# Large-scale CP violation from cosmic acceleration: Completing the analogy between gauge and reference frame transformations and its physical consequences

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**Abstract:** This discussion demonstrates a theorem that, if some hypothetical metric $g_{\alpha\beta}$ for either field-space or spacetime exists which couples to spin-$\frac{1}{2}$ field/particles, one can define a class of four-indexed spin-$\frac{1}{2}$ fields $\varphi^{\alpha}(x)$ with which the standard model $SU(3)\times SU(2)\times U(1)$ gauge group is automatically associated due to topological and geometric considerations, regardless of the nature of the field equation by which any specific field $\varphi^{\alpha}(x)$ is defined. Specifically, for this class of fields $\varphi^{\alpha}(x)$, which reduces to a physically equivalent unindexed field $\varphi(x)$ in flat space where the metric $g_{\alpha\beta}$ reduces to the Minkowski metric $\eta_{\alpha\beta}$, gauge transformations become exactly identified with covariant transformations under general reference frame transformations. This identification is used to construct a novel source of CP violation which may help to explain the degree to which the symmetry between matter and antimatter observed in the universe is broken. © *2009 Physics Essays Publication*. [DOI: 10.4006/1.3050302]

**Résumé:** Cette communication démontre un théorème qui, si l'on suppose dans un espace-champ ou dans un espace-temps l'existence d'une métrique $g_{\alpha\beta}$ qui se couple à des champs ou des particules de spin-$\frac{1}{2}$, alors on peut définir une classe de champs $\varphi^{\alpha}(x)$ de spin-$\frac{1}{2}$ à quatre indices avec lesquels le groupe de jauge $SU(3)\times SU(2)\times U(1)$ du modèle standard est automatiquement associé par des considérations topologiques et géométriques, indépendamment la nature de l'équation du champ par laquelle tout champ spécifique $\varphi^{\alpha}(x)$ est défini. Plus spécifiquement, pour cette classe de champs $\varphi^{\alpha}(x)$, qui se réduit à un champ physiquement équivalent non indicé $\varphi(x)$ dans un espace plat où la métrique $g_{\alpha\beta}$ se réduit à la métrique $\eta_{\alpha\beta}$ de Minkowski, les transformations de jauge s'identifient exactement aux transformations covariantes dans les transformations générales du cadre de référence. Cette identification est utilisée pour construire une nouvelle source de violation CP qui peut aider à expliquer le degré avec lequel la symétrie observée dans l'univers entre la matière et l'antimatière est brisée.

Key words: CP-violation; Yang–Mills Theory; Standard Model in Curved Spacetime; Gauge Transformations; Covariant Transformations; Reference Frame Transformations; Dirac Equation.

## I. INTRODUCTION

One of the outstanding problems in quantum cosmology arises from the broken symmetry between field/particles and antifield/particles, i.e., the abundance of ordinary matter as opposed to the near absence of antimatter.[1] Normally, in a vacuum one excites particle events in the form of pair production,[2] leading to equal numbers of field/particles and anti-field/particles in what is termed CP symmetry.[1,2] At some stage of the early universe, this symmetry was broken, and radically so;[1] all known classical objects in the universe are constructed from field/particles, not from antifield/particles. This symmetry-breaking process has been described as a phase transition[3] or a "see-saw mechanism."[4] The exact details of this process remain mysterious in that all the known particle-producing processes which violate CP symmetry taken together can only account for a small percentage of the observable mass in the universe, given the age of the universe.[5] This discussion presents a previously unrecognized physical mechanism, by which cosmic expansion,[6] i.e., vacuum expansion, produces field/particles but not antifield/particles in violation of CP symmetry, after developing the underlying formalism.

That formalism introduces a class of four-indexed fields $\varphi^{\alpha}(x)$, a spinor[7] field with one timelike and three spacelike components, and demonstrates that the $SU(3)\times SU(2)\times U(1)$ gauge group structure of the standard model[8] arises from geometric and topological restrictions on this class of fields regardless of the specific field equation of which the field $\varphi^{\alpha}(x)$ is a solution. One advantage of this class of fields $\varphi^{\alpha}(x)$ then lies in the fact that, simply by writing any given field equation in terms of fields $\varphi^{\alpha}(x)$, a certain degree of physicality is assured because of the field $\varphi^{\alpha}(x)$'s automatic association with the $SU(3)\times SU(2)\times U(1)$ gauge group. If one considers for example a $\lambda\varphi^4$ field theory[2,9] where

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$$[\partial_\alpha \varphi(x)][\partial^\alpha \varphi(x)] - \frac{\lambda}{4}\varphi(x)^4 = 0, \tag{1}$$

the fields $\varphi(x)$ may or may not have an associated $SU(3) \times SU(2) \times U(1)$ gauge group structure equivalent to that of the standard model. One must establish that this sort of gauge invariance applies in order to establish that degree of physicality to the given $\lambda\varphi^4$ field theory. However, if one uses fields $\varphi^\alpha(x)$ so that the $SU(3)\times SU(2)\times U(1)$ gauge group structure equivalent to that of the standard model follows automatically, then that level of physicality is assured because the Lagrangian (density) which takes the form

$$L[\varphi^\alpha(x), \partial_\beta \varphi^\alpha(x)] = [\partial_\beta \varphi^\alpha(x)][\partial^\beta \varphi_\alpha(x)] - \frac{1}{2}m^2 \varphi_\alpha(x)\varphi^\alpha(x) - \frac{\lambda}{4}[\varphi_\alpha(x)\varphi^\alpha(x)]^2 \tag{2}$$

is written in terms of fields $\varphi^\alpha(x)$.

At the same time, though, the manner in which the $SU(3)\times SU(2)\times U(1)$ gauge group structure arises also establishes that the analogy between gauge transformations and covariant transformations under general reference frame transformations is exact. The similarity has been noted before, but the usual contention is that the analogy breaks down due to the lack of an underlying field.[10] This discussion demonstrates that an underlying field does exist and is the same in both classes of transformations. Thus, since one demonstrates that the underlying field is the same for both classes of transformations and that the analogy between the two classes of transformations—namely, gauge transformations and covariant transformations under general reference frame transformations—is exact, these two classes of transformations must be equivalent physically. This is not just a mathematical nicety, but has direct physical consequences. Once the physical equivalence between gauge transformations and covariant transformations—under general reference frame transformations—has been established, one can to some degree interchange covariant and gauge transformations. They then constitute two manifestations of the same thing. One demonstrates this usage of a covariant transformation in lieu of a gauge transformation with the Dirac equation[2,8]—as modified to accommodate fields $\varphi^\alpha(x)$—in order to develop a previously unrecognized mechanism for CP violation. Namely, from the viewpoint of a comoving observer, meaning an observer at rest with respect to local expansion of the vacuum,[8,11] a point expanding away from the observer is boosted by the process of expansion; this nonconstant boost frees (potential) energy from the vacuum which then produces field/particle excitations. However, in violation of CP symmetry, according to which field/particles and antifield/particles interact similarly, only field/particles are excited in this process.

Discussion begins with a rigorous definition of four-indexed fields. Next comes a note on the nature of a general but unspecified spacetime and/or field-space metric[2] $g_{\alpha\beta}$, the existence of which is assumed throughout. The exact physical nature of this hypothetical metric $g_{\alpha\beta}$ in the present discussion remains in general unspecified except that four-indexed fields $\varphi^\alpha(x)$ couple to it in the sense that the coordinate index $\alpha$ on a field $\varphi^\alpha(x)$ can be lowered and then again raised by means of that metric field. The fundamental notion of the discussion is to demonstrate a theorem that, if one can define a metric $g_{\alpha\beta}$ which couples with four-indexed fields $\varphi^\alpha(x)$—however, this may be done,— then association of the familiar $SU(3)\times SU(2)\times U(1)$ standard model gauge group with four-indexed field $\varphi^\alpha(x)$ follows automatically, regardless of the governing field equation. After these preliminaries, the treatment of gauge symmetries and transformations[8,13] begins, starting with a general treatment of gauge symmetries in the absence of a specific field equation. A gauge condition is constructed from conservation of probability, which is then shown to be equivalent to the usual discussion based on the Lagrangian[2] which in turn is based on path integral formalism.[14] This lays the groundwork for addressing specific gauge symmetries. First, the $U(1)$ gauge symmetry[8] is constructed from the physical arbitrariness of the placement of the origin; this symmetry is related to a global (constant) reference frame transformation. Second, the $SU(2)$ gauge symmetry[8] is constructed from analytic (or holomorphic) conditions[15] on a four-index spinor field $\varphi^\alpha(x)$ in either spacetime or a metricized field space. This leads to incidental treatment of massless and massive fields, symmetry breaking and field handedness; these issues[8] are suggestive concerning the nature of leptons and quarks or hadrons. The usual covariant derivative[8] is constructed and shown to be a true covariant derivative. Only fields which are massive even without symmetry-breaking effects[8] (aside from the action of a Higgs[8,16] field) are subject to $SU(3)$ gauge symmetry which is constructed in connection with velocity-related degrees of freedom. Again, the usual covariant derivative[8] is constructed and shown to be a true covariant derivative.[12,1]

This formalism is then applied in order to explain a previously unrecognized mechanism by which particles may be produced in the process of vacuum expansion without at the same time producing antiparticles, thus breaking CP symmetry. Specifically, one first modifies the Dirac equation accordingly and interprets this field equation in terms of a simple harmonic oscillator (SHO).[17] One then uses the $SU(3) \times SU(2)\times U(1)$ gauge group symmetry properties to construct an external potential $V^\alpha_\beta(x)$ related to reference frame transformation. The resulting field equation, which corresponds to a driven oscillator, is applied to an expanding vacuum; expansion drives the harmonic oscillator exciting field/particles in the process without exciting antifield/particles.

## II. CONVENTIONS

Throughout this discussion, one uses natural units in which $\hbar = c = 1$. The summation convention used assumes repeated indices summed upon unless otherwise stated. Greek

[1]Although a classical metric does not simply carry over in general to a quantum field theoretical context, such a classical metric does physically serve as the classical limit of any hypothetical metric which is definable in a quantum field theoretical context. Definition of the usual classical metric-space, including the metric and covariant derivatives, associated with gravitation can be found in Ref. 12.

indices range from 0 to 3. Roman indices range from 1 to 3 when capitalized. The spacetime position $x^\alpha$ is however most often written as $x$, in which the index has simply been suppressed. States, in general bras $\langle\psi|$ and kets $|\psi\rangle$, are constants. Finally, the spacetime signature throughout this discussion is taken as $(+,-,-,-)$.

## III. GAUGE AND COVARIANT TRANSFORMATIONS FOR FOUR-INDEXED FIELDS REF. 18

### A. Nature of four-indexed fields

A four-indexed spinor field $\varphi^\alpha(x)$, which should not be confused with Dirac spinor notation,[8] is defined to have four components, one timelike component $\varphi^0(x)$ and three spacelike components $\varphi^1(x)$, $\varphi^2(x)$, and $\varphi^3(x)$. Each component is itself a spinor in the same sense that each component of a vector is itself a one component vector, not a scalar. The index $\alpha$ is thus a coordinate index. In principle, the timelike field component $\varphi^0(x)$ lies along a differential timelike coordinate axis $dx^0$, the spacelike field component $\varphi^1(x)$ similarly lies along a differential coordinate axis $dx^1$ and so forth. The existence of such differentials is implicit in the existence of a metric field.[12] Naturally, restrictions of simultaneous measurability[8,17] come into play. The result is that although the field $\varphi^\alpha(x)$ is well-defined, one cannot in principle treat its components entirely separately. A field $\varphi^\alpha(x)$ with a coordinate index $\alpha$ cannot be resolved into four independent fields.

