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How Really Massive are the Super-Massive Rotating Black Holes in the Milky Way's Bulge?

based on the Maxwell Analogy for Gravitation.

T. De Mees - thierrydemees @ pandora.be

Abstract

The centre of the Milky Way is populated with so-called *super-massive* black holes. In most of the papers and books about black holes at the centre of galaxies, the mass is said to be gigantic.
In this paper, we will see how to calculate the mass of these *super-massive* black holes out of observational data, by using the Maxwell Analogy for Gravitation, and we see how to make the difference between real physical mass and apparent (fictive) mass.
We discover that so-called 'super-massive black holes' do not have huge masses at all but that they have an apparent mass that can be thousands times the real mass. This suggests that the energy of such black-holes could decrease very fast in relative terms.

Keywords: *black hole – horizon – spinning star – super-massive – Maxwell*
Method: *analytic* Notations: *metric with comma*

1. Basic gyro-gravitation physics for a rotating sphere.

Rotating objects have a velocity-dependent property that is the following. Imagine a sphere that is rotating with an angular velocity ω. The gravitational field of the sphere is the steady reference field with a velocity that is locally zero. But the rotation of the particles at a certain velocity, depending from its orbit's radius, will undergo a second field that is entirely comparable with the magnetic field in electromagnetism. I call this field *gyrotation*, but several other names exist in literature, such as co-gravitation field, gravito-magnetic field, etc. This orbital velocity is locally an absolute velocity. In "A Coherent Dual Vector Field Theory for Gravitation" , I explained that this second field is generated by the motion of masses.

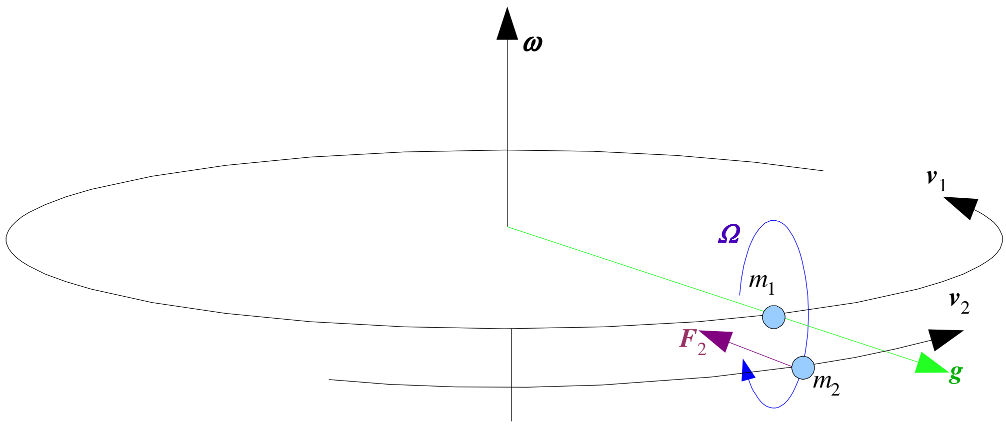

Fig. 1. If two particles inside a spherical object rotate at their corresponding circular velocity, they will influence each-other by a *gyrotational* force.

In fig 1, the mass m_1 (a particle of the sphere) orbits at a velocity $\boldsymbol{v}_1$ in the gravitation field $\boldsymbol{g}$ of the sphere. This motion generates a second field $\boldsymbol{\Omega}$ that is perpendicular on the velocity $\boldsymbol{v}_1$. This second field will influence any second mass m_2 that travels with a velocity $\boldsymbol{v}_2$ by generating a force $\boldsymbol{F}_2$ that will cause a deviation of the mass m_2. This force is perpendicular to both the velocity $\boldsymbol{v}_2$ and the gyrotation field $\boldsymbol{\Omega}$. Mutatis mutandis, the mass m_2 will also deviate any mass m_1 in a similar way.

