simple assumptions. In the energy approach it is sufficient to know only one function with the dimension of energy, in order to find all the equations of the theory with the help of it. The examples of such functions are Lagrangian and Hamiltonian.

The covariant theory of gravitation (CTG) appeared in 2009 [1], as a consequence of the relativistic generalization of the Lorentz-invariant theory of gravitation (LITG). LITG equations are similar by their form to Maxwell's equations and can be derived on the basis of axioms [2]. Recently derivation of CTG equations was made based on the principle of least action [3]. Based on the resulting form of the Lagrangian now it is possible to make the next step and go to the Hamiltonian corresponding to the CTG theory.

After a brief presentation of the Euler-Lagrange equations we use them to describe the motion of a small test particle, as well as in the case of continuously distributed matter. Then we find the Hamiltonian in its two forms, with the help of 4-velocity and the generalized momentum, and substitute the Hamiltonian into Hamilton equations to verify the motion equations. At the end of this paper we introduce for consideration the four-dimensional generalized velocity to simplify the expressions for the Lagrangian and Hamiltonian. The transition was done from the 4-vector of the generalized velocity to a new 4-vector of the Hamiltonian, specifying the energy and the momentum of substance in fundamental fields. The comparison with the Lagrangian approach is made, in which the energy and the momentum are calculated through energy-momentum tensors. The problem of mass is analyzed with the help of formulas for the energy. In the last part, we describe the action function as a function having an independent meaning in physics – it can help to determine the effects of time dilation, arising from the change of velocity of bodies' motion or under the influence of fields.

The principle of least action

In this section we shall write down known relations for the Lagrange function and the principle of least action for the covariant theory of gravitation (CTG). According to the latter, the equations of motion of substance and fields can be found by varying the action function $S=\int L\,dt$. In the coordinates $x^{\mu}=(ct,x,y,z)$ the Lagrangian depends on the coordinates x^{μ}, on the 4-velocity of substance motion $u^{\mu}=\frac{cdx^{\mu}}{ds}$ (where c – the speed of light, ds indicates the interval for the moving substance unit), on 4-potential D_{μ} of gravitational field and 4-potential A_{μ} of electromagnetic field and on metric tensor $g_{\mu\nu}$ of the reference frame. If to move on from x^{μ} and u^{μ} to the three-dimensional coordinates, time and velocity, then the Lagrangian function with these variables can be written in the form: $L=L(t,x,y,z,\dot{x},\dot{y},\dot{z},D_{\mu},A_{\mu},g_{\mu\nu})$. Here the quantities $\dot{x}=\frac{dx}{dt}$, $\dot{y}=\frac{dy}{dt}$, $\dot{z}=\frac{dz}{dt}$ are the components of 3-vector of coordinate velocity $\mathbf{v}=(\dot{x},\dot{y},\dot{z})$. When moving along a certain trajectory the current coordinates x,y,z of a substance unit, and its velocities $\dot{x},\dot{y},\dot{z}$ are functions of coordinate time t. In general 4-potentials D_μ and A_μ, which act on the substance, and the metric tensor $g_{\mu\nu}$ depend on the coordinates and time. If we take the coordinates of the substance along the trajectory as a function of time, then D_μ , A_μ and $g_{\mu\nu}$ at the trajectory can be considered as functions of time too. This allows us to consider the Lagrange function as a function of time, and the integral $S=\int_1^2 L\,dt$ between the spacetime points 1 and 2 – as a number. Theoretically, under variations of the coordinates we can understand small in magnitude functions of time, due to adding of which the shape of trajectory of the substance motion change, and respectively, change the value of the action function. From the principle of least action it follows, that the action S on the true trajectory has to be extreme (usually S has a minimum).

Variation of the action function along the trajectory, when all the variables are varying except the time, gives the following:

$$\delta S=\int_1^2 \delta L\,dt=\int_1^2\left(\frac{\partial L}{\partial x}\delta x+\frac{\partial L}{\partial y}\delta y+\frac{\partial L}{\partial z}\delta z+\frac{\partial L}{\partial \dot{x}}\delta \dot{x}+\frac{\partial L}{\partial \dot{y}}\delta \dot{y}+\frac{\partial L}{\partial \dot{z}}\delta \dot{z}+\right.$$
$$\left.+\frac{\partial L}{\partial D_\mu}\delta D_\mu+\frac{\partial L}{\partial A_\mu}\delta A_\mu+\frac{\partial L}{\partial g_{\mu\nu}}\delta g_{\mu\nu}\right)dt=0.$$

The term with the variation of the velocity $\delta\dot{x}$ can be integrated by parts:

$$\int_1^2\frac{\partial L}{\partial \dot{x}}\delta\dot{x}\,dt=\int_1^2\frac{\partial L}{\partial \dot{x}}\delta\left(\frac{dx}{dt}\right)dt=\int_1^2\frac{\partial L}{\partial \dot{x}}d\left(\delta x\right)=\frac{\partial L}{\partial \dot{x}}\delta x\Big|_1^2-\int_1^2\delta x\,d\left(\frac{\partial L}{\partial \dot{x}}\right)=-\int_1^2\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\delta x\,dt\ .$$

It was considered that the variation δx at the initial time point 1 and in the final time point 2 is zero according to the condition of varying trajectory. Integrating by parts also for terms with $\delta\dot{y}$ and $\delta\dot{z}$, for the variation of the action we obtain:

$$\delta S=\int_1^2\left(\left[\frac{\partial L}{\partial x}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\right]\delta x+\left[\frac{\partial L}{\partial y}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)\right]\delta y+\left[\frac{\partial L}{\partial z}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right)\right]\delta z\right)dt+$$
$$+\int_1^2\left(\frac{\partial L}{\partial D_\mu}\delta D_\mu+\frac{\partial L}{\partial A_\mu}\delta A_\mu+\frac{\partial L}{\partial g_{\mu\nu}}\delta g_{\mu\nu}\right)dt=0. \qquad (1)$$

Variations δx, δy, δz, δD_μ, δA_μ and $\delta g_{\mu\nu}$ in (1) are independent from each other and are not equal to zero on the true path, except for the initial and final points of the trajectory. From this we obtain the following Euler-Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\frac{\partial L}{\partial x}, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)=\frac{\partial L}{\partial y}, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right)=\frac{\partial L}{\partial z}. \qquad (2)$$

$$\frac{\partial L}{\partial D_\mu}=0, \qquad \frac{\partial L}{\partial A_\mu}=0, \qquad \frac{\partial L}{\partial g_{\mu\nu}}=0. \qquad (3)$$

We shall remind that the principle of least action is usually applied to conservative systems for which precise potential functions are given, from which acting forces can be found. We shall consider physical systems with substance and the fundamental fields, which include the gravitational and electromagnetic fields. These systems are conservative, and for them the law of conservation of energy-momentum can be found, which has the same form in all frames of reference. If the reference frame is fixed and is not accelerated, the total energy and total momentum remain separately for each moment of time, with the possible exchange of energy and momentum between substance and field.

Lagrange function and equations of motion

In the case of continuously distributed substance throughout the entire volume of space in the gravitational and electromagnetic fields, we shall use the Lagrangian function L, which in the covariant theory of gravitation (CTG) has the form [3]:

$$L=\int\left(kc\,(R-2\Lambda)-c\sqrt{J_\mu J^\mu}-D_\mu J^\mu+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-A_\mu j^\mu-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1\,dx^2\,dx^3\,, \qquad (4)$$

where $k = -\frac{c^3}{16\pi\gamma\beta}$ – proportionality factor, β – low coefficient of order of unity that depends on the properties of the reference frame, γ – gravitational constant,

c – speed of light, as the measure of velocity of electromagnetic and gravitational interactions propagation,

R – scalar curvature,

Λ – constant for the system (in the case when (4) is applied to cosmology, the constant Λ is called the cosmological constant),

$D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D}\right)$ – 4-potential of gravitational field which is described through scalar potential ψ and vector potential $\boldsymbol{D}$ of this field,

$J^\mu = \rho_0 u^\mu$ – 4-vector of mass current density,

ρ_0 – density of substance mass in reference frame in which the substance is at rest,

u^μ – 4-velocity of the substance unit,

$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$ – gravitational tensor (gravitational field strength tensor),

$\Phi^{\alpha\beta} = g^{\alpha\mu} g^{\nu\beta} \Phi_{\mu\nu}$ – definition of the gravitational tensor with contravariant indices by means of the metric tensor $g^{\alpha\mu}$,

$A_\mu = \left(\frac{\varphi}{c}, -\boldsymbol{A}\right)$ – 4-potential of electromagnetic field, set by scalar potential φ and vector potential $\boldsymbol{A}$ of the field,

$j^\mu = \rho_{0q} u^\mu$ – 4-vector of electric current density,

ρ_{0q} – charge density of substance in reference frame in which the charge is at rest,

μ_0 – vacuum permeability,

$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$ – electromagnetic tensor (electromagnetic field strength tensor),

$\sqrt{-g}$ – the square root of determinant g of metric tensor, taken with the negative sign,

$dx^1 dx^2 dx^3$ – product of differentials of spatial coordinates, which can be viewed as a spatial coordinate volume of the moving substance unit in the used reference frame.

Further we shall use international system of units, basic coordinates in the form of coordinates with contravariant indices (x^0, x^1, x^2, x^3), metric signature (+, –, –, –), metric tensor $g_{\mu\nu}$. The presence of repeated indices in formulas implies Einstein summation convention, which is a separate summation for each repeated index. The symbol ∇_μ denotes covariant derivative with respect to coordinates (in this case the coordinates x^μ). Similarly, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ is an operator of partial derivative with respect to coordinates, or 4-gradient.

We can assume that the quantities R, ρ_0, D_μ, $\Phi_{\mu\nu}$, ρ_{0q}, A_μ, $F_{\mu\nu}$ in the location of the substance unit are functions of its coordinates x^μ, as well as the functions of the coordinates and velocities of other substance units. However, the specified quantities in the first approximation are independent from the 4-velocity of the substance unit. This is possible if the substance unit is so small that the propagation delay of its own field within the volume of the substance unit can be neglected even at relativistic speeds. The smallness of the volume, mass and charge of the substance unit leads to the fact that the motion of this substance unit is determined only by the gradients of the external fields (in the form of superposition of fields from all the external substance units), and the substance unit itself does not contribute to the average gradient of the field inside the unit. With these assumptions in (4) only 4-velosity u^μ, as a part of J^μ and j^μ, will depend on the 3-velocity of the substance unit.

If we consider that the tensor of gravitational field depends on the 4-potential D_μ under the definition $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$, then the relation $\frac{\partial L}{\partial D_\mu} = 0$ of (3) for the Lagrangian (4) provides:

$$\nabla_\alpha \Phi^{\alpha\beta} = -\frac{4\pi\gamma}{c^2} J^\beta, \qquad \text{or} \qquad \nabla_\nu \Phi^{\mu\nu} = \frac{4\pi\gamma}{c^2} J^\mu. \tag{5}$$

Similarly, we obtain for the relation $\frac{\partial L}{\partial A_\mu} = 0$ in (3):

$$\nabla_\alpha F^{\alpha\beta} = \frac{1}{c^2 \varepsilon_0} j^\beta, \qquad \text{or} \qquad \nabla_\nu F^{\mu\nu} = -\frac{1}{c^2 \varepsilon_0} j^\mu = -\mu_0 j^\mu, \tag{6}$$

The relations (5) and (6) set the equations of gravitational and electromagnetic fields, respectively, carrying out the connection between the 4-potentials of fields and the sources of fields in the form of 4-currents of mass and charge. According to (5) and (6), the larger 4-currents are, the higher are the covariant derivatives of the variables $\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ ($\Phi^{\alpha\beta}$ and $F^{\alpha\beta}$ are 4-rotors of the 4-potentials of field).

As was shown in [3], the relation $\frac{\partial L}{\partial g_{\mu\nu}} = 0$ in (3) leads to the following:

$$R^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta} R = \frac{8\pi\gamma\beta}{c^4} \left(\phi^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} \right), \tag{7}$$

provided that:

$$c\rho_0 \sqrt{u_\mu u^\mu} + \rho_0 D_\mu u^\mu + \rho_{0q} A_\mu u^\mu = \frac{c^4 \Lambda}{8\pi\gamma\beta} = \rho'_0 c^2 . \tag{8}$$

In the equation for the metric (7) the quantity $R^{\alpha\beta}$ is Ricci tensor, so that the left side of (7) gives the Hilbert-Einstein tensor. The right side of (7) contains the stress-energy tensor of substance $\phi^{\alpha\beta}$, the stress-energy tensor of gravitational field $U^{\alpha\beta}$, as well as the stress-energy tensor of electromagnetic field $W^{\alpha\beta}$. The tensor $U^{\alpha\beta}$ is expressed through the tensor of gravitational field by the formula:

$$U^{\alpha\beta} = \frac{c^2}{4\pi\gamma} \left(g^{\alpha\nu} \Phi_{\kappa\nu} \Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right) = -\frac{c^2}{4\pi\gamma} \left(\Phi^{\alpha}{}_{\kappa} \Phi^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} \Phi_{\mu\nu} \Phi^{\mu\nu} \right). \tag{9}$$

Equation (8) states that there is a connection between the cosmological constant Λ and energy density $\rho'_0 c^2$ of the system's substance when the substance is dispersed to infinity and there it is still. In this case, the 4-potentials D_μ and A_μ in (8) are equal to zero. As a result of further interaction the substance merges into a smaller size system, and the substance density varies from ρ'_0 to ρ_0 , and there is the potential energy of interaction between the substance and the field due to the 4-potentials of the field.

In the interpretation of the constant Λ two approaches are possible. In the first, the difference between ρ_0 and ρ'_0 arises only from the macroscopic gravitational and electromagnetic fields. In the second case we can assume that to the 4-potentials of fields D_μ and A_μ the strong gravitation and electromagnetic fields make contribution which act at the level of elementary particles and alter the mass of the particles [2]. In this case, the density ρ'_0 should be composed of a certain density ρ''_0 and of additives from the macroscopic and microscopic fields, and the mass of bodies is described as a characteristic that defines the interaction of substance with field quanta – gravitons and electromagnetic quanta, acting at all levels of matter [4]. It should be noted that since the 4-potentials D_μ and A_μ of fields are defined up to gauge transformation, the cosmological constant Λ will be determined with the same precision.