A four-indexed field $\varphi^\alpha(x)$ is in general defined by the specific associated field equation. Nevertheless, if one assumes that any given field equation written in terms of a four-indexed field $\varphi^\alpha(x)$ has an analogous, i.e., physically equivalent (within certain restrictions developed immediately below), field equation written in terms of a conventional (unindexed) field $\varphi(x)$, any four-indexed (spin-$\frac{1}{2}$) field $\varphi^\alpha(x)$ can be constructed from a physically equivalent field $\varphi(x)$, the solution of some general field equation, in the following manner. Just as the field $\varphi(x)$ can be written as a linear combination of free fields $\varphi_n(x)$ of the form[2]

$$\varphi(x) = a_n \varphi_n(x) \equiv a_n \exp(-ik_{n\mu}x^\mu), \tag{3}$$

where wave vector $k_{n\mu}$, where integer $n \in (-\infty,\infty)$, represents the four-momentum of the $n$th field component in Hilbert space. (See the discussion of vectors below, according to which the second index of vector linear four-momentum is dropped to construct a spinor "wave vector." This means the vector is diagonalized and mapped onto a spinor.) The physically equivalent field $\varphi^\alpha(x)$ can be written as a similar linear combination of the form

$$[\varphi^\alpha(x)] = a^\alpha_{n\beta}[\varphi^\beta_n(x)] \equiv \frac{a^\alpha_{n\beta}}{2}\begin{bmatrix} \exp(ik_{n0}x^0) \\ \exp(-ik_{n1}x^1) \\ \exp(-ik_{n2}x^2) \\ \exp(-ik_{n3}x^3) \end{bmatrix}^\beta . \tag{4}$$

(The factor $\frac{1}{2}$ is a normalization.) The field $\varphi(x)$ in a sense represents[8] the probability (amplitude) that the particle associated with that field will occur at the spacetime location $x$. So does the field $\varphi^\alpha(x)$; this is what is meant by saying that the two fields $\varphi(x)$ and $\varphi^\alpha(x)$ are physically equivalent. In terms of probability (amplitude), the field representation $\varphi(x)$ takes the form of the multiplicative total probability (amplitude) of four events which must simultaneously occur in order to produce a physically observable particle, whereas the field $\varphi^\alpha(x)$ represents the total probability (amplitude) of the same four simultaneous events in terms of a superposition. Representation of the total probability (amplitude) of simultaneous events as a product or a superposition remains an arbitrary choice based upon convenience when applied to any given physical situation. Definition of the proper frame of reference for any given field $\varphi^\alpha(x)$ follows immediately from the definition. This is the frame of reference in which the three spacelike field components vanish and the field becomes entirely a function of the proper time $\tau$. The field then reduces to the form

$$\varphi^\alpha_{\text{proper}}(x) \equiv a^\alpha_n \exp(-ik_{0n\text{proper}}\tau) \equiv \begin{cases} \varphi_{\text{proper}}(\tau), & \alpha = 0 \\ 0, & \alpha \neq 0. \end{cases} \tag{5}$$

The timelike field component $\varphi^0_{\text{proper}}(x)$, which again is itself a spinor, becomes in the proper reference frame the field solution $\varphi_{\text{proper}}(\tau)$, i.e., the solution in the proper reference frame of the general field equation which does not relate to four-indexed fields $\varphi^\alpha(x)$ but rather to fields $\varphi(x)$. The physically equivalent equation for unindexed field $\varphi(x)$ should then be viewed as a special case of the field equation for field $\varphi^\alpha(x)$, namely the case where one considers a rest frame—meaning a reference frame physically equivalent to the proper frame of reference—so that the metric $g_{\alpha\beta}$ becomes the Minkowski metric $\eta_{\alpha\beta}$.

The spinor nature of indices does not present a problem of definition in general. One may always choose coordinates so that a classical spacetime position vector

$$[x^A] \equiv \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} \tag{6}$$

becomes replaced by a vector

$$[x^\alpha_\beta] \equiv \begin{bmatrix} x^0 & 0 & 0 & 0 \\ 0 & x^1 & 0 & 0 \\ 0 & 0 & x^2 & 0 \\ 0 & 0 & 0 & x^3 \end{bmatrix}. \tag{7}$$

The second index on vectors would seem artificial, except that it both lends itself to cases such as the quantum Hall

[2]This is fundamentally a Fourier series representation and physically describes a superposition of free fields.

effect[19] in which linear four-momentum becomes directionally dependent and provides the correct transformation properties. Even the physical utility of the latter taken alone should not be underestimated. The distinction between a vector such as a vector current density

$$[j^{\alpha}_{\beta}] \equiv \begin{bmatrix} j^0 & 0 & 0 & 0 \\ 0 & j^1 & 0 & 0 \\ 0 & 0 & j^2 & 0 \\ 0 & 0 & 0 & j^3 \end{bmatrix} \quad (8)$$

and an axial vector[8] current density such as

$$[\tilde{j}^{\alpha}_{\beta}] \equiv \begin{bmatrix} \tilde{j}^0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{j}^1 & 0 \\ 0 & 0 & 0 & \tilde{j}^2 \\ 0 & \tilde{j}^3 & 0 & 0 \end{bmatrix} \quad (9)$$

becomes transparent. Some arbitrariness exists in these definitions but this does not pose a difficulty so long as definitions remain consistent.

From a practical stand-point therefore, in order to construct basis fields $\varphi^{\beta}_{n}(x)$ as described above, one uses the $n$th linear four-momentum basis, i.e., wave vector $k^{\alpha}_{n\beta}$, and the spacetime position vector $x^{\beta}_{\gamma}$ to which the field space is tangent, eliminating the index $\gamma$ after applying the exponential operator. Summation with an appropriate constant $C^{\gamma}$ on index $\gamma$ accomplishes this latter as

$$\varphi^{\beta}_{n}(x) \equiv \exp\{ik^{\alpha}_{n\beta}x^{\beta}_{\gamma}\}C^{\gamma}, \quad (10)$$

where the exponential is defined by its Taylor series representation and where the first term in the series representation of basis field $\varphi^{\beta}_{n}(x)$ is defined as

$$[k^{\alpha}_{n\beta}x^{\beta}_{\gamma}C^{\gamma}] = \begin{bmatrix} k^0_n & 0 & 0 & 0 \\ 0 & -k^1_n & 0 & 0 \\ 0 & 0 & -k^2_n & 0 \\ 0 & 0 & 0 & -k^3_n \end{bmatrix} \begin{bmatrix} x^0 & 0 & 0 & 0 \\ 0 & x^1 & 0 & 0 \\ 0 & 0 & x^2 & 0 \\ 0 & 0 & 0 & x^3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \quad (11)$$

Other terms in the summation are defined accordingly. From these basis fields $\varphi^{\beta}_{n}(x)$, one constructs fields $\varphi^{\alpha}(x)$ as indicated above (4).

## B. Note on the metric

As stated in the introduction, the present discussion presumes the existence of some general but hypothetical metric $g_{\alpha\beta}$, the nature of which remains unspecified. The only assumptions thus made are that the form of the metric $g_{\alpha\beta}$ may in any given reference frame vary from that of the Minkowski metric $\eta_{\alpha\beta}$ where

$$[\eta_{\alpha\beta}] \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and that this metric $g_{\alpha\beta}$ couples to a four-indexed field $\varphi^{\alpha}(x)$ in the sense that the metric acts as a raising and lowering operator[12]

$$\varphi^{\alpha}(x) = g^{\alpha}_{\beta}\varphi^{\beta}(x) = g^{\alpha\beta}\varphi_{\beta}(x) = g^{\alpha\beta}g_{\beta\gamma}\varphi^{\gamma}(x). \quad (12)$$

The physical meaning and definition of such a metric $g_{\alpha\beta}$ constitutes an issue which would require a full discussion in and of itself.[20,3] For the present purposes, one need only imagine either the existence of some hypothetical field-space $g_{\alpha\beta}$ or a spacetime metric so defined that the metric $g_{\alpha\beta}$ may contract both four-indexed spinors and four vectors. The latter apparently simpler possibility would use the fact that any given field space is tangent to spacetime at some position $x$ and would use the general metric $g_{\alpha\beta}$ associated with that spacetime position. Nevertheless, however, the metric $g_{\alpha\beta}$ may be defined, the current discussion assumes primarily that such a general metric $g_{\alpha\beta}$ exists. Given its existence, the metric in lowered form $g_{\alpha\beta}$ or in raised form $g^{\alpha\beta}$ and mixed form $g^{\alpha}_{\beta}$ then acts as a raising and lowering operator and as a contraction operator

$$\langle\varphi_{\alpha}(x)|\varphi'^{\alpha}(x)\rangle \equiv \langle 0|\varphi^{\dagger}_{\alpha}(x)\varphi'^{\alpha}(x)|0\rangle \equiv \langle 0|g_{\alpha\beta}\varphi^{\dagger\alpha}(x)\varphi'^{\beta}(x)|0\rangle. \quad (13)$$

## C. Symmetry of fields $\varphi^{\alpha}(x)$ and conservation of probability

If one allows the metric $g_{\alpha\beta}$ to be fully general, then in order to transform fields $\varphi^{\alpha}(x)$ from one reference frame representation to another, one must expand upon the usual homogeneous Lorentz group to the full Poincaré group[12] where one still defines transformations of the form

$$\varphi'^{\alpha}(x) = \Lambda^{\alpha}_{\beta}(x)\varphi^{\beta}(x), \quad (14)$$

but the transformation operator $\Lambda^{\alpha}_{\beta}(x)$ now may include local, i.e., position dependent, transformations due to the fact that the metric $g_{\alpha\beta}$ in principle also depends on position. The field-gradient therefore becomes

$$\partial_{\mu}\varphi'^{\alpha}(x) = [\partial_{\mu}\Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x) + \Lambda^{\alpha}_{\beta}(x)\partial_{\mu}\varphi^{\beta}(x). \quad (15)$$

No field equation has been specified and so one may not follow the usual procedure of direct substitution into the field equation in order to establish gauge invariance.[8] However, one may instead invoke conservation of probability. The probability $P(\varphi'^{\alpha}(x)|\Omega)$ of observation of a field event $\varphi'^{\alpha}(x)$ in a given region $\Omega$ takes a form

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[3]The notion behind the use of a metric in association with quantum fields would most likely be a geometrized form of the standard model. Although the nature of such a metric formulation of the standard model lies beyond the purview of the present discussion, a geometric view of the standard model is not a new idea. See Ref. 20.

$$P[\varphi'^{\alpha}(x)|\Omega] \equiv \langle 0| \int_{\Omega} d^3x \varphi'^{\dagger}_{\alpha}(x) a \varphi'^{\alpha}(x)|0\rangle, \tag{16}$$

where $a$ is a constant operator (such as Dirac's $\gamma^0$ for example). Since the four-gradient $\partial_\mu$ is a Hermitian operator, conservation of probability demands

$$\partial_\mu[\varphi'^{\dagger}_{\alpha}(x)\varphi'^{\alpha}(x)] = 0. \tag{17}$$

One can always choose a reference frame where[4]

$$\partial_\mu \varphi'^{\alpha}(x) = 0. \tag{18}$$

This demands, however,

$$[\partial_\mu \Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x) + \Lambda^{\alpha}_{\beta}(x)\partial_\mu \varphi^{\beta}(x) = 0, \tag{19}$$

as well. Expression (19) acts as a gauge condition. This gauge condition (19) is identically that condition associated with gauge transformations constructed with respect to a Lagrangian (density) $\mathcal{L}$, which[8] is sufficient to demonstrate that the transformations described below constitute gauge transformations. Admittedly, in principle this condition may or may not leave equations of motion invariant, depending on one's choice of a field equation and hence a Lagrangian $\mathcal{L}$, but Lagrangians for which the equations of motion are not invariant under this type of gauge transformation are nonphysical.