The force $\boldsymbol{F}_2$ can be found by the vector expression :

$$\boldsymbol{F}_2 \Leftarrow m_2\,(\boldsymbol{v}_2 \times \boldsymbol{\Omega}) \qquad (1.1)$$

which can be completed by adding the gravitational force :

$$\boldsymbol{F}_{2,\,\mathrm{tot}} \Leftarrow m_2\,(\boldsymbol{g} + \boldsymbol{v}_2 \times \boldsymbol{\Omega}) \qquad (1.2)$$

When this scenario is repeated for all the particles of the sphere, the global gravitation is found and the global second field (the gyrotational field) is found.

For a spinning sphere with rotation velocity ω, the results for the gyrotation at a point p outside the sphere with mass m is given by the equation (1.3) [5]:

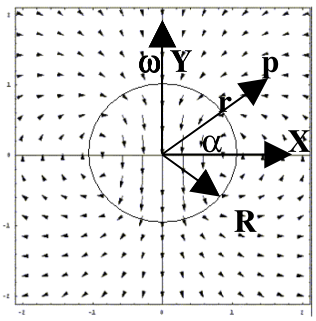

Fig. 1.2

$$\Omega_{ext} \Leftarrow \frac{-G\,m\,R^2}{5\,r^3\,c^2}\left(\omega - \frac{3\,r\,(\omega \bullet r)}{r^2}\right) \qquad (1.3)$$

wherein • means the scalar product of vectors.

(Reference: adapted from E. Negut). The drawing shows equipotentials of $-\Omega$.

For the level of the equatorial plane, the last term of (1.3) vanishes, and we get a simple expression that is dependent of the inverse cube of the distance, and, compared with the pure gravitational field, dependent from the square of the sphere's radius, from the inverse square of the light velocity, and from the sphere's angular velocity.

The result of the expression (1.3) can be put in (1.2) in order to find the total acceleration acting on a arbitrary mass in motion.

For the equatorial plane, equation (1.2) can generally also be written as :

$$a = -\frac{G\,m}{r^2}\left(1 + \frac{v_2\,\omega\,R^2}{5\,r\,c^2}\right) \qquad (1.4)$$

This equation shows that the second term can have some considerable relative importance, even if the mass is small. Indeed, the second term can be much larger than 1, depending from the values of the variables. This means that even if the mass is small, it is still possible to have the *impression* that we are coping with a massive object.

The equation (1.4) can be expressed more generally, when we consider I the inertial momentum of the central object:

Since $I_{\mathrm{sphere}} = \frac{2}{5}\,m\,R^2$, we get, more generally : $a = -\frac{G\,m}{r^2}\left(1 + \frac{v_2\,I\,\omega}{2\,m\,r\,c^2}\right)$ (1.5)

In the next chapter, we will analyse these parameters in detail.

2. Gyrotational centripetal forces.

The orbit velocity is non-Keplerian nearby fast spinning stars.

When the orbital velocity of an object surrounding an invisible star (or black hole) appears to be non-Keplerian, this central black hole is said to be super-massive. Therefore, let us use the velocity v_2 , used in the former equations as an orbital velocity of objects nearby the central black hole.

In "On the orbital velocities nearby rotary stars and black holes" I found the velocity of orbiting objects when taking in account the gyrotation field of the central star.

In relation to the Keplerian orbital velocity $v_k = \sqrt{\dfrac{G\,m}{r}}$, we found the following real orbit velocity :

$$v_2 = \sqrt{\frac{G\,m}{r}}\sqrt{1+\frac{G\,m}{r}\left(\frac{I\,\omega}{4\,m\,r\,c^2}\right)^2} + \frac{G\,m}{r}\frac{I\,\omega}{4\,m\,r\,c^2} \qquad (2.1)$$

Herein, I is the inertial momentum of the sphere, or in general, of the central celestial object. The orbiting object will experience a larger velocity than the Keplerian orbital velocity only, because of the spinning of the star.

We define the notations a_g , v_k and v_k as follows : the gravitational acceleration as $a_g = -\dfrac{G\,m}{r^2}$, the *angular spread* as $s_\Omega = \dfrac{I\,\omega}{4\,m\,r\,c^2}$ and the Keplerian orbital velocity as $v_k = \sqrt{\dfrac{G\,m}{r}}$.