Now we shall turn to the relations (2). We shall preselect in the Lagrangian (4) only those terms which directly depend on the coordinates and the velocities, and substitute the relations $J^\mu = \rho_0 u^\mu$ and $j^\mu = \rho_{0q} u^\mu$:

$$L' = \int \left(-c\rho_0 \sqrt{u_\mu u^\mu} - \rho_0 D_\mu u^\mu - \rho_{0q} A_\mu u^\mu \right) \sqrt{-g}\, dx^1 dx^2 dx^3 \,, \qquad (10)$$

$$L = L' + \int \left(kc(R - 2\Lambda) + \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3 \,.$$

We shall integrate (10) for the three-dimensional volume, assuming that $dx^0 = cdt$, taking into account the following relations [5]:

$$-g = bg_{00}, \qquad d\tau = \frac{dx^0}{c}\sqrt{g_{00}} = dt\sqrt{g_{00}} \,, \qquad \sqrt{-g} = \sqrt{bg_{00}} = \frac{d\tau}{dt}\sqrt{b} \,, \qquad (11)$$

where g – determinant of the metric tensor $g_{\mu\nu}$,

$d\tau$ – differential of the proper time at the point of reference frame, through which the substance unit passes,

dt – differential of the coordinate time of the used reference frame,

b – determinant of the three-dimensional metric tensor b_{ik}, with components $b_{ik} = -g_{ik} + \frac{g_{0i}\, g_{0k}}{g_{00}}$, $b^{ik} = -g^{ik}$, $i, k = 1, 2, 3$.

The invariant of three-dimensional volume is the product $\sqrt{b}\, dx^1 dx^2 dx^3$, and the factor $\sqrt{b}$ provides transition from a moving coordinate volume $dx^1 dx^2 dx^3$ to moving local volume in terms of the local observer at the point in space, through which at the moment τ of its proper (local) time the substance unit passes. This gives in (10): $\sqrt{-g}\, dx^1 dx^2 dx^3 = \frac{d\tau}{dt} \sqrt{b}\, dx^1 dx^2 dx^3 = \frac{d\tau}{dt} dV = \sqrt{g_{00}}\, dV$, where dV is the differential of the moving local volume. For the moving substance unit 4-velocity equals to $u^\mu = \frac{c dx^\mu}{ds}$, as well as:

$$\frac{c d\tau}{ds} \rho_0 dV = \frac{c dt}{ds} \rho_0 \sqrt{-g}\, dx^1 dx^2 dx^3 = \frac{\rho_0 \sqrt{-g}\, d\Sigma}{ds} = dm = \rho_0 dV_0 = \rho dV \, ,$$

$$\frac{c d\tau}{ds} \rho_{0q} dV = \frac{c dt}{ds} \rho_{0q} \sqrt{-g}\, dx^1 dx^2 dx^3 = \frac{\rho_{0q} \sqrt{-g}\, d\Sigma}{ds} = dq = \rho_{0q} dV_0 = \rho_q dV \, ,$$

where dV_0 is the differential of volume of substance unit in the co-moving reference frame, $\sqrt{-g}\, d\Sigma = \sqrt{-g}\, c\, dt\, dx^1 dx^2 dx^3$ – an invariant of moving 4-volume, provided $dx^0 = cdt$.

This implies the expression for the mass density ρ and charge density ρ_q of the moving substance:

$$\rho = \frac{c d\tau}{ds} \rho_0 = \frac{ds'}{ds} \rho_0 \, , \qquad \rho_q = \frac{c d\tau}{ds} \rho_{0q} = \frac{ds'}{ds} \rho_{0q} \, ,$$

where ds denotes the interval for the moving substance unit, and ds' is an interval for a stationary observer, by which the substance passes.

With the formulas for dm and dq, L' in (10) will equal to:

$$L' = \int \left(-c\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}} - D_\mu \frac{dx^\mu}{dt} \right) dm - \int A_\mu \frac{dx^\mu}{dt} dq = \\ = -mc\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}} - mD_\mu \frac{dx^\mu}{dt} - qA_\mu \frac{dx^\mu}{dt}. \qquad (12)$$

In (12) m and q are the mass and the charge of a small substance unit, moving as a whole with the coordinate velocity $\frac{dx^\mu}{dt}$, and this velocity is not a 4-vector. 4-potentials D_μ and A_μ in the result of integrating by volume are considered to be effective averaged by volume potentials acting on the substance unit. In the coordinates $x^\mu = (ct, x, y, z)$ the quantity $\frac{dx^\mu}{dt} = (c, \dot{x}, \dot{y}, \dot{z}) = (c, \boldsymbol{v})$, hence the product is $D_\mu \frac{dx^\mu}{dt} = \left(\frac{\psi}{c}, -\boldsymbol{D} \right)(c, \boldsymbol{v}) = \psi - \boldsymbol{v} \cdot \boldsymbol{D}$. Similarly for the electromagnetic potential is: $A_\mu \frac{dx^\mu}{dt} = \left(\frac{\varphi}{c}, -\boldsymbol{A} \right)(c, \boldsymbol{v}) = \varphi - \boldsymbol{v} \cdot \boldsymbol{A}$.

We shall note that the coordinate velocity $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}$ is different from the velocity of the substance unit, which is measured by the local observer. This is due to the fact that the local observer's proper time τ does not coincide with the coordinate time t (the coordinate time t is common for the reference frame as a whole, and the proper time τ is measured by stationary electromagnetic clocks in each specific point of reference frame, or by the clock associated with the moving substance, and depends on the actions on the clocks of existing gravitational and electromagnetic fields at the time of measurement).

Three-dimensional vector potential of gravitational field has its components along the spatial axes of the coordinate system: $\boldsymbol{D} = (D_x, D_y, D_z)$, as well for the vector potential of electromagnetic field it can be written down: $\boldsymbol{A} = (A_x, A_y, A_z)$.

Taking it into account for (12) we have:

$$L' = -mc\left[c(g_{00}c + g_{01}\dot{x} + g_{02}\dot{y} + g_{03}\dot{z}) + \dot{x}(g_{10}c + g_{11}\dot{x} + g_{12}\dot{y} + g_{13}\dot{z}) + \\ + \dot{y}(g_{20}c + g_{21}\dot{x} + g_{22}\dot{y} + g_{23}\dot{z}) + \dot{z}(g_{30}c + g_{31}\dot{x} + g_{32}\dot{y} + g_{33}\dot{z}) \right]^{1/2} - \\ - m(\psi - \dot{x}D_x - \dot{y}D_y - \dot{z}D_z) - q(\varphi - \dot{x}A_x - \dot{y}A_y - \dot{z}A_z). \qquad (13)$$

In the simplest case, we can assume that for an arbitrary reference frame the velocities $\dot{x}, \dot{y}, \dot{z}$ do not depend explicitly on the coordinates x, y, z, and are time-dependent; the mass

m and the charge q can be dependent on t, x, y, z and independent on $\dot{x}, \dot{y}, \dot{z}$; the scalar potentials ψ and φ, the vector potentials $\boldsymbol{D}$ and $\boldsymbol{A}$, the metric tensor $g_{\mu\nu}$ do not depend directly on $\dot{x}, \dot{y}, \dot{z}$, but depend on t, x, y, z. The assumption of independence $\dot{x}, \dot{y}, \dot{z}$ in an explicit form on the coordinates x, y, z means that the velocity field is free, and not the bound vector field. An example of the bound field is the velocity field in the liquid flowing in the volume bounded by a surface. Due to the interaction of the liquid with the surface and the liquid particles with each other there is a clear dependence of the velocity field on the coordinates. If we consider quasi-free motion of continuously distributed substance with weak gravitational and electromagnetic fields, the velocity will depend weakly on the spatial coordinates.

Under these conditions from (12) and (13) we find:

$$\frac{\partial L}{\partial \dot{x}} = \frac{\partial L'}{\partial \dot{x}} = -\frac{mc}{\sqrt{g_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt}}} (g_{10} c + g_{11} \dot{x} + g_{12} \dot{y} + g_{13} \dot{z}) + m D_x + q A_x = \\ = -m g_{1\mu} u^{\mu} + m D_x + q A_x . \tag{14}$$

In (14) it was taken into account that $\sqrt{g_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt}} = \frac{ds}{dt}$, where ds is the interval, and the relation $\frac{cdt}{ds} \frac{dx^{\mu}}{dt} = \frac{cdt}{ds} (c, \dot{x}, \dot{y}, \dot{z}) = u^{\mu} = \frac{cdx^{\mu}}{ds}$ was used. We shall note that from the definition of 4-velocity $u^{\mu} = \frac{cdx^{\mu}}{ds}$ and of the interval $ds = \sqrt{g_{\mu\nu} dx^{\mu} dx^{\nu}}$ follows the standard relation $u_{\mu} u^{\mu} = c^2$.

The full time derivative of (14) gives:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}\left(\frac{\partial L'}{\partial \dot{x}}\right) = -\frac{d(m g_{1\mu} u^{\mu})}{dt} + \frac{dm}{dt} D_x + m \frac{dD_x}{dt} + \frac{dq}{dt} A_x + q \frac{dA_x}{dt} . \tag{15}$$

The first spatial component of the gradient from L' will be equal to:

$$\frac{\partial L'}{\partial x}=-c\frac{ds}{dt}\frac{\partial m}{\partial x}-\frac{m u^{\mu}}{2}\frac{dx^{\nu}}{dt}\frac{\partial g_{\mu\nu}}{\partial x}-\frac{\partial m}{\partial x}(\psi-\dot{x}D_x-\dot{y}D_y-\dot{z}D_z)-$$
$$-m\left(\frac{\partial \psi}{\partial x}-\dot{x}\frac{\partial D_x}{\partial x}-\dot{y}\frac{\partial D_y}{\partial x}-\dot{z}\frac{\partial D_z}{\partial x}\right)-\frac{\partial q}{\partial x}(\varphi-\dot{x}A_x-\dot{y}A_y-\dot{z}A_z)-q\left(\frac{\partial \varphi}{\partial x}-\dot{x}\frac{\partial A_x}{\partial x}-\dot{y}\frac{\partial A_y}{\partial x}-\dot{z}\frac{\partial A_z}{\partial x}\right).$$

In view of (10) we have:

$$\frac{\partial L}{\partial x}=\frac{\partial L'}{\partial x}+\frac{\partial}{\partial x}\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1dx^2dx^3 . \qquad (16)$$

The Euler-Lagrange equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\frac{\partial L}{\partial x}$ from (2) requires that the equations (15) and (16) should be equal to each other:

$$-\frac{d(m g_{1\mu}u^{\mu})}{dt}+\frac{dm}{dt}D_x+m\frac{dD_x}{dt}+\frac{dq}{dt}A_x+q\frac{dA_x}{dt}=$$
$$=-c\frac{ds}{dt}\frac{\partial m}{\partial x}-\frac{mu^{\mu}}{2}\frac{dx^{\nu}}{dt}\frac{\partial g_{\mu\nu}}{\partial x}-\frac{\partial m}{\partial x}(\psi-\dot{x}D_x-\dot{y}D_y-\dot{z}D_z)-$$
$$-m\left(\frac{\partial \psi}{\partial x}-\dot{x}\frac{\partial D_x}{\partial x}-\dot{y}\frac{\partial D_y}{\partial x}-\dot{z}\frac{\partial D_z}{\partial x}\right)-\frac{\partial q}{\partial x}(\varphi-\dot{x}A_x-\dot{y}A_y-\dot{z}A_z)-q\left(\frac{\partial \varphi}{\partial x}-\dot{x}\frac{\partial A_x}{\partial x}-\dot{y}\frac{\partial A_y}{\partial x}-\dot{z}\frac{\partial A_z}{\partial x}\right)+$$
$$+\frac{\partial}{\partial x}\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1dx^2dx^3. \qquad (17)$$

With the help of 3-vector $\boldsymbol{u}=-(g_{1\mu}u^{\mu},g_{2\mu}u^{\mu},g_{3\mu}u^{\mu})$ we shall introduce the 3-vector of generalized momentum with the following components:

$$\begin{aligned}\boldsymbol{P}&=(-m g_{1\mu}u^{\mu}+mD_x+qA_x,-m g_{2\mu}u^{\mu}+mD_y+qA_y,-m g_{3\mu}u^{\mu}+mD_z+qA_z)=\\&=(mu_x+mD_x+qA_x,\ mu_y+mD_y+qA_y,\ mu_z+mD_z+qA_z).\end{aligned} \qquad (18)$$

In view of (18) instead of (17) it can be written in the 3-vector form:

$$\boldsymbol{F}=\frac{d\boldsymbol{P}}{dt}=-\nabla\left(mc\frac{ds}{dt}+m\psi-m\boldsymbol{v}\cdot\boldsymbol{D}+q\varphi-q\boldsymbol{v}\cdot\boldsymbol{A}\right)+$$
$$+\nabla\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1dx^2dx^3. \tag{19}$$

According to (19) for continuously distributed matter the rate of change of the generalized momentum of substance and the field is determined by gradients from the following quantities: the energy of the substance unit in gravitational and electromagnetic fields that can be found through the velocity $\boldsymbol{v}$ and the scalar and vector potentials; the integral by volume of the term with scalar spacetime curvature; the integral by volume of energy invariants of the gravitational and electromagnetic fields, which are in the volume of the substance unit, as well as those of their proper fields, which are generated by this substance and interact with it. Generalized force $\boldsymbol{F}$ in (19) also depends on the constant Λ and the term $mc\frac{ds}{dt}$ associated with the relativistic energy of the mass m.

We shall remind that deriving (17) and (19), we assumed that the velocity of the substance does not depend on spatial coordinates. In this regard, in (17) and (19) there are no gradients of the velocity components that appear in the case of the velocity field in some way connected with the points in space.

