

Matter Waves in a Static Gravitational Field

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The new approach of the matter waves concept initiated by Louis de Broglie in 1924 leads to a new model – the vacuum matter waves. By using the physics of time dilation, one can calculate a gravitational cutoff frequency, with respect to the spectrum of vacuum fluctuations, which is the upper bound of the frequency range of this new model. We described the properties of these new waves and outlined the connection between them and the dimensions at Planck length level. The calculation of velocity in circular orbits by using this new concept complies with the well-known equation $V_{orb} = \sqrt{-\Phi}$, thus confirming this study. One of its consequences is that the structure of the physical vacuum is modified on the radial direction with the gravity source, but not modified on its tangential direction. This study places the space-time structure from subquantum level into a new light and makes the connection between this level and that of the planetary systems.

1. INTRODUCTION

In 1924, Louis de Broglie in his doctoral thesis *On the Theory of Quanta* [1], claimed the existence of matter waves and formulated the equation that carries his name $\lambda = h / p$. This hypothesis says that particles of matter have wave-like properties. The concept of matter waves was experimentally confirmed for the first time in 1927 at Bell Labs, by Clinton Davisson and Lester Germer, by using electrons [2]. Recent experiments confirmed the de Broglie equation even for large molecules and macromolecules [3], [4].

The current study is grounded in the basic ideas of de Broglie, but using a new approach. Thus we are led to a new concept – vacuum matter waves – having a deterministic role in the genesis and motion of matter. I shall analyze the influence of gravity on these waves and on physical vacuum.

2. VACUUM MATTER WAVES

In modern physics, the concept of time dilation is a well-documented physical phenomenon in both theory and experiments. This concept originates in the experiments for ether detection (luminiferous ether), which were initiated in 1887 by Albert Michelson and Edward Morley [5] and continued until recently with the use of lasers and high accuracy devices [6]. The Principle of Invariant Light Speed used by Einstein in his Special Relativity (SR), motivated by Maxwell's theory of electromagnetism and the lack of evidence for luminiferous ether has as consequences time dilation and length contraction. Velocity and gravitational time dilation have been experimentally confirmed a number of times [7], [8], [9], [10], [11], and [12].

As is well known in SR, if the clock has a frequency ω_0 (angular frequency) in the rest frame of the particle, its frequency, according to the so-called time dilation, when it is moving with velocity V_0 is:

$$\omega = \omega_0 \sqrt{1 - \frac{V_0^2}{c_0^2}} \quad (1)$$

where c_0 denotes the speed of light in a vacuum in the absence of a gravitational field. The above equation can be rewritten as follows:

$$V_0 = V_0(\omega) = c_0 \sqrt{1 - \frac{\omega^2}{\omega_0^2}} \quad (2)$$

De Broglie assumed that ω is not the frequency of a clock moving with the particle, but the frequency of a wave accompanying the particle, propagating with velocity V_p (phase velocity) in the direction of motion (suppose we simplify our 3-dimensional universe to a 1-dimensional universe where we also put in an x-axis).

Moreover, in this study I will consider ω as a frequency from the spectrum of vacuum fluctuations.

Also according to de Broglie's theory, velocity V_0 is the group velocity of a wave packet composed of plane waves of a narrow bandwidth. The relation between phase velocity V_p and group velocity V_0 is given in the Rayleigh equation:

$$V_0 = V_p + \omega \frac{dV_p}{d\omega} \quad (3)$$

From equations (2) and (3) results the following differential equation:

$$V_p + \omega \frac{dV_p}{d\omega} = c_0 \sqrt{1 - \frac{\omega^2}{\omega_0^2}} \quad (4)$$

The solution of this equation is:

$$V_p(\omega) = \frac{1}{2} c_0 \left(\sqrt{1 - \frac{\omega^2}{\omega_0^2}} + \frac{\omega_0}{\omega} \arcsin \frac{\omega}{\omega_0} \right) + \frac{k}{\omega} \quad (5)$$

For avoiding an infinite value, but also for satisfying equation $|V_p(\omega)| = |V_p(-\omega)|$, we shall consider the solution for which $k=0$. For inserting the influence of the gravitational field in the equation (5), we shall analyze the relativistic fall (Fig. 1) of a massive particle with mass m in a static gravitational field Φ generated by mass M ($m \ll M$).

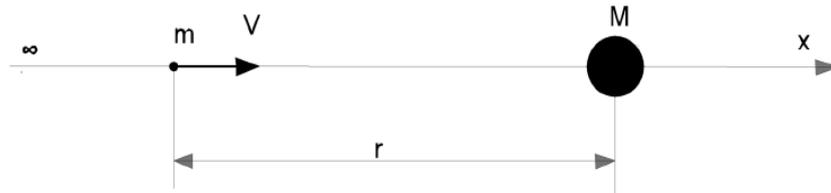


Fig. 1: Relativistic fall in a static gravitational potential.