Then, the equations (2.1) and (1.5) can be more simply written as :

$$v_2 = v_k\sqrt{1+v_k^2\,s_\Omega^2} + v_k^2\,s_\Omega \quad \text{and} \quad a = a_g\left[1+2\,s_\Omega\left(v_k\sqrt{1+v_k^2\,s_\Omega{}^2} + v_k^2\,s_\Omega\right)\right] \qquad (2.2)\ (2.3)$$

In the case of a large Keplerian orbital velocity v_k and of a considerable *angular spread* s_Ω (which has the dimension of the inverse velocity [s/m]) , we get a total centripetal force that can be many times larger than the gravitation force alone, but without therefore having a larger mass.
The main factor that will determine the gyrotation force (the second, large term in (2.3)) , is the angular velocity ω of the central star or black hole.

The apparent mass is caused by the non-Keplerian part of the orbital velocity.

Therefore, let us examine again the characteristics of rotary black holes. In "The Kerr-metric, Mass and Light-Horizons, and Black Holes' Radii" I explained the shape and other characteristics of black holes.

When someone is not aware that the velocity v_2 is not Keplerian, he will say that the mass of the central black hole can be found out of the Keplerian equation :

$$v_2 = \sqrt{\frac{G\,m_{bh}}{r}} \qquad (2.4)$$

But since we know better, we can say that the mass contains partly real mass and partly apparent mass, due to the wrong idea that v_2 would be Keplerian.

So, we get: $$v_2 = \sqrt{\frac{G\left(m + m_{\mathrm{app}}\right)}{r}} \tag{2.5}$$

The total mass (real and apparent) of the rotating star is given by the following expression :

$$m + m_{\mathrm{app}} = \frac{v_2^2 r}{G} \tag{2.6}$$

where the first term is the gravitational real mass and the second term the apparent mass.

The equation (2.6) can be written as following, when using (2.2) :

$$\frac{m_{\mathrm{app}}}{m} = \frac{v_2^2\, r}{m\,G} - 1 = \frac{v_k^2\left(\sqrt{1+v_k^2\, s_\Omega^2} + v_k\, s_\Omega\right)^2 r}{m\,G} = \left(\sqrt{1+v_k^2\, s_\Omega^2} + v_k\, s_\Omega\right)^2 \tag{2.7.a}$$

Remember that the first part of the equation can be written as $$\frac{m_{app}}{m} = \frac{v_2^2}{v_k^2} - 1 \tag{2.7.b}$$

In the next section, we will analyse the equations (2.7) closer.

Analysis of equations (2.7) and simplification.

Three cases are considered.

Let us consider the case where $v_k\, s_\Omega \approx 1$. Then the following approximations can be made :

$$\frac{m_{\mathrm{app}}}{m} \approx \left(1+\sqrt{2}\right)^2 \tag{2.8.a}$$

$$v_2 \approx \left(\sqrt{2}+1\right) v_k \tag{2.8.b}$$

The apparent mass is already considerable here.

When we consider the case where $v_k\, s_\Omega >> 1$, the following approximations can be made, since $\sqrt{1+x} \approx 1 + x/2$:

$$\frac{m_{\mathrm{app}}}{m} \approx \left[v_k\, s_\Omega\left(2+\frac{1}{2\,v_k^2\, s_\Omega^2}\right)\right]^2 \tag{2.9.a}$$

$$v_2 \approx 2\,v_k^2\, s_\Omega + \frac{1}{2\,s_\Omega} \tag{2.9.b}$$

The apparent mass is very important here. This is the case that will be studied further in this paper.

To a certain extend, for $v_k\, s_\Omega >>> 1$, it is even possible to reduce the equations to a more simplified version :

$$\frac{m_{app}}{m} \approx \left(2\, v_k\, s_\Omega\right)^2 \tag{2.10.a}$$

$$\frac{v_2}{v_k} \approx 2\, v_k\, s_\Omega \tag{2.10.b}$$

And when $v_k\, s_\Omega << 1$, the following approximations can be made :

$$\frac{m_{app}}{m} \approx \left(1 + v_k\, s_\Omega + \frac{v_k^2\, s_\Omega^2}{2}\right)^2 \qquad (2.11.a) \qquad\qquad v_2 \approx v_k + v_k^2\, s_\Omega + \frac{1}{2} v_k^3\, s_\Omega^2 \qquad (2.11.b)$$

Even here, the apparent mass can be of a noticeable importance, if $v_k\, s_\Omega$ is not too small.