The case of a small test particle outside a massive charged body

The equation of motion (17) can be simplified by using the operator equality: $\frac{d}{dt}=\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla$. This gives the following:

$$\frac{dD_x}{dt}=\frac{\partial D_x}{\partial t}+\dot{x}\frac{\partial D_x}{\partial x}+\dot{y}\frac{\partial D_x}{\partial y}+\dot{z}\frac{\partial D_x}{\partial z}, \qquad \frac{dA_x}{dt}=\frac{\partial A_x}{\partial t}+\dot{x}\frac{\partial A_x}{\partial x}+\dot{y}\frac{\partial A_x}{\partial y}+\dot{z}\frac{\partial A_x}{\partial z},$$

$$\frac{dm}{dt}=\frac{\partial m}{\partial t}+\dot{x}\frac{\partial m}{\partial x}+\dot{y}\frac{\partial m}{\partial y}+\dot{z}\frac{\partial m}{\partial z}, \qquad \frac{dq}{dt}=\frac{\partial q}{\partial t}+\dot{x}\frac{\partial q}{\partial x}+\dot{y}\frac{\partial q}{\partial y}+\dot{z}\frac{\partial q}{\partial z}.$$

Next, we shall introduce the vector of gravitational acceleration strength $\boldsymbol{G}$ and the vector of torsion field strength $\boldsymbol{\Omega}$ (gravitomagnetic field) according to the formulas:

$$\boldsymbol{G}=-\nabla\psi-\frac{\partial\boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega}=\nabla\times\boldsymbol{D}.$$

It is seen that these definitions of $\boldsymbol{G}$ and $\boldsymbol{\Omega}$ are written in generally covariant form, since these quantities with accuracy up to a constant factor, constitute the components of the gravitational tensor $\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu$. Similarly the strength of the electric field $\boldsymbol{E}$ and the induction of the magnetic field $\boldsymbol{B}$ are defined:

$$\boldsymbol{E} = -\nabla \varphi - \frac{\partial \boldsymbol{A}}{\partial t}, \qquad\qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}.$$

As far as $m[\boldsymbol{v}\times\boldsymbol{\Omega}]_x = m\dot{y}\Omega_z - m\dot{z}\Omega_y$, $q[\boldsymbol{v}\times\boldsymbol{B}]_x = q\dot{y}B_z - q\dot{z}B_y$, then using the previous equations for (17) we find:

$$\begin{aligned}
&-\frac{d(m\, g_{1\mu} u^\mu)}{dt} + \frac{m u^\mu}{2}\frac{dx^\nu}{dt}\frac{\partial g_{\mu\nu}}{\partial x} = -\frac{\partial m}{\partial t} D_x - \frac{\partial q}{\partial t} A_x - c\frac{ds}{dt}\frac{\partial m}{\partial x} - \frac{\partial m}{\partial x}\psi - \frac{\partial q}{\partial x}\varphi + \\
&\quad + \left[\boldsymbol{v}\times[\nabla m \times \boldsymbol{D}]\right]_x + \left[\boldsymbol{v}\times[\nabla q\times \boldsymbol{A}]\right]_x + mG_x + m[\boldsymbol{v}\times\boldsymbol{\Omega}]_x + qE_x + q[\boldsymbol{v}\times\boldsymbol{B}]_x + \\
&\quad + \frac{\partial}{\partial x}\int \left(kc(R-2\Lambda) + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0} \right)\sqrt{-g}\, dx^1 dx^2 dx^3.
\end{aligned} \tag{20}$$

Equation (20) is the equation of motion of the substance unit in the direction of the first spatial axis of the reference system, and it corresponds to the equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$ in (2). For other spatial axes the equations of motion will differ only by replacing the indices in the derivatives and the components of vectors. If we enter the 3-vector $\boldsymbol{u} = -(g_{1\mu}u^\mu, g_{2\mu}u^\mu, g_{3\mu}u^\mu)$, then instead of (20) we can write the equation of motion in 3-vector form:

$$\begin{aligned}
&\frac{d(m\boldsymbol{u})}{dt} + \frac{m u^\mu}{2}\frac{dx^\nu}{dt}\nabla g_{\mu\nu} = -\frac{\partial m}{\partial t}\boldsymbol{D} - \frac{\partial q}{\partial t}\boldsymbol{A} - c\frac{ds}{dt}\nabla m - \psi\nabla m - \varphi\nabla q + \\
&\quad + \left[\boldsymbol{v}\times[\nabla m\times\boldsymbol{D}]\right] + \left[\boldsymbol{v}\times[\nabla q\times\boldsymbol{A}]\right] + m\boldsymbol{G} + m[\boldsymbol{v}\times\boldsymbol{\Omega}] + q\boldsymbol{E} + q[\boldsymbol{v}\times\boldsymbol{B}] + \\
&\quad + \nabla\int \left(kc(R-2\Lambda) + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0} \right)\sqrt{-g}\, dx^1 dx^2 dx^3.
\end{aligned} \tag{21}$$

3-vector $\nabla g_{\mu\nu}$ in the left side of (21) is equivalent in its meaning to action of Christoffel symbols, which are used to write the equations of motion in Riemannian space in four-dimensional notation, both in the general theory of relativity and in the covariant theory of gravitation.

Since we consider a small test particle outside a massive charged body, then the contribution to the curvature R and the constant Λ is made only by the test particle itself. The terms $\frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma}$ and $\frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0}$ in (21) are associated with the energy density of gravitational and electromagnetic fields, respectively. If the test particle is small enough and has low density of mass and charge, then the main contribution to the energy density of the fields in the volume of the particle will be made by the external fields of the massive charged body. In addition, in (21) the gradient of the integral over the volume is taken, which in some cases can be close to zero due to symmetry and homogeneity of the distribution of field energy within the test particle. One of such cases is the approximate spatial homogeneity of the external field.

In Minkowski space we have: $\boldsymbol{u} = \frac{\boldsymbol{v}}{\sqrt{1-v^2/c^2}}$, $\nabla g_{\mu\nu} = 0$. If we also assume the constancy of the mass and charge with the time, zero gradients of the mass, charge, curvature and zero gradients in the distribution of field energy within the volume of the particle, then (21) takes the form of the equations of motion of the test particle in gravitational and electromagnetic fields in Lorentz-invariant theory of gravitation [2]:

$$\frac{d}{dt}\left(\frac{m\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right) = m\boldsymbol{G} + m[\boldsymbol{v}\times\boldsymbol{\Omega}] + q\boldsymbol{E} + q[\boldsymbol{v}\times\boldsymbol{B}]. \qquad (22)$$

The left side of (22) is the rate of change with the time of the relativistic particle momentum, while in the right side there is the two-component gravitational force and similar to it the two-component electromagnetic Lorentz force. Thus, from the variation of action (1) with the Lagrangian (4) in the framework of the covariant theory of gravitation (CTG), we can obtain the equation of motion of a particle (22), which is valid in the special theory of relativity (SRT). This means that the equations of CTG and SRT are linked by the correspondence principle, when after the aspiration of the curvature of spacetime to zero the equations of CTG turn into the equations of special relativity.

In contrast, the equations of general relativity do not have such a direct transition to the equations of special relativity. Indeed, in general relativity Lagrangian differs from (4) by the absence of gravitational terms of the form: $-\rho_0 D_\mu u^\mu + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}$. As a result, in (21) there are no gravitational terms, only the following remains:

$$\frac{d(m\boldsymbol{u})}{dt} + \frac{mu^\mu}{2}\frac{dx^\nu}{dt}\nabla g_{\mu\nu} = -\frac{\partial q}{\partial t}\boldsymbol{A} - c\frac{ds}{dt}\nabla m - \varphi\nabla q + [\boldsymbol{v}\times[\nabla q\times\boldsymbol{A}]] + q\boldsymbol{E} + q[\boldsymbol{v}\times\boldsymbol{B}] + \\ +\nabla\int\left(kc(R-2\Lambda) - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1 dx^2 dx^3. \tag{23}$$

In order that gravitation could appear in general relativity as an effective force of gravitation in the weak field limit, in (23) the decomposition of $\nabla g_{\mu\nu}$ should be carried out, and the appearing terms should be transferred to the right side are considered as a gravitational force. The difference between the positions of the general relativity and CTG is due to the fact that in general relativity gravitation is simply the curvature of spacetime (without specifying the reasons for this curvature), and in CTG gravitation is a real physical force which is substantiated by the mechanism of Le Sage gravitation [6]. In this case the scalar potential ψ of the gravitational field in CTG is the characteristic of scalar field associated with the flow of gravitons, and is proportional to the difference between the energy density of the graviton flux at the point where the potential is determined, and the energy density of the graviton flux at infinity. The gradients of the energy density of graviton flux in this case can be considered as gravitational field strengths. In the assumption that some gravitons are tiny charged particles, in [1] the scheme of appearance the electromagnetic force and the electric potential φ is derived. If scalar potentials are known in a fixed frame of reference, then after conversion into a moving frame of reference vector potentials of gravitational $\boldsymbol{D}$ and electromagnetic $\boldsymbol{A}$ of fields appear, as a consequence of field retardation effects due to the limited speed of their propagation. Thus we can understand why the fields are described by 4-potentials $D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D}\right)$ and $A_\mu = \left(\frac{\varphi}{c}, -\boldsymbol{A}\right)$.

The relation between the Lagrange and Hamilton functions

Describing the principle of least action, we recorded the Lagrange function in the general form: $L = L(t, x, y, z, \dot{x}, \dot{y}, \dot{z}, D_\mu, A_\mu, g_{\mu\nu})$, where the quantities $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$, $\dot{z} = \frac{dz}{dt}$ are the components of 3-vector of coordinate velocity $\boldsymbol{v} = (\dot{x}, \dot{y}, \dot{z})$ of the substance unit motion. Variation of the action function leads to the Euler-Lagrange equations (2) and (3) and requires variation of the Lagrangian, which has the form:

$$\delta L = \frac{\partial L}{\partial x}\delta x + \frac{\partial L}{\partial y}\delta y + \frac{\partial L}{\partial z}\delta z + \frac{\partial L}{\partial \dot{x}}\delta \dot{x} + \frac{\partial L}{\partial \dot{y}}\delta \dot{y} + \frac{\partial L}{\partial \dot{z}}\delta \dot{z} + \\ + \frac{\partial L}{\partial D_\mu}\delta D_\mu + \frac{\partial L}{\partial A_\mu}\delta A_\mu + \frac{\partial L}{\partial g_{\mu\nu}}\delta g_{\mu\nu}. \qquad (24)$$

We shall introduce the Hamiltonian $H = H(t, x, y, z, P_x, P_y, P_z, D_\mu, A_\mu, g_{\mu\nu})$, where the quantities P_x, P_y, P_z are the components of the 3-vector of the so-called conjugate generalized momentum $\boldsymbol{P} = (P_x, P_y, P_z)$ (conjugate with respect to the coordinates x, y, z). The Hamiltonian in the simplest case is determined by the Legendre transformation through the components of the conjugate momentum, the velocity components of the substance unit and the Lagrange function:

$$H = P_x \dot{x} + P_y \dot{y} + P_z \dot{z} - L = \boldsymbol{P} \cdot \boldsymbol{v} - L. \qquad (25)$$

With the vanishing of the variation in time, as it is required for the Lagrange function in the principle of least action, for the variation of the Hamiltonian we have:

$$\delta H = \frac{\partial H}{\partial x}\delta x + \frac{\partial H}{\partial y}\delta y + \frac{\partial H}{\partial z}\delta z + \frac{\partial H}{\partial P_x}\delta P_x + \frac{\partial H}{\partial P_y}\delta P_y + \frac{\partial H}{\partial P_z}\delta P_z + \\ + \frac{\partial H}{\partial D_\mu}\delta D_\mu + \frac{\partial H}{\partial A_\mu}\delta A_\mu + \frac{\partial H}{\partial g_{\mu\nu}}\delta g_{\mu\nu}. \qquad (26)$$

The result of the variation (25) is:

$$\delta H = \delta P_x \dot{x} + P_x \delta \dot{x} + \delta P_y \dot{y} + P_y \delta \dot{y} + \delta P_z \dot{z} + P_z \delta \dot{z} - \delta L. \qquad (27)$$

Substituting (24) and (26) in (27) gives the following relations:

$$\frac{\partial H}{\partial x}=-\frac{\partial L}{\partial x}, \qquad \frac{\partial H}{\partial y}=-\frac{\partial L}{\partial y}, \qquad \frac{\partial H}{\partial z}=-\frac{\partial L}{\partial z}, \tag{28}$$

$$\frac{\partial H}{\partial D_\mu}=-\frac{\partial L}{\partial D_\mu}, \qquad \frac{\partial H}{\partial A_\mu}=-\frac{\partial L}{\partial A_\mu}, \qquad \frac{\partial H}{\partial g_{\mu\nu}}=-\frac{\partial L}{\partial g_{\mu\nu}}, \tag{29}$$

$$\dot{x}=\frac{\partial H}{\partial P_x}, \quad \dot{y}=\frac{\partial H}{\partial P_y}, \quad \dot{z}=\frac{\partial H}{\partial P_z}, \quad P_x=\frac{\partial L}{\partial \dot{x}}, \quad P_y=\frac{\partial L}{\partial \dot{y}}, \quad P_z=\frac{\partial L}{\partial \dot{z}}. \tag{30}$$

After determining $\frac{\partial L}{\partial \dot{x}}$ through P_x in accordance with (30), and substituting in (2), taking into account (28) we have: $\frac{dP_x}{dt}=\frac{\partial L}{\partial x}=-\frac{\partial H}{\partial x}$. In general, we can write down:

$$\frac{d\boldsymbol{P}}{dt}=\nabla L=-\nabla H. \tag{31}$$

We shall find the components of the generalized momentum from (30), given that the velocity components $\dot{x},\dot{y},\dot{z}$ are directly included in the Lagrangian (4) according to (12) and (13) only in three terms, forming part of the Lagrangian L'. From (14) and analogous relations with the help of (30) can be obtained for the generalized momentum the same as in (18):

$$\boldsymbol{P}=(P_x,P_y,P_z), \qquad P_x=\frac{\partial L}{\partial \dot{x}}=\frac{\partial L'}{\partial \dot{x}}=-m\,g_{1\mu}u^\mu+m\,D_x+q\,A_x, \tag{32}$$

$$P_y=\frac{\partial L}{\partial \dot{y}}=\frac{\partial L'}{\partial \dot{y}}=-m\,g_{2\mu}u^\mu+m\,D_y+q\,A_y, \qquad P_z=\frac{\partial L}{\partial \dot{z}}=\frac{\partial L'}{\partial \dot{z}}=-m\,g_{3\mu}u^\mu+m\,D_z+q\,A_z.$$

The scalar product of the generalized momentum $\boldsymbol{P}$ and the velocity $\boldsymbol{V}$, taking into account the relation $\frac{cdt}{ds}\frac{dx^\mu}{dt}=\frac{cdt}{ds}(c,\dot{x},\dot{y},\dot{z})=u^\mu=\frac{cdx^\mu}{ds}$, gives:

$$\begin{aligned}\boldsymbol{P}\cdot\boldsymbol{V}=P_x\dot{x}+P_y\dot{y}+P_z\dot{z}&=-m\,g_{1\mu}u^\mu\dot{x}-m\,g_{2\mu}u^\mu\dot{y}-m\,g_{3\mu}u^\mu\dot{z}+m\boldsymbol{V}\cdot\boldsymbol{D}+q\boldsymbol{V}\cdot\boldsymbol{A}=\\&=mc\,g_{0\mu}u^\mu-mc\frac{ds}{dt}+m\boldsymbol{V}\cdot\boldsymbol{D}+q\boldsymbol{V}\cdot\boldsymbol{A}.\end{aligned} \tag{33}$$

Substituting this expression into (25) in view of (4), (10), (12), (13) allows us to find the Hamiltonian for the solid-state motion of the substance unit with the mass m and the charge q:

$$H = mc\, g_{0\mu} u^{\mu} + m\psi + q\varphi - \int \left(kc(R-2\Lambda) + \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3 \,. \qquad (34)$$

In mechanics the Hamiltonian is usually associated with the energy of a body (a substance unit). The first term in (34) is connected with the rest energy and kinetic energy of substance. Products $m\psi$ and $q\varphi$ give the potential energy of mass and charge in the gravitational and electromagnetic fields associated with scalar potentials. The volume integral in (34) defines the additional energies, depending on the curvature of spacetime R, the constant Λ, and the field strengths. If the volume of the test particle is small, the volume integral in (34) can be neglected compared to the first three terms. In this case the energy of the test particle includes the relativistic energy of motion and energy of the particle in field potentials.