Conservation of probability constitutes the more fundamental consideration. For the present purposes, one need not go into great depth and detail of the technicalities, but a short description will help to address any doubts that the transformations to be discussed do indeed constitute gauge transformations. The usual form of the symmetry demanded[8] is

$$\mathcal{L}'[\varphi'^{\alpha}(x)] = \mathcal{L}[\varphi^{\alpha}(x)] - \varepsilon \partial_\mu J^\mu, \tag{20}$$

where the parameter $\varepsilon$ is some constant and $J^\mu$ is defined as some conserved Noether current. Yet, this is a condition more strict than necessitated by a demand that the equation of motion

$$\partial_\mu \frac{\partial \mathcal{L}[\varphi^{\alpha}(x)]}{\partial[\partial_\mu \varphi^{\alpha}(x)]} - \frac{\partial \mathcal{L}[\varphi^{\alpha}(x)]}{\partial \varphi^{\alpha}(x)} = 0 \tag{21}$$

remain invariant. For example, a Lagrangian scaled by some constant $b$ such as

$$\mathcal{L}'[\varphi'^{\alpha}(x)] = b\mathcal{L}[\varphi^{\alpha}(x)] - \varepsilon \partial_\mu J^\mu \tag{22}$$

would also leave the equations of motion invariant, since the scale factor would cancel. Yet, such a transformation is indeed physically precluded as a valid symmetry. The reason is usually stated as that scaling of the Lagrangian leads to scaling of the action[8]

$$S[\varphi^{\alpha}(x)] \equiv \int \mathcal{L}[\varphi^{\alpha}(x)] d^4x, \tag{23}$$

by definition. From a physical point of view, one may ask why this scaling of the action is a problem. The answer lies in the connection between the action and probability amplitude, as most clearly shown in the construction of path integrals.[14] In the standard formulation, propagation of a field/particle with Hamiltonian $H$ from a position $x_a$ to a position $x_b$ in time $t$ is described by a propagation amplitude

$$\langle x_b|\exp(-iHt)|x_a\rangle = \int Dx(t)\exp\{iS[\varphi^{\alpha}(x)]\}, \tag{24}$$

where the specific nature of the Feynman propagator $Dx(t)$, other than to notice it involves only repeated integration, does not matter for the present purposes. The Feynman propagator $Dx(t)$ can be ignored. What does matter is the implicit but clear relationship between probability amplitude and the action $S[\varphi^{\alpha}(x)]$ in this quite general expression. Conversely, if probability is conserved, the action must be invariant due to the above expression. If the action $S[\varphi^{\alpha}(x)]$ is invariant, then the remainder of the usual construction of gauge invariance must follow. So, conservation of probability does indeed form a foundation on which to construct valid gauge symmetries; therefore, the transformations to be described are actual gauge symmetries.

Nonetheless, they are constructed from reference frame transformation[12] of fields. Under this topic comes covariant derivatives

$$D_\mu \varphi^{\alpha}(x) = \partial_\mu \varphi^{\beta}(x) - [\partial_\mu \Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x), \tag{25}$$

such as those familiar from the usual discussion of gauge symmetries. One difference exists however. Usually, one speaks of "so-called" covariant derivatives which are not in the strict sense of the term regarded as actual covariant derivatives.[8] Mathematically, a covariant derivative is defined as a generalized derivative $\partial_\mu \rightarrow D_\mu$ which keeps a locally constant field, such as the field $\varphi^{\alpha}(x)$ where $\partial_\mu \varphi^{\alpha}(x) = 0$, constant with respect to the defined covariant derivative $D_\mu$ regardless of position $x$ at which one takes the derivative.[12] Classically, covariant derivatives are by convention associated only with gravitational fields. Nevertheless, the covariant derivatives associated with quantum fields in the current discussion are constructed to be invariant under reference frame transformation. This is the defining characteristic of actual physical covariant derivatives. Therefore, in the present discussion, one constructs actual covariant derivatives as one simultaneously treats gauge symmetries, and this is the case even though these gauge symmetries are not in and of themselves associated with gravitational fields in any way. Admittedly, the general form of the metric $g_{\alpha\beta}$ may be associated with a gravitational field, which may in turn have an effect on the specific nature of the transformation operator $\Lambda^{\alpha}_{\beta}(x)$, but this fact is irrelevant to the general nature of the symmetries involved because the general form of the metric $g_{\alpha\beta}$ may also not be associated with a gravitational field.

In principle, four classes of gauge transformations exist because transformations can be either global or local and either Abelian or non-Abelian.[8] In reality, this reduces to three classes because global non-Abelian transformations, i.e., those involving global rotations, can be reconstructed in

[4]This does not require a massless field $\varphi^{\alpha}(x)$, although $m^2 = P_\alpha P^\alpha$ and the four-momentum operator is generally $P_\alpha = i\partial_\alpha$, because by definition four-momentum $P_\alpha$ vanishes in the proper frame of reference.

principle as Abelian transformations,[5] although this is not necessarily a simple procedure. However, local rotations cannot in general be deconstructed into Abelian transformations. One therefore proceeds to construct the $U(1)$, $SU(2)$, and $SU(3)$ symmetries from geometric and topological considerations to show that these respectively correspond to global Abelian, local Abelian, and local non-Abelian gauge transformations.[8]

### D. Origin of $U(1)$ symmetry

Construction of the $U(1)$ group symmetry for fields $\varphi^{\alpha}(x)$ begins with construction of an effective trajectory or world line. Of course, propagation of fields does not occur along a single unique path, nor is such a situation necessary in order to construct such an effective trajectory. Rather, one makes use of expectation values; defined in terms of the isotropic vacuum state $|0\rangle$, one writes the expectation value $\langle A\rangle$ of some Schrödinger picture (physical) operator $A$ or Heisenberg picture (physical) operator $A(x)$ as

$$\langle A\rangle \equiv \langle 0|\varphi_{\alpha}(x)A\varphi^{\alpha}(x)|0\rangle \equiv \langle 0|A(x)|0\rangle. \tag{26}$$

One then uses the expectation value of the four-momentum operator $P^{\alpha}_{\beta}$ (in which the second spacetime index reflects the possibility of a directional dependence of the four-momentum as noted above[19]) to construct as effective trajectory $X^{\alpha}_{\beta}$ associated with a field $\varphi^{\alpha}(x)$. For purposes of clarity, one uses coordinates which allow one to diagonalize these vectors and so suppress one index; this can always be done for nonpathological topologies. For massive fields (of mass $m$), topologically definable as fields for which four-velocity (tangent) $\beta^{\alpha}\neq g^{\alpha}_{0}$, one obtains a differential equation with respect to proper time $\tau$ as

$$\langle P^{\alpha}_{\beta=\alpha}\rangle = m\frac{dX^{\alpha}_{\beta=\alpha}}{d\tau}. \tag{27}$$

For massless fields, one must use an alternate proscription such as

$$g^{\alpha}_{\beta=\alpha} = \frac{dX^{\alpha}_{\beta=\alpha}}{d\tau}. \tag{28}$$

In either case, one solves for the effective trajectory $X^{\alpha}_{\beta=\alpha}$ using a boundary condition of the form

$$X^{\alpha}_{\beta=\alpha}(\tau=0) \equiv x^{\alpha}_{(0)}. \tag{29}$$

One could equally as well have constructed such differential equations for each physical path and summed over all possible paths, but this is by definition equivalent.

The $U(1)$ group symmetry arises from the arbitrariness of the boundary condition. Different choices of boundary conditions lead to a relative phase

$$\delta^{\alpha} \equiv k_{\alpha}(x^{\alpha}_{(0)} - x'^{\alpha}_{(0)}), \tag{30}$$

no summation on index $\alpha$, when applied to the definition of fields $\varphi^{\alpha}(x)$ above. In terms of the inhomogeneous Lorentz group,[12] this $U(1)$ symmetry describes the relative displacement of the origin. Such a displacement of the origin represents a global transformation

$$\varphi'^{\alpha}(x) = \Lambda^{\alpha}_{\beta}\varphi^{\beta}(x) \equiv \exp(-i\delta^{\alpha})\varphi^{\alpha}(x), \tag{31}$$

with again no summation on index $\alpha$. The transformation operator $\Lambda^{\alpha}_{\beta}(x)\equiv\Lambda^{\alpha}_{\beta}$ is constant, i.e.,

$$\partial_{\mu}\Lambda^{\alpha}_{\beta}(x) \equiv \partial_{\mu}\exp(-i\delta^{\alpha}) = 0, \tag{32}$$

and so the gauge condition (19) established above is trivially fulfilled. The generator of the group is the phase $\delta^{\alpha}$ itself.

### E. Origin and implications of $SU(2)$ symmetry

The $SU(2)$ group structure associated with electroweak interactions[8] arises in an interesting but related context, that of analytic (or holomorphic) conditions.[15] In any frame of reference other than the proper frame, the spacetime position $x$ (a parameter of the configuration space as usual) has at least two components, one timelike component and at least one spacelike component. The same is therefore true of the field $\varphi^{\alpha}(x)$. This leads to analytic (holomorphic) restrictions exactly analogous to Cauchy–Riemann restrictions on a complex function in complex space because a (1-1) mapping exists between a spacetime manifold of the form $M\sim\times\mathfrak{R}^3$ and a hypercomplex manifold (with three imaginary axes) of the form $C\sim\mathfrak{R}\times\mathfrak{I}^3$. Using the effective trajectory [described above Eqs. (27)–(29)] $X^{\alpha}_{\beta=\alpha}[\varphi^{\alpha}(x)]$—a functional of the associated field—to define coordinates such that two spacelike indices vanish (arbitrarily chosen as $x^2$ and $x^3$), these restrictions reduce to the form

$$\frac{\partial\varphi^{0}(x^{0})}{\partial x^{0}} = \frac{\partial\varphi^{1}(x^{1})}{\partial x^{1}}, \tag{33}$$

$$\frac{\partial\varphi^{0}(x^{0}[x^{1}])}{\partial x^{1}} = -\frac{\partial\varphi^{1}(x^{1}[x^{0}])}{\partial x^{0}}. \tag{34}$$

(One must treat spacetime coordinates in the second expression (34) as functionally dependent in order to define the derivatives.) For basis fields $\varphi^{\alpha}_{n}(x)$, defined by Eq. (4), these restrictions can be combined into the form

$$\left[\pm\frac{\varphi^{0}_{n}(x^{0})}{\varphi^{1}_{n}(x^{1})}\right]^{2} = 1. \tag{35}$$

This leads to basis fields of the form

$$[\varphi^{\alpha}_{n}(x)] = \pm\frac{\psi_n}{\sqrt{2}}\begin{bmatrix}\pm 1\\ 1\end{bmatrix}^{\alpha}, \tag{36}$$

where $\psi_n=\varphi^{0}_{n}(x^{0})$ for operators of the form $O=O(x^{0})$ and $\psi_n=\varphi^{1}_{n}(x^{1})$ for operators of the form $O=O(x^{1})$. Components $\varphi^{2}_{n}\equiv\varphi^{3}_{n}\equiv 0$ have been suppressed. The overall minus sign is used for antiparticles.