For $v_k\, s_\Omega <<< 1$, we can reduce the equations to a more simplified version :

$$\frac{m_{app}}{m} \approx \left(1 + v_k\, s_\Omega\right)^2 \qquad (2.12.a) \qquad\qquad \frac{v_2}{v_k} \approx 1 + v_k\, s_\Omega \qquad (2.12.b)$$

3. Study of the case $v_k\, s_\Omega >> 1$.

When the case $v_k\, s_\Omega >> 1$ is considered, the first thing to do is to verify what the physical balance conditions are that can comply with the several parameters of the expression $v_k\, s_\Omega$. We have to check the physical equilibrium between the so-called centripetal forces (gravitation and gyrotation) and the centrifugal forces (inertia of mass). In this paper we will study stars that are stable while spinning.

To obtain the condition $v_k\, s_\Omega >> 1$, it is sufficient to choose the orbit radius r small enough, but still $r > R$.

The condition of non-explosion of the star.

In "On the geometry of rotary stars and black holes" I found the critical radius at which a fast rotating spheric object will not fall apart at latitudes smaller than 35°16', even at very fast rotation speeds. The generalization for some other shapes than the sphere are worked out in "The Kerr-metric, Mass and Light-Horizons, and Black Holes' Radii", chapter 3, section "*Are Pure Black Holes explosion-free ?*". I found the equilibrium between the corresponding accelerations, and the corresponding explosion-free *Critical Radius* R_C :

$$R_C = \frac{\lambda\, R_S}{4} \qquad (3.1)$$

wherein the symbol R_s is the Schwarzschild radius $R_s = \frac{2Gm}{c^2}$ and the dimensionless factor λ is found out of $I = \lambda\, m\, R^2$. For a sphere, $\lambda = 2/5$ and for a thin ring with radius R we have $\lambda = 1$. The radius of the black hole must be equal or less than R_C .

Minimum spinning velocity for the validity of equation (3.1).

Remark that the condition for the non-explosion of fast spinning stars is independent from the spinning speed.

However, the expression (3.1) is not applicable for slowly rotating stars, because during the deduction of the equation (3.1) in the latter mentioned paper, I have supposed that the gravitational part is negligible versus the gyrotational part. When the gravitational part is not negligible, the star will even better be kept together, and the critical radius can be considerably larger without any risk for falling apart.

The condition for which (3.1) is precise enough and applicable in this paper as explained in the Appendix at the end of this paper, where a general study of the explosion-free equilibrium of stars is given.

Also in the mentioned papers, I find that the final shape of fast spinning stars and black holes must be tiny and ring-shaped.

Quotient of apparent mass and real mass.

Let us now compare the apparent mass with the real mass. The equation (2.6) , combined with the definition of $v_k s_\Omega$, and in which we replace R by the expression of R_C that we found in (3.1) , gives us the quotient of the apparent mass and the real mass :

$$\frac{m_{app}}{m} \approx \left[\frac{\lambda G^{5/2} m^{5/2} \omega}{8 r^{3/2} c^6} + \frac{32 r^{3/2} c^6}{\lambda G^{5/2} m^{5/2} \omega} \right]^2 \tag{3.2.a}$$

This equation is only valid for fast rotating stars that do not explode, due to the gyrotation force that keeps the star together, whatever the rotation speed might be.
One might be very surprised to see such huge powers in this equation. However, this is due because we have assembled several conditions in the same equation. The first condition is that we have considered that the apparent mass can only be observed by observing the orbit velocity of an orbiting mass about the star or black hole. Thus, we consider the apparent mass at the level of that orbiting object. One power of the real mass is accounting for the orbit velocity of the orbiting object, caused by gyrotation. The second condition is that the chosen type of star is an explosion-free one. A fourth power of the real mass is accounting for this condition!