If we consider the formulas for dm and dq, given before the relation (12), then the mass and the charge can be expressed in terms of the volume integral of the density of mass and charge:

$$m = \int \frac{c\,dt}{ds} \rho_0 \sqrt{-g}\, dx^1 dx^2 dx^3 \,, \qquad\qquad q = \int \frac{c\,dt}{ds} \rho_{0q} \sqrt{-g}\, dx^1 dx^2 dx^3 \,,$$

where ρ_0 – the substance density in the reference frame at rest relative to the substance unit;

ds – the interval;

ρ_{0q} – the charge density in the reference frame at rest relative to the substance unit.

In view of this the Hamiltonian for a continuously distributed matter would have the following form:

$$H = \int \left(\frac{cdt}{ds} (\rho_0 c\ g_{0\mu} u^{\mu} + \rho_0 \psi + \rho_{0q} \varphi) - kc(R - 2\Lambda) - \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} + \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3 .$$

(35)

In Minkowski space we have the following relations:

$$mc\, g_{0\mu} u^{\mu} = \frac{mc^2}{\sqrt{1 - v^2/c^2}} , \qquad \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} = -\frac{1}{8\pi\gamma} \left(G^2 - c^2 \Omega^2 \right) ,$$

$$\frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} = -\frac{\varepsilon_0}{2} (E^2 - c^2 B^2) ,$$

where $\boldsymbol{G}$ – the gravitational acceleration, $\boldsymbol{\Omega}$ – the vector of gravitational torsion field, $\boldsymbol{E}$ – the electric field strength, $\boldsymbol{B}$ – the magnetic induction, ε_0 – the vacuum permittivity.

Substituting these relations into (34) for the case of a small test particle, when one can neglect the term with the scalar curvature R :

$$H = \frac{mc^2}{\sqrt{1 - v^2/c^2}} + m\psi + q\varphi + \int \left(\frac{1}{8\pi\gamma} \left(G^2 - c^2 \Omega^2 \right) - \frac{\varepsilon_0}{2} (E^2 - c^2 B^2) \right) dx^1 dx^2 dx^3 + const .$$

(36)

For external fields it is necessary in (36) to integrate over the volume of the particle, and for the fields generated by the substance of the particle, it is necessary to integrate over the volume both inside and outside the particle. The Hamiltonian (36) as the energy of a small test particle is determined up to a constant, which arises from integration over the volume of constant Λ (for the meaning of this constant see our discussion after relation (9)). In the Minkowski space metric does not depend on the coordinates and time, and therefore the term with the constant Λ in variation of Lagrangian disappears and does not contribute to the equations of motion. However, due to the definition of the Hamiltonian (25), where the Lagrange function L is included as a whole, the constant Λ appears in (36) as additional constant.

The expression of the Hamiltonian through the generalized momentum

In (34) and (35) the Hamiltonian is expressed through the 4-velocity u^{μ}, depending on the 3-vector of velocity $\boldsymbol{v}=(\dot{x},\dot{y},\dot{z})$. However, in the canonical form the Hamiltonian is defined by the components of generalized momentum: $H=H(t,x,y,z,P_x,P_y,P_z,D_{\mu},A_{\mu},g_{\mu\nu})$. We express the components of the 3-velocity through components of the generalized momentum $\boldsymbol{P}=(P_x,P_y,P_z)$, for which, taking into account the expressions $\frac{cdt}{ds}\frac{dx^{\mu}}{dt}=\frac{cdt}{ds}(c,\dot{x},\dot{y},\dot{z})=u^{\mu}=\frac{cdx^{\mu}}{ds}$, we rewrite (32) in another form:

$$g_{11}\dot{x}+g_{12}\dot{y}+g_{13}\dot{z}=-\frac{P_x-mD_x-qA_x}{m}\frac{ds}{cdt}-g_{10}c\,. \qquad (37)$$

$$g_{21}\dot{x}+g_{22}\dot{y}+g_{23}\dot{z}=-\frac{P_y-mD_y-qA_y}{m}\frac{ds}{cdt}-g_{20}c\,. \qquad (38)$$

$$g_{31}\dot{x}+g_{32}\dot{y}+g_{33}\dot{z}=-\frac{P_z-mD_z-qA_z}{m}\frac{ds}{cdt}-g_{30}c\,. \qquad (39)$$

In view of (32), we introduce the following notation:

$$\frac{P_x-mD_x-qA_x}{m}=C_x=-g_{1\mu}u^{\mu}\,, \qquad \frac{P_y-mD_y-qA_y}{m}=C_y=-g_{2\mu}u^{\mu}\,,$$

$$\frac{P_z-mD_z-qA_z}{m}=C_z=-g_{3\mu}u^{\mu}\,, \qquad (40)$$

as components of a 3-vector, normalized to unit mass.

We also need the following minors:

$M_{\alpha\beta}$ – minors of the matrix of the components of the metric tensor $g_{\alpha\beta}$, where $\alpha,\beta=0,1,2,3$;

m_{ik} – minors of the spatial submatrix of the components of the metric tensor g_{ik}, where $i,k=1,2,3$. As the examples of such minors, taking into account the symmetry of the metric tensor $g_{\alpha\beta}$ we can write down:

$$M_{01}=g_{10}(g_{22}g_{33}-g_{23}g_{32})-g_{20}(g_{12}g_{33}-g_{13}g_{32})+g_{30}(g_{12}g_{23}-g_{13}g_{22})\,, \qquad (41)$$

$$M_{02} = g_{01}(g_{21}g_{33} - g_{23}g_{31}) - g_{02}(g_{11}g_{33} - g_{13}g_{31}) + g_{03}(g_{11}g_{23} - g_{13}g_{21}),$$

$$M_{03} = g_{01}(g_{21}g_{32} - g_{22}g_{31}) - g_{02}(g_{11}g_{32} - g_{12}g_{31}) + g_{03}(g_{11}g_{22} - g_{12}g_{21}),$$

$$m_{11} = g_{22}\,g_{33} - g_{23}\,g_{32}, \qquad m_{12} = g_{21}\,g_{33} - g_{23}\,g_{31}, \qquad m_{13} = g_{21}\,g_{32} - g_{22}\,g_{31}.$$

We shall also use the following relations:

$$\begin{aligned} &g_{12}m_{21} - g_{13}m_{31} - g_{11}m_{11} = -M_{00}. \\ &g_{12}m_{12} + g_{23}m_{32} - g_{22}m_{22} = -M_{00}. \\ &-g_{13}m_{13} + g_{23}m_{23} - g_{33}m_{33} = -M_{00}. \end{aligned} \tag{42}$$

$$g_{13}m_{12} - g_{23}m_{22} + g_{33}m_{32} = 0, \qquad g_{12}m_{23} - g_{13}m_{33} - g_{11}m_{13} = 0,$$

$$-g_{12}m_{11} - g_{23}m_{31} + g_{22}m_{21} = 0, \qquad -g_{12}m_{13} - g_{23}m_{33} + g_{22}m_{23} = 0,$$

$$-g_{13}m_{11} + g_{23}m_{21} - g_{33}m_{31} = 0, \qquad -g_{12}m_{22} + g_{13}m_{32} + g_{11}m_{12} = 0.$$

With these notations from (37), (38) and (39) we have:

$$M_{00}\,\dot{x} = \frac{ds}{cdt}\left(-m_{11}C_x + m_{12}C_y - m_{13}C_z\right) - M_{01}\,c. \tag{43}$$

$$M_{00}\,\dot{y} = \frac{ds}{cdt}\left(m_{21}C_x - m_{22}C_y + m_{23}C_z\right) + M_{02}\,c. \tag{44}$$

$$M_{00}\,\dot{z} = \frac{ds}{cdt}\left(-m_{31}C_x + m_{32}C_y - m_{33}C_z\right) - M_{03}\,c. \tag{45}$$

Dividing (44) and (45) by (43), $\dot{y}$ and $\dot{z}$ can be expressed by $\dot{x}$:

$$\dot{y} = \frac{(M_{00}\,\dot{x} + M_{01}\,c)\left(m_{21}C_x - m_{22}C_y + m_{23}C_z\right)}{M_{00}\left(-m_{11}C_x + m_{12}C_y - m_{13}C_z\right)} + \frac{M_{02}\,c}{M_{00}}. \tag{46}$$

$$\dot{z} = \frac{(M_{00}\,\dot{x} + M_{01}\,c)\left(-m_{31}C_x + m_{32}C_y - m_{33}C_z\right)}{M_{00}\left(-m_{11}C_x + m_{12}C_y - m_{13}C_z\right)} - \frac{M_{03}\,c}{M_{00}}. \tag{47}$$

From (43) we find:

$$\left(\frac{ds}{cdt}\right)^2 = \frac{(M_{00}\dot{x} + M_{01}c)^2}{\left(-m_{11}C_x + m_{12}C_y - m_{13}C_z\right)^2} . \tag{48}$$

On the other hand, $\frac{dx^\mu}{dt} = (c, \dot{x}, \dot{y}, \dot{z})$, and for the square of the interval $(ds)^2 = g_{\alpha\beta}dx^\alpha dx^\beta$. In view of this, we have:

$$\left(\frac{ds}{cdt}\right)^2 = \frac{g_{\alpha\beta}}{c^2}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = \frac{1}{c^2}(g_{00}c^2 + 2g_{01}c\dot{x} + 2g_{02}c\dot{y} + 2g_{03}c\dot{z} + \\ + 2g_{12}\dot{x}\dot{y} + 2g_{13}\dot{x}\dot{z} + 2g_{23}\dot{y}\dot{z} + g_{11}\dot{x}^2 + g_{22}\dot{y}^2 + g_{33}\dot{z}^2). \tag{49}$$

From equations (48) and (49) it follows:

$$\frac{c^2(M_{00}\dot{x} + M_{01}c)^2}{\left(-m_{11}C_x + m_{12}C_y - m_{13}C_z\right)^2} = g_{00}c^2 + 2g_{01}c\dot{x} + 2g_{02}c\dot{y} + 2g_{03}c\dot{z} + \\ + 2g_{12}\dot{x}\dot{y} + 2g_{13}\dot{x}\dot{z} + 2g_{23}\dot{y}\dot{z} + g_{11}\dot{x}^2 + g_{22}\dot{y}^2 + g_{33}\dot{z}^2. \tag{50}$$

If we substitute $\dot{y}$ and $\dot{z}$ from (46) and (47) in (50), we obtain a quadratic equation for the velocity component $\dot{x}$. However, this equation is too cumbersome to write. Equation (50) can be simplified by introducing a new variable:

$$\frac{M_{00}\dot{x} + M_{01}c}{-m_{11}C_x + m_{12}C_y - m_{13}C_z} = X , \qquad \dot{x} = \frac{X(-m_{11}C_x + m_{12}C_y - m_{13}C_z) - M_{01}c}{M_{00}} . \tag{51}$$

Using in (50) relations (46), (47) and (51), after lengthy calculations we find:

$$X^2 = \frac{c^2(-g)}{Z} . \tag{52}$$

where g is the determinant of the metric tensor $g_{\alpha\beta}$, and g is negative:

$$g = g_{00}M_{00} - g_{01}M_{01} + g_{02}M_{02} - g_{03}M_{03} ,$$

and the following abbreviation is used:

$$Z=-M_{00}c^2-C_x(-m_{11}C_x+m_{12}C_y-m_{13}C_z)-C_y\left(m_{21}C_x-m_{22}C_y+m_{23}C_z\right)-$$
$$-C_z(-m_{31}C_x+m_{32}C_y-m_{33}C_z). \tag{53}$$

From (52) and (51) we find $\dot{x}$, and then from (46) and (47) define $\dot{y}$ and $\dot{z}$:

$$\dot{x}=\frac{c\sqrt{-g}(-m_{11}C_x+m_{12}C_y-m_{13}C_z)-\sqrt{Z}M_{01}c}{\sqrt{Z}M_{00}}, \tag{54}$$

$$\dot{y}=\frac{c\sqrt{-g}\left(m_{21}C_x-m_{22}C_y+m_{23}C_z\right)+\sqrt{Z}M_{02}c}{\sqrt{Z}M_{00}},$$

$$\dot{z}=\frac{c\sqrt{-g}\left(-m_{31}C_x+m_{32}C_y-m_{33}C_z\right)-\sqrt{Z}M_{03}c}{\sqrt{Z}M_{00}}.$$

From (54) and (43) we derive the quantity $\frac{ds}{cdt}$:

$$\frac{ds}{cdt}=\frac{c\sqrt{-g}}{\sqrt{Z}}. \tag{55}$$

We can calculate $g_{0\mu}u^\mu$ using (54), (55) and the expression $\frac{cdt}{ds}\frac{dx^\mu}{dt}=\frac{cdt}{ds}(c,\dot{x},\dot{y},\dot{z})=u^\mu=\frac{cdx^\mu}{ds}$:

$$g_{0\mu}u^\mu=g_{0\mu}\frac{cdt}{ds}(c,\dot{x},\dot{y},\dot{z})=\frac{cdt}{ds}(g_{00}c+g_{01}\dot{x}+g_{02}\dot{y}+g_{03}\dot{z})=$$
$$=\frac{-C_xM_{01}+C_yM_{02}-C_zM_{03}}{M_{00}}-\frac{\sqrt{Z}\sqrt{-g}}{M_{00}}. \tag{56}$$

In (56) using the previously introduced in (40) notations $\frac{P_x-mD_x-qA_x}{m}=C_x$, $\frac{P_y-mD_y-qA_y}{m}=C_y$, $\frac{P_z-mD_z-qA_z}{m}=C_z$, we can move from C_x, C_y and C_z to the generalized momenta P_x, P_y and P_z. After multiplying (56) by mc the result will be equal to:

$$mc\,g_{0\mu}u^{\mu}=\frac{-cM_{01}(P_x-mD_x-qA_x)+cM_{02}(P_y-mD_y-qA_y)-cM_{03}(P_z-mD_z-qA_z)}{M_{00}}-$$
$$-\frac{mc\sqrt{Z}\sqrt{-g}}{M_{00}}.$$

Let us substitute this into the formula for the Hamiltonian (34):

$$H=\frac{-cM_{01}(P_x-mD_x-qA_x)+cM_{02}(P_y-mD_y-qA_y)-cM_{03}(P_z-mD_z-qA_z)}{M_{00}}-$$
$$-\frac{mc\sqrt{Z}\sqrt{-g}}{M_{00}}+m\psi+q\varphi-\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1dx^2dx^3.$$

(57)

In Minkowski space, i.e. in the special theory of relativity when the curvature of spacetime is absent, $M_{00}=-1$, $M_{01}=M_{02}=M_{03}=0$, $\sqrt{-g}=1$, and taking into account the expressions (53) for Z and (40) for C_x, C_y and C_z, the Hamiltonian will be expressed through the 3-vector of the generalized momentum $\boldsymbol{P}$, through the scalar potentials ψ, φ, and vector potentials $\boldsymbol{D}$, $\boldsymbol{A}$:

$$H=c\sqrt{m^2c^2+(\mathbf{P}-m\mathbf{D}-q\mathbf{A})^2}+m\psi+q\varphi-\int\left(\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)dx^1dx^2dx^3+const.$$

(57’)

Similarly to (36) in the expression for the Hamiltonian (57’) there is some constant. In this case the gravitational tensor $\Phi_{\mu\nu}$ and electromagnetic tensor $F_{\mu\nu}$ are differential functions of the potentials of fields in the form of derivatives of coordinates and time. The resulting expression (57’) for H, but without taking into account the gravitational field, that is, without terms with the potentials ψ and $\boldsymbol{D}$, and without taking into account the integral with the tensors $\Phi_{\mu\nu}$ and $F_{\mu\nu}$, we can find in [5].