[5]This is equivalent to saying that one may construct mutually independent coordinates in flat space.

One should notice that the overall factor $\psi_n$ makes the field $\varphi_n^\alpha(x)$ remain a spinor, not a vector, because the components of the field $\varphi_n^\alpha(x)$ transform as single-component spinors; the overall transformation properties of the field $\varphi_n^\alpha(x)$ are therefore those of a multicomponent spinor. To understand this, one ought recall that components of an ordinary vector transform as one-component vectors, not truly as scalars. One just functionally treats them as scalars in most cases. Nevertheless, a choice of coordinates so that some arbitrary vector becomes a one-component vector does not fundamentally change that vector's transformation properties. Similar reasoning applies in this instance as well.

The bases $\left[\begin{smallmatrix}\pm 1\\ 1\end{smallmatrix}\right]$ may look familiar as eigenvectors of the Pauli matrices,[15] which ought be no surprise due to the association of these matrices with intrinsic spin-$\frac{1}{2}$ fields.[8] For convenience (both physical and mathematical as will be seen), these bases can be rotated to new bases

$$\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}, \quad \begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}. \tag{37}$$

Consequently, any general field $\varphi^\alpha(x)$ can be written as a linear combination of such bases

$$\begin{aligned}[\varphi^\alpha(x)] &= \psi \exp\left(-\frac{i}{2}a^\alpha_{\beta M}\sigma^M\right)\begin{bmatrix}p\\ n\end{bmatrix}^\beta \\ &= \psi \exp\left(-\frac{i}{2}a'^\alpha_{\beta M}\sigma^M\right)\begin{bmatrix}n-p\\ n+p\end{bmatrix}^\beta,\end{aligned} \tag{38}$$

for which parameters $p$ and $n$ may for now be regarded as arbitrary. The resemblance of the former to isospin bases[8] will be seen however to be purposeful, as that physical basis can be regarded as one form these bases may take.

If one considers this $SU(2)$ symmetry from the viewpoint of symmetry-breaking, one notices a fundamental difference between bases $\left[\begin{smallmatrix}1\\ 1\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}-1\\ 1\end{smallmatrix}\right]$. Under inversion of axes, i.e., $x^0 \to x^1$ and $x^1 \to x^0$, these bases are, respectively, symmetric and antisymmetric. These are not, respectively, bosons and fermions in this case, since the linear combination must ensure the transformation properties of the field $\varphi^\alpha(x)$ remain the same. Nevertheless, clearly two fundamental classes of basis fields have arisen. The exact symmetry between spacelike and timelike components remains for basis fields $\varphi_n^\alpha(x) \propto \left[\begin{smallmatrix}1\\ 1\end{smallmatrix}\right]$, but this same symmetry has as clearly become broken for basis fields $\varphi_n^\alpha(x) \propto \left[\begin{smallmatrix}-1\\ 1\end{smallmatrix}\right]$. When one considers the field components of basis fields $\varphi_n^\alpha(x)$ simply as two separate fields, some physical consequences of this sort of symmetry breaking are well-known in that this class of physical situation is usually associated with the construction of massive gauge bosons.[8] So, in the case of massless fields, the symmetric basis field $\varphi_n'^\alpha(x) \propto \left[\begin{smallmatrix}1\\ 1\end{smallmatrix}\right]$ remains massless, but the antisymmetric basis field $\varphi_n^\alpha(x) \propto \left[\begin{smallmatrix}-1\\ 1\end{smallmatrix}\right]$ does not. In fact, any arbitrary equation of motion must compensate for the latter's components' difference of sign. Therefore, if the field were chargeless before symmetry breaking, it acquires charge.[6] If the field were charged, the same reasoning leads to a difference in charges. Again, in the case of fields which are massive before symmetry breaking, the two classes of fields acquire a mass difference.

This situation corresponds exactly to the doublets $\left(\begin{smallmatrix}e^-\\ \nu_e\end{smallmatrix}\right)$, etc., for leptons, $\left(\begin{smallmatrix}p^+\\ n^0\end{smallmatrix}\right)$, etc., for baryons and $\left(\begin{smallmatrix}d\\ u\end{smallmatrix}\right)$, etc., for quarks.[8] One therefore defines leptons in this description as fields which are massless before any symmetry-breaking effects. Hadrons are associated with fields which are massive even before symmetry-breaking effects. The intrinsic mass of hadrons may be associated with the binding energy of quarks, which are themselves intrinsically massless apart from symmetry-breaking effects. Quark and hadron fields are however discussed in more detail in the construction of the $SU(3)$ symmetry below.

In a sense, however, an element of arbitrariness exists in the identification of basis vector $\left[\begin{smallmatrix}1\\ 1\end{smallmatrix}\right]$ with the neutral leptons and hadrons or the charge $+\frac{2}{3}e$ quarks and of basis vector $\left[\begin{smallmatrix}-1\\ 1\end{smallmatrix}\right]$ with the charged leptons and hadrons and the charge $-\frac{1}{3}e$ quarks. One could have as easily reversed the associations, whatever may be the aesthetic reasons for the convention chosen. If standard usage had chosen to also use left-handed coordinate systems rather than only right-handed coordinate systems, one could relate coordinate systems of differing handedness by the transformation $x^0 \to -x^0$, the spacelike component remaining untransformed. Then the basis vectors would reverse $\left[\begin{smallmatrix}\pm 1\\ 1\end{smallmatrix}\right] \to \left[\begin{smallmatrix}\mp 1\\ 1\end{smallmatrix}\right]$. Physically, from the reference frame of a charged field/particle, an uncharged field/particle is charged and of course vice-versa. Therefore, one includes this type of "handedness" in the definition of any frame of reference. A massless field with unbroken symmetry (a neutrino) can only have contributions from one class of basis fields and so it must have a single, unique handedness, in spite of arbitrary standard usage left-handedness. Handedness of massive fields, even with unbroken symmetry, can always be viewed from a boosted frame of reference such that a momentum vector, for example, parallel the spacelike axis becomes antiparallel, which is equivalently a change of handedness.[8] Finally, antifield/particles, as equivalent to negative energy solutions, have in a sense "flipped" the timelike axis (i.e., $x^0 \to -x^0$) and so would have opposite handedness. This only has especially meaningful consequences for the massless antifield/particle with unbroken symmetry, the antineutrino, since only the neutrino of the ordinary field/particles has a unique handedness. Thus, all antineutrinos must be right-handed since all neutrinos are left-handed, as is observed.[8]

If one returns to the above-mentioned representation of the field $\varphi^\alpha(x)$ in form

[6]This view associates charge with both a sign-dependent deflection of a field/particle from the effective path of a similar neutral field/particle under certain conditions and association of this deflection with a term in the field equation. This type of field term would be analogous to the classical Lorentz term in the geodesic equation associated with a charged classical particle. In the standard model, this type of field term would be a charge coupling term.

$$[\varphi^{\alpha}(x)] = \psi \exp\left(-\frac{i}{2}a^{\alpha}_{\beta M}\sigma^{M}\right)\begin{bmatrix} p \\ n \end{bmatrix}^{\beta}, \tag{39}$$

from Eq. (38), one may notice that setting $\psi=1$ implicitly selects a certain class of field as the only class of field to which the resultant expression then applies. Use of the nucleonic isospin basis[8]

$$[\varphi^{\alpha}(x)]_{\text{nucleonic isospin}} = \exp\left(-\frac{i}{2}a^{\alpha}_{\beta M}\sigma^{M}\right)\begin{bmatrix} p \\ n \end{bmatrix} \tag{40}$$

excludes fields which cannot be represented as a superposition of nucleons. Isospin is thus only one manifestation of a more general type of symmetry.

One can also use the general form above to define a transformation

$$\varphi'^{\alpha}(x) = \exp\left(-\frac{i}{2}a^{\alpha}_{\beta M}(x)\sigma^{M}\right)\varphi^{\beta}(x). \tag{41}$$

The coefficients $a^{\alpha}_{\beta M}(x)$ must be local, i.e., dependent on spacetime position $x$, because the symmetry relates to analytic (holomorphic) conditions which are intrinsically local conditions. No rotations of spacetime coordinates are involved in satisfying analytic (holomorphic) conditions and so the transformation operator

$$\Lambda^{\alpha}_{\beta}(x) \equiv \exp\left[-\frac{i}{2}a^{\alpha}_{\beta M}(x)\sigma^{M}\right] \tag{42}$$

and its generator

$$f^{\alpha}_{\beta}(x) \equiv \frac{1}{2}a^{\alpha}_{\beta M}(x)\sigma^{M} \tag{43}$$

must be Abelian. To satisfy the gauge condition above, one must therefore construct an associated covariant derivative.

As usual,[8,12] a covariant derivative operator $D_{\mu}$ is defined to replace an ordinary derivative operator $\partial_{\mu}$. Therefore, one demands definitions

$$D_{\mu}\varphi^{\beta}(x) = \partial_{\mu}\varphi^{\beta}(x) - [\partial_{\mu}\Lambda^{\beta}_{\alpha}(x)]\varphi^{\alpha}(x), \tag{44}$$

$$D_{\mu}\Lambda^{\alpha}_{\beta}(x) \equiv \partial_{\mu}\Lambda^{\alpha}_{\beta}(x), \tag{45}$$

so that the product rule expression

$$[D_{\mu}\Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x) + \Lambda^{\alpha}_{\beta}(x)D_{\mu}\varphi^{\beta}(x) = 0 \tag{46}$$

reduces to the original untransformed condition

$$\partial_{\mu}\varphi^{\beta}(x) = 0 \tag{47}$$

stated above Eq. (18). One can associate the covariant derivative $D_{\mu}$ with the photon field $A^{\alpha}_{\mu\beta}$ and charge $q$ in the usual manner[8] so that

$$\partial_{\mu}\Lambda^{\alpha}_{\beta}(x) = iqA^{\alpha}_{\mu\beta}. \tag{48}$$

The two seemingly additional indices are added to the photon field $A^{\alpha}_{\mu\beta}$, as opposed to the more familiar form of the photon field $A_{\mu}$, in order to allow coupling with the four-indexed field $\varphi^{\beta}(x)$.