This particle comes from an infinite distance ($\Phi = 0$) with initial velocity V_0 . Taking into account the gravitational potential $\Phi = -\frac{GM}{r}$, the relativistic velocity satisfies the following equation:

$$V = c_0 \sqrt{1 - \left(1 - \frac{V_0^2}{c_0^2}\right) \left(1 + \frac{\Phi}{c_0^2}\right)^2} \quad (6)$$

For the multibody potential, Φ represents the sum of the Newtonian gravitational potentials due to the masses in the neighborhood.

$$\Phi = \Phi_1 + \Phi_2 + \dots + \Phi_n = -\left(\frac{GM_1}{r_1} + \frac{GM_2}{r_2} + \dots + \frac{GM_n}{r_n}\right) \quad (7)$$

If considering equation (1), equation (6) is written as follows:

$$V = V(\omega, \Phi) = c_0 \sqrt{1 - \frac{\omega^2}{\omega_0^2} \left(1 + \frac{\Phi}{c_0^2}\right)^2} \quad (8)$$

The phase velocity can likewise be computed in the way we obtained equation (5):

$$V_p(\omega, \Phi) = \frac{1}{2} c_0 \left(\sqrt{1 - \frac{\omega^2}{\omega_0^2} \left(1 + \frac{\Phi}{c_0^2}\right)^2} + \frac{\omega_0}{\omega \left(1 + \frac{\Phi}{c_0^2}\right)} \arcsin \frac{\omega}{\omega_0} \left(1 + \frac{\Phi}{c_0^2}\right) \right) \quad (9)$$

This equation can be rewritten in a simplified form as follows:

$$V_p(\omega, \Phi) = \frac{1}{2} c_0 \left(\sqrt{1 - \frac{\omega^2}{\omega_G^2}} + \frac{\omega_G}{\omega} \arcsin \frac{\omega}{\omega_G} \right) \quad (10)$$

in which we made the following denotation:

$$\omega_G = \frac{\omega_0}{1 + \frac{\Phi}{c_0^2}} \quad (11)$$

where ω_G is the frequency from the spectrum of vacuum fluctuations, in which the group velocity is null.

As can be seen in Fig. 2, the range of frequencies $0 \dots \omega_G$ does cover the entire range that is necessary for generating the group velocities $V = 0 \dots c_0$.

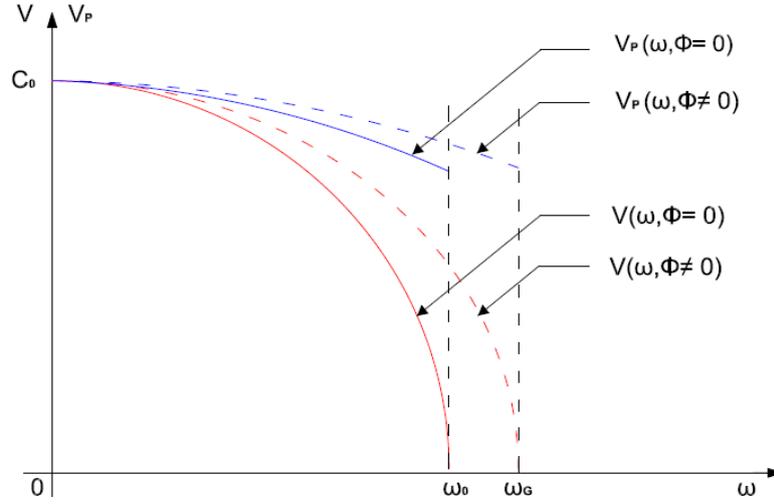


Fig. 2: Phase (V_p) and group (V) velocities versus frequency (ω) and gravitational potential (Φ).

The value ω_G can be interpreted as a gravitational cutoff frequency, such that the frequency spectrum of vacuum fluctuations that can participate in the structure of the massive particles is $0 \dots \omega_G$. Observe that the higher the intensity of the gravitational field, the wider this frequency spectrum. We can interpret this fact as under the influence of gravity, a part of the frequency spectrum that is present in vacuum fluctuations is extracted and used in the genesis and motion of the matter. In the absence of gravity, the cutoff frequency has an initial value ω_0 . Further, we call these waves having the phase velocity $V_p(\omega, \Phi)$ and frequencies in the range of $0 \dots \omega_G$ as *vacuum matter waves*.