Hamilton's equations according to (30) and (31), with the components of 3-vector coordinate velocity $\mathbf{v} = (\dot{x}, \dot{y}, \dot{z})$, and the components of 3-vector of the generalized momentum $\mathbf{P} = (P_x, P_y, P_z)$ (32) have the following form:

$$\dot{x} = \frac{\partial H}{\partial P_x}, \qquad \dot{y} = \frac{\partial H}{\partial P_y}, \qquad \dot{z} = \frac{\partial H}{\partial P_z}, \quad \text{or} \quad \mathbf{v} = \frac{\partial H}{\partial \mathbf{P}}. \tag{58}$$

$$\frac{d\mathbf{P}}{dt} = \nabla L = -\nabla H . \tag{59}$$

In order to verify the validity of equations (58) the quantity Z of (53) should be substituted into (57), and the quantities C_x, C_y and C_z should be expressed in terms of generalized momenta P_x, P_y and P_z, using (40). If we then take the partial derivatives from the Hamiltonian H according to (58) we shall obtain expressions (54) for the components of velocity. The physical meaning of equation (59) lies in the fact that the gradient of the Hamiltonian as the energy of the system, taken with opposite sign, is equal to the rate of change of the generalized momentum with time.

Now we shall write (57) in four-dimensional form, for which we shall use the following expressions:

$$-\frac{\sqrt{-g}}{M_{00}} = \frac{g}{M_{00}\sqrt{-g}} = \frac{1}{g^{00}\sqrt{-g}}, \qquad M_{00} = g^{00} g, \tag{60}$$

$$-\frac{M_{01}}{M_{00}} = -\frac{M_{01}}{g^{00}g} = \frac{g^{01}}{g^{00}}, \qquad \frac{M_{02}}{M_{00}} = \frac{M_{02}}{g^{00}g} = \frac{g^{02}}{g^{00}}, \qquad -\frac{M_{03}}{M_{00}} = -\frac{M_{03}}{g^{00}g} = \frac{g^{03}}{g^{00}}.$$

For the first term in (57) with the help of (32) it gives:

$$\frac{-cM_{01}(P_x - mD_x - qA_x) + cM_{02}(P_y - mD_y - qA_y) - cM_{03}(P_z - mD_z - qA_z)}{M_{00}} =$$

$$= \frac{-mcg^{01} g_{1\mu}u^{\mu} - mcg^{02} g_{2\mu}u^{\mu} - mcg^{03} g_{3\mu}u^{\mu}}{g^{00}} =$$

$$= \frac{-mcu^{\mu}(g^{00}g_{0\mu}+g^{01}g_{1\mu}+g^{02}g_{2\mu}+g^{03}g_{3\mu})}{g^{00}}+mcg_{0\mu}u^{\mu}=\frac{-mcu^{0}}{g^{00}}+mcg_{0\mu}u^{\mu}. \qquad (61)$$

We shall make further transformations of the following auxiliary quantities with the help of (41) and (42):

$$u^{\mu}(m_{11}g_{1\mu}-m_{12}g_{2\mu}+m_{13}g_{3\mu})=u^{0}(m_{11}g_{10}-m_{12}g_{20}+m_{13}g_{30})+$$
$$+u^{1}(m_{11}g_{11}-m_{12}g_{21}+m_{13}g_{31})=u^{0}M_{01}+u^{1}M_{00}.$$

$$u^{\mu}\left(-m_{21}g_{1\mu}+m_{22}g_{2\mu}-m_{23}g_{3\mu}\right)=-u^{0}M_{02}+u^{2}M_{00}.$$
$$u^{\mu}(m_{31}g_{1\mu}-m_{32}g_{2\mu}+m_{33}g_{3\mu})=u^{0}M_{03}+u^{3}M_{00}. \qquad (62)$$

From (40) it follows that $C_x=-g_{1\mu}u^{\mu}$, $C_y=-g_{2\mu}u^{\mu}$, $C_z=-g_{3\mu}u^{\mu}$. Then, using (62) and the equality $g_{\nu\mu}u^{\mu}u^{\nu}=c^2$ the expression (53) for Z can be transformed as follows:

$$Z=-M_{00}c^2-C_x(-m_{11}C_x+m_{12}C_y-m_{13}C_z)-C_y\left(m_{21}C_x-m_{22}C_y+m_{23}C_z\right)-$$
$$-C_z(-m_{31}C_x+m_{32}C_y-m_{33}C_z)=$$
$$=-M_{00}c^2+g_{1\mu}u^{\mu}u^{\mu}(m_{11}g_{1\mu}-m_{12}g_{2\mu}+m_{13}g_{3\mu})+g_{2\mu}u^{\mu}u^{\mu}\left(-m_{21}g_{1\mu}+m_{22}g_{2\mu}-m_{23}g_{3\mu}\right)+$$
$$+g_{3\mu}u^{\mu}u^{\mu}(m_{31}g_{1\mu}-m_{32}g_{2\mu}+m_{33}g_{3\mu})=$$
$$=-M_{00}c^2+g_{1\mu}u^{\mu}(u^{0}M_{01}+u^{1}M_{00})+g_{2\mu}u^{\mu}(-u^{0}M_{02}+u^{2}M_{00})+g_{3\mu}u^{\mu}(u^{0}M_{03}+u^{3}M_{00})=$$
$$=-M_{00}c^2+g_{1\mu}u^{\mu}u^{0}M_{01}-g_{2\mu}u^{\mu}u^{0}M_{02}+g_{3\mu}u^{\mu}u^{0}M_{03}+g_{\nu\mu}u^{\mu}u^{\nu}M_{00}-g_{0\mu}u^{\mu}u^{0}M_{00}=$$
$$=g_{1\mu}u^{\mu}u^{0}M_{01}-g_{2\mu}u^{\mu}u^{0}M_{02}+g_{3\mu}u^{\mu}u^{0}M_{03}-g_{0\mu}u^{\mu}u^{0}M_{00}.$$

Now we shall use (60):

$$Z=g_{1\mu}u^{\mu}u^{0}M_{01}-g_{2\mu}u^{\mu}u^{0}M_{02}+g_{3\mu}u^{\mu}u^{0}M_{03}-g_{0\mu}u^{\mu}u^{0}M_{00}=$$
$$=-g_{1\mu}u^{\mu}u^{0}\,\frac{g^{01}M_{00}}{g^{00}}-g_{2\mu}u^{\mu}u^{0}\,\frac{g^{02}M_{00}}{g^{00}}-g_{3\mu}u^{\mu}u^{0}\,\frac{g^{03}M_{00}}{g^{00}}-g_{0\mu}u^{\mu}u^{0}\,\frac{g^{00}M_{00}}{g^{00}}=$$
$$=-\frac{M_{00}}{g^{00}}u^{\mu}u^{0}(g_{1\mu}g^{01}+g_{2\mu}g^{02}+g_{3\mu}g^{03}+g_{0\mu}g^{00})=-\frac{M_{00}}{g^{00}}u^{\mu}u^{0}\delta^{0}_{\mu}=-\frac{M_{00}}{g^{00}}u^{0}u^{0}=-g\,u^{0}u^{0}.$$
$$(63)$$

In (63) we used Kronecker delta $\delta_{\mu}^{\nu} = \begin{Bmatrix} 1, \mu = \nu \\ 0, \mu \neq \nu \end{Bmatrix}$. In view of (63) for the second term in (57) we find:

$$-\frac{mc\sqrt{Z}\,\sqrt{-g}}{M_{00}} = \frac{mc\sqrt{Z}}{g^{00}\sqrt{-g}} = \frac{mc\sqrt{-g\,u^0 u^0}}{g^{00}\sqrt{-g}} = \frac{mcu^0}{g^{00}} .$$

We substitute this expression and the result from (61) into (57):

$$H = mc\,g_{0\mu}u^{\mu} + m\psi + q\varphi - \int \left(kc(R-2\Lambda) + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\,dx^1 dx^2 dx^3 . \qquad (64)$$

The Hamiltonian (64) coincides with the expression for the Hamiltonian (34). Thus, we made a circle: first, by introducing the generalized momentum ***P*** (32) we made the transition from (34) to the Hamiltonian in the form of (57), and then by other way, we got back to (34).

To check the validity of equations (59) for the Hamiltonian in the form of (64), we find the quantity $-\frac{\partial H}{\partial x}$:

$$-\frac{\partial H}{\partial x} = -\frac{\partial}{\partial x}(mc\,g_{0\mu}u^{\mu}) - \frac{\partial}{\partial x}(m\psi) - \frac{\partial}{\partial x}(q\varphi) + $$
$$+\frac{\partial}{\partial x}\int \left(kc(R-2\Lambda) + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\,dx^1 dx^2 dx^3 .$$

From (59) it follows:

$$-\frac{\partial H}{\partial x} = \frac{dP_x}{dt} = \frac{d}{dt}(-m\,g_{1\mu}u^{\mu} + m\,D_x + q\,A_x).$$

From the last two equations we obtain:

$$\frac{d}{dt}(-m\,g_{1\mu}u^{\mu}+m\,D_x+q\,A_x)=-\frac{\partial}{\partial x}(mc\,g_{0\mu}u^{\mu})-\frac{\partial}{\partial x}(m\psi)-\frac{\partial}{\partial x}(q\varphi)+$$
$$+\frac{\partial}{\partial x}\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1dx^2dx^3. \qquad (65)$$

In Minkowski space: $-m\,g_{1\mu}u^{\mu}=\frac{m\,\dot{x}}{\sqrt{1-v^2/c^2}}$, $mc\,g_{0\mu}u^{\mu}=\frac{m\,c^2}{\sqrt{1-v^2/c^2}}$. If we consider the situation for a small test particle outside the massive charged body and apply the relations:

$$\frac{dD_x}{dt}=\frac{\partial D_x}{\partial t}+\dot{x}\frac{\partial D_x}{\partial x}+\dot{y}\frac{\partial D_x}{\partial y}+\dot{z}\frac{\partial D_x}{\partial z}, \qquad \frac{dA_x}{dt}=\frac{\partial A_x}{\partial t}+\dot{x}\frac{\partial A_x}{\partial x}+\dot{y}\frac{\partial A_x}{\partial y}+\dot{z}\frac{\partial A_x}{\partial z},$$

$$\boldsymbol{G}=-\nabla\psi-\frac{\partial\boldsymbol{D}}{\partial t}, \qquad \boldsymbol{\Omega}=\nabla\times\boldsymbol{D},$$

$$\boldsymbol{E}=-\nabla\varphi-\frac{\partial\boldsymbol{A}}{\partial t}, \qquad \boldsymbol{B}=\nabla\times\boldsymbol{A},$$

then with constant mass m and charge q of the particle, and assuming that the velocity $\boldsymbol{v}$ and the scalar products $\boldsymbol{v}\cdot\boldsymbol{D}$ and $\boldsymbol{v}\cdot\boldsymbol{A}$ do not directly depend on the coordinates, the equation (65) turns into (22) for the component of the momentum $\frac{m\,\dot{x}}{\sqrt{1-v^2/c^2}}$.

The four-dimensional generalized velocity

We shall introduce 4-vector of the generalized velocity with the covariant index:

$$s_{\mu}=\frac{c\,J_{\mu}}{\sqrt{J_{\lambda}J^{\lambda}}}+D_{\mu}+\frac{\rho_{0q}}{\rho_0}A_{\mu}. \qquad (66)$$

where $D_{\mu}=\left(\frac{\psi}{c},-\boldsymbol{D}\right)$ – 4-potential of gravitational field,

$A_{\mu}=\left(\frac{\varphi}{c},-\boldsymbol{A}\right)$ – 4-potential of electromagnetic field.

The ratio $\frac{\rho_{0q}}{\rho_0}$ in (66) is the ratio of the densities of charge and mass of the substance unit in the reference frame in which the substance is at rest. The scalar $s_\mu J^\mu$ will be equal to:

$$s_\mu J^\mu = \frac{c\, J_\mu J^\mu}{\sqrt{J_\lambda J^\lambda}} + D_\mu J^\mu + \frac{\rho_{0q}}{\rho_0} A_\mu J^\mu = c\sqrt{J_\mu J^\mu} + D_\mu\, J^\mu + A_\mu\, j^\mu\,, \tag{67}$$

where $j^\mu = \rho_{0q} u^\mu$ is the 4-vector of electric current density.

Taking it into account we can rewrite the Lagrangian (4) as follows:

$$L = \int \left(kc\,(R - 2\Lambda) - s_\mu J^\mu + \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3\,, \tag{68}$$

and $S = \int L\, dt$ is the function of the action, and $\sqrt{-g}\, d\Sigma = \sqrt{-g}\, c\, dt\, dx^1 dx^2 dx^3$ – an invariant 4-volume, provided that $dx^0 = cdt$. With the help of (11) and the subsequent relations we can write down:

$$\sqrt{-g}\, d\Sigma = \frac{d\tau}{dt} \sqrt{b}\; c\, dt\, dx^1 dx^2 dx^3 = c\, d\tau\, dV = ds'\, dV = ds\, dV_0\,.$$

Thus, the invariance of the 4-volume $\sqrt{-g}\, d\Sigma$ with respect to the change of coordinates is expressed in the invariance of the interval ds of the moving substance unit, and in the invariance of the three-dimensional volume dV_0 of the substance unit in the co-moving frame of reference.