### F. Origin and nature of *SU*(3) symmetry

Fields which are massive also differ in one clearly fundamental respect from massless fields; they have velocity related degrees of freedom, whereas for massless fields velocity $\beta^{\alpha}=g^{\alpha}_{0}$. In the case of leptons, although half of these acquire mass in symmetry breaking, one may always choose a field-space reference frame in which that particular lepton remains massless as discussed above, and so these degrees of freedom are not physically significant in most respects. Indeed, except for artifices due to arbitrary choice of frame of reference, leptons can always be treated as massless field/particles, i.e., in the chiral limit,[8] by definition in the proposed description. For massive fields however, velocity $\beta^{\alpha} \neq g^{\alpha}_{0}$ represents true degrees of freedom. Therefore, one may describe massive fields $\varphi^{\alpha}(x)$ in terms of a functional dependence

$$\varphi^{\alpha}(x) = \varphi^{\alpha}[x, \beta^{M}(x)]. \tag{49}$$

Since the component $\beta^{0}$ is a constant at any spacetime location $x$, this effectively leads to dependence on only the spacelike components of velocity $\beta^{\alpha}$. In an exact analogy with the procedure described above, Eqs. (3) and (4), in which one constructs field $\varphi^{\alpha}(x)$ from field $\varphi(x)$, one constructs a field $\varphi^{\alpha M}(x)$ from field $\varphi^{\alpha}(x)$, for which the index $M$ describes the dependence on the spacelike components of velocity $\beta^{\alpha}$, namely $\beta^{M}$. Explicitly, one writes the field $\varphi^{\alpha}_{M}(x)$ for velocity $\beta^{M}$ nonuniaxial as

$$[\varphi^{\alpha}_{M}(x)] \equiv a^{\alpha N}_{n\mu M}[\varphi^{\mu}_{nN}(x)] \equiv a^{\alpha N}_{n\mu M}\begin{bmatrix} \exp(-im\beta_{1n}x^{0}) & \exp(-im\beta_{2n}x^{0}) & \exp(-im\beta_{3n}x^{0}) \\ \exp(-im\beta_{1n}x^{1}) & \exp(-im\beta_{2n}x^{1}) & \exp(-im\beta_{3n}x^{1}) \\ \exp(-im\beta_{1n}x^{2}) & \exp(-im\beta_{2n}x^{2}) & \exp(-im\beta_{3n}x^{2}) \\ \exp(-im\beta_{1n}x^{3}) & \exp(-im\beta_{2n}x^{3}) & \exp(-im\beta_{3n}x^{3}) \end{bmatrix}^{\mu}_{N}. \tag{50}$$

The index $M$ lends itself to interpretation as a field-index, specifically an index among three fields constituent to the total observable field $\varphi^{\alpha}(x)$. However, whenever the velocity $\beta^{M}$ is uniaxial, mathematically Fourier series representation or physically ordinary quantum mechanical considerations demand a superposition of the form

$$[\varphi^{\alpha}_{M}(x)] \equiv \frac{a^{\alpha 1}_{n\mu M}}{\sqrt{2}} \left\{ \begin{bmatrix} \exp(-\,im\beta_{1n}x^{0}) \\ \exp(-\,im\beta_{1n}x^{1}) \\ \exp(-\,im\beta_{1n}x^{2}) \\ \exp(-\,im\beta_{1n}x^{3}) \end{bmatrix}^{\mu}_{1(n>0)} + \begin{bmatrix} \exp(im\beta_{1n}x^{0}) \\ \exp(im\beta_{1n}x^{1}) \\ \exp(im\beta_{1n}x^{2}) \\ \exp(im\beta_{1n}x^{3}) \end{bmatrix}^{\mu}_{1(n>0)} + \begin{bmatrix} \exp(-\,im\beta_{10}x^{0}) \\ \exp(-\,im\beta_{10}x^{1}) \\ \exp(-\,im\beta_{10}x^{2}) \\ \exp(-\,im\beta_{10}x^{3}) \end{bmatrix}^{\mu}_{1(n=0)} \right\}. \qquad (51)$$

This is a superposition of a field/particle, its antifield/particle, and its vacuum field.[8] Such a decomposition of a general field $\varphi^{\alpha}(x)$, respectively, into particle $\varphi^{\alpha}_{n}(x)$, antiparticle $\varphi^{\alpha}_{-n}(x)$, and vacuum $\varphi^{\alpha}_{n=0}(x)$ field contributions

$$\varphi^{\alpha}(x) = a^{\alpha}_{n\beta}\varphi^{\beta}_{n}(x) = \{\varphi^{\alpha}_{n}(x) + \varphi^{\alpha}_{-n}(x)\}_{(n>0)} + \varphi^{\alpha}_{n=0}(x) \qquad (52)$$

applies to any species of field, not just quarks. [One may notice that the vacuum field $\varphi^{\alpha}_{n=0}(x)$ remains in general both nontrivial and even in principle nonisotropic.] Therefore, observable intrinsically massive fields (i.e., hadrons) come in two varieties, those made up of three constituent fields which are mutually orthogonal and those made up of two constituent fields, a field/particle and its antifield/particle. The observable field/particles are respectively defined as baryons and mesons, the constituent field/particles (each $M$th component of the field $\varphi^{\alpha M}(x)$ for baryons and partial sum $\varphi^{\alpha M}_{(n>0)}(x)$ for mesons) which are not independently observable are defined as quarks.

The usual $SU(3)$ symmetry[8] arises from rotations in the (spacelike) velocity three-space and so any field $\varphi^{\alpha M}(x)$ can be written as a superposition which is represented in matrix form as

$$[\varphi^{\alpha M}(x)] = \psi \exp\left(-\frac{i}{2} b^{M}_{N} \cdot \lambda^{\alpha N}_{\beta K}\right) \begin{bmatrix} r \\ b \\ g \end{bmatrix}^{\beta K}. \qquad (53)$$

Transformation among such fields $\varphi^{\alpha M}(x)$ then must take the form

$$\varphi'^{\alpha M}(x) = \exp\left(-\frac{i}{2} b^{M}_{N}(x) \cdot \lambda^{\alpha N}_{\beta K}\right) \varphi^{\beta K}(x), \qquad (54)$$

where $\lambda^{K}_{N}$ takes the form of the usual gluon-associated $SU(3)$ basis matrices.[8] The color labels red, blue, and green therefore label directions in the (spacelike) velocity three space, but such labels remain arbitrary and so the choice of color labels do also. The associated group $SU(3)$ is non-Abelian, since the group geometrically represents rotations in a Euclidean three-space which are of course non-Abelian. In principle, a caveat should, however, be associated with this discussion. A velocity-associated three space constitutes a subspace. To contract three vectors, one must in principle define the timelike component for three vectors to be identically zero and use a metric $g_{\alpha\beta}$ written in terms of spacelike axes labeled red, blue, and green. Nevertheless, no physical reason precludes the choice of coordinates so that red, blue, and green axes form a Euclidean basis, and moreover no apparent advantage is associated with not doing so. The caveat therefore does not really apply.

Again, in order to satisfy the original gauge condition (19), one must replace the ordinary derivative by a covariant derivative

$$D_{\mu}\varphi^{\alpha}(x) = \partial_{\mu}\varphi^{\alpha}(x) - [\partial_{\mu}\Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x), \qquad (55)$$

$$D_{\mu}\Lambda^{\alpha}_{\beta}(x) = \partial_{\mu}\Lambda^{\alpha}_{\beta}(x). \qquad (56)$$

The essential part of the generator of the symmetry $\frac{1}{2}b^{M}_{N}(x) \cdot \lambda^{\alpha N}_{\beta K}$ is the matrix $\lambda^{\alpha N}_{\beta K}$, which may itself loosely be termed the generator. One then defines the gauge field $B^{\alpha}_{\mu\beta}$ and coupling $a$ (analogous to electrostatic charge $q$ in quantum electrodynamics) as

$$-\,iaB^{\alpha}_{\mu\beta} \equiv \partial_{\mu} \exp\left[-\frac{i}{2} b^{M}_{N}(x) \cdot \lambda^{\alpha N}_{\beta K}\right]. \qquad (57)$$

The usual notation $g$ for the coupling constant is avoided to prevent confusion with the modulus of the metric. The commutation properties of this gauge field and its generators are as usually associated with QCD.

## IV. THE DIRAC EQUATION, VACUUM EXPANSION, AND SYMMETRY BREAKING

### A. Nature of the example

In order to demonstrate the power and implications of the above formalism, one applies that formalism to description of expansion of the vacuum, i.e., cosmic expansion.[21] In short, one considers two points in empty space, i.e., vacuum, $x_{0}(t)$ and $x'(t)$. Initially, at time $t=0$, these spacetime locations are not resolvable so that the separation

$$a(t) \equiv |x_{0}(t) - x'(t)| \qquad (58)$$

has the boundary condition

$$a(t=0) = 0. \qquad (59)$$

The separation increases with time so that

$$\frac{\partial_{0}a(t)}{a(t)} > 0. \qquad (60)$$

Similarly, the vacuum expansion rate, as per current physical results,[21] is accelerating so that cosmic acceleration is characterized as

$$\frac{(\partial_{0})^{2}a(t)}{a(t)} > 0, \qquad (61)$$

as well. This situation will be described using the Dirac equation modified for four-indexed fields $\varphi^{\alpha}(x)$ and interpreted as a harmonic oscillator. The process of expansion drives this oscillator leading to the excitation of field/particles.

### B. Dirac equation for four-indexed fields

Construction of the Dirac equation as modified for four-indexed fields $\varphi^\alpha(x)$ mainly requires algebra. A Lagrangian for the Dirac equation has been formulated,[2] but that Lagrangian was constructed to lead to the desired form of the field equation, rather than the field equation deriving originally from it. So, to construct the form of the Dirac equation for four-indexed fields $\varphi^\beta(x)$ in the absence of an external potential, one uses the conventional Dirac equation

$$[i\gamma^\alpha\partial_\alpha - m]\varphi(x) = 0. \tag{62}$$

One first factors each basis field $\varphi_n(x)$, in terms of which the ordinary Dirac field $\varphi(x)$ takes the form

$$\varphi(x) \equiv a_n\varphi_n(x), \tag{63}$$

to construct the field

$$\varphi^\alpha(x) \equiv a^\alpha_{n\beta}\varphi^\beta_n(x). \tag{64}$$

The additional indices introduced on the coefficients $a^\alpha_{n\beta}$, with respect to $a_n$, allow components of basis fields $\varphi^\beta_n(x)$ in principle to couple. Where one can apply separation of variables to the field $\varphi^\alpha(x)$ directly, the coefficients $a^\alpha_{n\beta}\equiv a_n g^\alpha_\beta$. In general though, one obtains the expression

$$\left\{ i\begin{bmatrix} \gamma^0\partial_0 & 0 & 0 & 0 \\ 0 & \gamma^1\partial_1 & 0 & 0 \\ 0 & 0 & \gamma^2\partial_2 & 0 \\ 0 & 0 & 0 & \gamma^3\partial_3 \end{bmatrix}^\alpha_\beta - m[g^\alpha_\beta] \right\}[\varphi^\beta(x)] = 0, \tag{65}$$

if

$$\partial_\alpha a^\alpha_{n\beta} = 0, \tag{66}$$

as is assumed to be at least locally valid. The condition $\partial_\alpha a^\alpha_{n\beta}=0$ will always be true in some frame of reference and since the coefficients are essentially arbitrary can always be constructed to be true. When such a condition is fulfilled, the nature of the field $\varphi^\alpha(x)$ makes this equivalent to

$$\left\{ i\begin{bmatrix} \gamma^0\partial_0 & \gamma^0\partial_1 & \gamma^0\partial_2 & \gamma^0\partial_3 \\ \gamma^1\partial_0 & \gamma^1\partial_1 & \gamma^1\partial_2 & \gamma^1\partial_3 \\ \gamma^2\partial_0 & \gamma^2\partial_1 & \gamma^2\partial_2 & \gamma^2\partial_3 \\ \gamma^3\partial_0 & \gamma^3\partial_1 & \gamma^3\partial_2 & \gamma^3\partial_3 \end{bmatrix}^\alpha_\beta - m[g^\alpha_\beta] \right\}[\varphi^\beta(x)] = 0, \tag{67}$$

which is more compactly written as

$$[i\gamma^\alpha\partial_\beta - m g^\alpha_\beta]\varphi^\beta(x) = 0. \tag{68}$$

If an external potential $V^\alpha_\beta(x)$ is present, one modifies this expression to

$$[i\gamma^\alpha\partial_\beta - m g^\alpha_\beta]\varphi^\beta(x) = V^\alpha_\beta(x)\varphi^\beta(x). \tag{69}$$

The form of potential may be $V^\alpha_\beta(x)=V(x)g^\alpha_\beta$, but this may not necessarily be the case. Also, the Dirac matrix operator $\gamma^\alpha$ in these expressions has the well-known definition[8]

$$\frac{1}{2}\{\gamma^\alpha,\gamma^\beta\} \equiv \frac{1}{2}[\gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha] \equiv g^{\alpha\beta}, \tag{70}$$

but here the metric $g^{\alpha\beta}$ is general so that usual forms of the matrices may serve of bases but not as general forms. Mathematically, one has constructed a completely equivalent field equation, which when one lowers indices can also take the form

$$[ig_{\alpha\mu}\gamma^\mu\partial_\beta - m g_{\alpha\beta}]\varphi^\beta(x) \equiv [i\gamma_\alpha\partial_\beta - m g_{\alpha\beta}]\varphi^\beta(x) = V_{\alpha\beta}(x)\varphi^\beta(x). \tag{71}$$

Physically, this modified Dirac equation couples the field $\varphi^\beta(x)$ with a general metric $g_{\alpha\beta}$ explicitly and allows components of the field $\varphi^\beta(x)$ to couple with the external potential $V_{\alpha\beta}(x)$ independently.