3. PROPERTIES OF VACUUM MATTER WAVES

We can split the range of frequencies $0 \dots \omega_G$ by the value of the group velocity V relative to the escape velocity V_{esc} . Thus, for a frequency $\omega = \omega_0$, the group velocity is equal to the escape velocity. For the range of frequencies lower than ω_0 , the group velocity is higher than the escape velocity, while for the range of $\omega_0 \dots \omega_G$ the group velocity is lower than the escape velocity.

$$\left. \begin{aligned}
 V(\omega, \Phi) &= c_0 \sqrt{1 - \frac{\omega^2}{\omega_0^2} \left(1 + \frac{\Phi}{c_0^2}\right)^2} \\
 \omega &= [0 \dots \omega_0) \quad \rightarrow \quad V > V_{esc} \\
 \omega &= \omega_0 \quad \rightarrow \quad V = V_{esc} \\
 \omega &= (\omega_0 \dots \omega_G] \quad \rightarrow \quad V < V_{esc}
 \end{aligned} \right\} \quad (12)$$

For vacuum matter waves, the upper bound of the phase velocity is c_0 , and this holds for relatively low frequencies:

$$\lim_{\omega \rightarrow 0} V_p(\omega, \Phi) = c_0 \quad (13)$$

We can also observe that for a null group velocity, the phase velocity has a fixed value, regardless of the intensity of the gravitational field:

$$V_p(\omega_0, 0) = V_p(\omega_G, \Phi) = \frac{\pi}{4} c_0 \quad (14)$$

De Broglie - Einstein velocity equation (an equivalent equation) is:

$$V(\omega, \Phi) V_p(\omega, \Phi) = \frac{1}{2} c_0^2 \left(1 - \frac{\omega^2}{\omega_G^2} + \sqrt{1 - \frac{\omega^2}{\omega_G^2}} \frac{\omega_G}{\omega} \arcsin \frac{\omega}{\omega_G} \right) \quad (15)$$

and has the following property:

$$\lim_{\omega \rightarrow 0} V(\omega, \Phi) V_p(\omega, \Phi) = c_0^2 \quad (16)$$

In a static gravitational field the refractive index $n(\omega, \Phi)$ for vacuum matter waves is:

$$n(\omega, \Phi) = \frac{V_p(\omega, 0)}{V_p(\omega, \Phi)} = \frac{\sqrt{1 - \frac{\omega^2}{\omega_0^2} + \frac{\omega_0}{\omega} \arcsin \frac{\omega}{\omega_0}}}{\sqrt{1 - \frac{\omega^2}{\omega_G^2} + \frac{\omega_G}{\omega} \arcsin \frac{\omega}{\omega_G}}} \quad (17)$$

We can define the global time dilation TD as follows:

$$TD = \frac{\omega}{\omega_0} = \left[0, \dots, \frac{\omega_G}{\omega_0} \right] \quad (18)$$

We can also define the gravitational time dilation TD_G as the ratio between the frequencies for which the group velocities are null, as follows:

$$TD_G = \frac{\omega_G}{\omega_0} = \frac{1}{1 + \frac{\Phi}{c_0^2}} \geq 1 \quad (19)$$

Velocity time dilation TD_V can be deduced as follows:

$$TD_V = \frac{TD}{TD_G} = \frac{\omega}{\omega_G} = \frac{\omega}{\omega_0} \left(1 + \frac{\Phi}{c_0^2} \right) \leq 1 \quad (20)$$

In these definitions, ω_0 represents the clock frequency in the rest frame and in the absence of the gravitational field.

The graphical form of the phase velocity $V_p(\omega, \Phi)$ around the gravitational cutoff frequency (Fig. 2) suggests that there exists a minimal value of the wavelength λ_0 for which vacuum matter waves do exist. Thus, we can deduce a value for ω_0 from the following equation:

$$\lambda_0 = 2\pi \frac{V_p(\omega_0, 0)}{\omega_0} = \frac{\pi^2 c_0}{2 \omega_0} \quad (21)$$

$$\omega_0 = \frac{\pi^2 c_0}{2 \lambda_0} \quad (22)$$

Equations (11) and (14) yield the minimal value of the wavelength λ_G as a function of the gravitational potential:

$$\lambda_G = \lambda_0 \left(1 + \frac{\Phi}{c_0^2} \right) \quad (23)$$

From this equation, it follows that the gravitational potential "deforms" the structure of the physical vacuum, leading to a minimal wavelength λ_G . The vacuum matter waves do not exist below this value.