We shall designate $L_2 = \int \left(-s_\mu J^\mu \sqrt{-g} \right) dx^1 dx^2 dx^3$ in (68) and find the variation δL_2, associated with variation of part the action function $S_2 = \int L_2\, dt$:

$$\delta S_2 = \int \delta L_2\, dt\,, \qquad\qquad \delta L_2 = \int \delta\left(-s_\mu J^\mu \sqrt{-g} \right) dx^1 dx^2 dx^3\,. \tag{69}$$

$$\delta\left(-s_\mu J^\mu \sqrt{-g}\right) = -s_\mu \delta\left(J^\mu \sqrt{-g}\right) - J^\mu \sqrt{-g}\, \delta s_\mu = $$
$$= -s_\mu \sqrt{-g}\, \delta J^\mu - s_\mu J^\mu \delta\sqrt{-g} - J^\mu \sqrt{-g}\, \delta\left(\frac{c\, J_\mu}{\sqrt{J_\lambda J^\lambda}} + D_\mu + \frac{\rho_{0q}}{\rho_0} A_\mu \right). \qquad (70)$$

We shall use the following standard formulas:

$$\delta\sqrt{-g} = \frac{\sqrt{-g}}{2} g^{\mu\nu}\, \delta g_{\mu\nu}, \qquad J_\mu J^\mu = g_{\mu\nu} J^\mu J^\nu,$$
$$\delta J^\mu = \nabla_\sigma \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) = \frac{1}{\sqrt{-g}} \partial_\sigma \left[\sqrt{-g} \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) \right], \qquad (71)$$
$$\delta\rho_0 = -\nabla_\sigma \left(\rho_0 \xi^\sigma \right) + \frac{\rho_0}{c^2} u_\sigma u^\nu \nabla_\nu \xi^\sigma, \qquad \delta\rho_{0q} = -\nabla_\sigma \left(\rho_{0q} \xi^\sigma \right) + \frac{\rho_{0q}}{c^2} u_\sigma u^\nu \nabla_\nu \xi^\sigma,$$

where the variations δJ^μ, $\delta\rho_0$, $\delta\rho_{0q}$ are taken from [7], [8], and displacement ξ^μ are variations of the coordinates, due to of which arise the variation of mass 4-current δJ^μ, the variation of mass density $\delta\rho_0$ and the variation of charge density $\delta\rho_{0q}$.

We shall transform the first term in (70) in view of (71):

$$-s_\mu \sqrt{-g}\, \delta J^\mu = -s_\mu \partial_\sigma \left[\sqrt{-g} \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) \right] =$$
$$= -\partial_\sigma \left[s_\mu \sqrt{-g} \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) \right] + \sqrt{-g} \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) \partial_\sigma s_\mu .$$

In this expression the term with the total divergence in the integration over the 4-volume in the function of the action will not make any contribution. The remaining term will be transformed further:

$$\sqrt{-g} \left(J^\sigma \xi^\mu - J^\mu \xi^\sigma \right) \partial_\sigma s_\mu = \left(\partial_\sigma s_\mu - \partial_\mu s_\sigma \right) J^\sigma \xi^\mu \sqrt{-g},$$

where the value $\partial_\sigma s_\mu - \partial_\mu s_\sigma$ is the rotor of 4-vector of generalized velocities s_μ.

We shall transform the expression in the third term in (70):

$$-J^{\mu}\sqrt{-g}\,\delta\left(\frac{cJ_{\mu}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)=-cJ^{\mu}\sqrt{-g}\,\delta\left(\frac{g_{\mu k}J^{k}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}\right)=$$

$$=-cJ^{\mu}\sqrt{-g}\left(\frac{g_{\mu k}\delta J^{k}+J^{k}\delta g_{\mu k}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}-\frac{2g_{\mu k}J^{k}g_{\alpha\beta}J^{\alpha}\delta J^{\beta}+g_{\mu k}J^{k}J^{\alpha}J^{\beta}\delta g_{\alpha\beta}}{2\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}\right)=$$

$$=-c\sqrt{-g}\left(\frac{J^{\mu}g_{\mu k}\delta J^{k}+J^{\mu}J^{k}\delta g_{\mu k}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}-\frac{g_{\alpha\beta}J^{\alpha}\delta J^{\beta}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}-\frac{J^{\alpha}J^{\beta}\delta g_{\alpha\beta}}{2\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}\right)=-\frac{c\sqrt{-g}\,J^{\alpha}J^{\beta}\delta g_{\alpha\beta}}{2\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}.$$

With the help of (71) we shall find the variation $\delta\left(\frac{\rho_{0q}}{\rho_0}\right)$:

$$\delta\left(\frac{\rho_{0q}}{\rho_0}\right)=\frac{\rho_0\,\delta\rho_{0q}-\rho_{0q}\delta\rho_0}{(\rho_{0)})^2}=-\xi^{\sigma}\nabla_{\sigma}\left(\frac{\rho_{0q}}{\rho_0}\right).$$

Substitution in (70) and (69) of the obtained above expressions gives:

$$\delta S_2=\int\delta\left(-s_{\mu}J^{\mu}\sqrt{-g}\right)dx^1\,dx^2\,dx^3\,dt=$$

$$=\int\left[-s_{\mu}\sqrt{-g}\,\delta J^{\mu}-s_{\mu}J^{\mu}\,\delta\sqrt{-g}-J^{\mu}\sqrt{-g}\,\delta\left(\frac{cJ_{\mu}}{\sqrt{J_{\nu}J^{\nu}}}+D_{\mu}+\frac{\rho_{0q}}{\rho_0}A_{\mu}\right)\right]dx^1\,dx^2\,dx^3\,dt=$$

$$=\int\left[\left(\partial_{\sigma}s_{\mu}-\partial_{\mu}s_{\sigma}\right)J^{\sigma}\xi^{\mu}-\frac{\delta g_{\mu\nu}}{2}\left(s_{\alpha}J^{\alpha}g^{\mu\nu}+\frac{cJ^{\mu}J^{\nu}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}\right)-J^{\mu}\,\delta D_{\mu}-\right.$$

$$\left.-j^{\mu}\,\delta A_{\mu}+J^{\sigma}A_{\sigma}\xi^{\mu}\nabla_{\mu}\left(\frac{\rho_{0q}}{\rho_0}\right)\right]\sqrt{-g}\,dx^1\,dx^2\,dx^3\,dt.$$

(72)

We shall designate $L_1=\int\left(kc(R-2\Lambda)+\frac{c^2\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma}-\frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\,dx^1\,dx^2\,dx^3$ in (68) and take in [3] the variation δL_1, associated with the variation of the action function $S_1=\int L_1\,dt$. This gives the following:

$$\delta S_1 = \int \delta L_1 \, dt \,,$$

$$\delta L_1 = \int \delta \left(kc(R-2\Lambda)\sqrt{-g} + \frac{c^2 \Phi_{\mu\nu}\Phi^{\mu\nu}\sqrt{-g}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}\sqrt{-g}}{4\mu_0} \right) dx^1 dx^2 dx^3 \;.$$

$$\delta S_1 = \int \left[kc\left(-R^{\mu\nu} + \frac{1}{2} g^{\mu\nu} R - \Lambda g^{\mu\nu} \right)\delta g_{\mu\nu} - \frac{c^2}{4\pi\gamma}\nabla_\alpha \Phi^{\alpha\mu}\,\delta D_\mu - \frac{U^{\mu\nu}}{2}\delta g_{\mu\nu} + \right. \\ \left. + \frac{1}{\mu_0}\nabla_\alpha F^{\alpha\mu}\,\delta A_\mu - \frac{W^{\mu\nu}}{2}\delta g_{\mu\nu} \right]\sqrt{-g}\, dx^1 dx^2 dx^3 \, dt \,, \qquad (73)$$

where $U^{\mu\nu}$ is the stress-energy tensor of gravitational field (9), and the stress-energy tensor $W^{\mu\nu}$ of electromagnetic field has the form:

$$W^{\alpha\beta} = \varepsilon_0 c^2 \left(-g^{\alpha\nu} F_{\kappa\nu} F^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right) = \varepsilon_0 c^2 \left(F^{\alpha}_{\ \kappa} F^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} \right). \qquad (74)$$

By the principle of least action, the variation of the action must be equal to zero: $\delta S = \int \delta L \, dt = \delta S_1 + \delta S_2 = 0$. We shall substitute here (73) and (72), and equate to zero all the terms inside the integrals, placed before the variations $\delta g_{\mu\nu}$, δD_μ, δA_μ, ξ^μ:

$$\delta g_{\mu\nu}: \; kc\left(-R^{\mu\nu} + \frac{1}{2} g^{\mu\nu} R - \Lambda g^{\mu\nu} \right) - \frac{U^{\mu\nu}}{2} - \frac{W^{\mu\nu}}{2} - \frac{1}{2}\left(s_\alpha J^\alpha g^{\mu\nu} + \frac{c J^\mu J^\nu}{\sqrt{g_{\sigma\lambda} J^\sigma J^\lambda}} \right) = 0 \,, \qquad (75)$$

$$\delta D_\mu: \; -\frac{c^2}{4\pi\gamma}\nabla_\alpha \Phi^{\alpha\mu} - J^\mu = 0 \,, \qquad \delta A_\mu: \; \frac{1}{\mu_0}\nabla_\alpha F^{\alpha\mu} - j^\mu = 0 \,, \qquad (76)$$

$$\xi^\mu: \; \left(\partial_\sigma s_\mu - \partial_\mu s_\sigma \right) J^\sigma + J^\sigma A_\sigma \nabla_\mu \left(\frac{\rho_{0q}}{\rho_0} \right) = 0 \,. \qquad (77)$$

Equations (76) are equivalent to the gravitational (5) and electromagnetic (6) field equations. The first term in equation (77) can be expanded by using the operator of proper-time-derivative $\frac{D}{D\tau}=u^{\mu}\nabla_{\mu}$ according to [1], and the 4-vector of generalized velocity (66):

$$\left(\partial_{\sigma}s_{\mu}-\partial_{\mu}s_{\sigma}\right)J^{\sigma}=\rho_0 u^{\sigma}\nabla_{\sigma}s_{\mu}-J^{\sigma}\nabla_{\mu}s_{\sigma}=\rho_0\frac{Ds_{\mu}}{D\tau}-J^{\sigma}\nabla_{\mu}s_{\sigma}=$$
$$=\rho_0\frac{Ds_{\mu}}{D\tau}-J^{\sigma}\nabla_{\mu}\left(\frac{cJ_{\sigma}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)-J^{\sigma}\nabla_{\mu}D_{\sigma}-J^{\sigma}A_{\sigma}\nabla_{\mu}\left(\frac{\rho_{0q}}{\rho_0}\right)-j^{\sigma}\nabla_{\mu}A_{\sigma}.$$

Taking into account (77) it follows:

$$\rho_0\frac{Ds_{\mu}}{D\tau}=J^{\sigma}\nabla_{\mu}\left(\frac{cJ_{\sigma}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)+J^{\sigma}\nabla_{\mu}D_{\sigma}+j^{\sigma}\nabla_{\mu}A_{\sigma}. \qquad (78)$$

As far as according to (66):

$$\rho_0\frac{Ds_{\mu}}{D\tau}=\rho_0 u^{\sigma}\nabla_{\sigma}s_{\mu}=J^{\sigma}\nabla_{\sigma}\left(\frac{cJ_{\mu}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)+J^{\sigma}\nabla_{\sigma}D_{\mu}+j^{\sigma}\nabla_{\sigma}A_{\mu}+J^{\sigma}A_{\mu}\nabla_{\sigma}\left(\frac{\rho_{0q}}{\rho_0}\right),$$

so comparing with (78) we find:

$$J^{\sigma}\nabla_{\sigma}\left(\frac{cJ_{\mu}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)+J^{\sigma}\nabla_{\sigma}D_{\mu}+j^{\sigma}\nabla_{\sigma}A_{\mu}+J^{\sigma}A_{\mu}\nabla_{\sigma}\left(\frac{\rho_{0q}}{\rho_0}\right)=$$
$$=J^{\sigma}\nabla_{\mu}\left(\frac{cJ_{\sigma}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)+J^{\sigma}\nabla_{\mu}D_{\sigma}+j^{\sigma}\nabla_{\mu}A_{\sigma}. \qquad (79)$$

We shall apply the following relations:

$$J^{\sigma}\nabla_{\mu}D_{\sigma}-J^{\sigma}\nabla_{\sigma}D_{\mu}=J^{\sigma}(\nabla_{\mu}D_{\sigma}-\nabla_{\sigma}D_{\mu})=J^{\sigma}\Phi_{\mu\sigma}, \qquad J^{\sigma}\nabla_{\mu}A_{\sigma}-J^{\sigma}\nabla_{\sigma}A_{\mu}=J^{\sigma}F_{\mu\sigma},$$

$$J^{\sigma}\nabla_{\sigma}\left(\frac{cJ_{\mu}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)=\rho_0 u^{\sigma}\nabla_{\sigma}u_{\mu}=\rho_0\frac{Du_{\mu}}{D\tau}, \qquad J^{\sigma}\nabla_{\mu}\left(\frac{cJ_{\sigma}}{\sqrt{J_{\lambda}J^{\lambda}}}\right)=\rho_0 u^{\sigma}\nabla_{\mu}u_{\sigma}=0.$$

This gives in (79):

$$\rho_0\frac{Du_{\mu}}{D\tau}=J^{\sigma}\Phi_{\mu\sigma}+j^{\sigma}F_{\mu\sigma}-J^{\sigma}A_{\mu}\nabla_{\sigma}\left(\frac{\rho_{0q}}{\rho_0}\right). \qquad (80)$$

Above it was assumed that the mass and the charge of substance unit in the variation does not change. In this case, the density ratio $\frac{\rho_{0q}}{\rho_0}$ will be unchanged, the covariant derivative $\nabla_{\sigma}\left(\frac{\rho_{0q}}{\rho_0}\right)$ is zero, and (80) turns into the equation of motion of substance in gravitational and electromagnetic fields, taken in the covariant theory of gravitation under these conditions (see the equation (35) in [3]).