### C. Construction as a SHO

Because of the SHO's association with a number operator,[7] transformation of the Dirac equation into a harmonic oscillator facilitates discussion. If one defines a Hermitian generalized momentum operator

$$\pi^\alpha_\beta = i\gamma^\alpha\partial_\beta, \tag{72}$$

the form of the Dirac equation above can alternately be written quadratically as

$$[\pi^\alpha_\mu - m g^\alpha_\mu]^\dagger[\pi^\mu_\delta - m g^\mu_\delta]\varphi^\delta(x) = [\pi^\alpha_\beta\pi^\beta_\delta - m^2 g^\alpha_\beta g^\beta_\delta]\varphi^\delta(x) = V^\alpha_\beta V^\beta_\delta\varphi^\delta(x), \tag{73}$$

subject to restrictions

$$\pi^\alpha_\beta\pi^\beta_\delta = m\pi^\alpha_\delta, \tag{74}$$

$$V^\alpha_\beta V^\beta_\delta = mV^\alpha_\delta, \tag{75}$$

a form which facilitates interpretation as a SHO.

The Dirac equation is a Lagrangian based expression, by definition. If one defines a generalized Hamiltonian operator $H^\alpha_\beta$, one writes the equivalent Hamiltonian expression

$$H^\alpha_\beta m\varphi^\beta(x) \equiv \frac{1}{2}[\pi^\alpha_\beta\pi^\beta_\delta + V^\alpha_\beta V^\beta_\delta + m^2 g^\alpha_\beta g^\beta_\delta]\varphi^\delta(x) \equiv \frac{V^\mu_\mu}{2}\left[a^{\dagger\alpha}_\beta a^\beta_\delta + \frac{1}{2}g^\alpha_\delta\right]\varphi^\delta(x) \equiv \frac{V^\mu_\mu}{8}\left[N + \frac{1}{2}\right]\varphi^\alpha(x). \tag{76}$$

One reads off from this creation and annihilation operators, respectively,

$$a^\alpha_\beta \equiv \sqrt{\frac{m}{V^\mu_\mu}}\left\{g^\alpha_\beta + \frac{1}{m}[i\pi^\alpha_\beta - V^\alpha_\beta]\right\}, \tag{77}$$

$$a^{\dagger\alpha}_\beta \equiv \sqrt{\frac{m}{V^\mu_\mu}}\left\{g^\alpha_\beta - \frac{1}{m}[i\pi^\alpha_\beta + V^\alpha_\beta]\right\}. \tag{78}$$

Definition of the number operator

$$N \equiv \frac{a^{\dagger\alpha}_{\beta} a^{\beta}_{\alpha}}{4} \tag{79}$$

is implied if one absorbs a constant into the characteristic frequency $\omega^{\alpha}_{\beta}$ (in natural units the energy of the $\beta$th field component as measured with respect to the $\alpha$th local coordinate axis) as

$$\omega^{\alpha}_{\beta} = \frac{V^{\mu}_{\mu}}{2m} g^{\alpha}_{\beta}. \tag{80}$$

In the usual manner,[8] one is able to associate separate creation and annihilation operators with each field/particle by treating these operators as functions of mass and linear four momentum. Notably, these operators cannot be defined for either zero mass or zero external potential. The former restriction $m \neq 0$ is not problematic, even with respect to massless neutrinos, because the symmetry associated with leptons $l$ and associated neutrinos $\nu_l$ described above implies an effective association of the lepton $l$'s mass with the neutrino $\nu_l$. This effective mass is physically a mass difference between a lepton $l$ and an associated neutrino $\nu_l$. The latter restriction $V^{\mu}_{\mu} \neq 0$ coincides with the usual definition of the ground of any field, i.e., as being the vacuum field. In short, in the absence of an external potential, fields remain at ground and therefore no particles are produced.[8]

### D. Vacuum expansion as driving the SHO

One now returns to the specific physical problem at hand namely vacuum or cosmic expansion. Initially, no physical difference arises if one defines the origin of one's frame of reference at either position $x_0(t)$ or position $x'(t)$, since the initial separation $a(t=0)=0$ by definition. One chooses a reference frame with respect to position $x_0(t)$. The governing field equation

$$[i\gamma^{\alpha}\partial_{\beta} - m g^{\alpha}_{\beta}]\varphi^{\beta}(x_0) = 0 \tag{81}$$

remains unchanged for position $x_0(t)$. No external fields are present with which a spin-$\frac{1}{2}$ field $\varphi^{\alpha}(x)$ interacts as far as an observer at position $x_0(t)$ is concerned. This is only initially true for an observer at position $x'(t)$; in general, the field equation must be transformed, using the covariant derivative

$$D_{\mu}\varphi^{\alpha}(x') = \partial_{\mu}\varphi^{\alpha}(x') - [\partial_{\mu}\Lambda^{\alpha}_{\beta}(x')]\varphi^{\beta}(x'). \tag{82}$$

The transformation operator $\Lambda^{\alpha}_{\beta}(x')$ transforms from the reference frame with respect to position $x'(t)$ back to the position $x_0(t) \equiv x_0$. Direct substitution of covariant derivative $D_{\mu}$ for ordinary derivative $\partial_{\mu}$, as required for covariant transformation, leads to the field equation

$$[i\gamma^{\alpha}D_{\beta} - m g^{\alpha}_{\beta}]\varphi^{\beta}(x_0) = 0, \tag{83}$$

$$[i\gamma^{\alpha}\partial_{\beta} - m g^{\alpha}_{\beta}]\varphi^{\beta}(x_0) = i\gamma^{\alpha}[\partial_{\mu}\Lambda^{\mu}_{\beta}(x')]\varphi^{\beta}(x') \equiv V^{\alpha}_{\beta}(x')\varphi^{\beta}(x') \tag{84}$$

at position $x'(t)$ as observed from position $x_0$. As seen from position $x_0$, a potential exists at position $x'(t)$. That potential increases as the relative separation $a(t)$ does because, as per Hubble's law,[12] the relative velocity $\partial_0 a(t)$ increases with distance. At each given moment, an observer at position $x'(t)$ can be described as having received a boost with respect to an observer at position $x_0$ so that the transformation operator takes the form

$$\{\Lambda^{\mu}_{\beta}[t,a(t)]\} = \begin{pmatrix} \{1-[\partial_0 a(t)]^2\}^{-1/2} & -[\partial_0 a(t)]\{1-[\partial_0 a(t)]^2\}^{-1/2} & 0 & 0 \\ -[\partial_0 a(t)]\{1-[\partial_0 a(t)]^2\}^{-1/2} & \{1-[\partial_0 a(t)]^2\}^{-1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{\mu}_{\beta}, \tag{85}$$

in spherical polar coordinates (where angular coordinates $x^2$ and $x^3$ do not matter) with respect to position $x_0$, but the amount of that boost continuously increases. Accordingly, the four gradient of this transformation operator $\partial_{\mu}\Lambda^{\mu}_{\beta}(x')$ takes the form

$$[\partial_{\mu}\Lambda^{\mu}_{\beta}(x')] = \begin{bmatrix} \{1-[\partial_0 a(t)]^2\}^{-3/2}[\partial_0 a(t)][(\partial_0)^2 a(t)] \\ -[(\partial_0)^2 a(t)](\{1-[\partial_0 a(t)]^2\}^{-1/2} + [\partial_0 a(t)]^2\{1-[\partial_0 a(t)]^2\}^{-3/2}) \\ 0 \\ 0 \end{bmatrix}_{\beta}. \tag{86}$$

From this one constructs the potential

$$V^{\alpha}_{\beta}(x') \equiv i\gamma^{\alpha}\partial_{\mu}\Lambda^{\mu}_{\beta}(x') \tag{87}$$

as defined above [Eq. (84)].

The time-dependence of the number operator $N$ can then be determined from the expectation-valued expression

$$\varphi_{\alpha}(x)\gamma^0 H^{\alpha}_{\beta}\varphi^{\beta}(x) \equiv \varphi_{\alpha}(x)\gamma^0 H\varphi^{\alpha}(x) = \varphi_{\alpha}(x)\gamma^0 \frac{V^{\mu}_{\mu}}{2m}\left[N + \frac{1}{2}\right]\varphi^{\alpha}(x), \tag{88}$$

since the generalized Hamiltonian operator $H$ is constant, by definition. This leads to the operator expression

$$\partial_0 N = -\frac{[\partial_0 V^{\mu}_{\mu}]}{2V^{\alpha}_{\alpha}}, \tag{89}$$

since initially no particles are excited so that one applies the boundary condition $N(t=0)=0$. One assumes the operator $\gamma^{\alpha}$ constant hereafter, since this has no effect on the physical results; one may always describe spacetime as locally flat.[12] The potential in this case leads to the trace of the potential as

$$V^{\alpha}_{\alpha} = i\frac{[(\partial_0)^2 a(t)]}{\{1-[\partial_0 a(t)]^2\}^{1/2}}[\gamma^0\{1-[\partial_0 a(t)]^2\}^{-1}[\partial_0 a(t)] + \gamma^1(1+[\partial_0 a(t)]^2\{1-[\partial_0 a(t)]^2\}^{-1})]. \tag{90}$$

One will assume $(\partial_0)^2 a(t)$ constant as well, i.e., a time-independent cosmic acceleration,[21] so that the time dependence of the trace of the potential becomes

$$\partial_0 V^{\alpha}_{\alpha} = i[(\partial_0)^2 a(t)]^2\{1-[\partial_0 a(t)]^2\}^{-5/2}\{2\gamma^0[\partial_0 a(t)]^2 + \gamma^0 + 3\gamma^1[\partial_0 a(t)]\} > 0. \tag{91}$$

In this case, the trace of the potential $V^{\alpha}_{\alpha}$ takes the form of an operator so that definition of the inverse operator $(V^{\alpha}_{\alpha})^{-1}$ would be long and tedious. However, one may notice that if one defines operator

$$V^{\alpha}_{\alpha} = i \operatorname{Im} V^{\alpha}_{\alpha}, \tag{92}$$

then the imaginary portion $\operatorname{Im} V^{\alpha}_{\alpha}$ of the operator $V^{\alpha}_{\alpha}$ is positive definite. The inverse of the operator $\operatorname{Im} V^{\alpha}_{\alpha}$ must therefore also be positive definite but the factor $i$ in the original operator leads to a factor $-i$ in its inverse; inverse operator $(V^{\alpha}_{\alpha})^{-1}$ is negative definite. In short, the time dependence of the number operator is positive definite as

$$\partial_0 N = -\frac{1}{2}(V^{\alpha}_{\alpha})^{-1}[\partial_0 V^{\mu}_{\mu}] = \frac{1}{2}|(V^{\alpha}_{\alpha})^{-1}|[\partial_0 V^{\mu}_{\mu}] > 0. \tag{93}$$

As one expects with a driven oscillator, excitations are produced. In this case, those excitations are excitations of field/particles. Excitation of antifield/particles would decrease the expectation value of number operator $N$. Therefore, field/particles must be excited in this process in greater numbers than antifield/particles, in violation of CP symmetry.