We could speculate that λ_0 has the Planck length, defined as:

$$l_P = \sqrt{\frac{\hbar G}{c_0^2}} = 1.616199(97) \cdot 10^{-35} \text{ m} \quad (24)$$

In this case, we can compute the initial value of the cutoff frequency by using equation (22):

$$\omega_0 = \frac{\pi^2 c_0}{2 l_P} = 2\pi \left(1.4568 \cdot 10^{43} \text{ Hz} \right) \quad (25)$$

A brief characterization of the vacuum matter waves in a 1-dimensional universe can be done through the following equations:

Table 1: Vacuum matter waves properties

1	Wave equation	$\Psi(x, t) = A(\omega) e^{-i \omega \left(t \pm \frac{x}{V_p} \right)}$ <ul style="list-style-type: none"> - $A(\omega)$, waves amplitude - t, time in the rest frame and in the absence of a gravitational field
2	Phase velocity	$V_p(\omega, \Phi) = \frac{1}{2} c_0 \left(\sqrt{1 - \frac{\omega^2}{\omega_G^2}} + \frac{\omega_G}{\omega} \arcsin \frac{\omega}{\omega_G} \right)$
3	Gravitational cutoff frequency	$\omega_G = \frac{\omega_0}{1 + \frac{\Phi}{c_0^2}}$
4	Refractive index	$n(\omega, \Phi) = \frac{V_p(\omega, 0)}{V_p(\omega, \Phi)}$
5	Frequency range	$\omega = [0 \dots \omega_G]$
6	Default cutoff frequency	$\omega_0 = \frac{\pi^2 c_0}{2 \lambda_0}$ <p>if $\lambda_0 = l_P \rightarrow \omega_0 = 2\pi \left(1.4568 \cdot 10^{43} \text{ Hz} \right)$</p>

4. ESCAPE VELOCITY

For the escape velocity V_{esc} , the wave packet is centered on frequency $\omega = \omega_0$. The value of this velocity can be computed through equation (8) as follows:

$$V_{esc} = V(\omega_0, \Phi) = c_0 \sqrt{1 - \left(1 + \frac{\Phi}{c_0^2}\right)^2} = \sqrt{-2\Phi - \frac{\Phi^2}{c_0^2}} \quad (26)$$

According to definitions (18, 19, and 20), we can compute the global time dilation on a free-fall trajectory

$$TD_{esc} = \frac{\omega_0}{\omega_0} = 1 \quad (27)$$

and the gravitational and velocity time dilation (TD_{Gesc} and TD_{Vesc} respectively):

$$TD_{Gesc} = \frac{1}{1 + \frac{\Phi}{c_0^2}} \geq 1 \quad (28)$$

$$TD_{Vesc} = \frac{TD_{esc}}{TD_{Gesc}} = 1 + \frac{\Phi}{c_0^2} \leq 1 \quad (29)$$

5. CIRCULAR ORBIT VELOCITY

Denote by ω_{orb} the central frequency of a wave packet moving on a circular orbit with velocity $V_{orb} = V(\omega_{orb}, \Phi)$. The wavelength of vacuum matter waves corresponding to this frequency is:

$$\lambda_{orb} = \frac{2\pi V_p(\omega_{orb}, \Phi)}{\omega_{orb}} \quad (30)$$

We impose the condition of stability of the circular orbit,

$$\lambda_{orb} = N\lambda_0, \quad N = 1, 2, \dots \quad (31)$$

which leads us to the following equation:

$$\frac{2\pi V_p(\omega_{orb}, \Phi)}{\omega_{orb}} = N \frac{\pi^2 c_0}{2 \omega_0} \quad (32)$$

This equation can be solved through approximating methods, for instance by using the program *Wolfram Mathematica* [13]. For $N = 1$ and $-\Phi \ll c_0^2$, one of these approximated solutions is:

$$\omega_{orb} \approx \omega_0 \left(1 - \frac{\Phi}{2c_0^2}\right) \quad (33)$$

We can compute the circular orbit velocity by using equation (8):

$$V_{orb} = V(\omega_{orb}, \Phi) = c_0 \sqrt{1 - \frac{\omega_{orb}^2}{\omega_0^2} \left(1 + \frac{\Phi}{c_0^2}\right)^2} = c_0 \sqrt{1 - \left(1 - \frac{\Phi}{2c_0^2}\right)^2 \left(1 + \frac{\Phi}{c_0^2}\right)^2} \approx \sqrt{-\Phi} \quad (34)$$

that is the well-known nonrelativistic value of this velocity. Obtaining the equation of the orbital velocity is of the utmost importance, because it confirms this study. The stability condition (31) has been considered as a necessity of a synchronism between the wavelength λ_{orb} corresponding to the velocity of the motion and the wavelength λ_0 characterizing the physical vacuum. A consequence of obeying this condition is the fact that the gravitational potential "deforms" the physical vacuum on the radial direction with the gravity source, but not on its tangential direction.