Now we shall consider the equation for the metric (75). If we separate out the terms $\Lambda g^{\mu\nu}$ and $s_{\alpha}J^{\alpha}g^{\mu\nu}$, then with condition $k=-\frac{c^3}{16\pi\gamma\beta}$ (75) is divided into two equations:

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{8\pi\gamma\beta}{c^4}\left(\frac{cJ^{\mu}J^{\nu}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}+U^{\mu\nu}+W^{\mu\nu}\right), \qquad (81)$$

$$\frac{c^4\Lambda}{8\pi\gamma\beta}=s_{\alpha}J^{\alpha}. \qquad (82)$$

In view of (67), expression (82) coincides with (8). As for (81), from the comparison with (7) it follows that it should equal to:

$$\phi^{\mu\nu}=\frac{cJ^{\mu}J^{\nu}}{\sqrt{g_{\sigma\lambda}J^{\sigma}J^{\lambda}}}. \qquad (83)$$

Equation (82) can be considered as the gauge of the cosmological constant, with which it is possible to use equation (81) to find the metric.

We shall remind that the variations δJ^{μ}, $\delta\rho_0$, $\delta\rho_{0q}$ in (70) found in [7], [8], were determined from the condition that the mass and charge of substance unit are constants during variation. This leads to the equation of motion of the type (80), in which instead of the proposed total derivative $\frac{D(\rho_0 u_{\mu})}{D\tau}$ (the rate of change of mass 4-current) the quantity $\rho_0 \frac{D u_{\mu}}{D\tau}$ appears as the product of the mass density and the 4-acceleration.

The Hamiltonian and the problem of mass

The Hamiltonian (64) can be represented in another form by using the generalized 4-velocity (66). If we assume that $\frac{\rho_{0q}}{\rho_0}$ sets in (66) the charge to the mass ratio, and considering that $g_{0\mu} s^{\mu} = s_0$, for the Hamiltonian we have:

$$H = mcs_0 - \int \left(kc(R - 2\Lambda) + \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3 . \qquad (84)$$

From here it follows that the contribution to the energy of substance unit with mass m is made by the timelike component of 4-vector of generalized velocity with the covariant index s_0, and the energy of fields, found by the integral over the volume of space. In addition, the amount of energy is corrected by the curvature of spacetime (the term with curvature R), and is determined up to a constant (the term with Λ). Hamiltonian H sets the energy in such a way that the energy in each reference frame is different. This applies to the value of the generalized 4-velocity of the substance unit, and the total momentum of the substance and fields. So it should be, because in the theory of relativity only a definite combination of energy and momentum can be maintained invariant and preserved in each reference frame.

The Hamiltonian (84) looks like it should be the timelike component of a 4-vector of energy-momentum H_{μ} , written with a lower (covariant) index. In this case, the timelike component of this 4-vector is associated with the energy and the spatial component should be connected with the momentum of substance unit. We shall make the notation:

$$\int \left(kc(R - 2\Lambda) + \frac{c^2 \Phi_{\mu\nu} \Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu} F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\, dx^1 dx^2 dx^3 = \frac{N}{c} u_0 , \qquad (85)$$

where N is an invariant associated with the energy of fields and with amendments to the energy arising from the curvature R and from the constant Λ,

$\frac{u_0}{c}$ – the timelike component of the dimensionless 4-velocity $\frac{u_\mu}{c}$, and the 4-velocity $\frac{u_\mu}{c}$ is a simplest 4-vector of unit length.

With this definition, the integral (85) is assumed to be equal to the timelike component of a 4-vector. Then, taking into account (66) we have:

$$H = H_0 = mcs_0 - \frac{N}{c} u_0 = \left(mc - \frac{N}{c} \right) u_0 + mcD_0 + mc \frac{\rho_{0q}}{\rho_0} A_0 = \left(mc - \frac{N}{c} \right) u_0 + m\psi + q\varphi .$$

(86)

Equation (86) in view of (85) coincides with the expression for the Hamiltonian (34). Now we shall write the 4-vector of the Hamiltonian in the contravariant form:

$$H^\mu = mcs^\mu - \frac{N}{c} u^\mu = \left(mc - \frac{N}{c} \right) u^\mu + mcg^{\mu\nu} D_\nu + qcg^{\mu\nu} A_\nu . \quad (87)$$

As there is the 4-vector of generalized velocity s^μ in (87), the 4-vector of the Hamiltonian contains the 4-vector of the generalized momentum in the form ms^μ. The timelike component of the 4-vector H^μ must specify the relativistic energy E, and the spatial components – multiplied by the speed of light momentum $\boldsymbol{p}$. This follows from the conventional expression of the 4-vector energy-momentum of a free particle without taking into account of the action of fields on it: $p^\mu = mcu^\mu$. This vector in the flat Minkowski space, i.e., in the special theory of relativity, is expressed as follows: $p^\mu = mcu^\mu = \left(\frac{mc^2}{\sqrt{1 - v^2/c^2}}, \frac{mc\boldsymbol{v}}{\sqrt{1 - v^2/c^2}} \right) = (E, c\boldsymbol{p})$. Fields and interactions with other particles can vary quantities E and $\boldsymbol{p}$, but when the particle becomes free, from the invariance of the mass m, the speed of light c and the equality

$\frac{1}{c^2} p_\mu p^\mu = m^2 u_\mu u^\mu = m^2 c^2 = E^2 - c^2 p^2$ should follow the well-known formula for the relationship between mass, energy and momentum for a particle in relativistic physics, valid in any inertial frame of reference. According to this formula, one can find the momentum of the particle at certain energy and rest mass of the particle, or determine the rest mass and the type of the particle by its momentum and energy.

By analogy with the 4-vector energy-momentum $p^\mu = (E, c\boldsymbol{p})$ from the components of the 4-vector H^μ (87) we obtain:

$$E = \left(mc - \frac{N}{c} \right) u^0 + mcg^{0\nu} D_\nu + qcg^{0\nu} A_\nu . \qquad p_x = \left(m - \frac{N}{c^2} \right) u^1 + mg^{1\nu} D_\nu + qg^{1\nu} A_\nu ,$$

$$p_y = \left(m - \frac{N}{c^2} \right) u^2 + mg^{2\nu} D_\nu + qg^{2\nu} A_\nu , \qquad p_z = \left(m - \frac{N}{c^2} \right) u^3 + mg^{3\nu} D_\nu + qg^{3\nu} A_\nu . \tag{87'}$$

For the case of substance without its direct interaction with another substance (other bodies), located only in its own gravitational and electromagnetic fields, energy E and momentum $\boldsymbol{p}$ of the substance unit at constant mass and charge can not change, and must be equal to some constant for the energy and constant vector for the momentum. This can be represented by the equation $H^\mu = const$, describing the conservation laws of energy and momentum of a closed system.

If in (85) we neglect the term with the curvature R and determine the constant equal to zero needed for the energy calibration, which arises due to the constant Λ, then in the weak field limit, at the transition to the special theory of relativity, for the energy and the momentum in (87') we obtain:

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} + m\psi + q\varphi + \int \left(\frac{1}{8\pi\gamma} \left(G^2 - c^2 \Omega^2 \right) - \frac{\varepsilon_0}{2} (E^2 - c^2 B^2) \right) dx^1 dx^2 dx^3 . \tag{88}$$

$$\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1 - v^2/c^2}} + m\boldsymbol{D} + q\boldsymbol{A} + \frac{\boldsymbol{v}}{c^2} \int \left(\frac{1}{8\pi\gamma} \left(G^2 - c^2 \Omega^2 \right) - \frac{\varepsilon_0}{2} (E^2 - c^2 B^2) \right) dx^1 dx^2 dx^3 .$$

From (88) it is seen that the term $\frac{mc^2}{\sqrt{1-v^2/c^2}}$ plays the role of kinetic energy, and other terms belong to the potential energy. In this case the potential energy includes not only the energy of the field strengths, but also the energy associated with the scalar field potentials.

From the substance unit we can proceed to a separate moving body, for which in case of straight-line motion with constant velocity in the absence of external fields, the relations $\mathbf{D}=\frac{\psi}{c^2}\mathbf{v}$, $\mathbf{A}=\frac{\varphi}{c^2}\mathbf{v}$ are valid. In this case for the momentum we have:

$$\mathbf{p}=\frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}}+\frac{m\psi}{c^2}\mathbf{v}+\frac{q\varphi}{c^2}\mathbf{v}+\frac{\mathbf{v}}{c^2}\int\left(\frac{1}{8\pi\gamma}\left(G^2-c^2\Omega^2\right)-\frac{\varepsilon_0}{2}(E^2-c^2B^2)\right)dx^1dx^2dx^3=\frac{E}{c^2}\mathbf{v}.$$

(89)

Here the gravitational scalar potential ψ and the electromagnetic scalar potential φ are understood as the averaged potentials inside the body, arising from its own fields. To find the rest mass of the body, taking into account the fields we should write the ratio $M=\frac{E}{c^2}$ with $\mathbf{v}=0$. We shall use (88) to determine the rest mass with the help of volume integral:

$$M=\frac{m}{\sqrt{1-v'^2/c^2}}+\frac{1}{c^2}m\psi+\frac{1}{c^2}q\varphi+\frac{1}{c^2}\int\left(\frac{1}{8\pi\gamma}\left(G^2-c^2\Omega^2\right)-\frac{\varepsilon_0}{2}(E^2-c^2B^2)\right)dV_0=$$
$$=\frac{1}{c^2}\int\left(\frac{\rho_0c^2}{\sqrt{1-v'^2/c^2}}+\rho_0\psi+\rho_{0q}\varphi+\frac{1}{8\pi\gamma}\left(G^2-c^2\Omega^2\right)-\frac{\varepsilon_0}{2}(E^2-c^2B^2)\right)dV_0.$$

(90)

The rest mass M of the body differs from the mass m of its substance due to the contribution from the field energies and energy of internal motion. If the body as a whole is at rest, but its substance is in some internal motion with speed v', it contributes to the overall mass due to the kinetic energy, as well as due to the emerging field of gravitational torsion Ω, and due to the magnetic field B. Determining the mass the terms with field strengths should be integrated over volume both inside and outside the body.

Now we shall use the relation (8) and apply it to (90) in case of stationary and not rotating solid body:

$$M_0 = \frac{1}{c^2}\int\left(\rho_0' c^2 + \frac{1}{8\pi\gamma}G^2 - \frac{\varepsilon_0}{2}E^2\right)dV_0 = m' + \frac{1}{c^2}\int\left(\frac{1}{8\pi\gamma}G^2 - \frac{\varepsilon_0}{2}E^2\right)dV_0 \,, \qquad (91)$$

where ρ_0' is constant mass density associated with the cosmological constant Λ. The density ρ_0' is obtained by excluding all the fields in the substance. For example, if the body is divided into pieces and spread to infinity with zero velocity, then the normal field of gravitation and the electromagnetic field will not be making large contribution to the density of the substance parts, and the total mass of these parts will be equal to m'.

According to (91), the mass of the whole body becomes greater than the total mass of its parts, due to the contribution of the gravitational energy with density $\frac{1}{8\pi\gamma}G^2$. Simultaneously, the electrical energy of the body reduces its mass. These findings are consistent with results obtained by another way in [3], [4], [9]. In the cosmic bodies the gravitational energy is generally higher than the electromagnetic energy, so as we move from small to large bodies the body mass should increase, as well due to the potential energy of gravitation.

We shall note that instead of using the 4-vector of Hamiltonian (87) to estimate the energy, momentum and mass, we can use another approach based on integration over volume of the timelike components of the stress-energy tensors of substance $\phi^{\alpha\beta}$ (83), the gravitational field $U^{\alpha\beta}$ (9), as well as the electromagnetic field $W^{\alpha\beta}$ (74). From the properties of the left side of the equation for the metric (81) it follows that the covariant derivative of the right side is equal to zero:

$$\nabla_\nu(\phi^{\mu\nu} + U^{\mu\nu} + W^{\mu\nu}) = \nabla_\nu \tau^{\mu\nu} = \partial_\nu \tau^{\mu\nu} + \Gamma^{\mu}_{\nu\alpha}\tau^{\alpha\nu} + \Gamma^{\beta}_{\beta\alpha}\tau^{\mu\alpha} = 0 \,. \qquad (92)$$

This equation is equivalent to the equation of motion of substance in the gravitational and electromagnetic fields (80), in which it is considered that $\nabla_\sigma\left(\frac{\rho_{0q}}{\rho_0}\right) = 0$.

Then we shall use the procedure, which was used in [5] and many other works on the theory of gravitation, to simplify the integration of (92) over 4-volume. If we introduce a frame of reference relative to which the substance unit at a given time is moving like it should move according to the special theory of relativity, in this reference frame the Christoffel symbols $\Gamma^{\mu}_{\nu\alpha}$ and $\Gamma^{\beta}_{\beta\alpha}$ in (92) are equal to zero. Then the covariant derivative ∇_ν of the tensor $\tau^{\mu\nu} = \phi^{\mu\nu} + U^{\mu\nu} + W^{\mu\nu}$ is equal to the ordinary derivative ∂_ν, which is the 4-divergence of the tensor $\tau^{\mu\nu}$ due to minimizing by the index ν. Instead of (92) we obtain the equality $\partial_\nu \tau^{\mu\nu} = 0$, the left part of which can be integrated over the 4-volume, taking into account the Gauss theorem, and in this case $\sqrt{-g} = 1$:

$$P^{\mu} = \int \partial_\nu \tau^{\mu\nu} c\, dt\, dx^1 dx^2 dx^3 = \int \tau^{\mu\nu} dS_\nu ,$$

where dS_ν is the element of an infinite hypersurface surrounding the 4-volume. The projection of this hypersurface at the hyperplane $x^0 = const$ gives a three-dimensional volume element $dS_0 = dx^1 dx^2 dx^3 = dV$, and for the 4-vector energy-momentum we can write down:

$$P^{\mu} = \int \tau^{\mu 0} dS_0 = \int \tau^{\mu 0} dV = \int (\phi^{\mu 0} + U^{\mu 0} + W^{\mu 0})\, dV . \qquad (93)$$

In contrast to (87), the expression (93) does not contain the energy of substance in its proper field, that is, the energy associated with scalar potentials ψ and φ. Despite this, for a stationary homogeneous ball in its proper gravitational field the mass-energies of this field according to (90) and (93) coincide. This follows from the next equation:

$$\frac{1}{c^2}\int \rho_0 \psi\, dV_b + \frac{1}{c^2}\int \left(\frac{1}{8\pi\gamma} G^2 \right) dV_0 = \frac{1}{c^2}\int U^{00}\, dV_0 = -\frac{1}{c^2}\int \left(\frac{1}{8\pi\gamma} G^2 \right) dV_0 , \qquad (94)$$

where dV_b is the differential of volume of the ball, dV_0 – the differential of volume of space inside and outside of the ball.