## V. CONCLUSION

The foregoing discussion has simultaneously constructed gauge transformations[8] and covariant transformations, under general reference frame transformations,[12] showing at each step how each transformation under discussion can be described as either class of transformation. The class of fields $\varphi^{\alpha}(x)$ considered are defined as solutions of the form

$$[\varphi^{\alpha}(x)] = \frac{a^{\alpha}_{n\beta}}{2}\begin{bmatrix} \exp(ik_{n0}x^0) \\ \exp(-ik_{n1}x^1) \\ \exp(-ik_{n2}x^2) \\ \exp(-ik_{n3}x^3) \end{bmatrix}^{\beta} \tag{94}$$

to some general but unspecified field equation. The index $\alpha$ on fields $\varphi^{\alpha}(x)$ is a coordinate index in the sense that it is raised and lowered by means of a metric field and that $\alpha=0$ denotes a timelike field component and $\alpha=1,2,3$ denotes spacelike field components. One assumes the field equation to be physically meaningful, but no details of its form are discussed. Conservation of probability is used to construct a gauge condition

$$\partial_{\mu}\varphi'^{\alpha}(x) = [\partial_{\mu}\Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x) + \Lambda^{\alpha}_{\beta}(x)\partial_{\mu}\varphi^{\beta}(x) \tag{95}$$

for any field transformation of the general form

$$\varphi'^{\alpha}(x) = \Lambda^{\alpha}_{\beta}(x)\varphi^{\beta}(x). \tag{96}$$

The field/particle interpretation of any field equation—as opposed to a single particle interpretation of that same equation—necessitates a local reference frame transformation operator $\Lambda^{\alpha}_{\beta}(x)$. The operator $\Lambda^{\alpha}_{\beta}(x)$ transforms the field $\varphi^{\beta}(x)$ not from the reference frame of one localized particle to that of some other localized particle but from the reference frame of one multiparticle field to that of some other multiparticle field. In effect, the operator $\Lambda^{\alpha}_{\beta}(x)$ represents the set of all possible transformations between pairs of all possible field/particle excitations. Even within the class of rest frames, the elements of such a set of transformations will only be constant within a very restrictive set of physical circumstances.

Although no Lagrangian is specified, one has demonstrated that this class of transformations, subject to the above condition, does indeed constitute a gauge transformation. The same condition above used as a gauge condition also constitutes a condition for covariance. Thus, when one constructs a covariant derivative

$$D_{\mu}\varphi^{\alpha}(x) \equiv \partial_{\mu}\varphi^{\alpha}(x) - [\partial_{\mu}\Lambda^{\alpha}_{\beta}(x)]\varphi^{\beta}(x), \tag{97}$$

$$D_{\mu}\Lambda^{\alpha}_{\beta}(x) \equiv \partial_{\mu}\Lambda^{\alpha}_{\beta}(x), \tag{98}$$

this is an actual—not an effective—covariant derivative.

The first class of transformations considered above was those where the transformation operator $\Lambda^{\alpha}_{\beta}(x)=\Lambda^{\alpha}_{\beta}$ is constant with respect to spacetime location $x$, termed global transformations. This involves the usual $U(1)$ gauge symmetry associated with the arbitrary nature of the choice of a coordinate system's (reference frame's) origin. The covariant derivative is this case is trivial in that $D_{\mu}\varphi^{\alpha}(x)=\partial_{\mu}\varphi^{\alpha}(x)$ since $\partial_{\mu}\Lambda^{\alpha}_{\beta}(x)=0$. The generator of the symmetry group is the phase $\delta^{\alpha}$ in each $\alpha$th field component $\varphi^{\alpha}(x)$ associated with displacement of the origin with respect to which one describes fields $\varphi^{\alpha}(x)$. Only Abelian global transformation operators need be treated because non-Abelian global transformation operators, such as those involving rotations, can be constructed from Abelian operators in the global case.

The second class of transformations remains Abelian but allows the transformation operator $\Lambda^{\alpha}_{\beta}(x)$ to depend on spacetime position $x$, so that the operator is locally defined as discussed above. This leads, via analytic (holomorphic) restrictions[15] of fields $\varphi^{\alpha}(x)$, to the $SU(2)$ symmetry most familiar from isospin, but the symmetry is also used to construct lepton doublets $\binom{l}{\nu_l}$. Essentially, analytic (holomorphic) restrictions lead to decomposition of fields $\varphi^{\alpha}(x)$ into superpositions of two classes of basis fields, with and without symmetry breaking. These classes of basis fields are respectively mappable as proportional to $\left[\begin{smallmatrix} -1 \\ 1 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$. That these matrices form a basis for the Pauli group[15] should be no surprise. Particle handedness is viewed as a property of the given field/particle or equivalently its proper frame of reference. Similarly, determination of which form of field $\varphi^{\alpha}(x)$ arises through symmetry breaking is associated with a choice of reference frame. These properties are described by the transformation operator $\Lambda^{\alpha}_{\beta}(x)$ where

$$\partial_{\mu}\Lambda^{\alpha}_{\beta}(x) \equiv -\frac{i}{2}\{\partial_{\mu}a^{\alpha}_{\gamma M}(x)\sigma^{M}\}\exp\left(-\frac{i}{2}a^{\gamma}_{\beta N}(x)\sigma^{N}\right)$$
$$\equiv iqA^{\alpha}_{\mu\beta}. \tag{99}$$

The additional indices on the photon field $A^{\alpha}_{\mu\beta}$ allow it to couple with the field $\varphi^{\alpha}(x)$. The Pauli matrices $\sigma^{A}$ should be expected given the basis matrices delineated just above. The covariant derivatives then becomes

$$D_{\mu}\varphi^{\beta}(x) = \partial_{\mu}\varphi^{\beta}(x) - iqA^{\beta}_{\mu\alpha}\varphi^{\alpha}(x), \tag{100}$$

where the photon field $A^{\alpha}_{\mu\beta}$ takes the form

$$qA^{\alpha}_{\mu\beta} \equiv -\frac{1}{2}\{\partial_{\mu}a^{\alpha}_{\gamma M}(x)\sigma^{M}\}\exp\left[-\frac{i}{2}a^{\gamma}_{\beta N}(x)\sigma^{N}\right]. \tag{101}$$

The $SU(2)\times U(1)$ gauge symmetry is associated with fields $\varphi^{\alpha}(x)$ whether the field $\varphi^{\alpha}(x)$ is massive or massless apart from symmetry-breaking effects.

Only fields $\varphi^{\alpha}(x)$ which are massive even apart from symmetry-breaking effects truly possess velocity-related degrees of freedom. This can be shown in at least either of two ways. One can argue from the fact that symmetry breaking is a reference frame phenomenon but the speed of light (to which massless field/particles are constrained) is constant in any frame of reference. Then, since either excitation of the field $\varphi^{\alpha}(x)$, i.e., either member of the $SU(2)$ doublet, can be seen as the portion that travels at lightspeed,[7] neither can possess true velocity degrees of freedom. Alternatively, since the field $\varphi^{\alpha}(x)$ possesses no velocity degrees of freedom aside from symmetry-breaking effects, an excitation of the field $\varphi^{\alpha}(x)$ which "becomes" massive due to symmetry-breaking effects cannot "gain" true velocity-related degrees of freedom due to continuity restrictions. However, one argues the fact, only those fields which are massive even apart from symmetry-breaking effects can possess velocity-related degrees of freedom. (Specifically, this represents three degrees of freedom, since timelike velocity $\beta^{0}\equiv 1$.) This velocity dependence allows one to functionally describe inherently massive fields

$$\varphi^{\alpha}(x) \equiv \varphi^{\alpha}[x^{\mu},\beta^{N}]. \tag{102}$$

In a decomposition process similar to that by which one constructed four-indexed fields $\varphi^{\alpha}(x)$ from nonindexed but physically equivalent fields $\varphi(x)$, one constructs a field

[7]By convention, electromagnetic effects are excluded from spacetime curvature. Were this not the case, one could in principle define both charged and uncharged photonlike events. In the proposed view of leptons, leptonic field/particles are excitations of charged (and therefore in the conventional view massive) photonlike field events.

$$[\varphi^{\alpha}_{M}(x,\beta)] \equiv \frac{a^{\alpha}_{n\beta}}{2}\begin{bmatrix}\exp(imx^{0})\\ \exp(-im\beta_{nM}x^{1})\\ \exp(-im\beta_{nM}x^{2})\\ \exp(-im\beta_{nM}x^{3})\end{bmatrix}^{\beta}$$

$$\text{where } \beta^{\mu} \neq \begin{bmatrix}1\\ \beta\\ 0\\ 0\end{bmatrix}, \tag{103}$$

$$[\varphi^{\alpha}_{M}(x,\beta)] \equiv \frac{a^{\alpha}_{n\beta}}{2}\begin{bmatrix}\exp(imx^{0})+\exp(-imx^{0})\\ \exp(-im\beta_{nM}x^{1})+\exp(im\beta_{nM}x^{1})\\ 0\\ 0\end{bmatrix}^{\beta},$$

$$\text{where } \beta^{\mu} = \begin{bmatrix}1\\ \beta\\ 0\\ 0\end{bmatrix}. \tag{104}$$

These are, respectively, termed hadrons and mesons. Each $M$th component field $\varphi^{\alpha}_{M}(x,\beta)$ represents a quark. This leads directly to confinement because quarks are not constructed as independent fields, only functionally independent fields. Alternately, this leads to strict quark confinement because mass $m$ acts as a proportionality constant which relates velocity $\beta_{\alpha}$ and momentum $P_{\alpha}$ as

$$P_{\alpha} = m\beta_{\alpha}. \tag{105}$$

Quarks fields are constructed parallel spacelike components of velocity $\beta_{\alpha}$ and therefore of momentum $P_{\alpha}$. If any one quark could be physically separated to occur as an independent physical event, one would be able to specify a hypersurface to which the remaining quarks are confined, but this violates physical restrictions on simultaneous measurability. Color labels the axes of velocity three-space so that chromatic transformations become rotations within a (Euclidean) three space, leading to asymptotic freedom. Velocity however remains a local, i.e., spacetime position $x$ dependent, quantity. Thus, one obtains a local non-Abelian symmetry which applies only to fields $\varphi^{\alpha}(x)$ which are massive even aside from symmetry-breaking effects, not to those fields $\varphi^{\alpha}(x)$ which are intrinsically massless. On this basis the latter are identified with leptons, the former with baryons. The usual covariant derivative, adapted for four-indexed fields $\varphi^{\alpha}(x)$, then applies

$$D_{\mu}\varphi^{\alpha}(x) = \partial_{\mu}\varphi^{\beta}(x) - \left\{\partial_{\mu}\exp\left[-\frac{i}{2}b^{M}_{N}(x)\cdot\lambda^{\alpha N}_{\beta K}\right]\right\}\varphi^{\beta}(x)$$
$$\equiv \partial_{\mu}\varphi^{\beta}(x) + iaB^{\alpha}_{\mu\beta}\varphi^{\beta}(x). \tag{106}$$

Gluons matrices $\lambda^{\alpha N}_{\beta K}$ take the usual forms. Again, this is a true covariant derivative, not an effective one, for the same reasons as in the case of $SU(2)$ symmetry.