The case of $v_k s_\Omega$ >>> 1 allows us to maintain only the first term of the right hand of equation (3.2). The second term becomes negligible.

The equation (3.2.a) can then be simplified as :

$$\frac{m_{app}}{m} \approx \frac{\lambda G^5 m^5 \omega^2}{64 r^3 c^{12}} \tag{3.2.b}$$

We see that the speed of light is present to the power minus sixth, which induces that there are needed very high values for the black hole's mass in order to get significant values for the apparent mass.

Observational limitations due to the Light Horizon.

The radius of the Light Horizon r_{LH} of black holes has been calculated in "The Kerr-metric, Mass and Light-Horizons, and Black Holes' Radii", chapter 2, section "*What specifies the light-horizon of black holes?*" and is given by :

$$r_{LH} = R_s + \frac{G I \omega^2}{2 c^4} \tag{3.3}$$

This value is the minimum distance r from the black hole that has to be used in (3.2), because it is not possible to observe phenomena that are closer to it, at the level of the equatorial plane, wherefore equation (3.3) is applicable.

The condition for non-explosion (3.1) should still be applicable, although it has not yet combined in the equation (3.3).
Since $I = \lambda m R^2$, and combined with the equation (3.1) , we get as a limit for the light horizon r_{LH} (by replacing R by R_C) :

$$r_{LH} = R_s + \frac{\lambda G m R_C^2 \omega^2}{2 c^4} = R_s + \frac{\lambda R_s R_C^2 \omega^2}{4 c^2} = R_s + \frac{\lambda^3 R_s^3 \omega^2}{64 c^2} = \frac{2 G m}{c^2} + \frac{\lambda^3 G^3 m^3 \omega^2}{8 c^8} \tag{3.4}$$

wherein the effective radius of the black hole must be equal or less than R_C . In (3.4) , we expressed the light horizon in different ways.

Orders of magnitude.

Let us apply the equation (3.2.b) with (3.3) for a ring-shaped black hole of one hundred solar masses.

When we suppose it is rotating at 1000 rpm, we get in fig.3.1, for a given distance r , the apparent mass versus the black hole's mass.
Based on the equation (3.1) for the non-explosion of the star, and as far as we can trust the values of the natural constants c and G at that position in space, this stable ring-shaped super-massive black hole has a radius of 73 km only ! Nearby that radius, the apparent mass is hundreds of times the black hole's real mass (Fig.3.1).
In this example, the light horizon is at 298 km.

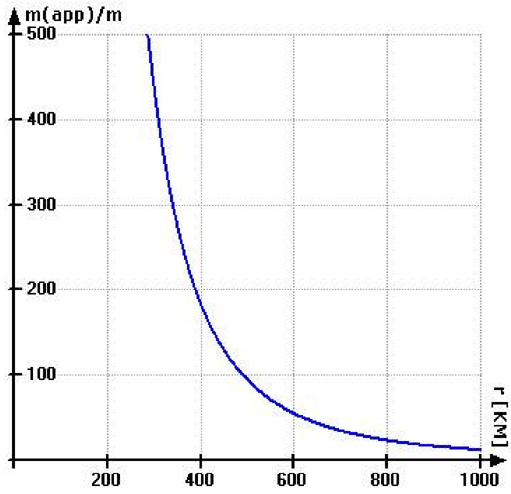

Fig. 3.1

In the case of a ring-shaped black hole of a thousand solar masses, at the same rotation rate of 1000 rpm.
Then, we get in fig.3.2, at a given distance r , the apparent mass versus the the black hole's mass.
Based on the equation (3.1) for the non-explosion of the star, the stable ring-shaped super-massive black hole has a radius of less than 733 km ! Close to that radius, the apparent mass is ten thousands times the black hole's real mass (Fig.3.2) !
In this example, the light horizon is at 7315 km.