According to (94), the potential energy of the ball in its proper gravitational field associated with the scalar potential is two times greater than the potential energy associated with the field strengths. The same is true for the electromagnetic field, as in case of uniform arrangement of charges in the volume of the ball, and at their location only on the surface.

Equation (94) in its meaning resembles the virial theorem for a stationary system of particles bound by its proper gravitational field – in this system the absolute value of the total potential energy is approximately equal to double kinetic energy of all particles.

For the relativistic energy of substance from (88), and respectively, from (93) we also obtain the equality:

$$E_{sub} = \frac{mc^2}{\sqrt{1-v^2/c^2}}\,, \qquad E_{sub} = \int \phi^{00}\, dV = \int \frac{\rho_0 c^2}{1-v^2/c^2}\sqrt{1-v^2/c^2}\,dV_0 = \frac{mc^2}{\sqrt{1-v^2/c^2}}\,.$$

One aspect of the application (93) is the discrepancy between the mass-energy field of the moving bodies that are found either through the field strengths in the potential energy, or through the energy flux density and the momentum of the field (the so-called problem of 4/3). An attempt to solve this problem was made in [9] on the basis of the contribution of the field mass-energy into the total body mass. At the same time taking into account (94) we obtain the equality of the momentum in (89) and the total momentum of substance and field contained in (93) in the spatial components of 4-vector P^{μ}.

We now turn our attention to the mass ratio of the substance unit contained in (90) and (91), for the case when the contribution to the mass of the mass-energy of the electromagnetic field in comparison with the mass-energy of the gravitational field is small. Taking into account (94), then for the masses of rest substance the relation must be valid: $m' < M < m$, where the mass m is a part of the rest energy mc^2; the mass M determines the total mass of substance together with the field; the mass m', as it follows from (8), is the substance mass scattered to infinity, where all fields are set to zero. Which of these masses determine proper potentials and strengths of the gravitational field of the considered substance unit? In our opinion, the observed mass is the mass M, it must specify both the inert and the gravitational properties of the mass. This mass should be included in the formulas for the potential and field strength, and in the potential energy. Then for a homogeneous stationary ball we can write down:

$$m' = m + \frac{1}{c^2}\int \psi\, dM + \frac{1}{c^2}\int \varphi\, dq = m - \frac{6\gamma M^2}{5Rc^2} + \frac{2}{c^2}\int \left(\frac{\varepsilon_0}{2} E^2 \right) dV_0\,.$$

$$M = m' + \frac{1}{c^2}\int\left(\frac{1}{8\pi\gamma}G^2 - \frac{\varepsilon_0}{2}E^2\right)dV_0 = m - \frac{3\gamma M^2}{5Rc^2} + \frac{1}{c^2}\int \varphi dq - \frac{1}{c^2}\int\left(\frac{\varepsilon_0}{2}E^2\right)dV_0 =$$

$$= m - \frac{3\gamma M^2}{5Rc^2} + \frac{1}{c^2}\int\left(\frac{\varepsilon_0}{2}E^2\right)dV_0 .$$

Since the observed mass is M, then the mass m can be determined from the last equation, and then the mass m' can be calculated from the first equation. The mass density of substance ρ_0 through the 4-vector of mass current density $J^\mu = \rho_0 u^\mu$ is included in the Lagrangian (4) for a substance unit, and is also included in the Hamiltonian (35). In the integration over three-dimensional volume of the term $\frac{cdt}{ds}\rho_0 c\, g_{0\mu}\, u^\mu$ in (35), the mass m appears, and the integration over the volume of the term $\frac{cdt}{ds}\rho_0 \psi$ leads in the result of the integration to appearing of the mass M. The difference between the masses m and M is due to the fact that at the addition of substance units into a coherent body the 4-velocity u^μ is assumed constant, whereas the scalar potential ψ in itself is a function of mass (more precisely, at the constant density of the substance the potential ψ within the body depends on the characteristic size of the body, or the amount of mass). Changing of the potential ψ while the summation of the substance units into a single body in the course of integration over volume instead of m gives the mass M, which is used to calculate the energy of the field.

The stated above reveals the difference of forms of writing, and complementarity of Hamiltonian and Lagrangian approaches in finding the mass, energy and momentum of the moving substance.

Action as the function to determine the effect of time dilation

In view of (10) and (12), we shall write the differential of the action function for a substance unit with the mass m and the charge q:

$$dS = L\,dt = -mc\sqrt{g_{\mu\nu}\,dx^\mu dx^\nu} - mD_\mu\,dx^\mu - q A_\mu\,dx^\mu +$$
$$+ dt\int\left(k\,c(R-2\Lambda) + \frac{c^2\,\Phi_{\mu\nu}\Phi^{\mu\nu}}{16\pi\gamma} - \frac{F_{\mu\nu}F^{\mu\nu}}{4\mu_0}\right)\sqrt{-g}\;dx^1 dx^2 dx^3. \qquad (95)$$

From (95) it is seen that the action is a scalar quantity. In addition, the differential of the action can be decomposed by the differentials of the interval $ds = \sqrt{g_{\mu\nu}\, dx^{\mu} dx^{\nu}}$, the 4-vector of displacement dx^{μ}, and the coordinate (global) time dt, taken with the relevant factors.

Now we shall turn to the results obtained in [1]. It was shown there that the expression $d\eta = D_{\mu}\, dx^{\mu}$ contains a specific gauge function of gravitational field, which equals to $\eta = \int D_{\mu}\, dx^{\mu} = \int (\psi - \boldsymbol{D} \cdot \boldsymbol{v})\, dt$, provided $d\boldsymbol{r} = \boldsymbol{v}\, dt$. A similar specific calibration function for electromagnetic field is equal to $\vartheta = \int A_{\mu}\, dx^{\mu} = \int (\varphi - \mathbf{A} \cdot \boldsymbol{v})\, dt$. We shall remind that the fundamental field potentials are defined up to the coordinate and time derivatives from an arbitrary gauge function. If we replace the 4-potentials for the gravitational field as follows:

$$D'_{\mu} = D_{\mu} - F_{\mu}\,, \tag{96}$$

where we introduce the 4-vector $F_{\mu} = \partial_{\mu} \eta = \left(\frac{1}{c} \frac{\partial \eta}{\partial t}, \nabla \eta \right)$, then the strengths of the gravitational field and the equations of motion of substance in the field will not change. The same is true for the electromagnetic field and its specific gauge function ϑ. The gauge transformation (96) in the case where the specific gauge function is selected in the form $\eta = \int D_{\mu}\, dx^{\mu} = \int (\psi - \boldsymbol{D} \cdot \boldsymbol{v})\, dt$, actually clears the existing potentials of the gravitational field. Although it seems that the system has not changed, it is not so. In fact, it turns out that when comparing two systems, in one of which some gauge transformation is made by changing the potentials, there are different rates of time flow. For gravitational and electromagnetic fields the difference of a clock indications in weak field approximation is described by the following formulas:

$$\tau_1 - \tau_2 = \frac{m}{mc^2} \int_1^2 D_{\mu}\, dx^{\mu}\,, \qquad\qquad \tau_1 - \tau_2 = \frac{q}{mc^2} \int_1^2 A_{\mu}\, dx^{\mu}\,. \tag{97}$$

The clock 2, which measures the time τ_2, is check one and the clock 1 measures the time τ_1 and is under the influence of additional 4-field potentials D_{μ} or A_{μ}. Time points 1 and 2 within the integrals indicate the beginning and the end of the field action. If there is only a static gravitational field with zero vector potential, then $D_\mu \, dx^\mu = \psi \, d\tau_2$. Assuming then, that initially all the clocks had zero indication, for the difference in the clocks' indications from (97) we obtain the effect of gravitational time dilation: $\tau_1 - \tau_2 = \frac{1}{c^2} \int_1^2 \psi \, d\tau_2 = \frac{\psi}{c^2} \tau_2$.

From the time difference (97) we can move to the phase shift for the same type of processes in different points of the field. To do this, in (97) in the denominators it is necessary to replace mc^2 by the value of the characteristic angular momentum. In quantum mechanics this value is the Dirac constant $\hbar$ (this value is equal to Planck constant h, divided by 2π), which allows to take into account the appropriate phase shift which is inversely proportional to this constant:

$$\theta_1 - \theta_2 = \frac{m}{\hbar} \int_1^2 D_\mu \, dx^\mu , \qquad \theta_1 - \theta_2 = \frac{q}{\hbar} \int_1^2 A_\mu \, dx^\mu . \tag{98}$$

Phase shift in (98), obtained due to the electromagnetic 4-potential A_μ, is proved by the Aharonov-Bohm effect.

If we divide the first part in (95) by mc^2 and take the integral, we can obtain the standard time dilation effect due to the clock motion with the speed v:

$$\tau_1 - \tau_2 = \frac{mc}{mc^2} \int_0^t \sqrt{g_{\mu\nu} \, dx^\mu dx^\nu} - \tau_2 = \frac{1}{c} \int_0^t ds - \tau_2 = \int_0^t \sqrt{g_{00}} \sqrt{1 - v^2/c^2} \, dt - \int_0^t \sqrt{g_{00}} \, dt , \tag{99}$$

here the clock speed v is measured by the local observer at the point with the timelike component of the metric g_{00}; the moving clock measures time τ_1, and the fixed clock – the time $\tau_2 = \int_0^t \sqrt{g_{00}} \, dt$ of the local observer, expressed by the coordinate time t.

In (95), there is one more, the last term in the integral form, which in our opinion should also influence the effect of time dilation. Any gauge transformation of 4-potentials does not affect the values of field strengths, which are part of the tensors $\Phi_{\mu\nu}$ and $F_{\mu\nu}$. The energy of fields associated with the substance mass m, depends not only on the absolute value of the 4-potentials, but also on the rates of their changes in spacetime, that is, the field strengths.

Each additional energy must affect the intrinsic properties of substance, including the flow rate of proper time. From this we deduce:

$$\tau_1 - \tau_2 = -\frac{1}{mc^2} \int_0^t \left[\int \left(k\, c\, (R - 2\Lambda) + \frac{c^2\, \Phi_{\mu\nu}\, \Phi^{\mu\nu}}{16\pi\, \gamma} - \frac{F_{\mu\nu}\, F^{\mu\nu}}{4\mu_0} \right) \sqrt{-g}\; dx^1 dx^2 dx^3 \right] dt \, .$$

From the stated above it follows that the action is not only a function by which from the principle of least action the equations of motion are obtained, through the Legendre transformation the Hamiltonian, or the Hamilton-Jacobi equations are defined. The action function has also a direct physical meaning as the function describing the change in some intrinsic properties of physical bodies. These include the intrinsic properties of the rate of the time flow, and consequently the rate of increase of the phase angle of periodic processes depending on time. The special role of time in relation to spatial size as a characteristic property of physical bodies is due to the fact that the time shift during motion and in the fundamental fields is an absolute effect, whereas the change of the observed size is only relative.

Summary

Based on the principle of least action and Euler-Lagrange equations, we presented in (17) the relativistic equation of motion of a substance unit in fundamental fields (for motion along the axis OX of the Cartesian reference frame). This equation is written for the case when the velocity depends only on time, and can be specified for the general case by introducing into the equation the dependence of the velocity on the spatial coordinates. After determining the generalized momentum for the substance and the field we obtain vector equation (19), which expresses the dependence of the generalized force on different physical variables for the substance in the field.

Difference in positions of the covariant theory of gravitation (CTG) and the general theory of relativity (GTR) describing the motion of a small test particle in an external field is demonstrated in equations (21) and (23). In weak fields, the equation of CTG (21) exactly transforms into the Lorentz-covariant equation of motion (22) which is used in the special theory of relativity. In contrast, for appearance of the gravitational force in GTR, not only the weak-field approximation is required, but also preliminary calculation of the gradients of the metric tensor. This is due to the fact that in GTR the gravitational field potentials are related to the metric tensor components and are not independent quantities. We shall note

also that in contrast to CTG, in GTR there is no definite limit transition into special theory of relativity, that is, into the case of weak fields, based on the principle of conformity and conservation laws for such quantities as energy, momentum and angular momentum [10].

Hamiltonian, expressed through the 4-velocity and characterizing the energy of the particle (substance unit) with mass m, is given by (34). For continuously distributed substance Hamiltonian is determined by an integral over the 4-volume in relation (35). After simplification of these expressions we obtain formula (36) for the relativistic energy of the particle taking into account the energy of fields in the framework of the special theory of relativity. The expression for Hamiltonian through generalized momenta is given in (57), and relation (57’) sets the energy of the particle for flat Minkowski space.

In relation (66) we introduced into theory the 4-vector of generalized velocity s_μ and wrote with the help of it the Lagrangian (68). After applying the principle of least action to this Lagrangian we obtain equations for the gravitational and electromagnetic fields (76), the equation of substance motion (80), and the equation for the metric (81) and the relation for the cosmological constant (82). In addition, the timelike component of the 4-vector s_μ is directly included into Hamiltonian (84), and the product of the particle mass and the contravariant 4-vector s^μ sets the 4-vector of the generalized momentum in the form ms^μ. As a result, Hamiltonian (84) is the timelike component of a 4-vector with covariant index H_μ associated with the energy and momentum. We denoted it as a 4-vector of the Hamiltonian (4-energy), in contravariant form it is determined in (87) and according to (87’) it sets the energy and momentum of the particle through the mass, charge, 4-velocity, 4-potentials and field strengths. The alternative expression of energy and momentum of the particle through the energy-momentum tensors of substance and fields is given in (93) in the form of a 4-vector of energy-momentum.

The mass of the particle can be determined from (87’) by calculating the energy E in the limit of zero velocity and dividing this energy by the square of the light speed. In the weak-field approximation formula (90) holds for the mass of a body at rest. The relation for three masses associated with the body follows from (91) – (94): $m' < M < m$, where the mass m is part of the rest energy mc^2; the mass M determines the relativistic mass of the body substance with the proper fields as the measure of inertia and gravitational mass; the mass m' is the mass of the substance scattered at infinity, where all fields are zero. In this case, we see that the relation holds: $m - M \approx M - m'$.

From the analysis of the differential of action function (95) we find a change in the rate of time (97) in physical objects due to the action of potentials of the gravitational and electromagnetic fields. For quantum objects the corresponding phase shift is expressed by relations (98). The motion of clocks changes the rate of time according to (99). The action function also includes the field strengths (through tensors $\Phi_{\mu\nu}$ and $F_{\mu\nu}$), and we assume that they, as well as each component of the action function, should lead to some change in the rate of clock and the phase of processes. Perhaps this idea can be proved experimentally by measuring the quantum-mechanical phase shift under the influence of field strengths in those cases when the field strengths rather change the energy of the field than influence the motion of particles.

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