The significance of these results can be assessed at a few levels. If one seeks immediate practical consequences without looking for deeper implications, one finds that a certain high degree of physicality [namely, the $SU(3)\times SU(2)\times U(1)$ gauge group structure associated with the standard model] is assured to any field theory consistently constructed in terms of four-indexed fields $\varphi^\alpha(x)$. Thus, if one returns to the example given in the introduction of a $\lambda\varphi^4$ field theory with Lagrangian

$$\mathcal{L}[\varphi^\alpha(x),\partial_\beta\varphi^\alpha(x)]=[\partial_\beta\varphi^\alpha(x)][\partial^\beta\varphi_\alpha(x)] - \frac{1}{2}m^2\varphi_\alpha(x)\varphi^\alpha(x)-\frac{\lambda}{4}[\varphi_\alpha(x)\varphi^\alpha(x)]^2, \tag{107}$$

a driven quantum oscillator familiar from both Higgs theory and statistical mechanics,[9] one can immediately write down a transformed Lagrangian

$$\mathcal{L}'[\varphi'^\alpha(x),\partial_\beta\varphi'^\alpha(x)]=[D_\beta\varphi'^\alpha(x)][D^\beta\varphi'_\alpha(x)] - \frac{1}{2}m^2\varphi'_\alpha(x)\varphi'^\alpha(x) - \frac{\lambda}{4}[\varphi'_\alpha(x)\varphi'^\alpha(x)]^2, \tag{108}$$

without any need to supply additional justification that this Lagrangian has the gauge properties of the standard model Lagrangian. [No primes are needed on the mass $m$ and coupling $\lambda$ because the modulus $\varphi_\alpha(x)\varphi^\alpha(x)=\varphi'_\alpha(x)\varphi'^\alpha(x)$ is not changed by transformation, since it is a scalar quantity.] Any analytic field solution $\varphi^\alpha(x)$ of the associated Euler–Lagrange equation

$$\left[\partial_\beta\partial^\beta+m^2+\frac{\lambda}{2}\varphi_\mu(x)\varphi^\mu(x)\right]\varphi_\alpha(x) = \left[D_\beta D^\beta+m^2+\frac{\lambda}{2}\varphi'_\mu(x)\varphi'^\mu(x)\right]\varphi'_\alpha(x)=0 \tag{109}$$

must transform in this manner, a constrain which acts as a powerful tool when solving the associated field equation. The difference of a factor $\frac{1}{3}$ from the equation associated with a Lagrangian $\mathcal{L}[\varphi(x),\partial_\beta\varphi(x)]$ for fields $\varphi(x)$ which are not four indexed in the final term can in this case be absorbed into the coupling $\lambda$, but this serves to illustrate an important point; the equations of motion obtained when one employs four-indexed fields $\varphi^\alpha(x)$ may differ by more than writing an index on each field.

At a more fundamental level, association of standard model gauge symmetries with actual, rather than effective, covariant derivatives $D_\mu$ eliminates one of the many technical difficulties which must be surmounted if anyone is to ever construct an eventual physically meaningful quantum theory of gravitation. Likewise, the demonstration that a four-indexed field $\varphi^\alpha(x)$ reduces to an unindexed but otherwise physically equivalent field $\varphi(x)$ when restricted to rest frames so that the metric $g_{\alpha\beta}$ becomes the Minkowski metric $\eta_{\alpha\beta}$ (as discussed above) suggests that any truly generally relativistic quantum field theory must be written in terms of four-indexed fields $\varphi^\alpha(x)$.

This formalism has been additionally clarified by the consideration of the excitation of field/particles by expansion of the vacuum using the Dirac equation.[8] The Dirac equation when modified for four-indexed fields becomes

$$[i\gamma^\alpha\partial_\beta - mg^\alpha_\beta]\varphi^\beta(x)=V^\alpha_\beta(x)\varphi^\beta(x). \tag{110}$$

The form of the Dirac matrix operator $\gamma^\alpha$, defined implicitly as

$$\frac{1}{2}\{\gamma^\alpha,\gamma^\beta\}\equiv\frac{1}{2}[\gamma^\alpha\gamma^\beta+\gamma^\beta\gamma^\alpha]\equiv g^{\alpha\beta}, \tag{111}$$

varies when one allows a general form of the metric $g^{\alpha\beta}$, but the example used considers the vacuum as locally flat so that one uses the Minkowski metric as usual. If one considers a point $x'(t)$ expanding away from a fixed point $x_0$ and separated by a radius $a(t)$, one defines the potential with respect to the stationary point vacuum so that

$$V^\alpha_\beta(x_0)=0, \tag{112}$$

but an observer at point $x_0$ sees a nonzero and time dependent potential

$$[V^\alpha_\beta(x')]=i\gamma^\alpha\partial_\mu\begin{pmatrix}\{1-[\partial_0 a(t)]^2\}^{-1/2} & -[\partial_0 a(t)]\{1-[\partial_0 a(t)]^2\}^{-1/2} & 0 & 0\\ -[\partial_0 a(t)]\{1-[\partial_0 a(t)]^2\}^{-1/2} & \{1-[\partial_0 a(t)]^2\}^{-1/2} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}^\mu_\beta \tag{113}$$

at point $x'(t)$, due to the relative motion of the two points. Because both the relative velocity $\partial_0 a(t)$ and relative acceleration $(\partial_0)^2 a(t)$ of the two points are positive quantities, in accordance with the observed accelerating vacuum expansion rate, field/particles are excited. In fact, in terms of a number operator $N$, one finds that

$$\partial_0 N>0. \tag{114}$$

Field/particles are excited at the point $x'(t)$ in clear violation of CP symmetry. This is similar to emission of a photon by an electron due to wave spreading phenomena. The vacuum state of the field $\varphi^\alpha(x)$ spreads as the vacuum itself expands.

This lowers the energy of the ground state of the vacuum, but energy must be conserved. At some critical point, a particle excitation of the field $\varphi^{\alpha}(x)$ occurs because this spreading lowers the excitation energy sufficiently. Clearly, this violates conservation of lepton and/or baryon number, but some have suspected for some time that these quantities are not strictly conserved. Indeed, if one assumes that the universe started out as vacuum, some process or processes must exist which violate particle number conservation laws quite badly. This process for field/particle production does just that. However, a physical process of this sort could only be recognized if one establishes the physical equivalence of gauge transformations and covariant transformations, under general coordinate transformations.

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## APPENDIX A: USAGE OF THE TERM "METRIC" IN THIS DISCUSSION

Whenever one refers in this discussion to a metric $g_{\alpha\beta}$, one refers to a symmetric bilinear objection (a tensor or a spin-tensor) $g_{\alpha\beta}$ like that used in a line-element

$$ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}. \tag{A1}$$

Loosely, the explicit line-element such as

$$ds^2 = \left(1 - \frac{r_S}{r}\right) dt^2 - \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 - r^2 (d\phi^2 + \sin^2 \theta d\theta^2) \tag{A2}$$

in the case of a Schwarzschild metric is sometimes also referred to as the metric. When metric $g_{\alpha\beta}$ in strictly diagonal, this looser usage leads to no confusion; one can read off the elements of metric $g_{\alpha\beta}$ with a minimum of effort if need be. Representation of the metric $g_{\alpha\beta}$ via the explicit line-element $ds^2$ is still possible but much more cumbersome when all ten independent components of a metric $g_{\alpha\beta}$ are in principle nonvanishing as in the case of a line-element

$$ds^2 = g_{00} dt^2 + 2 g_{01} dt dx^1 + \cdots + 2 g_{23} dx^2 dx^3 + g_{33} (dx^3)^2. \tag{A3}$$

The association of mixed differential coordinates and the factor 2 in the case of off-diagonal terms tends more to obscure the nature of the metric $g_{\alpha\beta}$ than otherwise. Yet, throughout this discussion, the possible existence of nonvanishing off-diagonal terms of a metric $g_{\alpha\beta}$ is central to the logic of the argument presented. Such nonvanishing off-diagonal metric components $g_{\alpha\beta}$ are most often encountered in the context of reference frame transformations, as when one boosts a strictly diagonal metric. The metric transformation rule

$$g'_{\alpha\beta} = g_{\mu\nu} \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} \tag{A4}$$

is simply not as clearly or succinctly expressed in terms of a line-element $ds^2$. For this reason, the sake of clarity, the more strictly rigorous usage of the term metric—which refers to a bilateral symmetric object $g_{\alpha\beta}$ by which one in principle specifies a line-element $ds^2$—is used throughout this discussion.

## APPENDIX B: CONSTRUCTION OF *SU*(2) BASES (REF. 18)

Construction of the $SU(2)$ bases cited above begins with the analytic restrictions

$$\frac{\partial \varphi^0(x^0)}{\partial x^0} = \frac{\partial \varphi^1(x^1)}{\partial x^1}, \quad \frac{\partial \varphi^0(x^0[x^1])}{\partial x^1} = - \frac{\partial \varphi^1(x^1[x^0])}{\partial x^0}, \tag{B1}$$

also cited above. (The field has been constructed to be independent of the remaining spacelike axes as explained in the main discussion.) In direct analogy with Cauchy–Riemann restrictions on a complex function in a complex domain, one derives these restrictions by insisting that definition of the divergence $\partial_{\alpha} \varphi^{\alpha}(x)$ be path independent. One plugs into these expressions the form of the bases defined above

$$\varphi_n(x) \equiv \exp(- i k_{n\mu} x^{\mu}). \tag{B2}$$

One combines expressions in the form

$$\frac{\partial \varphi^0(x^0)}{\partial x^0} \frac{\partial \varphi^0(x^0[x^1])}{\partial x^1} = \frac{\partial \varphi^1(x^1[x^0])}{\partial x^0} \frac{\partial \varphi^1(x^1)}{\partial x^1}, \tag{B3}$$

using the fact that spacelike components square negatively, and then applies definitions

$$\frac{dx^0}{dx^1} \equiv \left(\frac{dx^1}{dx^0}\right)^{-1}, \tag{B4}$$

$$\left(\frac{k_0}{k_1}\right)^2 \left(\frac{dx^1}{dx^0}\right)^{-2} \equiv 1. \tag{B5}$$

The general form

$$\left[ \pm \frac{\varphi_n^0(x^0)}{\varphi_n^1(x^1)} \right]^2 = 1 \tag{B6}$$

cited above then follows from algebra.

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