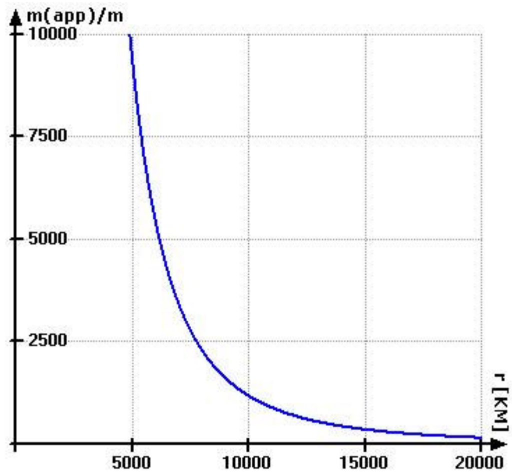

Fig. 3.2

We conclude that the apparent mass takes the main part of the total gyro-gravitational attraction for black holes. Non-Keplerian attraction is then observed. However, at very large distances, this apparent mass does not play a significant role and can be neglected. The choice of 1000 rpm has been observed and this value is not unusual. Due to the fact that matter can be transformed to gamma rays under high speed, such as with beaming black holes, the limitation of the black-hole's spin velocity is set by the speed of light of the disintegrated mass.

4. Discussion and conclusion.

Out of the equation (3.2) it is confirmed that fast-spinning super-massive black holes can generate incredibly huge apparent masses, if they are shaped at the critical radius that is necessary for non-explosion, which is given by the equation (3.1). The apparent mass, caused by gyrotation, however decreases with the inverse cubic power of the distance, see equation (1.5) , whereas the gravitational forces decrease with the inverse square power of the distance. This fast decrease of the gyrotation force with the distance preserves that more distant objects would be attracted and absorbed by these predator stars.
As known from the equation (3.1) that gives the radius' value of fast non-exploding rotating stars, these stars have very small shapes, in the order of magnitude of kilometres.
Finally, we conclude that the rotation speed of the star is not the main parameter for obtaining huge apparent masses. The parameter of the (real) mass of the star is much more important to the gyrotational mass due to its power 5/2 .

5. References.

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Appendix : Critical radius of a spinning star.

The critical radius at which a star will not fall apart even when spinning at a high rate, is deduced from the equilibrium equation for accelerations, containing gravitation, gyrotation and centripetal accelerations. In this Appendix, I analyze the outcome of equation (3.1) more generally.

In "On the geometry of rotary stars and black holes", chapter 3, I wrote the equation (3.3), which can be simplified for the equator by putting the latitude angle α to zero. The star does not fall apart if this radial acceleration is negative.

$$0 < \omega^2 R \left(1 - \frac{G I}{2 R^3 c^2}\right) - \frac{G m}{R^2} \qquad \text{(A.1)}$$

I generalize the case for any angular inertia of the type $I = \lambda m R^2$ and get :

$$0 < \omega^2 R^2 \left(R - \frac{\lambda G m}{2 c^2}\right) - G m \qquad \text{(A.2)}$$

Let us consider four cases.

Case 1 : $$G m << \omega^2 R^2 \left(R - \frac{\lambda G m}{2 c^2}\right) \quad . \qquad \text{(A.3)}$$

Here, the explosion at the equatorial zone can be avoided if $R < R_C = \dfrac{\lambda G m}{2 c^2}$. (A.4)

This is the most general situation for fast spinning stars, as we saw in an earlier paper. The gravitational part is negligible, and we get high spins and small star's shapes.

Case 2 : $$R << \frac{\lambda G m}{2 c^2} \quad . \qquad \text{(A.5)}$$

Then the total acceleration is always negative and this confirms the case 1.

Case 3 : $$R >> \frac{\lambda G\, m}{2\, c^2} \quad . \tag{A.6}$$

Falling apart of the star can be avoided if $$R < R_C = \sqrt[3]{\frac{G\, m}{\omega^2}} \quad . \tag{A.7}$$

This case is applicable to the sun and to the classic stars with a rather slow spin.

Case 4 : $$R \approx \frac{\lambda G\, m}{2\, c^2} \; . \tag{A.8}$$

Also here, the total acceleration is then always negative and this confirms again the case 1.

In fine, we can maintain two cases with their corresponding critical radii : cases 1 and 3. The cases 2 and 4 are only different aspects of the case 1.