According to definitions (18, 19, and 20), we can compute the global time dilation TD_{orb} on a circular orbit:

$$TD_{orb} = \frac{\omega_{orb}}{\omega_0} \quad (35)$$

and the gravitational and velocity time dilation (TD_{Gorb} and TD_{Vorb} respectively):

$$TD_{Gorb} = \frac{1}{1 + \frac{\Phi}{c_0^2}} \geq 1 \quad (36)$$

$$TD_{Vorb} = \frac{TD_{orb}}{TD_{Gorb}} = \frac{\omega_{orb}}{\omega_0} \left(1 + \frac{\Phi}{c_0^2}\right) \quad (37)$$

6. PROGRAM OF FURTHER RESEARCH

In this section, we list directions for further research and sketch approaches to the stated problems.

6.1 Default gravitational potential, extending to a multidimensional space and connecting to other theories

The semblance between equations (5) and (10) suggests the fact that as gravitational cutoff frequency ω_G is generated by a gravitational potential Φ , so ω_0 could be generated by a default gravitational potential Φ_0 . Thus, the total gravitational potential would have the following equation:

$$\Phi_T = \Phi_0 + \Phi$$

In this case, equation (11) would be just an approximation of the cutoff frequency ω_G around the value ω_0 and we should find a function of the form:

$$\omega_G = \omega_G(\Phi_T)$$

The existence of this initial potential would make possible the calculation of the escape and circular orbit velocities and we should find their physical interpretation. Also, we should study the existence of some connections between Φ_0 and the concepts of *dark matter* and *dark energy*.

Meanwhile, it is necessary to extend the study to a 3- or more dimensional space, analyzing in the same time the possibility of including Lorentz symmetry. We should also find a function for the amplitude $A(\omega)$ (Table 1.1), as well as its physical interpretation.

Because this study deals with dimensions of the Planck-length order, the analysis of a possible relation to string theory becomes necessary.

6.2 Inertia

If we define the space $(x, y, z, \omega, \Phi_T)$, a particle located in a certain place into this space will have a well-defined velocity. In other words, the particle velocity, which is given by the group velocity $V(\omega, \Phi_T)$, is a feature of this space. From this point of view, a correlation with Mach's principle should be analyzed.

6.3 Compton frequency and de Broglie equation

The nature of the Compton frequency should be the same as that of the vacuum matter waves. Should we consider it as having a different nature, it would render it complicated. One of the possibilities is that Compton frequency ω_C be given by a narrow bandwidth $\Delta\omega$ of a wave packet, centered on frequency ω , with the following equation:

$$\omega_C = \frac{\Delta\omega}{2}$$

Of course, ω_C is the frequency that "shows" in the reference frame of the particle. If the particle moves with velocity $V = V(\omega, \Phi)$, then this frequency changes according to the value:

$$\frac{\omega_C}{\sqrt{1 - \frac{V^2}{c^2}}}$$

The associated wavelength is given by the de Broglie equation $\lambda = h / p$. This way of interpreting the Compton frequency would also justify the equation of energy:

$$E_0 = h \omega_C = h \frac{\Delta\omega}{2}$$

given that the energy of a harmonic oscillator is proportional to the square of its frequency.

6.4 Improved software algorithm

Developing an improved algorithm of computing the velocity on circular orbits according to equation (32) for the various values of N is necessary. This algorithm could be based on several parts of this study and might become an important tool for future developments.

6.5 Temperature time dilation

We should analyze how temperature can be involved in this study. By equalling the kinetic energy that generates the velocity time dilation with the energy given by the increase in temperature, a new concept – the temperature time dilation – could be introduced.

7. CONCLUSION

The new approach of the matter waves concept initiated by Louis de Broglie in 1924 leads to a new model – the vacuum matter waves. By using the physics of time dilation, one can calculate a gravitational cutoff frequency, with respect to the spectrum of vacuum fluctuations, which is the upper bound of the frequency range of this new model. We described the properties of these new waves and outlined the connection between them and the dimensions at Planck length level. The calculation of velocity in circular orbits by using this new concept complies with the well-known equation $V_{orb} = \sqrt{-\Phi}$, thus confirming this study. One of its consequences is that the structure of the physical vacuum is modified on the radial direction with the gravity source, but not modified on its tangential direction. This study places the space-time structure from subquantum level into a new light and makes the connection between this level and that of the planetary systems.

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